1.6 Division of Rational Numbers

Learning Objectives

- Find multiplicative inverses.
- Divide rational numbers.
- Solve real-world problems using division.

Introduction – Identity elements

An **identity element** is a number which, when combined with a mathematical operation on a number, leaves that number unchanged. For addition and subtraction, the **identity element** is **zero**.

$$2+0=2$$
$$-5+0=-5$$
$$99-0=99$$

The inverse operation of addition is subtraction.

$x+5-5=x$ When we subtract what we have added, we get back to where we started!

When you add a number to its **opposite,** you get the identity element for addition.

$$5+(-5)=0$$

You can see that the **addition of an opposite is an equivalent operation to subtraction.**

For multiplication and division, the **identity element** is **one**.

$$2\times 1=2$$
$$-5\times 1=-5$$
$$99\div 1=99$$

In this lesson, we will learn about **multiplying by a multiplicative inverse** as an equivalent operation to division. Just as we can use **opposites** to turn a **subtraction** problem into an **addition** problem, we can use **reciprocals** to turn a **division** problem into a **multiplication** problem.

Find Multiplicative Inverses

The **multiplicative inverse** of a number, x, is the number when multiplied by x yields **one**. In other words, any number times the multiplicative inverse of that number equals one. The multiplicative inverse is commonly the reciprocal, and the multiplicative inverse of x is denoted by $\frac{1}{x}$.

Look at the following multiplication problem:

Simplify $\frac{2}{3} \times \frac{3}{2}$

We know that we can cancel terms that appear on both the numerator and the denominator. Remember we leave a one when we cancel all terms on either the numerator or denominator!

$\frac{2}{3} \times \frac{3}{2} = \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2}} = 1$

It is clear that $\frac{2}{3}$ is the multiplicative inverse of $\frac{3}{2}$. Here is the rule.

To find the multiplicative inverse of a rational number, we simply ***invert the fraction***.

The multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$, as long as $a \neq 0$

Example 1

Find the multiplicative inverse of each of the following.

a) $\frac{3}{7}$

b) $\frac{4}{7}$

c) $3\frac{1}{2}$

d) $-\frac{x}{y}$

e) $\frac{1}{11}$

a) **Solution**

The multiplicative inverse of $\frac{3}{7}$ is $\frac{7}{3}$.

b) **Solution**

The multiplicative inverse of $\frac{4}{9}$ is $\frac{9}{r}$.

c) To find the multiplicative inverse of $3\frac{1}{2}$ we first need to convert $3\frac{1}{2}$ to an **improper fraction**:

$$3\frac{1}{2} = \frac{3}{1} + \frac{1}{2} = \frac{6}{2} + \frac{1}{2} = \frac{7}{2}$$

Solution

The multiplicative inverse of $3\frac{1}{2}$ is $\frac{2}{7}$.

d) Do not let the negative sign confuse you. The multiplicative inverse of a negative number is also negative!

Solution

The multiplicative inverse of $-\frac{x}{y}$ is $-\frac{y}{x}$.

e) The multiplicative inverse of $\frac{1}{11}$ is $\frac{11}{1}$. Remember that when we have a denominator of one, we omit the denominator.

Solution

The multiplicative inverse of $\frac{1}{11}$ is 11.

Look again at the last example. When we took the multiplicative inverse of $\frac{1}{11}$ we got a whole number, 11. This, of course, is expected. We said earlier that the multiplicative inverse of x is $\frac{1}{x}$.

The multiplicative inverse of a whole number is one divided that number.

Remember the idea of the **invisible denominator**. The idea that every integer is actually a rational number whose denominator is one. $5 = \frac{5}{1}$.

Divide Rational Numbers

Division can be thought of as the inverse process of multiplication. If we multiply a number by seven, we can divide the answer by seven to return to the original number. Another way to return to our original number is to multiply the answer by the **multiplicative inverse of seven.**

In this way, we can simplify the process of dividing rational numbers. We can turn a division problem into a multiplication process by replacing the divisor (the number we are dividing by) with its multiplicative inverse, or **reciprocal**.

To divide rational numbers, invert the divisor and multiply $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

Also, $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$

Example 2

Divide the following rational numbers, giving your answer in the **simplest form.**

a) $\frac{1}{2} \div \frac{1}{4}$

b) $\frac{7}{3} \div \frac{2}{3}$

c) $\frac{x}{2} \div \frac{1}{4y}$

d) $\frac{11}{2x} \div \left(-\frac{x}{y}\right)$

a) Replace $\frac{1}{4}$ with $\frac{4}{1}$ and multiply. $\frac{1}{2} \times \frac{4}{1} = \frac{1}{\not{2}} \times \frac{\not{2}.2}{2} = \frac{1}{2}$.

Solution

$$\frac{1}{2} \div \frac{1}{4} = 2$$

b) Replace $\frac{2}{3}$ with $\frac{3}{2}$ and multiply. $\frac{7}{\not{3}} \times \frac{\not{3}}{2} = \frac{7}{2}$.

Solution

$$\frac{7}{3} \div \frac{2}{3} = \frac{7}{2}$$

c) Replace $\frac{1}{4y}$ with $\frac{4y}{1}$ and multiply. $\frac{x}{2} \times \frac{4y}{1} = \frac{x}{\not{2}} \times \frac{\not{2}.2.y}{1} = \frac{x.2y}{1}$

Solution

$$\frac{x}{2} \div \frac{1}{4y} = 2xy$$

d) Replace $\left(-\frac{x}{y}\right)$ with $\left(-\frac{y}{x}\right)$ and multiply. $\frac{11}{2x} \times \left(-\frac{y}{x}\right) = -\frac{11.y}{2x.x}$.

Solution

$$\frac{11}{2x}\left(-\frac{x}{y}\right) = -\frac{11y}{2x^2}$$

Solve Real-World Problems Using Division

Speed, Distance and Time

An object moving at a certain **speed** will cover a fixed **distance** in a set **time**. The quantities speed, distance and time are related through the equation:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Example 3

Andrew is driving down the freeway. He passes mile marker 27*at exactly mid-day. At 12:35 he passes mile marker* 69.*At what speed, in miles per hour, is Andrew traveling?*

To determine speed, we need the distance traveled and the time taken. If we want our speed to come out in miles per hour, we will need distance in **miles** and time in **hours**.

$$\text{Distance} = 69 - 27 = 42 \text{ miles}$$

$$\text{Time taken} = 35 \text{ minutes} = \frac{35}{60} = \frac{\cancel{5}.7}{\cancel{5}.12} = \frac{7}{12} \text{ hour}$$

We now *plug in* the values for distance and time into our equation for speed.

$$\text{Speed} = \frac{42}{\left(\frac{7}{12}\right)} = \frac{42}{1} \div \frac{7}{12} \qquad \text{Replace} \frac{7}{12} \text{ with} \frac{12}{7} \text{ and multiply.}$$

$$\text{Speed} = \frac{42}{1} \times \frac{12}{7} = \frac{\cancel{7}.6}{1}\frac{12}{\cancel{7}} = \frac{6.12}{1}$$

Solution

Andrew is driving at 72 miles per hour .

Example 4

Anne runs a mile and a half in a quarter hour. What is her speed in miles per hour?

We already have the distance and time in the correct units (miles and hours). We simply write each as a rational number and plug them into the equation.

$$\text{Speed} = \frac{\left(\frac{3}{2}\right)}{\left(\frac{1}{4}\right)} = \frac{3}{2} \div \frac{1}{4} \qquad \text{Replace} \frac{1}{4} \text{ with } \frac{4}{1} \text{ and multiply.}$$

$$\text{Speed} = \frac{3}{2} \times \frac{4}{1} = \frac{12}{2} = 6$$

Solution

Anne runs at 6 miles per hour.

Example 5 – Newton's Second Law

Newton's second law $(F = ma)$ *relates the force applied to a body* (F), *the mass of the body* (m) *and the acceleration* (a). *Calculate the resulting acceleration if a Force of* $7\frac{1}{3}$*Newtons is applied to a mass of* $\frac{1}{5}$ kg.

First, we rearrange our equation to isolate the acceleraion, a

$$a = \frac{F}{m}$$ Substitute in the known values.

$$a = \frac{\left(7\frac{1}{3}\right)}{\left(\frac{1}{5}\right)} = \left(\frac{7.3}{3} + \frac{1}{3}\right) \div \left(\frac{1}{5}\right)$$ Determine improper fraction, then invert $\frac{1}{5}$ and multiply.

$$a = \frac{22}{3} \times \frac{5}{1} = \frac{110}{3}$$

Solution

The resultant acceleration is $36\frac{2}{3}\ \mathrm{m/s^2}$.

Lesson Summary

- The **multiplicative inverse** of a number is the number which produces one when multiplied by the original number. The multiplicative inverse of x is the reciprocal $\frac{1}{x}$.
- To find the multiplicative inverse of a rational number, we simply **invert the fraction**: $\frac{a}{b}$ inverts to $\frac{b}{a}$.
- To divide rational numbers, invert the divisor and multiply $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

Review Questions

1. Find the multiplicative inverse of each of the following.
   (a) 100
   (b) $\frac{2}{8}$
   (c) $-\frac{19}{21}$
   (d) 7
   (e) $-\frac{z^3}{2xy^2}$
2. Divide the following rational numbers, be sure that your answer in the simplest form.
   (a) $\frac{5}{2} \div \frac{1}{4}$
   (b) $\frac{1}{2} \div \frac{7}{9}$
   (c) $\frac{5}{11} \div \frac{6}{7}$
   (d) $\frac{1}{2} \div \frac{1}{2}$
   (e) $-\frac{x}{2} \div \frac{5}{7}$
   (f) $\frac{1}{2} \div \frac{x}{4y}$
   (g) $\left(-\frac{1}{3}\right) \div \left(-\frac{3}{5}\right)$
   (h) $\frac{7}{2} \div \frac{7}{4}$
   (i) $11 \div \left(-\frac{x}{4}\right)$
3. The label on a can of paint states that it will cover 50 square feet per pint. If I buy a $\frac{1}{8}$ pint sample, it will cover a square two feet long by three feet high. Is the coverage I get more, less or the same as that stated on the label?
4. The world's largest trench digger, "Bagger 288", moves at $\frac{3}{8}$ mph. How long will it take to dig a trench $\frac{2}{3}$ mile long?
5. A $\frac{2}{7}$ Newton force applied to a body of unknown mass produces an acceleration of $\frac{3}{10}\ \mathrm{m/s^2}$. Calculate the mass of the body. Note: Newton $= \mathrm{kg\ m/s^2}$.

Review Answers

1. (a) $\frac{1}{101}$
   (b) $\frac{2}{8}$
   (c) $-\frac{21}{19}$
   (d) $\frac{1}{7}$
   (e) $-\frac{2xy^2}{z^3}$
2. (a) 10
   (b) $\frac{9}{14}$
   (c) $\frac{35}{66}$
   (d) 1
   (e) $-\frac{7x}{10}$
   (f) $\frac{2y}{x}$
   (g) $\frac{5}{9}$
   (h) 2
   (i) $-\frac{44}{x}$
3. At 48 square feet per pint I get less coverage.
4. Time $= \frac{16}{9}$ hour(1 hr46 min40 sec)
5. mass $= \frac{20}{21}$ kg