2.3 One-Step Equations

Learning Objectives

- Solve an equation using addition.
- Solve an equation using subtraction.
- Solve an equation using multiplication.
- Solve an equation using division.

Introduction

Nadia is buying a new mp3 player. Peter watches her pay for the player with a \$100 bill. She receives \$22.00 in change, and from only this information, Peter works out how much the player cost. How much was the player?

In math, we can solve problems like this using an **equation.** An **equation** is an algebraic expression that involves an **equals** sign. If we use the letter x to represent the cost of the mp3 player we could write the following equation.

$$x+22=100$$

This tells us that the value of the player **plus** the value of the change received is **equal** to the \$100 that Nadia paid.

Peter saw the transaction from a different viewpoint. He saw Nadia receive the player, give the salesperson \$100 then he saw Nadia receive \$22 change. Another way we could write the equation would be:

$$x=100-22$$

This tells us that the value of the player is **equal** to the amount of money Nadia paid $(100-22)$.

Mathematically, these two equations are equivalent. Though it is easier to determine the cost of the mp3 player from the second equation. In this chapter, we will learn how to solve for the variable in a one variable linear equation. Linear equations are equations in which each term is either a constant or the product of a constant and a single variable (to the first power). The term linear comes from the word line. You will see in later chapters that linear equations define lines when graphed.

We will start with simple problems such as the one in the last example.

Solve an Equation Using Addition

When we write an algebraic equation, equality means that whatever we do to one side of the equation, we have to do to the other side. For example, to get from the second equation in the introduction back to the first equation, we would add a quantity of 22 to both sides:

$$x=100-22$$

$x + 22 = 100 - 22 + 22$ or $x + 22 = 100$

Similarly, we can add numbers to each side of an equation to help solve for our unknown.

Example 1

Solve $x - 3 = 9$

Solution

We need to **isolate** x. Change our equation so that x appears by itself on one side of the equals sign. Right now our x has a 3 subtracted from it. To reverse this, we could add 3, but we must do this to **both sides.**

$$\begin{aligned} x - 3 &= 9 \\ x - 3 + 3 &= 9 + 3 \text{ The } +3 \text{ and } -3 \text{ on the left cancel each other. We evaluate } 9 + 3 \\ x &= 12 \end{aligned}$$

Example 2

Solve $x - 3 = 11$

Solution

To isolate x we need to add 3 to both sides of the equation. This time we will add vertically.

$$\begin{aligned} x - \cancel{3} &= 11 \\ +\cancel{3} &= +3 \\ x &= 14 \end{aligned}$$

Notice how this format works. One term will always cancel (in this case the three), but we need to remember to carry the x down and evaluate the sum on the other side of the equals sign.

Example 3

Solve $z - 9.7 = -1.026$

Solution

This time our variable is z, but dont let that worry you. Treat this variable like any other variable.

$$\begin{aligned} z - 9.7 &= -1.026 \\ +9.7 &= +9.7 \\ zb &= 8.674 \end{aligned}$$

Make sure that you understand the addition of decimals in this example!

Solve an Equation Using Subtraction

When our variable appears with a number added to it, we follow the same process, only this time to isolate the variable we **subtract** a number from both sides of the equation.

Example 4

Solve $x+6=26$

Solution

To isolate x we need to subtract six from both sides.

$$\begin{aligned} \cancel{x}+\cancel{6} &= 26 \\ -\cancel{6} &= -6 \\ x &= 20 \end{aligned}$$

Example 5

Solve $x+20=-11$

Solution

To isolate x we need to subtract 20 from both sides of the equation.

$$\begin{aligned} x+20 &= -11 \\ -20 &= -20 \\ x &= -31 \end{aligned}$$

Example 6

Solve $x+\frac{4}{7}=\frac{9}{5}$

Solution

To isolate x we need to subtract $\frac{4}{7}$ from both sides.

$$\begin{aligned} x+\frac{4}{7} &= \frac{9}{5} \\ -\frac{4}{7} &= -\frac{4}{7} \\ x &= \frac{9}{5}-\frac{4}{7} \end{aligned}$$

To solve for x, make sure you know how to subtract fractions. We need to find the lowest common denominator. 5 and 7 are both prime. So we can multiply to find the LCD, LCD = $5\cdot 7=35$.

$$\begin{aligned} x &= \frac{9}{5}-\frac{4}{7} \\ x &= \frac{7\cdot 9}{35}-\frac{4\cdot 5}{35} \\ x &= \frac{63-20}{35} \\ x &= \frac{43}{35} \end{aligned}$$

Make sure you are comfortable with decimals and fractions! To master algebra, you will need to work with them frequently.

Solve an Equation Using Multiplication

Suppose you are selling pizza for $1.50 a slice and you get eight slices out of a single pizza. How much do you get for a single pizza? It shouldnt take you long to figure out that you get $8 \times \$1.50 = \$12.00$. You solve this problem by multiplying. The following examples do the same algebraically, using the unknown variable x as the cost in dollars of the whole pizza.

Example 7

Solve $\frac{1}{8} \cdot x = 1.5$

Our x is being multiplied by one-eighth. We need to cancel this factor, so we multiply by the reciprocal 8. Do not forget to multiply **both sides** of the equation.

$$\begin{aligned} \cancel{8}\left(\frac{1}{\cancel{8}} \cdot x\right) &= 8(1.5) \\ x &= 12 \end{aligned}$$

In general, when x is multiplied by a fraction, we multiply by the reciprocal of that fraction.

Example 8

Solve $\frac{9x}{5} = 5$

$\frac{9x}{5}$ is equivalent to $\frac{9}{5} \cdot x$ so x is being multiplied by $\frac{9}{5}$. To cancel, multiply by the reciprocal, $\frac{5}{9}$.

$$\begin{aligned} \frac{5}{9}\left(\frac{9x}{5}\right) &= \frac{5}{9} \cdot 5 \\ x &= \frac{25}{9} \end{aligned}$$

Example 9

Solve $0.25x = 5.25$

0.25 is the decimal equivalent of one fourth, so to cancel the 0.25 factor we would multiply by 4.

$$\begin{aligned} 4(0.25x) &= 4(5.25) \\ x &= 21 \end{aligned}$$

Solve an Equation Using Division

Solving by division is another way to cancel any terms that are being multiplied x. Suppose you buy five identical candy bars, and you are charged $3.25. How much did each candy bar cost? You might just divide $3.25 by 5. Or you may convert to cents and divide 325 by 5. Lets see how this problem looks in algebra.

Example 10

Solve $5x = 3.25$*To cancel the* 5*we divide both sides by* 5.

$$\frac{\cancel{5}x}{\cancel{5}} = \frac{3.25}{5}$$
$$x = 0.65$$

Example 11

Solve $7x = \frac{5}{11}$*Divide both sides by* 7.

$$x = \frac{5}{7 \cdot 11}$$
$$x = \frac{5}{77}$$

Example 12

Solve $1.375x = 1.2$*Divide by* 1.375

$$x = \frac{1.2}{1.375}$$
$$x = 0.8\overline{72}$$

Notice the bar above the final two decimals. It means recurring or repeating: the full answer is $0.872727272\ldots$.

Solve Real-World Problems Using Equations

Example 13

In the year 2017, *Anne will be* 45*years old. In what year was Anne born?*

The unknown here is the year Anne was born. This is x. Here is our equation.

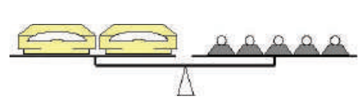
$$x + 45 = 2017$$
$$-45 = -45$$
$$x = 1972$$

Solution

Anne was born in 1972.

Example 14

A mail order electronics company stocks a new mini DVD player and is using a balance to determine the shipping weight. Using only one lb weights, the shipping department found that the following arrangement balances.



Knowing that each weight is one lb, calculate the weight of one DVD player.

Solution

We know that the system balances, so the weights on each side must be equal. We can write an algebraic expression based on this equality. The unknown quantity, the weight of the DVD player (in pounds), will be x. The combined weight on the right of the balance is $5 \times 1\ lb = 5lb$.

$2x = 5 Divide\ both\ sides\ by\ 2.$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{5}{2}$$

$x = 2.5$

Each DVD player weighs x 2.5 lbs.

Example 15

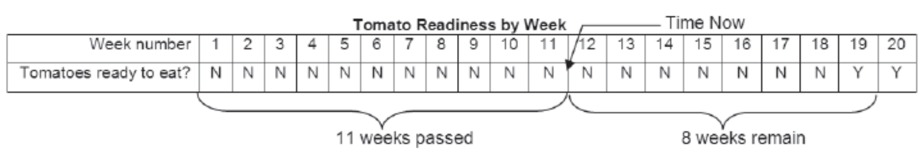
In good weather, tomato seeds can grow into plants and bear ripe fruit in as little as 19 weeks . *Lora planted her seeds* 11*weeks ago. How long must she wait before her tomatoes are ready to eat?*

Solution

We know that the total time to bear fruit is 19 weeks , and that the time so far is 19 weeks . Our unknown is the time in weeks remaining, so we call that x. Here is our equation.

$$\begin{aligned} x + 11 &= 19 \\ -11 &= -11 \\ x &= 8 \end{aligned}$$

Lora will have to wait another 8 weeks before her tomatoes are ready. We can show this by designing a table.


Tomato Readiness by Week

Week number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Tomatoes ready to eat?	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	Y	Y

Example 16

In 2004, *Takeru Kobayashi, of Nagano, Japan, ate* $53\frac{1}{2}$*hot dogs in* 12 minutes . *He broke his previous world record, set in* 2002, *by three hot dogs. Calculate:*

a) How many minutes it took him to eat one hot dog.

b) How many hot dogs he ate per minute.

c) What his old record was.

a) We know that the total time for 53.5 hot dogs is 12 minutes . If the time, in minutes, for each hot dog (the unknown) is x then we can write the following equation.

$$\begin{aligned} 53.5x &= 12 && \text{Divide both sides by 53.5} \\ x &= \frac{12}{53.5} = 0.224 && \text{minutes Convert to seconds, by multiplying by 60} \end{aligned}$$

.

Solution

The time taken to eat one hot dog is 0.224 minutes , or about 13.5 seconds .

Note: We round off our answer as there is no need to give our answer to an accuracy better than 0.1 (one tenth) of a second.

b) This time, we look at our data slightly differently. We know that he ate for 12 minutes . His **rate per-minute** is our new unknown (to avoid confusion with x, we will call this y). We know that the total number of hot dogs is 53.5 so we can write the following equation.

$$\begin{aligned} 12y &= 53.5 && \text{Divide both sides by 12} \\ y &= \frac{53.5}{12} = 4.458 \end{aligned}$$

Solution

Takeru Kobayashi ate approximately 4.5. hot dogs per minute.

c) We know that his new record is 53.5. and also that his new record is three more than his old record. We have a new unknown. We will call his old record z, and write the following equation.

$$\begin{aligned} x+3 &= 53.5 \\ -3 &= -3 \\ x &= 50.5 \end{aligned}$$

Solution

Takeru Kobayashis old record was $50\frac{1}{2}$ hot dogs in 12 minutes .

Lesson Summary

- An equation in which each term is either a constant or a product of a constant and a single variable is a **linear equation.**
- Adding, subtracting, multiplying, or dividing both sides of an equation by the same value results in an equivalent equation.
- To solve an equation, **isolate** the unknown variable on one side of the equation by applying one or more arithmetic operations to both sides.

Review Questions

1. Solve the following equations for x.
   (a) $x+11=7$
   (b) $x-1.1=3.2$

(c) $7x = 21$
(d) $4x = 1$
(e) $\frac{5x}{12} = \frac{2}{3}$
(f) $x + \frac{5}{2} = \frac{2}{3}$
(g) $x - \frac{5}{6} = \frac{3}{8}$
(h) $0.01x = 11$

2. Solve the following equations for the unknown variable.
   (a) $q - 13 = -13$
   (b) $z + 1.1 = 3.0001$
   (c) $21\text{ s} = 3$
   (d) $t + \frac{1}{2} = \frac{1}{3}$
   (e) $\frac{7f}{11} = \frac{7}{11}$
   (f) $\frac{3}{4} = -\frac{1}{2} \cdot y$
   (g) $6r = \frac{3}{8}$
   (h) $\frac{9b}{16} = \frac{3}{8}$
3. Peter is collecting tokens on breakfast cereal packets in order to get a model boat. In eight weeks he has collected 10 tokens. He needs 25 tokens for the boat. Write an equation and determine the following information.
   (a) How many more tokens he needs to collect, n.
   (b) How many tokens he collects per week, w.
   (c) How many more weeks remain until he can send off for his boat, r.
4. Juan has baked a cake and wants to sell it in his bakery. He is going to cut it into 12 slices and sell them individually. He wants to sell it for three times the cost of making it. The ingredients cost him \$8.50, and he allowed \$1.25 to cover the cost of electricity to bake it. Write equations that describe the following statements
   (a) The amount of money that he sells the cake for (u).
   (b) The amount of money he charges for each slice (c).
   (c) The total profit he makes on the cake (w).

Review Answers

1. (a) $x = -4$
   (b) $x = 4.3$
   (c) $x = 3$
   (d) $x = 0.25$
   (e) $x = 1.6$
   (f) $x = -\frac{11}{6}$
   (g) $x = \frac{29}{24}$
   (h) $x = 1100$
2. (a) $q = 0$
   (b) $z = 1.9001$
   (c) $s = 1/7$
   (d) $t = -\frac{1}{6}$
   (e) $f = 1$
   (f) $y = -1.5$
   (g) $r = \frac{1}{16}$
   (h) $b = \frac{2}{3}$
3. (a) $n + 10 = 25, n = 15$
   (b) $8w = 10, w = 1.25$

(c) $r \cdot w = 15$ or $1.25r = 15$, $r = 12$

4. (a) $u = 3(8.5 + 1.25)$
(b) $12v = u$
(c) $w = u - (8.5 + 1.25)$