1.2 Addition of Rational Numbers Learning Objectives

Learning Objectives

- Add using a number line.
- Add rational numbers.
- Identify and apply properties of addition.
- Solve real-world problems using addition of fractions.

Add Using a Number Line

In Lesson one, we learned how to represent numbers on a number line. When we perform addition on a number line, we start at the position of the first number, and then move to the right by the number of units shown in the sum.

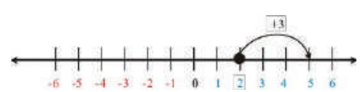
Example 1

Represent the sum $2+3$*on a number line.*

We start at the number 2, and then move 3 to the right. We end at the number 5.

Solution

$$2+3=5$$



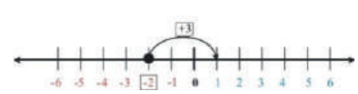
Example 2

Represent the sum $-2+3$*on a number line*.

We start at the number -2, and then move 3 to the right. We thus end at $+1$.

Solution

$$-2+3=1$$



Example 3

Represent the sum $2-3$*on a number line.*

We are now faced with a subtraction. When subtracting a number, an equivalent action is **adding a negative number**. Either way, we think of it, we are moving to the left. We start at the number 2, and then move 3 to the left. We end at -1.

Solution

$$2 - 3 = -1$$

If they have the <u>same sign</u> : Add the absolute value and use the common sign. The means a **positive + positive = positive** and a **negative + negative = negative**

3 + 5 = 8 -3 + (-5) = -8

If they have <u>different signs</u> : Subtract the smaller absolute value from the larger absolute value. The answer will have the sign of the number with the larger absolute value.

-3 + 5 = 2 3 + (-5) = -2

The money analogy Using a money analogy might be helpful when adding signed numbers. When you adding signed numbers you can think of a positive number a gaining money and a negative number as a loss of money. For example, -3 + 5 means you lose 3 dollars and then gain 5 dollars. This leaves you with 2 dollars so the answer is positive 2. 3 + (-5) means you gain 3 dollars and then lose 5 dollars. As a result, you owe 2 dollars so the result is -2.

We can use the number line as a rudimentary way of adding fractions. The enlarged number line below has a number of common fractions marked. The markings on a ruler or a tape measure follow the same pattern. The two shaded bars represent the lengths $\frac{1}{3}$ and $\frac{1}{4}$.

To find the difference between the two fractions look at the difference between the two lengths. You can see the red is $\frac{1}{12}$ longer than the yellow. You could use this as an estimate of the difference.

$$\text{equation} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}.$$

To find the sum of the two fractions, we can lay them end to end. You can see that the sum $\frac{1}{3} + \frac{1}{4}$ is a little over one half.

Adding Rational Numbers

We have already seen the method for writing rational numbers over a common denominator. When we add two fractions we need to ensure that the denominators match before we can determine the sum.

Rules

1. Add or subtract the numerators. 2. Keep the denominators the same. 3. Simplify if possible

$\frac{a}{c}+\frac{b}{c}=\frac{a+c}{c}$

If the

1. Factor each denominator to build the LCD. The LCD is the least common multiple of the denominators. 2. Rewrite each fraction as an equivalent fraction that has the LCD for its denominator.3. Add or subtract the numerators. The denominators stay the same.4. Simplify if possible.

While our goal to add fractions it to determine the **least common denominator**, the following procedure will work. However, it will more than likely require additional simplifying.

$\frac{a}{c}+\frac{b}{d}=\frac{a\cdot d}{c\cdot d}+\frac{b\cdot c}{d\cdot c}=\frac{a\cdot d+b\cdot c}{c\cdot d}$

Example 4

Simplify $\frac{3}{5}+\frac{1}{6}$

To combine these fractions, we need to rewrite them over a common denominator. We are looking for the **lowest common denominator** (LCD). We need to identify the **lowest common multiple** or **least common multiple** (LCM) of 5 and 6. That is the smallest number that both 5 and 6 divide into without remainder.

- The lowest number that 5 and 6 both divide into without remainder is 30. The LCM of 5 and 6 is 30, so the lowest common denominator for our fractions is also 30.

We need to rewrite our fractions as new **equivalent fractions** so that the denominator in each case is 30.

If you think back to our idea of a cake cut into a number of slices, $\frac{3}{5}$ means 3 slices of a cake that has been cut into 5 pieces. You can see that if we cut the same cake into 30 pieces (6 times as many) we would need 18 slices to have an equivalent share, since $18=3\times 6$.

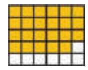
$\frac{3}{5}$ is equivalent to $\frac{18}{30}$

By a similar argument, we can rewrite the fraction $\frac{1}{6}$ as a share of a cake that has been cut into 30 pieces. If we cut it into 5 times as many pieces we require 5 times as many slices.

$\frac{1}{6}$ is equivalent to $\frac{5}{30}$

Now that both fractions have the same common denominator, we can add the fractions. If we add our 18 smaller pieces of cake to the additional 5 pieces you see that we get a total of 23 pieces. 23 pieces of a cake that has been cut into 30 pieces means that our answer is.

Solution



$$\frac{3}{5}+\frac{1}{6}=\frac{18}{30}+\frac{5}{30}=\frac{23}{30}$$

You should see that when we have fractions with a common denominator, we **add the numerators** but we **leave the denominators alone**. Here is this information in algebraic terms.

When adding fractions: $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

Example 5

Simplify $\frac{14}{11}+\frac{1}{9}$

The lowest common denominator in this case is 99. This is because the lowest common multiple of 9 and 11 is 99. So we write equivalent fractions for both $\frac{14}{11}$ and $\frac{1}{9}$ with denominators of 99.

11 divides into 99 nine times so $\frac{14}{11}$ is equivalent to $\frac{14\cdot 9}{11\cdot 9}=\frac{126}{99}$

We can multiply the numerator and denominator by 9 (or by any number) since $9/9=1$ and 1 is the multiplicative identity.

9 divides into 99 eleven times so $\frac{1}{9}$ is equivalent to $\frac{1\cdot 11}{9\cdot 11}=\frac{11}{99}$.

Now we simply add the numerators.

Solution

$$\frac{14}{11}+\frac{1}{9}=\frac{126}{99}+\frac{11}{99}=\frac{137}{99}$$

Example 6

Simplify

$$-\frac{1}{12}+\frac{2}{9}$$

The least common denominator in this case is 36. This is because the LCM of 12 and 9 is 36. We now proceed to write the equivalent fractions with denominators of 36.

12 divides into 36 three times so $-\frac{1}{12}$ is equivalent to $\frac{-1\cdot 3}{12\cdot 3}=\frac{-3}{36}$.

9 divides into 36 four times so $\frac{2}{9}$ is equivalent to $\frac{2\cdot 4}{9\cdot 4}=\frac{8}{36}$.

Solution

$$-\frac{1}{12}+\frac{2}{9}=\frac{-3}{36}+\frac{8}{36}=\frac{5}{36}$$

You can see that we quickly arrive at an equivalent fraction by multiplying the numerator and the denominator by the same non-zero number.

Example 7

Simplify $-\frac{2}{15}+\frac{2}{25}$

The least common denominator is 75. This is because the LCM of 15 and 25 is 75. We now proceed to write the equivalent fractions with the denominator of 75.

Solution

$\frac{-2}{3 \cdot 5} + \frac{3}{5 \cdot 5} = \frac{-2 \cdot 5}{3 \cdot 5 \cdot 5} + \frac{3 \cdot 3}{5 \cdot 5 \cdot 3} = \frac{-10+6}{75} = -\frac{4}{75}$

Identify and Apply Properties of Addition

The three mathematical properties which involve addition are the **commutative, associative,** and the **additive identity properties**.

- **Commutative property** When two numbers are added, the sum is the same even if the order of the items being added changes.

Example $3+2=2+3$

On a number line this means move 3 units*to the right then* 2 units*to the right. The commutative property says this is equivalent of moving* 2 units*to the right then* 3 units*to the right. You can see that they are both the same, as they both end at* 5.

- **Associative Property** When three or more numbers are added, the sum is the same regardless of how they are grouped.

Example $(2+3)+4=2+(3+4)$

- **Additive Identity Property** The sum of any number and zero is the original number.

Example $5+0=5$

Example 8

Nadia and Peter are building sand castles on the beach. Nadia built a castle two feet tall, stopped for ice-cream and then added one more foot to her castle. Peter built a castle one foot tall before stopping for a sandwich. After his sandwich, he built up his castle by two more feet. Whose castle is the taller?

Solution

Nadia's castle is $(2+1)$ feet tall. Peter's castle is $(1+2)$ feet tall. According to the **Commutative Property of Addition,** the two castles are the same height.

Example 9

Nadia and Peter each take candy from the candy jar. Peter reaches in first and grabs one handful. He gets seven pieces of candy. Nadia grabs with both hands and gets seven pieces in one hand and five in the other. The following day Peter gets to go first. He grabs with both hands and gets five pieces in one hand and six in the other. Nadia, grabs all the remaining candy, six pieces, in one hand. In total, who got the most candy?

Solution

On day one, Peter gets 7 candies, and on day two he gets $(5+6)$ pieces. His total is $7+(5+6)$. On day one, Nadia gets $(7+5)$ pieces. On day two, she gets 6. Nadia's total is therefore $(7+5)+6$. According to the **Associative Property of Addition** they both received exactly the same amount.

Solve Real-World Problems Using Addition

Example 10

Peter is hoping to travel on a school trip to Europe. The ticket costs $2400. *Peter has several relatives who have pledged to help him with the ticket cost. His parents have told him that they will cover half the cost. His grandma Zenoviea will pay one sixth, and his grandparents in Florida will send him one fourth of the cost. What fraction of the cost can Peter count on his relatives to provide?*

The first thing we need to do is extract the relevant information. Here is what Peter can count on.

$\left(\frac{1}{2}\right)$	From parents
$\left(\frac{1}{6}\right)$	From grandma
$\left(\frac{1}{4}\right)$	From grandparents in Florida

Here is out problem. $\frac{1}{2}+\frac{1}{6}+\frac{1}{4}$

To determine the sum, we first need to find the LCD. The LCM of 2, 6 and 4 is 12. This is our LCD.

2 divides into 12 six times :	$\frac{1}{2}=\frac{6\cdot 1}{6\cdot 2}=\frac{6}{12}$
6 divides into 12 two times :	$\frac{1}{6}=\frac{2\cdot 1}{2\cdot 6}=\frac{2}{12}$
4 divides into 12 six times :	$\frac{1}{4}=\frac{3\cdot 1}{3\cdot 4}=\frac{3}{12}$
So an equivalent sum for our problem is	$\frac{6}{12}+\frac{2}{12}+\frac{3}{12}=\frac{(6+2+3)}{12}=\frac{11}{12}$

Solution

Peter can count on eleven-twelfths of the cost of the trip ($\$2,200$ out of $\$2,400$).

Lesson Summary

- To add fractions, rewrite them over the **lowest common denominator (LCD)**. The lowest common denominator is the **lowest** (or **least**) **common multiple (LCM)** of the two denominators.
- When **adding fractions**: $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$
- The fractions $\frac{a}{b}$ and $\frac{a\cdot c}{b\cdot c}$ are **equivalent** when $c \neq 0$
- The **additive properties** are:
- **Commutative property** the sum of two numbers is the same even if the order of the items to be added changes.

Ex: $2+3=3+2$

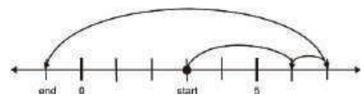
- **Associative Property** When three or more numbers are added, the sum is the same regardless of how they are grouped.

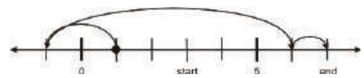
Ex: $(2+3)+4=2+(3+4)$

- **Additive Identity Property** The sum of any number and zero is the original number.

Review Question

1. Write the sum that the following moves on a number line represent.

(a) 

(b) 

2. Add the following rational numbers and write the answer in its **simplest form.**
   (a) $\frac{3}{7}+\frac{2}{7}$
   (b) $\frac{3}{10}+\frac{1}{5}$
   (c) $\frac{5}{16}+\frac{5}{12}$
   (d) $-\frac{3}{8}+\frac{9}{16}$
   (e) $\frac{8}{25}+\frac{7}{10}$
   (f) $\frac{1}{6}+\left(-\frac{1}{4}\right)$
   (g) $\frac{7}{15}+\left(-\frac{2}{9}\right)$
   (h) $\frac{5}{19}+\frac{2}{27}$
3. Which property of addition does each situation involve?
   (a) Whichever order your groceries are scanned at the store, the total will be the same.
   (b) However many shovel-loads it takes to move 1 ton of gravel the number of rocks moved is the same.
4. Nadia, Peter and Ian are pooling their money to buy a gallon of ice cream. Nadia is the oldest and gets the greatest allowance. She contributes half of the cost. Ian is next oldest and contributes one third of the cost. Peter, the youngest, gets the smallest allowance and contributes one fourth of the cost. They figure that this will be enough money. When they get to the check-out, they realize that they forgot about sales tax and worry there will not be enough money. Amazingly, they have exactly the right amount of money. What fraction of the cost of the ice cream was added as tax?

Review Answers

1. (a) $3+3+1-8=-1$
   (b) $1-2+7+1=7$
2. (a) $\frac{5}{7}$
   (b) $\frac{1}{2}$
   (c) $\frac{35}{48}$
   (d) $\frac{3}{16}$
   (e) $\frac{51}{50}$
   (f) $-\frac{1}{12}$
   (g) $\frac{11}{45}$
   (h) $\frac{173}{513}$
3. (a) Commutative and Associative
   (b) Associative
4. $\frac{1}{12}$ is added as tax.