Whole-Number Multiplication and Division

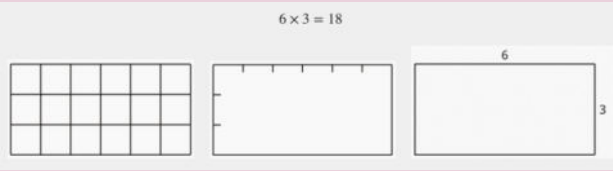
Grade 5: Unit 4

Standards addressed: 5.MD.C, 5.NBT.B, 5.OA.A.2, 5.NF.B.3, 5.OA.A.1, 5.OA.A.2, 5.NF.B.7

Unit 4 Progression Overview

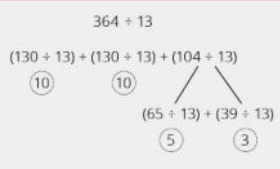
Section A
Lessons 1-9
5.MD.C.3, 5.MD.C.5, 5.NBT.B, 5.NBT.B.5, 5.NF.B.5, 5.OA.A.2

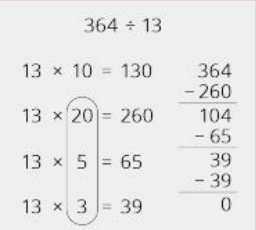
- Multiply multi-digit whole numbers using strategies based on place value and the properties of operations, including the standard algorithm.

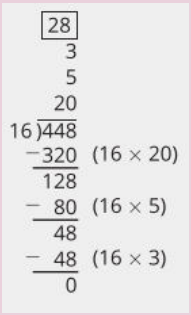


Section B
Lessons 10-17
5.NBT.B.5, 5.NBT.B.6, 5.NF.B.3, 5.OA.A.1, 5.OA.A.2

- Divide multi-digit whole numbers using strategies based on place value, properties of operations, and relationship between multiplication and division.







Section C
Lessons 18-21
5.MD.C, 5.MD.C.5, 5.NBT.B.5, 5.NBT.B.6, 5.NF.B.7

- Multiply and divide to solve real-world and mathematical problems involving volume.

Unit 4 Quick Links

Adapt 1

Adapt 2

Adapt 3

L1

L2

L3

L4

L5

L6

L7

L8

L9

Adapt 4

Adapt 5

L10

L11

L12

L13

L14

L15

L16

L17

L18

L19

L20

L21

Adaptation
Lesson 1

Multiply Two-Digit Numbers by One-Digit Numbers

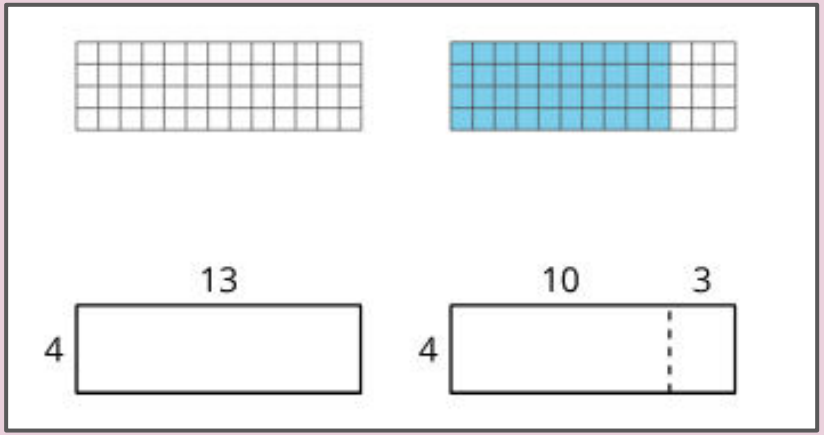
Let’s multiply two-digit and one-digit numbers

4.NBT.B.5

Warm up

Notice and Wonder: With and Without a Grid

What do you notice?

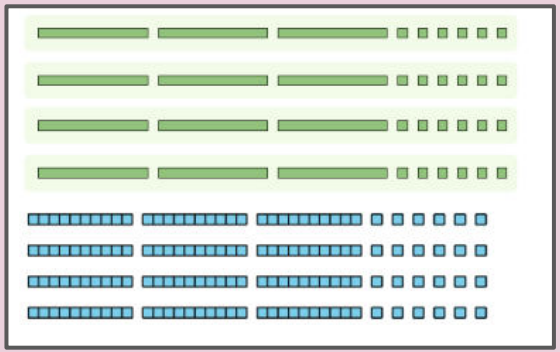


What do you wonder?

What was Tyler Thinking?

Activity #1

1. To find the value of 4 x 36, Tyler uses a base-ten diagram, as shown here.

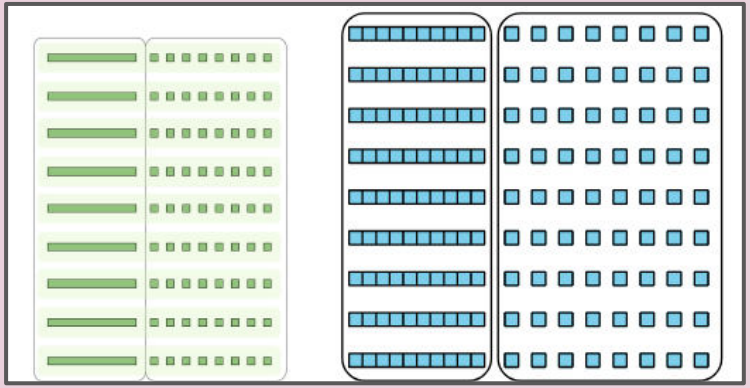


a. Where is the 36 in Tyler's diagram?

b. Where is the 4 in Tyler's diagram?

c. What is the value of 4 x 36?

2. To find 9 x 18, Tyler uses 9 tens blocks and 8 ones blocks. Here is a diagram of how he arranges the blocks.

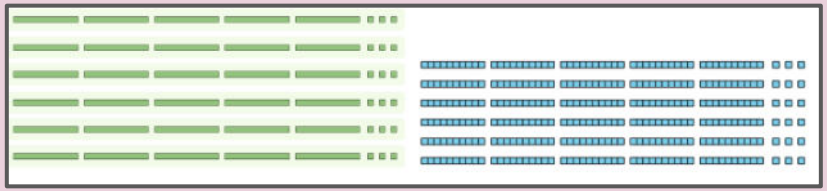


Explain or show how his arrangement helps him find the value of 9 x 18

Rectangular Diagrams

Activity #2

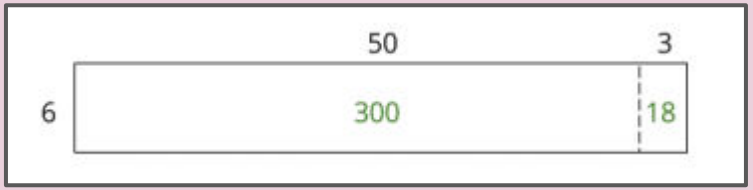
1. Priya drew a base-ten diagram to multiply 6 x 53. She said it shows that the product can be found by adding 300 and 18.



a. Where do you see 6 and 53 in her diagram?

b. Where do you see 300 and 18 in Priya's diagram? What do they represent?

2. Han multiplied 6 x 53 using this diagram:



Where do you see 300 and 18 in his diagram? What do they represent?

Rectangular Diagrams

Activity #2

3. Which way do you prefer for multiplying 6 x 53: Han's way or Priya's way? Explain your reasoning.

4. Find the value of 6 x 53.

5. Draw a diagram to represent each multiplication expression. Then, find the value of each product.

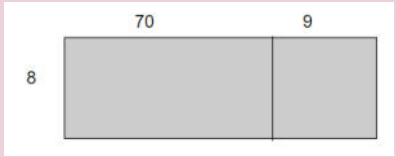
a. 6 x 48

b. 9 x 67

Lesson Synthesis

Today we used base-ten diagrams and rectangular diagrams to represent multiplication. We compared these representations and also used expressions to represent them.

8×79



What expression could we write to represent the diagram?

How could we use this rectangle to represent 8 x 79?

How does this diagram help us find the value of 8 x 79?

Adaptation
Lesson 2

Multiply Three- and Four-digit Numbers by One-digit Numbers

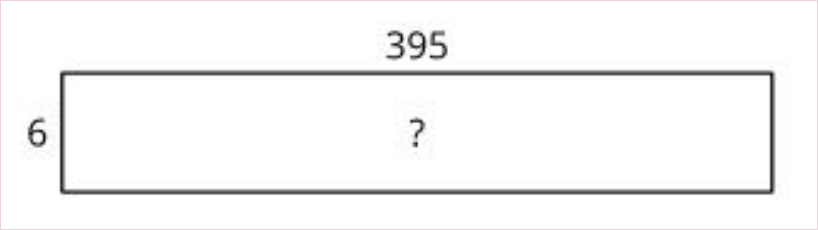
Let’s multiply three- and four-digit numbers by one-digit numbers

4.NBT.B.5

Warm up

Estimation Exploration: Multiplying Teens

What is the area of the rectangle?



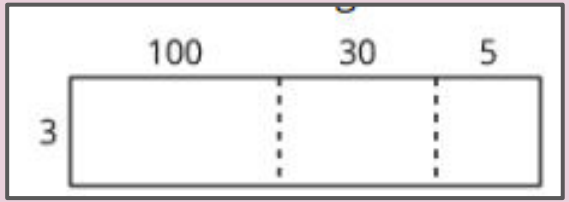
Record an estimate that is:

Too low	About right	Too high

Larger Numbers to Multiply

Activity #1

1. Clare drew this diagram.



a. What multiplication expression can be represented by the diagram?

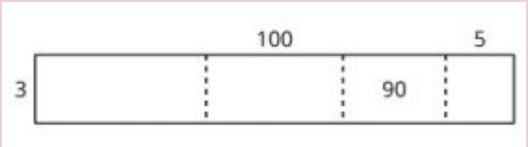
b. Find the value of the expression. Show your reasoning.

2. Consider the expression 6 x 252.

a. Draw a diagram to represent the expression.

b. Find the value of the expression. Show your reasoning.

3. Lin drew a diagram to represent 3 x 2,135.



a. Complete Lin's diagram.

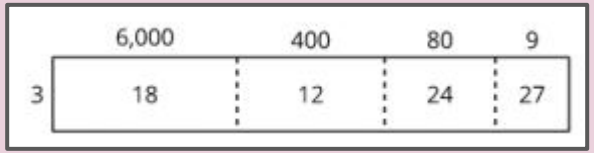
b. Write an expression to represent the value of each part of the diagram.

c. Find the value of 3 x 2,135. Show your reasoning.

Jada's Errors

Activity #2

1. Jada used a diagram to multiply 3 x 6,489 and made a few errors.



a. Explain the errors Jada made.

b. Find the value of 3 x 6,489. Show your reasoning

2. Find the value of each expression. Show your reasoning for each.

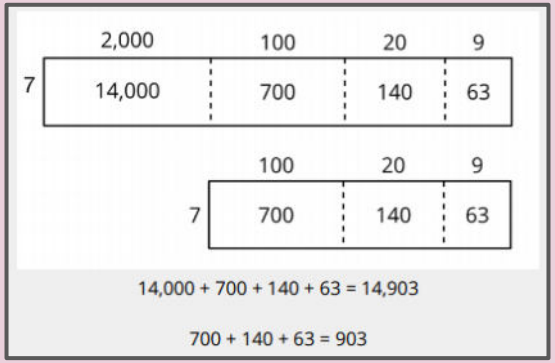
a. 5 x 699

b. 8 x 4,973

Lesson Synthesis

Today we used rectangular diagrams to multiply three- and four-digit numbers by one-digit numbers. Let's compare the diagrams for 7 x 2,129 and 7 x 129.

7 × 2,129 7 × 129



How are the representations alike and how are they different?

How would you find the value of 7 x 2,039?

Adaptation
Lesson 3

Multiply 2 Two-Digit Numbers

Let’s multiply 2 two-digit numbers.

4.NBT.B.5

Warm up

Number Talk: Extra Groups

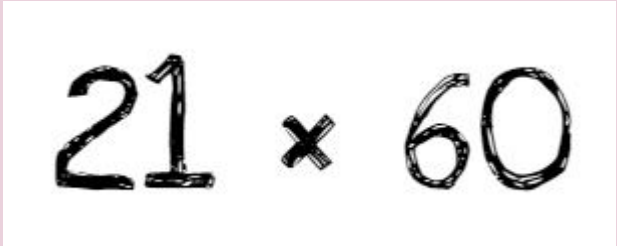
Find the value of each product mentally.

$$20 \times 60$$

Warm up

Number Talk: Extra Groups

Find the value of each product mentally.



Warm up

Number Talk: Extra Groups

Find the value of each product mentally.

$$20 \times 62$$

Warm up

Number Talk: Extra Groups

Find the value of each product mentally.

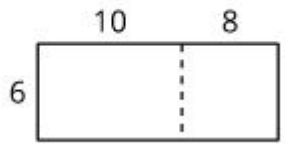
$$19 \times 60$$

Two by Two

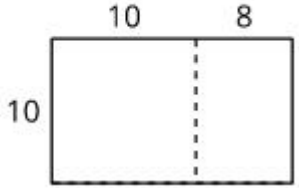
Activity #1

1. For each diagram, write a multiplication expression that the diagram can represent. Then, find the value of the expression. Use equations to show your reasoning.

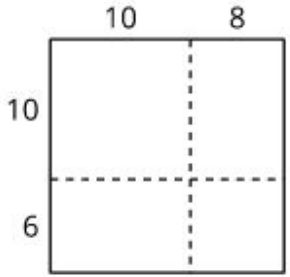
A.



B.



C.



2. How are the diagrams alike? How are they different? Discuss with your partner.

Two by Two

Activity #1

3. Use a diagram to find each product.

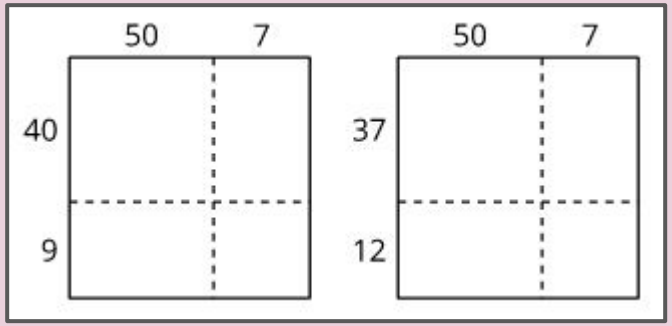
a. 13 x 21

b. 25 x 46

Decompose by Place Value

Activity #2

These diagrams could be used to find the value of 49 x 57.



1. Why is diagram A more helpful than diagram B when finding the value of 49 x 57?

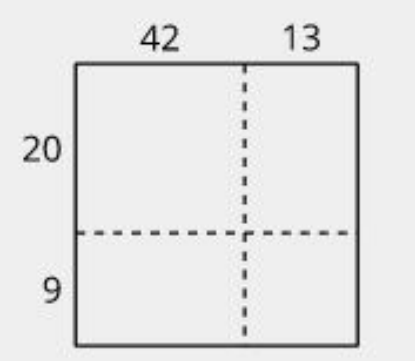
2. Use a diagram to find each product.

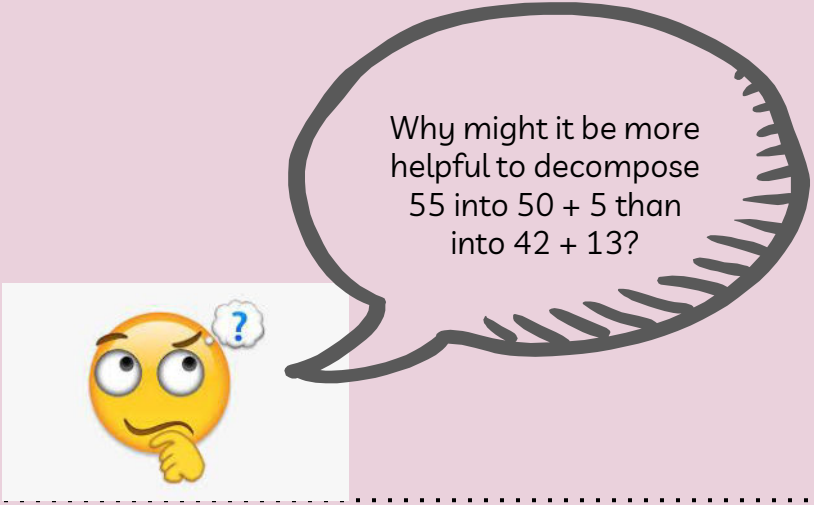
a. 49 x 57

b. 29 x 55

Lesson Synthesis

Today we learned how to represent the multiplication of 2 two-digit numbers using a rectangular diagram. We learned that we can decompose each factor by place value and show the tens and ones on each side of the rectangle, and that doing this can help us to multiply efficiently.





1

Lots of Milk

Let's make estimates with big numbers.

5.MD.C.5, 5.NBT.B

Warm up

Estimation Exploration: How Big is the Milk Carton?

What is the volume of the milk carton in cubic inches?

Record an estimate that is:

Too low	About right	Too high

Activity #1

Milk for Everyone

In each situation, estimate the volume of milk, in cubic inches, that you or the group would drink in one day. Explain your reasoning.

1. you
2. your class
3. your grade
4. your school
5. 10 schools

Activity #2

How Big is 1,000,000?

Estimate the number of days it would take each group to drink 1,000,000 cubic inches of milk. Explain your reasoning.

1. 10 local schools
2. your school
3. your grade
4. your class
5. you

Lesson Synthesis

In this lesson we estimated products and quotients.

How can you use multiplication to estimate how many days it would take your school to drink 1,000,000 ounces of milk?

Could you also make this estimate using division?

Throughout this unit we will be finding products and quotients of whole numbers and learning how they are related.

2

Multiplication Estimates and Evaluation

Let's estimate and calculate products.

Warm up

Number Talk:
A Multiple of 10

Find the value of each product mentally.

50×6

Warm up

Number Talk:
A Multiple of 10

Find the value of each product mentally.

$$50 \times 60$$

Warm up

Number Talk:
A Multiple of 10

Find the value of each product mentally.

$$50 \times 600$$

Warm up

Number Talk: A Multiple of 10

Find the value of each product mentally.

600×500

Reasonable Estimates

Activity #1

1. Which estimate for the product 18×149 is most reasonable? Be prepared to explain your reasoning to your partner.

a. 2,000

b. 4,000

c. 3,000

d. 1,500

2. Are any of the estimates unreasonable? Be prepared to explain your reasoning to your partner.

3. Do you think the actual product will be more or less than your estimate? Explain your reasoning.

Multiply by 18

Activity #2

Evaluate each expression. Explain your reasoning

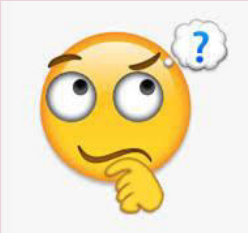
1. 18×9 2. 18×49 3. 18×149

Lesson Synthesis

Today we estimated the value of 18 × 149 and then found the exact value.

What was the same about estimating and finding the exact value?

What was different about estimating and finding the exact value?



In the next lessons, we are going to continue to investigate strategies for organizing calculations for products like 18 × 149.

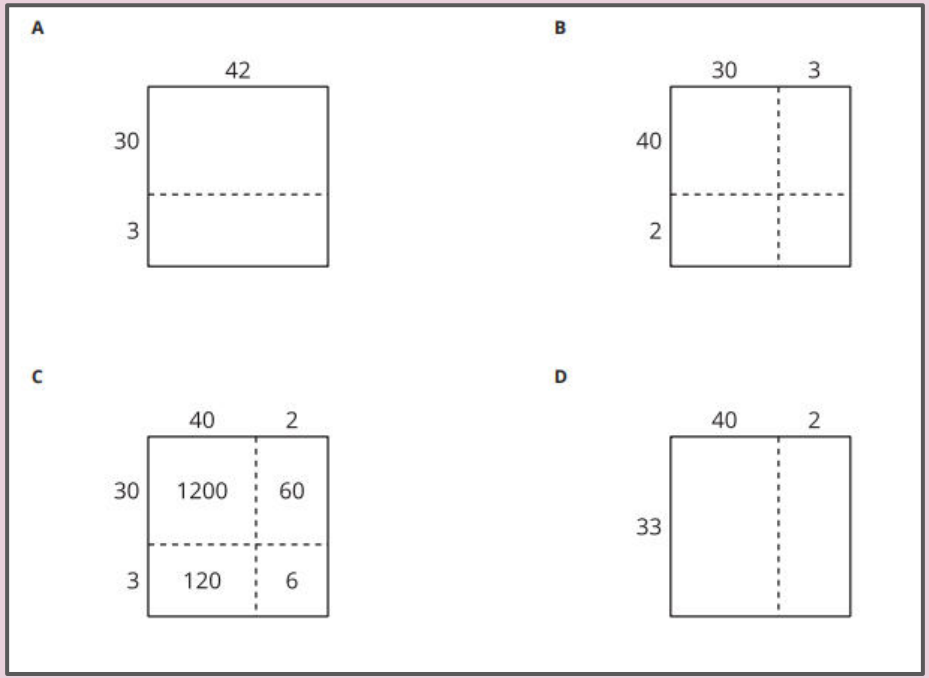
3

Partial Products with Diagrams

Let's use diagrams to help find products.

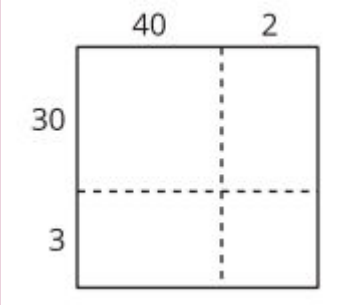
Warm up

Which One Doesn't Belong: Diagrams to Find Products



Decompose in Many Ways

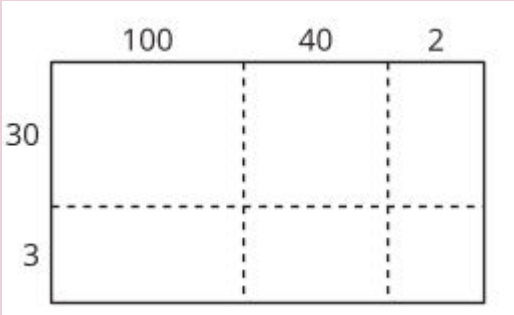
Activity #1



1. Write the value of each product inside the rectangles.

2. Find the value of 42 × 33.

3. This diagram represents 142 × 33.



Write the value of each product in the boxes

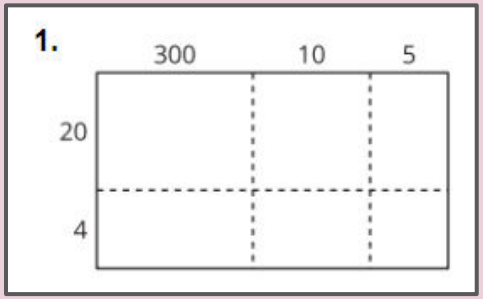
. 4. Find the value of 142 × 33.

Activity #2

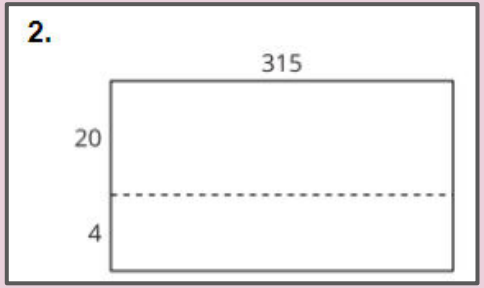
Calculating in Many Ways

Here are some different diagrams that represent 315 × 24 .
For each diagram, write a multiplication expression inside each rectangle to represent the product.

1.



2.



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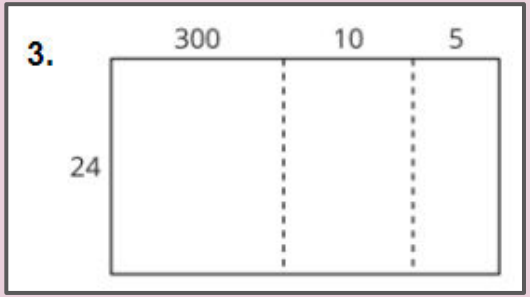
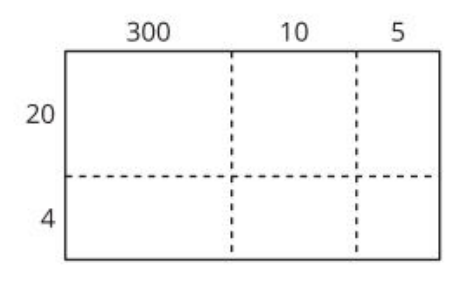


4. What are some advantages of each way to calculate 315 × 24 ? Explain your reasoning.

5. Use one of the diagrams to find 315 × 24.

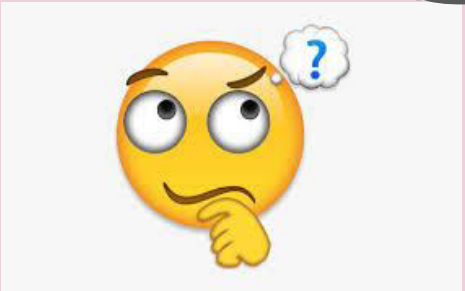
Lesson Synthesis

Today we multiplied numbers and represented these products with diagrams.



What do you need to know to find each product?

How does the diagram help visualize calculating 315 × 24?



Tomorrow we are going to work with partial products and organize them in a different way

4

Different Methods

Let's organize products with expressions.

5.NBT.B, 5.NBT.B.5, 5.OA.A.2

Warm up

Estimation Exploration: A Large Product

$$(5 + 40 + 600) \times (70 + 3)$$

Record an estimate that is:

Too low	About right	Too high

Two Ways to Multiply

Activity #1

Andre

$$
\begin{array}{rrrr}
 & 3 & 4 & 1 \\
\times & & 2 & 2 \\
\hline
 & 1 & 1 & \\
6, & 0 & 0 & 0 \\
 & 8 & 0 & 0 \\
 & & 2 & 0 \\
 & 6 & 0 & 0 \\
 & & 8 & 0 \\
+ & & & 2 \\
\hline
7, & 5 & 0 & 2
\end{array}
$$

Clare

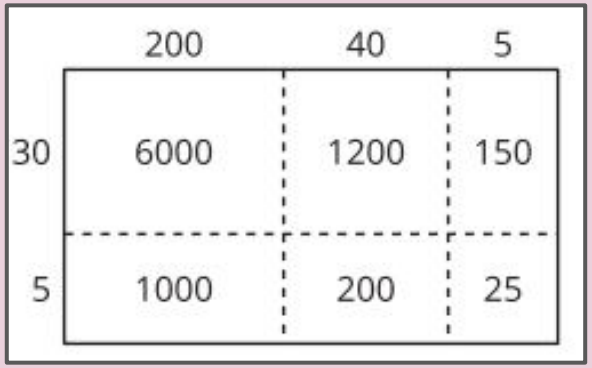
$$
\begin{array}{rrrr}
 & 3 & 4 & 1 \\
\times & & 2 & 2 \\
\hline
 & & & 2 \\
 & 1 & 8 & 0 \\
 & 6 & 0 & 0 \\
 & & 2 & 0 \\
1 & 8 & 0 & 0 \\
+ \ 6, & 0 & 0 & 0 \\
\hline
7, & 5 & 0 & 2
\end{array}
$$

1. How are Andre's and Clare's calculations the same? How are they different?

2. Create a list of expressions to match the partial products Andre and Clare found.

Partial Products Everywhere

Activity #2



1. Take turns picking out expressions that are equivalent to 245 × 35 when put together.

2. Explain how you know the sum of your expressions is equal to 245 × 35 .

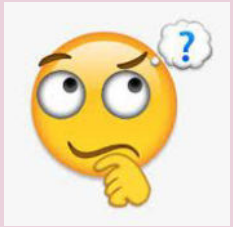
3. What is the value of 245 × 35 ? Show or explain your reasoning.

Lesson Synthesis

Today we found products of two-digit and three-digit numbers using partial products. Sometimes we listed each partial product and sometimes we put more than one partial product together.

How do you know that all the different ways to find the product give the same answer?

Which method did you like most?



What is helpful to remember when you are using partial products to determine a full product?

5

The Standard Algorithm

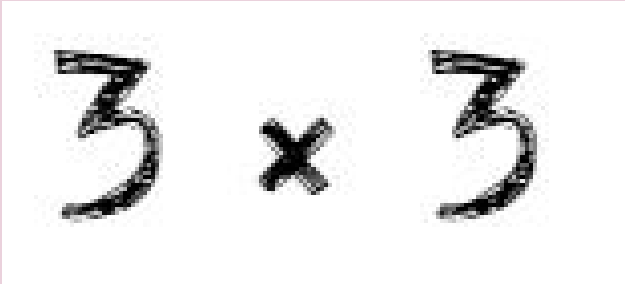
Let’s learn the standard algorithm to multiply multi-digit whole numbers.

5.NBT.B.5

Warm up

Number Talk: Partial Product

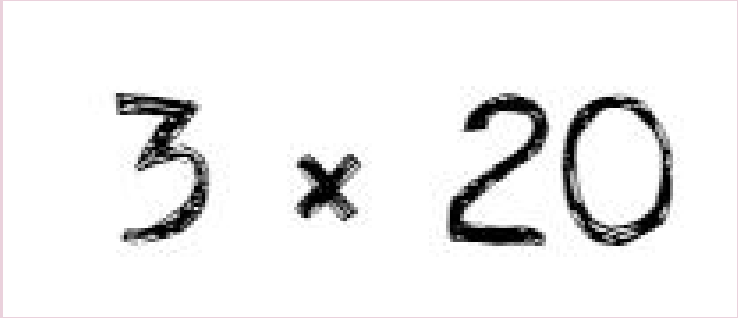
Find the value of each product mentally.



Warm up

Number Talk: Partial Product

Find the value of each product mentally.



Warm up

Number Talk: Partial Product

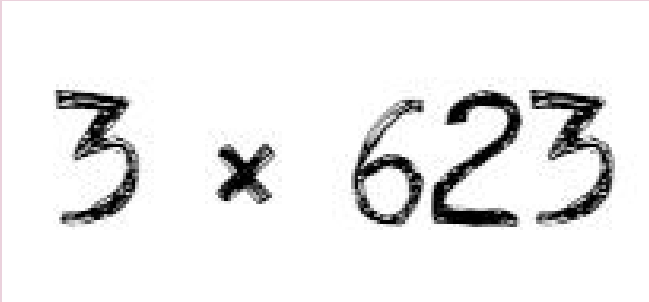
Find the value of each product mentally.

3×600

Warm up

Number Talk: Partial Product

Find the value of each product mentally.



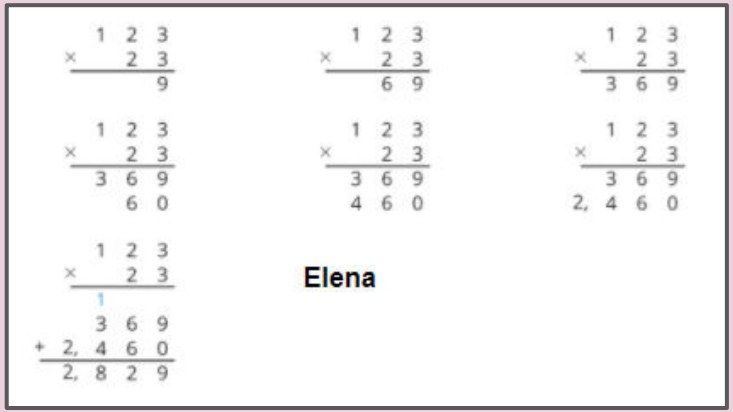
Compare Two Algorithms

Activity #1

Two algorithms for evaluating 123 × 23 are shown below.

Diego

$$
\begin{array}{rrrrl}
 & 1 & 2 & 3 & \\
\times & & 2 & 3 & \\
\hline
 & & & 9 & \text{step 1} \\
 & 1 & 6 & 0 & \text{step 2} \\
 & 3 & 0 & 0 & \text{step 3} \\
 & & 6 & 0 & \text{step 4} \\
 & 4 & 0 & 0 & \text{step 5} \\
+ & 2, & 0 & 0 & 0 \text{ step 6} \\
\hline
 & 2, & 8 & 2 & 9 \text{ step 7}
\end{array}
$$



- How are Diego's algorithm and Elena's algorithm the same? How are they different?
- Explain where you see each step from Diego's algorithm in Elena's algorithm.
- How do the final steps in the two algorithms compare?

Using the Standard Algorithm

Activity #1

Calculate each product using Elena's method.

1. 412 × 2

2. 412 × 32

3. 423 × 22

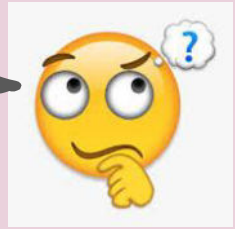
Lesson Synthesis

Today we learned a new algorithm to multiply whole numbers. We organized the partial products in a different way. It is called the standard algorithm for multiplication.

$$
\begin{array}{rrrrr}
 & & 4 & 1 & 2 \\
 & \times & & 3 & 2 \\
\hline
 & 1 & & & \\
 & & 8 & 2 & 4 \\
+ & 1 & 2, & 3 & 6 & 0 \\
\hline
 & 1 & 3, & 1 & 8 & 4
\end{array}
$$

Which part of the product does the partial product 824 represent? How do you know?

Which part of the product does the partial product 1,236 represent? How do you know?



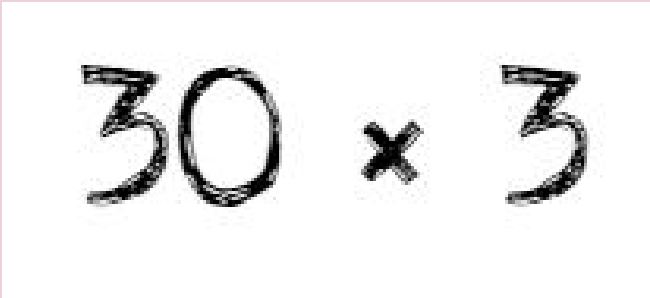
Compose a New Unit

Let’s use the standard algorithm when we have to compose a new unit.

Warm up

Number Talk: Partial Product

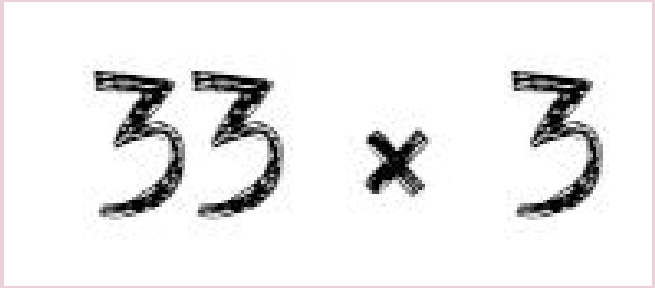
Find the value of each product mentally.



Warm up

Number Talk: Partial Product

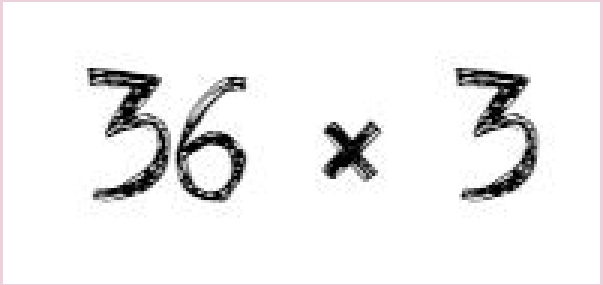
Find the value of each product mentally.



Warm up

Number Talk: Partial Product

Find the value of each product mentally.



Warm up

Number Talk: Partial Product

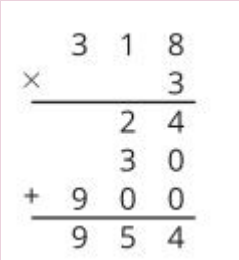
Find the value of each product mentally.

38×3

Carry on with the Standard Algorithm

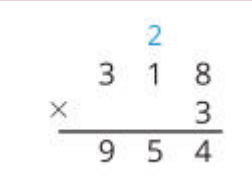
Activity #1

Here is how Han calculated 318 × 3



1. Show or explain where you see these partial products in Han's diagram:
   - 3 × 8
   - 3 × 10
   - 3 × 300

Here is how Elena calculated 318 × 3



2. What does the 2 in Elena's calculation represent? Show or explain your reasoning.

3. What does the 5 in Elena's solution represent? Show or explain your reasoning.

Using the Standard Algorithm with Carrying

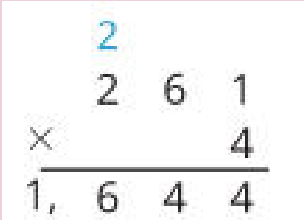
Activity #2

1. Find each product using the standard algorithm.

   a. 327 × 3

   a. 261 × 4

2. Tyler tried to use the algorithm to multiply 261 × 4 , but he made a mistake.

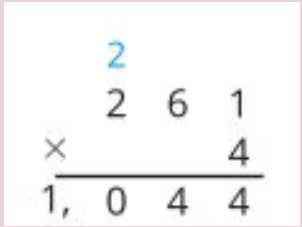


a. Describe the mistake Tyler made.

b. Explain what Tyler should do to fix his mistake.

Lesson Synthesis

Today we learned how to multiply numbers with the standard algorithm in which we need to compose a new unit, or use carrying notation to record part of a partial product in the place value to the left.



What does the 2 over the 2 in 261 represent?

Why is there 1 thousand in the product 261 × 4 ?

7

Compose More Units

Let’s learn how to record more newly composed units with the standard algorithm.

5.NBT.B.5

Warm up

Number Talk: Three Factors

Find the value of each product mentally.

$(2 \times 3) \times 10$

Warm up

Number Talk: Three Factors

Find the value of each product mentally.

$$(2 \times 40) \times 10$$

Warm up

Number Talk: Three Factors

Find the value of each product mentally.

$$(2 \times 200) \times 10$$

Warm up

Number Talk: Three Factors

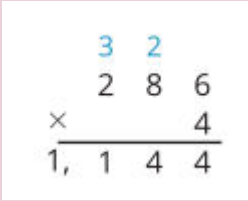
Find the value of each product mentally.

$$(2 \times 243) \times 10$$

Compose and Carry Again

Activity #1

1. Find the value of 286 × 4 . Here is how Tyler used the standard algorithm to calculate 286 × 4.



2. What does the 2 over the 8 represent? Explain your reasoning.

3. What does the 3 over the 2 represent? Explain your reasoning.

4. Use Tyler's method to calculate

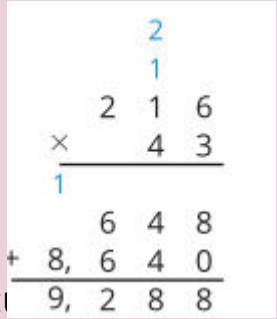
375 × 3

Units Upon Units

Activity #2

1. Find 216 × 3 using the standard algorithm.

2. Lin used the standard algorithm to calculate 216 × 3 . Here is her work:



calc

a. Where do you see 216 × 3 in Lin's work?

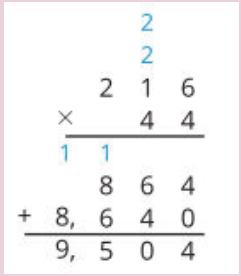
b. Where do you see 216 × 40 in Lin's work?

c. What does the blue 2 represent in Lin's

3. Use Lin's method to calculate 128 × 24.

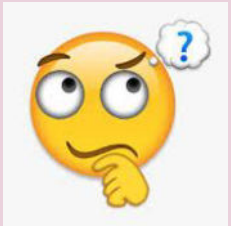
Lesson Synthesis

Today we learned how to use the standard multiplication algorithm when there are two new units composed.



$$40 \times 216 = 10 \times (4 \times 216).$$

Why are the digits in the second line of the calculation the same as those in the first?



Multiplying by 10 shifts each digit to the left one place because each place value is 10 times the value of the place to its right.

8

Build Multi-digit Multiplication Fluency

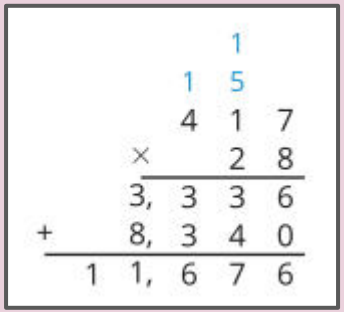
Let’s multiply multi-digit whole numbers using the standard algorithm.

5.NBT.B.5

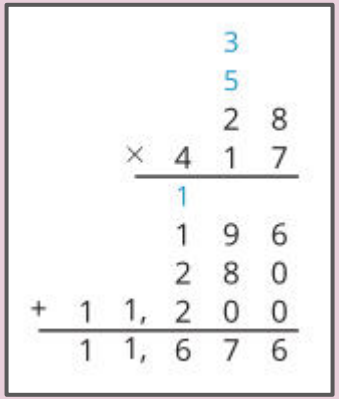
Warm up

Notice and Wonder: Same Solution

What do you notice?



$$
\begin{array}{r}
{\scriptstyle 1\ \ \ \ } \\
{\scriptstyle 1\ 5\ } \\
4\ 1\ 7 \\
\times\quad 2\ 8 \\
\hline
3,\ 3\ 3\ 6 \\
+\quad 8,\ 3\ 4\ 0 \\
\hline
1\ 1,\ 6\ 7\ 6
\end{array}
$$



$$
\begin{array}{r}
{\scriptstyle 3\ \ \ \ } \\
{\scriptstyle 5\ \ \ \ } \\
2\ 8 \\
\times\ 4\ 1\ 7 \\
\hline
{\scriptstyle 1\ \ \ \ } \\
1\ 9\ 6 \\
2\ 8\ 0 \\
+\ 1\ 1,\ 2\ 0\ 0 \\
\hline
1\ 1,\ 6\ 7\ 6
\end{array}
$$

What do you wonder?

Activity #1

All the Products

Find each product using the standard algorithm.

1. 647×9
2. 647×50
3. 647×59
4. 264×38

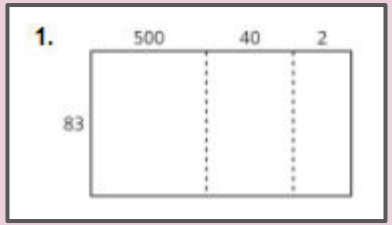
Match the algorithms to the diagrams.

Will it Always Work?

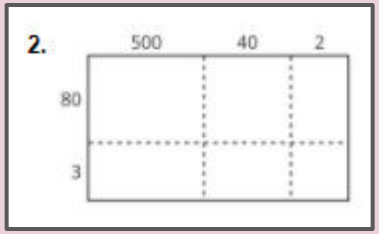
Activity #2

Diagrams and Algorithms for 542 × 83

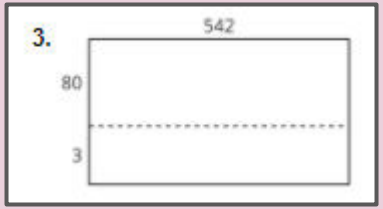
1.



2.



3.



A.

$$\begin{array}{rrrrr} & & 5 & 4 & 2 \\ & \times & & 8 & 3 \\ \hline 4 & 0, & 0 & 0 & 0 \\ & 3, & 2 & 0 & 0 \\ & & 1 & 6 & 0 \\ & 1, & 5 & 0 & 0 \\ & & 1 & 2 & 0 \\ + & & & & 6 \\ \hline & 4 & 4, & 9 & 8 & 6 \end{array}$$

B.

$$\begin{array}{rrrrr} & & {\scriptstyle 3}\ {\scriptstyle 1} & & \\ & & {\scriptstyle 1} & & \\ & & 5 & 4 & 2 \\ & \times & & 8 & 3 \\ \hline & 1, & 6 & 2 & 6 \\ + & 4 & 3, & 3 & 6 & 0 \\ \hline & 4 & 4, & 9 & 8 & 6 \end{array}$$

C.

$$\begin{array}{rrrrr} & & & {\scriptstyle 1} & \\ & & & {\scriptstyle 1} & \\ & & & 8 & 3 \\ & & \times\ 5 & 4 & 2 \\ \hline & & 1 & 6 & 6 \\ & 3, & 3 & 2 & 0 \\ + & 4 & 1, & 5 & 0 & 0 \\ \hline & 4 & 4, & 9 & 8 & 6 \end{array}$$

Do you think the algorithm will work to multiply any 2 whole numbers?
Show or explain your reasoning.

Desperately Seeking 9 New Units

Activity #2

Tyler notices that when he uses the standard algorithm and composes a new unit, sometimes there is 1 new unit, sometimes 2, all the way up to 8. He has not seen an example with 9 of the new unit.

1. For each of these products, how many of each new unit do you compose?

   a. 256 × 5

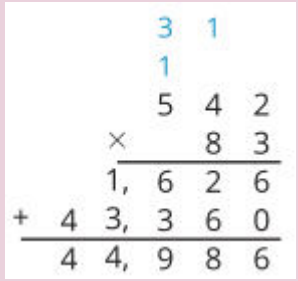
   b. 587 × 8

   c. 809 × 9

2. Do you think it is possible to compose 9 of a new unit with the standard multiplication algorithm?

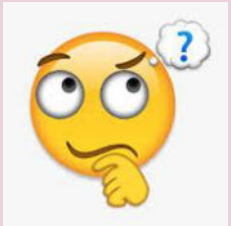
Lesson Synthesis

Today we used the standard algorithm to find products of numbers with no restriction on the number of newly composed units. We also reviewed some of the diagrams and procedures we used earlier in the unit.



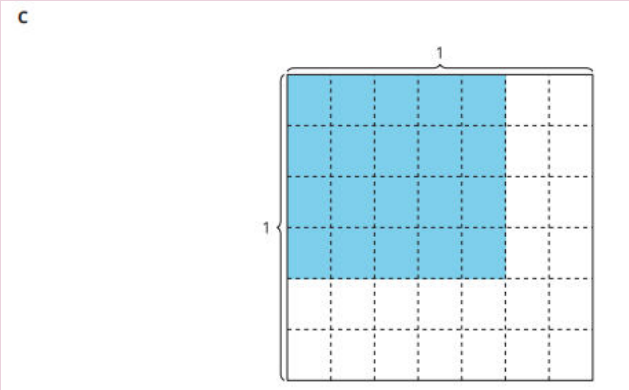
		3	1		
		1			
		5	4	2	
	×		8	3	
	1,	6	2	6	
+	4 3,	3	6	0	
	4 4,	9	8	6	

Do you think the algorithm for multiplying whole numbers will work for any and all whole numbers? Why or why not? Discuss with a partner.



What do you still wonder about the standard algorithm for multiplying whole numbers?

Section Summary



In diagram C, we can see that $\frac{4}{6} \times \frac{5}{7.} = \frac{20}{42}$ We can multiply the numerators, 4 x 5 to find the numerator in the product. We can multiply the denominators, 6 x 7 , to find the denominator in the product. We can represent this relationship with the equation: $\frac{(4 \times 5)}{(6 \times 7)} = \frac{20}{42}$ Diagram C represents a 4 by 5 array inside a 5 by 7 array. It also represents 20 pieces that are each 1/42 of the whole square.

9

The Birds

Let’s solve multiplication problems.

5.MD.C.3, 5.MD.C.5, 5.NBT.B.5

Warm up

Notice and Wonder: For the Birds

What do you notice?

What do you wonder?

Home is Where the Bird Lives

Activity #1

Different types of birds use different types of houses. The table gives you the recommended birdhouse box sizes for various species.

type of bird	dimensions of floor	height	volume estimate
chickadee	4 in. by 4 in.	6-10 inches	
wood duck	10 in. by 18 in.	10-24 inches	
barn owl	10 in. by 18 in.	15-18 inches	
red-headed woodpecker	6 in. by 6 in.	12-15 inches	
bluebird	5 in. by 5 in.	6-12 inches	
swallow	6 in. by 6 in.	6-8 inches	

Estimate a possible volume for each birdhouse.

What is the Volume?

Activity #2

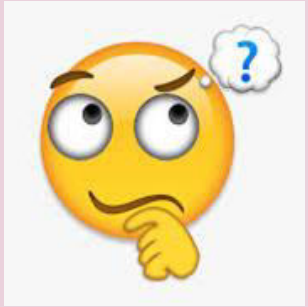
Use the criteria from the table to determine the possible range of volumes for each type of birdhouse.

type of bird	dimensions of floor	height	range of volume
chickadee	4 in. by 4 in.	6-10 inches	
wood duck	10 in. by 18 in.	10-24 inches	
barn owl	10 in. by 18 in.	15-18 inches	
red-headed woodpecker	6 in. by 6 in.	12-15 inches	
bluebird	5 in. by 5 in.	6-12 inches	
swallow	6 in. by 6 in.	6-8 inches	

Lesson Synthesis

Today we discussed when we would use the standard algorithm and when we would use other strategies

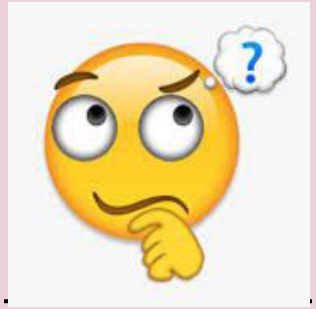
When is it most helpful to use the standard algorithm for multiplication?



Lesson Synthesis

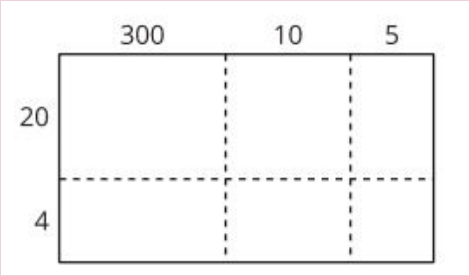
Take a minute to think about which of these problems you would use the standard algorithm to solve

45×6
20×200
143×67
125×9



Different problems call for different strategies, and we each might choose a different way to solve each of these problems. We could use the algorithm to solve all these problems, but we don't have to

Section Summary

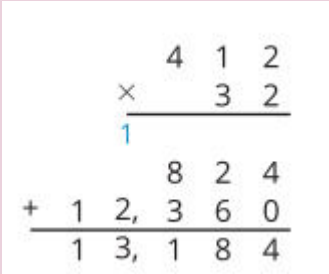


In this unit we multiplied multi-digit whole numbers by two-digit whole numbers. We first represented the multiplication with diagrams that help us visualize the product. This is a partial products strategy that helps us break down the multiplication by place value.

This diagram breaks up the product 315 × 24 by place value. Like we learned in previous grades, if we add up all of the products in each box, we will get the product of 315 × 24 .

Section Summary

Then we learned a new algorithm to multiply whole numbers—the standard algorithm for multiplication.

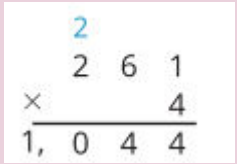


We can see the partial products are organized in different ways. 824 represents the partial product for 2×412 and 12,360 represents the partial product for 30×412 .

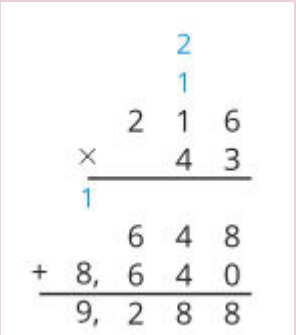
We noticed that sometimes we need to compose a new unit when we multiply, and represent that unit with carrying notation. Sometimes, we may have to compose more than one new unit.

Section Summary

In the first example, the 2 above the 2 in 261 represents 2 hundreds from the product 4×60 .



The second example shows multiple units being composed. The 1 above the 1 represents 1 ten from the product 3×6 and the 2 represents 2 hundreds from the product 40×6.



Section Summary

We also learned that the algorithm for multiplying whole numbers will work for any and all whole numbers, but sometimes it is more helpful to use other strategies. For example, it may be more helpful to use the standard algorithm for multiplication to evaluate the expression 143×67 and a mental strategy to evaluate the expression 20×200 .

Adaptation
Lesson 4

Division and Area of a Rectangle

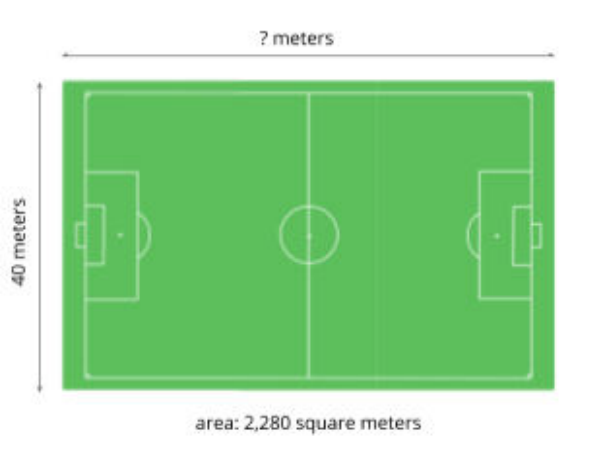
Let's divide to find the side-length of a rectangle.

4.NBT.B.6, 4.MD.A.3, 4.OA.A.3

Warm up

Estimation Exploration: Area of a Soccer Field

What is the length of the soccer field in meters?



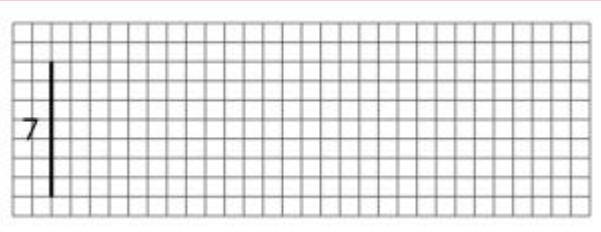
Record an estimate that is:

Too low	About right	Too high

Elena's Mural

Activity #1

Elena used 189 square tiles to create a rectangular mural for the art club. The mural is 7 tiles wide.



1. How many tiles long is Elena's mural? Be prepared to explain or show how you know.

2. Write one or more equations that show how you solved this problem

Tyler's Mural

Activity #2

Tyler is also creating a rectangular mural for the art club. He has 197 tiles for his mural. His mural is 6 tiles wide

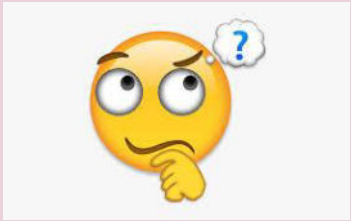
1. Will Tyler use all of his tiles in the mural? Explain your reasoning.

2. How many tiles long is Tyler's mural? Show your reasoning using numbers, pictures, or words.

Lesson Synthesis

Today we used division to find side lengths of rectangles. For each rectangle, we knew the area and the length of one side and we used division to find the length of the other.

What is the relationship between the side lengths and the area of a rectangle?



How do we find the missing side length?

Adaptation
Lesson 5

Area Diagrams to Represent Division

Let’s represent division using area diagrams.

Warm up

What Do You Know about Multiples of 9?

9, 18, 27, 36, 45, ..., 90, 99, 108, ..., 450, 459, 468, .

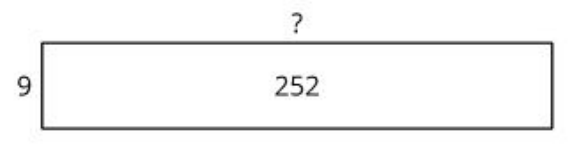
How Long is the Pool?

Activity #1

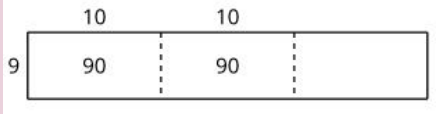
1. A rectangular pool has an area of 252 square meters. Its width is 9 meters and its length is a whole number of meters. What is the length of the pool?

a. Write a multiplication equation and a division equation to represent the situation.

b. To find the length of the pool, Diego drew the following rectangular diagrams.



He thought, “Nine times 10 is 90. Another 9 times 10 is another 90.



Complete Diego’s reasoning. What is the length of the pool?

How Long is the Pool?

Activity #1

2. A rectangular patio has a width of 8 feet and an area of 376 square feet. What is its length?

a. What equations can we write to represent this situation?

b. Use an area diagram to help answer the question.

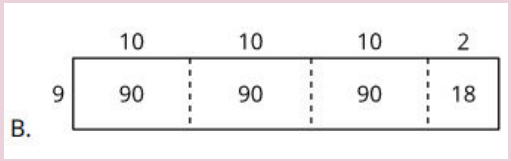
Which Diagram to Use?

Activity #2

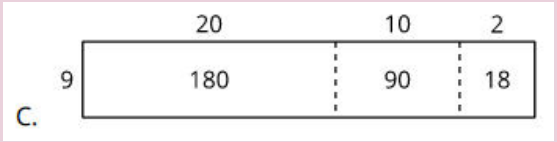
Here are three diagrams that can be used to help find the value of 288 ÷ 9.

A.

B.



C.



1. Where do you see the 288 in each diagram?
2. What does the 9 represent?
3. What does the value of 288 ÷ 9 represent?
4. Which diagram would you use to find the value of 288 ÷ 9? Explain your reasoning.

Lesson Synthesis

Today we saw that when we have the length of one side of a rectangle and its area, we could find the length of the other side. Some of us did this by using multiplication, and others used division. But all of us found the missing length in sections

$6 \times ? = 414$

$414 \div 6 = ?$

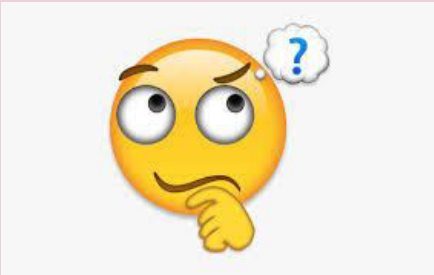
Suppose a rectangle has an area of 414 square units and a width of 6 units, and we want to know its length. Which of these representations describe the question?

Lesson Synthesis

Here are some ways to record our strategies for finding its length.

$6 \times ? = 414$	$414 \div 6 = ?$
$6 \times 50 = 300$	$300 \div 6 = 50$
$6 \times 10 = 60$	$60 \div 6 = 10$
$6 \times 5 = 30$	$30 \div 6 = 5$
$6 \times 4 = 24$	$24 \div 6 = 4$

	50	10	5	4
6	300	60	30	24



10

World's Record Noodle Soup

Let's explore division with large numbers.

5.NBT.B.6

Notice and Wonder: World Record Event

Warm up

A Chinese food company holds the Guinness World Record for making the longest noodle. The noodle measured about 10,119 ft and served 400 people.

What do you notice?

What do you wonder?

Activity #1

How Many Feet in One Serving?

A Chinese food company cooked a single noodle measuring about 10,119 ft. It served 400 people

1. If the noodle was shared equally, about how many feet of noodle did each person eat? Explain or show your thinking.

2. Is your estimate lower or higher than the actual length of noodle each person ate? Explain your reasoning without calculating the actual length.

Noodle Soup

Activity #2

Partner A:
Mai made noodle soup. She made a noodle that is 155 feet long. Five people share the noodle equally.

1. Write a division expression to represent the situation.

2. How many feet of noodle does each person eat? Show or explain your thinking.

3. Compare your work with your partner's. What is the same? What is different?

Partner B:
Han made noodle soup. He made a noodle that is 165 feet long. Five people share the noodle equally.

1. Write a division expression to represent the situation.

2. How many feet of noodle does each person eat? Show or explain your thinking.

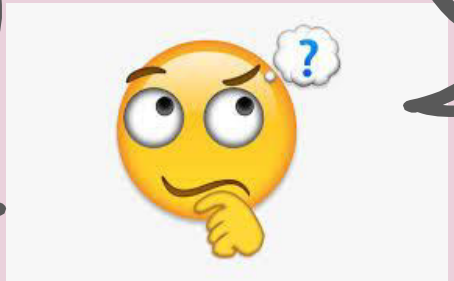
3. Compare your work with your partner's. What is the same? What is different?

Lesson Synthesis

Today, we solved problems using division. Some of us were able to solve the same problems using multiplication.

$$165 \div 5 = 33 \quad 5 \times 33 = 165$$

What do we know about the relationship between multiplication and division?



How did we use the relationship between multiplication and division today?

11

Fractions as Partial Quotients

Let’s use fractions to help us divide whole numbers.

5.NBT.B.6, 5.NF.B.3

What Do You Know About.....

Warm up

What do you know about $\frac{60}{6} + \frac{6}{6}$?

Select Expressions

Activity #1

1. Select **all** the expressions that are equal to the value of $\frac{78}{6}$. Show or explain your reasonin

a. $78 \div 6$

b. $\frac{66}{6} + \frac{12}{6}$

c. $\frac{60}{6} + \frac{18}{6}$

d. $(60 \div 6) + (18 \div 6)$

e. $\frac{77}{6} + \frac{8}{6}$

f. $(60 \div 6) + 18$

2. What is the value of $78 \div 6$? Show or explain your thinking.

Activity #2

Choose One Expression

For each problem, choose one expression and use it to find the value of the division expression. Show or explain your thinking.

1. $165 \div 15 =$ _____

   a. $\frac{45}{15} + \frac{20}{15} + \frac{100}{15}$

   b. $\frac{30}{15} + \frac{30}{15} + \frac{30}{15} + \frac{60}{15} + \frac{15}{15}$

   c. $\frac{150}{15} + \frac{15}{15}$

2. $540 \div 18 =$ _____

   a. $\frac{180}{18} + \frac{180}{18} + \frac{180}{18}$

   b. $\frac{500}{18} + \frac{40}{18}$

   c. $\frac{360}{18} + \frac{180}{18}$

3. Which expressions were most helpful? Show or explain your thinking.

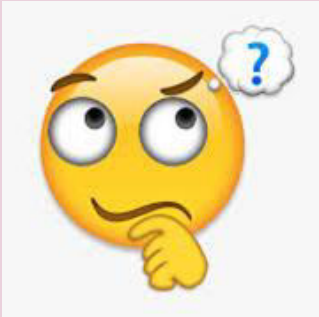
4. Why were some expressions more helpful than others? Show or explain your thinking.

Lesson Synthesis

$$540 \div 18 = \_\_\_\_\_$$

$$\frac{180}{18} + \frac{180}{18} + \frac{180}{18}$$

How do we know this expression is equal to 540/18 ?

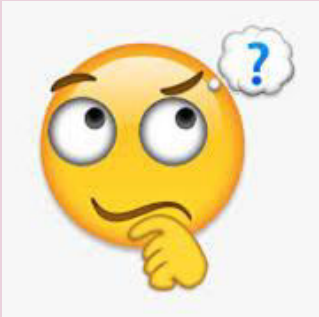


How can we use this expression to find the value of 540 ÷ 18?

Lesson Synthesis

$$\frac{360}{18} + \frac{180}{18}$$

How can we use this expression to help us find the value of 540 ÷ 18 ?



$$\frac{500}{18} + \frac{40}{18}$$

How do we know this expression is equal to 540 ÷ 18?

Why is this expression not as helpful as the others?

12

Interpret Division Expressions

Let’s interpret and evaluate division expressions.

5.NBT.B.6, 5.OA.A.1, 5.OA.A.2

Warm up

True or False: Lucky Number 7

Decide whether each statement is true or false.

Be prepared to explain your reasoning.

$$77 \div 7 = (70 \div 7) + (7 \div 7)$$

Warm up

True or False: Lucky Number 7

Decide whether each statement is true or false.

Be prepared to explain your reasoning.

$$77 \div 7 = 11$$

Warm up

True or False: Lucky Number 7

Decide whether each statement is true or false.

Be prepared to explain your reasoning.

$$84 \div 7 = (70 \div 7) + 14$$

Division Expressions

Activity #1

Take turns:

1. Choose a set of expressions that, when added together, is equal to $308 \div 14$. Not all expressions will be used.

2. Explain to your partner how you know that your cards represent a sum that is equal to $308 \div 14$.

   (Pause for teacher directions.)

3. Choose one of the sets of expressions that is equal to $308 \div 14$ and use it to find the value of $308 \div 14$.

Activity #2

Write Expressions

1. Fill in the blanks to write expressions that are equal to

$$299 \div 13$$

a. (_____ ÷ 13) + (_____ ÷ 13)

b. (_____ ÷ 13) + (_____ ÷ 13) + (_____ ÷ 13)

c. (_____ ÷ 13) + (_____ ÷ 13) + (_____ ÷ 13) + (_____ ÷ 13)

2. Use one of the expressions to find the value of

$$299 \div 13.$$

3. Explain why you chose the expression to find the value of

$$299 \div 13.$$

Lesson Synthesis

$299 \div 13$

$(130 \div 13) + (130 \div 13) + (26 \div 13) + (13 \div 13)$

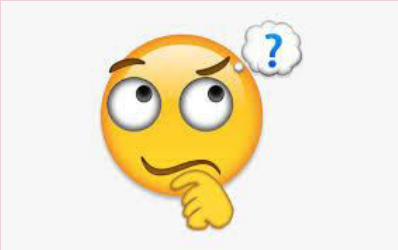
$(130 \div 13) + (130 \div 13) + (39 \div 13)$

$(130 \div 13) + (169 \div 13)$

$(260 \div 13) + (39 \div 13)$

What changes in the expressions?

What stays the same in the expressions?



The number 13 is the divisor. The divisor stays the same because we divide 299 into 13 groups. The dividend changes in the expressions because it is decomposed in different ways. Why is it helpful to decompose the dividend in these ways?

13

Partial Quotients

Let’s use partial quotients to divide.

5.NBT.B.6

Warm up

True or False: 192 ÷ 16

Decide whether each statement is true or false.

Be prepared to explain your reasoning.

$$192 \div 16 = 100/16 + 92/16$$

Warm up

True or False: 192 ÷ 16

Decide whether each statement is true or false.

Be prepared to explain your reasoning.

$$192 \div 16 = (16 \div 192) + (2 \div 16)$$

Warm up

True or False: 192 ÷ 16

Decide whether each statement is true or false.

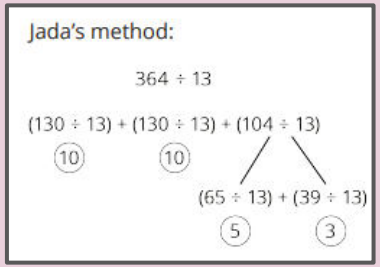
Be prepared to explain your reasoning.

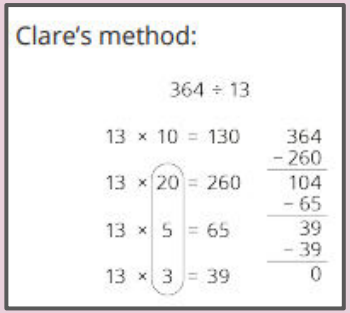
$$(160 \div 16) + (32 \div 16) = 192 \div 16$$

Two Different Methods

Activity #1

1. With your partner, discuss: What's different about Jada's and Clare's method? What's the same?



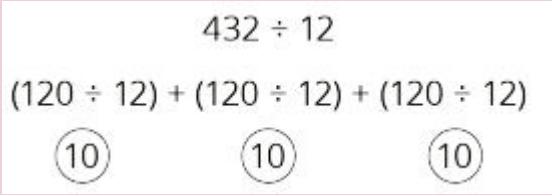


2. Find the value of 364 ÷ 13.

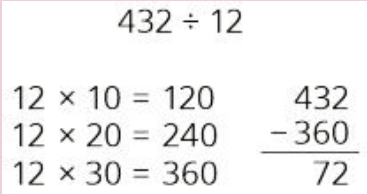
Two Different Methods

Activity #2

1. Jada started to find the value of 432 ÷ 12 . What could she do next?



2. Clare started to find the value of 432 ÷ 12 . What could she do next?



3. What is the value of 432 ÷ 12?

Lesson Synthesis

Diego said, ‘In order to solve division problems, you have to use all the operations.’ What does Diego mean? When did we use addition, subtraction, and multiplication to divide?

14

Partial Quotients Algorithm

Let's make sense of a partial quotient algorithm.

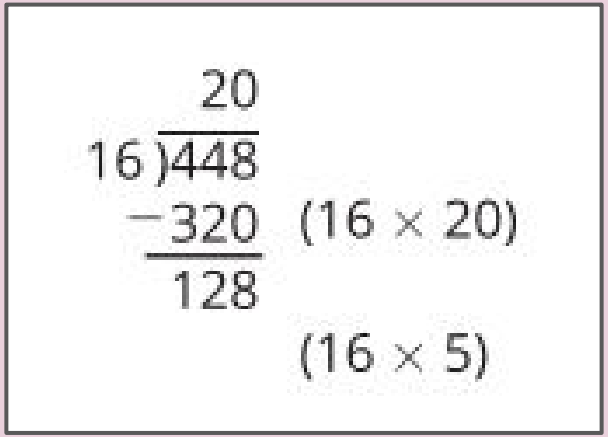
5.NBT.B.6

Warm up

Notice and Wonder: Incomplete Solution

What do you notice?

What do you wonder?

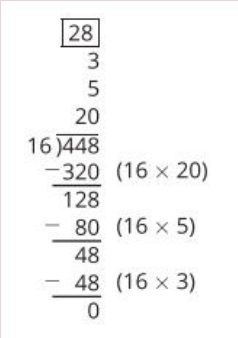


Elena's Work

Activity #1

1. Find the value of 448 ÷ 16 . Show your thinking. Organize it so it can be followed by others.

2. This is Elena's work. Describe the steps Elena took to find the value of 448 ÷ 16.



Activity #2

Complete the Solution

1. Use Elena's method to complete the following problems:

a.

$$
\begin{array}{r}
20 \\
20 \\
12\overline{)492} \\
-240 \quad (12 \times 20) \\
\hline
252 \\
-240 \quad (12 \times 20) \\
\hline
\end{array}
$$

b.

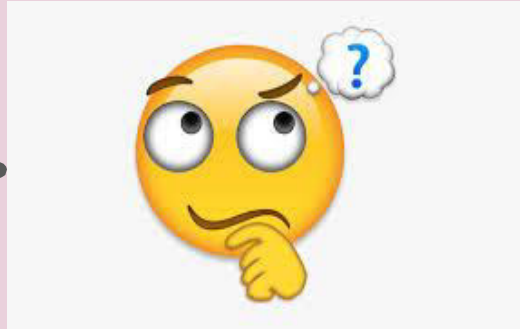
$$
\begin{array}{r}
40 \\
15\overline{)630} \\
\end{array}
\qquad (15 \times 40)
$$

c. $14\overline{)364}$

Lesson Synthesis

$$
\begin{array}{r}
\boxed{26} \\
1 \\
5 \\
20 \\
14\overline{)364} \\
-280 \quad (20 \times 14) \\
\hline
84 \\
-\ 70 \quad (5 \times 14) \\
\hline
14 \\
-\ 14 \quad (1 \times 14) \\
\hline
0
\end{array}
$$

Explain to your partner how multiplication, addition, and subtraction were used to find the value of 364 ÷ 14 .



15

Divide Using Partial Quotients

Lets use a partial quotients algorithm to divide three-digit dividends by two-digit divisors.

5.NBT.B.6

Warm up

Number Talk: Divide

Find the value of each expression mentally.

$55 \div 5$

Warm up

Number Talk: Divide

Find the value of each expression mentally.

$$99 \div 9$$

Warm up

Number Talk: Divide

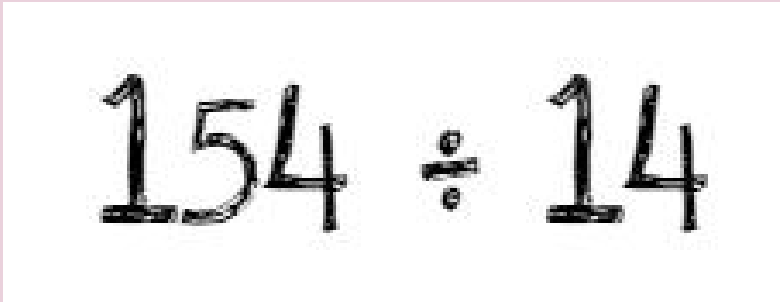
Find the value of each expression mentally.

$132 \div 12$

Warm up

Number Talk: Divide

Find the value of each expression mentally.



Compare Solutions

Activity #1

1. Use the partial quotients algorithm to evaluate one of the problems. Be prepared to explain how you found the quotient.

Partner 1:

$32\overline{)608}$

Partner 2:

$19\overline{)589}$

2. Explain to your partner how you found the quotient in your problem.

3. Pair up with another group and compare your work.

Activity #2

Estimate and Solve

For each problem, make a reasonable estimate.
Then, use the partial quotients algorithm to solve.

1. A reasonable estimate for $612 \div 34$ is _____.

$34\overline{)612}$

2. A reasonable estimate for $529 \div 23$ is _____.

$23\overline{)529}$

3. A reasonable estimate for $1{,}044 \div 29$ is _____.

$29\overline{)1044}$

Lesson Synthesis

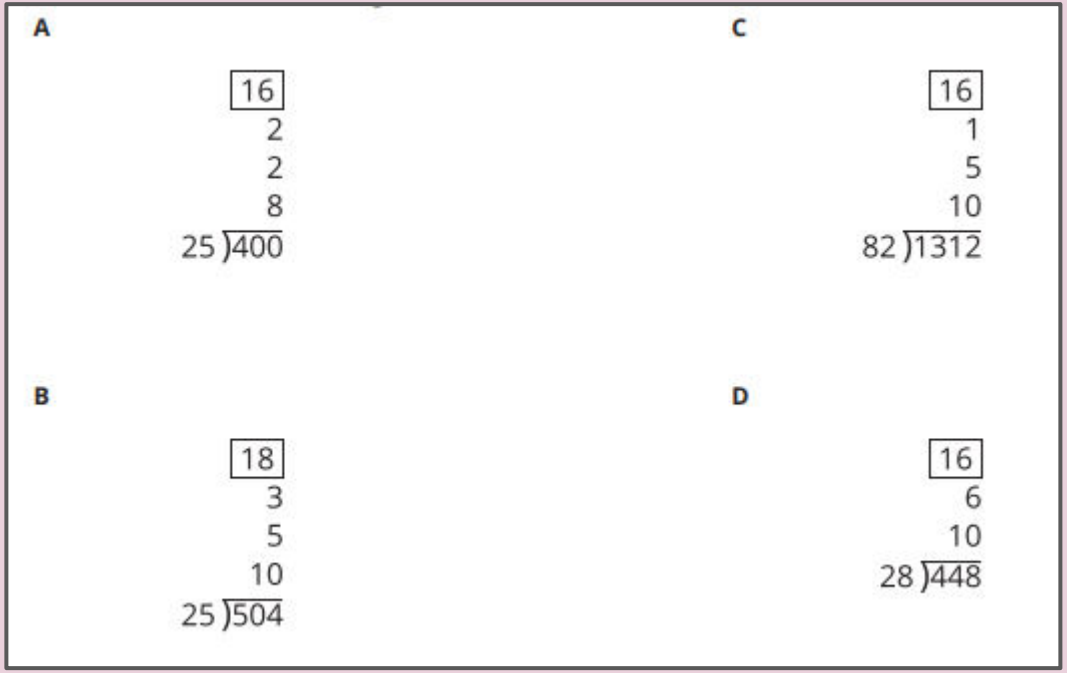
Today we used partial quotients to divide whole numbers. What makes sense to you about this procedure? What questions do you still have about using this procedure?

Practice a Partial Quotients Algorithm

Let’s practice using a partial quotients algorithm.

Warm up

Which One Doesn't Belong: Different Ways



Activity #1

Find the Mistake

For each problem, describe where you see an error in the calculation. Fix the error to find the whole number quotient.

1.

$$
\begin{array}{r}
\boxed{29} \\
4 \\
5 \\
20 \\
46\overline{)1656} \\
-920 \\
\hline
436 \\
-230 \\
\hline
206 \\
-184 \\
\hline
22
\end{array}
$$

2.

$$
\begin{array}{r}
\boxed{64} \\
4 \\
60 \\
18\overline{)972} \\
-900 \\
\hline
72 \\
-\ 72 \\
\hline
0
\end{array}
$$

3.

$$
\begin{array}{r}
\boxed{211} \\
1 \\
10 \\
200 \\
24\overline{)744} \\
-480 \\
\hline
264 \\
240 \\
\hline
24
\end{array}
$$

Activity #2

Practice Problems

Evaluate. Then check in with a partner to review your work.

1. $16\overline{)768}$

2. $29\overline{)1305}$

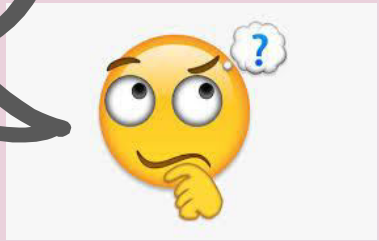
3. $48\overline{)1488}$

4. $45\overline{)810}$

Lesson Synthesis

Today, we practiced using the partial quotients algorithm to divide multi-digit numbers.

Turn and Talk: Describe how to use the partial quotients algorithm to evaluate 935 ÷ 85 .



- Think about the biggest multiple of 10 and 85 that can divide into 935. So I start with 10.
- Then multiply 85 x 10 and subtract the product from 935 to see how much more I have left to divide.
- Keep on doing this until I get zero as a difference.
- Multiply the quotient by the divisor to make sure I get back to the original dividend.

17

Find Missing Dimensions

Let’s use the relationship between multiplication and division to solve problems.

5.NBT.B.5, 5.NBT.B.6

Warm up

Estimation Exploration: The Garden

What is the area of one of the large rectangles in the garden?

Record an estimate that is:

Too low	About right	Too high

Find the Missing Dimension, Part 1

Activity #1

Complete the table.

area (square feet)	length (feet)	width (feet)
816	24	
1,248		48
	23	253
576		36

Activity #2

Find the Missing Dimension, Part 2

1. Complete the table to find the missing dimension for each rectangular prism.

volume (cubic feet)	base (square feet)	height (feet)
375	15	
1,176		28

2. Clare wants to find the height of a rectangular prism with the following measurements:

volume (cubic feet)	length (feet)	width (feet)	height (feet)
882	6	7	

a. First, Clare divides 882 ÷ 6. What should she do next?

b. Find the missing height to finish the problem for Clare.

Activity #2

Find the Missing Dimension, Part 2

3. Complete the table to find the missing dimensions for each rectangular prism.

volume (cubic feet)	length (feet)	width (feet)	height (feet)
936	8		9
1,536		48	2
1,008	36		

Lesson Synthesis

Today we found missing dimensions of rectangles and rectangular prisms using division.

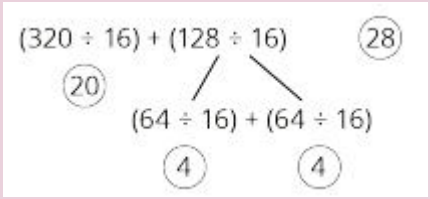
How did the partial quotients algorithm help you solve problems today?

Section Summary

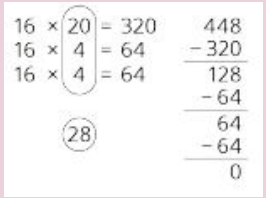
In this section, we learned how to divide multi-digit whole numbers. We used what we know about the relationship between fractions and division expressions to make sense of partial quotients.

$$165 \div 15 = \frac{150}{15} + \frac{15}{15}$$

Then, we used the relationship between multiplication and division to make sense of partial quotients.



Finally, we learned a new strategy for dividing multi-digit whole numbers.



18

Problem Solving: World's Largest Wagon

Let's solve problems about volume.

5.MD.C.5, 5.NBT.B.5, 5.NBT.B.6

Notice and Wonder: Radio Flyer

Warm up

What do you notice?

What do you wonder?

Anatomy of an Estimate

Activity #1

1. What measurements would you take of the wagon to accurately estimate its volume?

2. What units would you use to measure the wagon? Explain your reasoning.

3. Record an estimate for the volume of the wagon that is:

too low	about right	too high

4. What can you use about the picture to refine your estimate?

Estimating the Size of the Radio Flyer

Activity #1

Use the picture of the wagon to make a better estimate of the length, width, and height of the wagon bed.

Make sure to:

- explain how you estimated each measurement
- include how accurate you think each estimate is

Activity #3

Sand Wagon

The Radio Flyer wagon is 27 feet long 13 feet wide and 2 feet deep.

1. A 150-pound bag of sand will fill about 9 cubic feet. About how many bags of sand will it take to fill the wagon with sand?

2. A 150-pound bag of sand costs about $12. About how much will it cost to fill the wagon with sand?

3. Jada said the sand will cost approximately $120. Is Jada's estimate reasonable? Why or why not?

Lesson Synthesis

Today we made and refined estimates for the volume of the world's largest wagon.

What other question do you have about the wagon?

We are going to continue to investigate the wagon tomorrow

Problem Solving: Boxes of Toys

Let’s solve more problems about volume.

Notice and Wonder: Radio Flyer

Warm up

What do you notice?

What do you wonder?

Activity #1

How Many Boxes?

The Radio Flyer wagon is 27 feet long 13 feet wide and 2 feet deep

1. Each toy box is 1 foot by 1 foot by 1 foot. How many boxes will fit in the wagon?

2. If each toy box is 2 feet by 2 feet by 2 feet, how many boxes will fit in the wagon?

3. If each toy box is 3 feet by 3 feet by 3 feet, how many boxes will fit in the wagon?

Activity #2

More Boxes

The Radio Flyer wagon is 27 feet long 13 feet wide and 2 feet deep.

The wagon is being used to deliver 4,000 boxes that each have the dimensions 2 feet by 2 feet by 2 feet. How many trips will the wagon have to make? Be prepared to explain your reasoning.

Lesson Synthesis

What strategies for multiplication and division did you find most helpful today? Why were they helpful?

20

Problem Solving: More Wagon Problems

Let’s create and solve problems about the world’s largest wagon.

5.MD.C.5, 5.NBT.B.5, 5.NBT.B.6

Warm up

Estimation Exploration: Wagon People

How many people would fit in the big red wagon?
Record an estimate that is:

Record an estimate that is:

Too low	About right	Too high

The Radio Flyer wagon is 27 feet long 13 feet wide and 2 feet deep.

Activity #1

How Many Students?

The Radio Flyer wagon is 27 feet long 13 feet wide and 2 feet deep.

The world's largest wagon can hold more than 75 children. Is this statement reasonable? Be prepared to explain your reasoning.

Activity #2

Wonder Wagon

The Radio Flyer wagon is 27 feet long 13 feet wide and 2 feet deep.

The wagon is 9 times the size of the original wagon.

the wagon was built on December 20th, 2016

The wheels measure 8 feet across and weight 1,000 pounds each.

the wagon is on display in Chicago, Illinois.

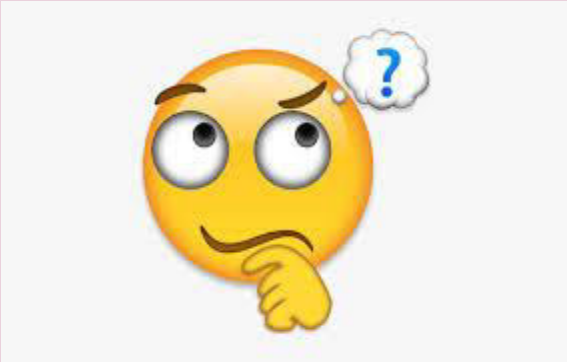
The overall weight of the wagon is 15,000 pounds.

Six tons of steel were used to build the wagon.

1. What else do you wonder about the wagon?
2. Write a problem about the world's largest wagon.
3. What information would you need in order to solve your problem?

Lesson Synthesis

We have been learning about multiplication and division algorithms in this unit. When did we use multiplication and division while we were solving problems about the world's largest wagon? Which problems did you use one of the algorithms to solve? Why did you use the algorithm to solve that problem?



Section Summary

In this section we solved problems using multiplication and division involving the world's largest wagon.

First, we made and refined estimates for the volume of the wagon. Then, we used multiplication to figure out how many boxes fit in the wagon. We used division to figure out how many trips the wagon has to take if it has to deliver a total of 4,000 boxes. Then, we created our own problems about the world's largest wagon.

Objects in the Real World (optional)

Let’s think more about area and volume in the real world.

Warm up

Estimation Exploration: Wooden Box

How wide is one wooden box?

Record an estimate that is:

Too low	About right	Too high

Activity #1

What Could it Be?

1. What object might each of the following represent? Be prepared to show or explain your reasoning.

a. Rectangle. Area: 816 sq ft

b. Rectangle. Width: 253 in

c. Rectangular prism. Volume: 375 cubic cm, base: 15 sq cm

d. Rectangular prism. Volume: 1,536 cubic ft

e. Rectangular prism. Length: 8 m, height: 9 m

2. Choose your own measurement and units. What object does this represent?

Activity #2

Who Wants to Know?

1. Make a sketch of your design that includes its length, width, height, and volume.

2. Describe your design.

    a. What did you design?

    b. What is it used for?

    c. Why did you choose these measurements?

    d. Why would the people in your community want this product?

Lesson Synthesis

Today you designed a rectangular prism that would be useful to the people in your community.

You saw another group's design. Why would the people in your community buy their product?

How might you change the measurements to make it more useful?