Fractions as Quotients and Fraction Multiplication

Grade 5: Unit 2

Standards addressed: 5.NF.B.3, 5.NF.B.4, 5.NF.B.7

Unit 2 Progression Overview

Section A
Lessons 1-5
5.NF.B.3

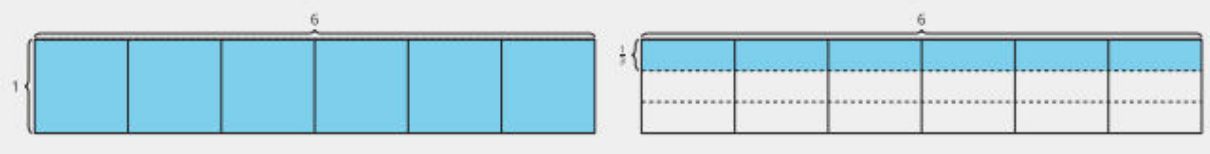
- Represent and explain the relationship between division and fractions.
- Solve problems involving division of whole numbers leading to answers that are fractions.

Section B
Lessons 6-8
5.NF.B.4

- Extend understanding of the relationship between multiplication and division to fractions.
- Represent and solve problems involving multiplication of a whole number by a fraction.
- Write and interpret numerical expressions using the relationship between multiplication and division.

Section C
Lessons 9-17
5.NF.B.4, 5.NF.B.7

- Find the area of a rectangle when one side length is a whole number and the other side length is a fraction or mixed number.
- Represent and solve problems involving the multiplication of a whole number by a fraction or mixed number.
- Write, interpret and evaluate numerical expressions that represent multiplication of a whole number by a fraction or mixed number.



Unit 2 Quick Links

Adapt 1

Adapt 2

L1 L2 L3 L4 L5 L6 L7 L8

L9 L10 L11 L12 L13 L14 L15 L16

TOY THEATER
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Adaptation
Lesson 1

Finding All Kinds of Quotients

Let’s solve problems involving division.

OA.B.4

Warm up

True or False: Multiples

Is each statement true or false? Be prepared to explain your reasoning.

1. 105 is a multiple of 2.

2. 105 is a multiple of 3.

3. 105 is a multiple of 5.

4. 105 is a multiple of 15.

Activity #1

Muffins & Seats

1. Two bakers at a bakery made 378 muffins. The muffins are put in boxes of 4.
   - The first baker says they will need 94 boxes for all the muffins.
   - The second baker says 95 boxes are needed.

   Who do you agree with? Explain or show your reasoning.

1. An auditorium seats 258 people. The seats are arranged in rows of 9, but there is one short row with fewer than 9 seats.

   How many regular rows are there? How many seats are in the shorter row?

Activity #2

Save for a Garden

1. A school needs $1,270 to build a garden. After saving the same amount each month for 8 months, the school is still short by $6.

   How much did they save each month? Explain or show your reasoning.

1. Choose one of the following division expressions.

   711 ÷ 3 3,128 ÷ 8

a. Write a situation to represent the expression.
b. Find the quotient. Show your reasoning.
c. Explain what the value means in the situation you invented.

Lesson Synthesis

Let's Put it All Together

What strategies did you find yourself using to divide numbers?

Adaptation
Lesson 2

Representations of Fractions (Part 1)

Let’s name some fractions and represent them visually

OA.B.4

Warm up

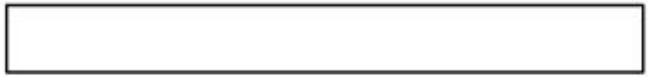
What Do you Know About ½ ?

What do you know about this number?

Fraction Strips

Activity #1

Your teacher will give you strips of paper. Each strip represents 1.



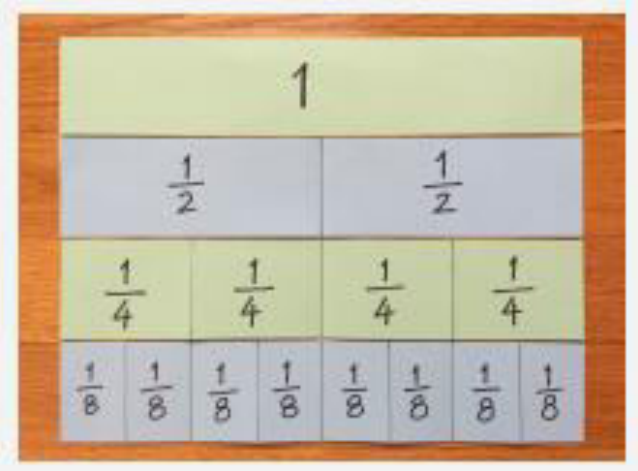
1. Use the strips to represent halves, fourths, and eighths.

   Use one strip for each fraction and label the fractional parts.

2. What do you notice about the number of pieces or the size of the pieces? Make at least two observations.

Synthesis

Fraction Strips

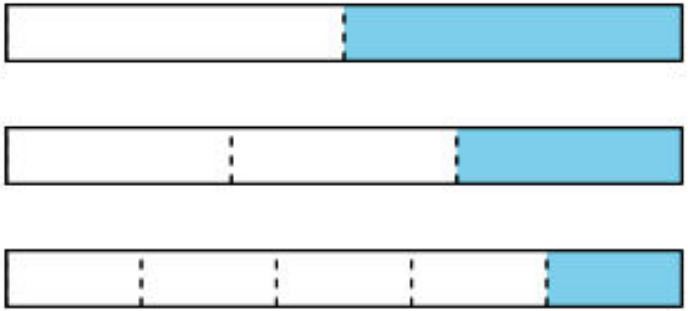


What do you notice about the number and size of the parts on the strips?

Activity #2

Fractions, Represented

1. If the entire diagram represents 1, what fraction does each shaded part represent?



Activity #2

Fractions, Represented

2. Here are four other blank diagrams. Each diagram represents 1. Partition each diagram and shade one part so that the shaded part represents the given fraction.

a. $\frac{1}{6}$

b. $\frac{1}{8}$

c. $\frac{1}{10}$

d. $\frac{1}{12}$

Activity #2

Fractions, Represented

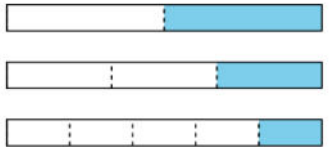
3. Suppose you are creating a representation of $\frac{1}{20}$ using the same tape diagram. Would the shaded part be larger or smaller than the shaded part in the diagram of $\frac{1}{10}$? Explain how you know.

Synthesis

Fractions, Represented

What relationships do you see between the fractions in this activity?”

1. If the entire diagram represents 1, what fraction does each shaded part represent?



2. Here are four other blank diagrams. Each diagram represents 1. Partition each diagram and shade one part so that the shaded part represents the given fraction.

a. $\frac{1}{6}$

b. $\frac{1}{8}$

c. $\frac{1}{10}$

d. $\frac{1}{12}$

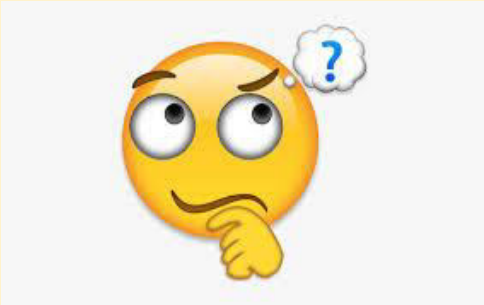
Lesson Synthesis

Let's Put it All Together

In general, what does the denominator in a fraction represent?

What does the fraction $\frac{1}{5}$ tell us?

What relationships did we see between the fractions that we studied today?



What did you notice about the size of a fraction as the denominator gets larger?

Today, we named fractions and represented them visually.

Next, let’s learn more about fractions and **tape diagrams.**

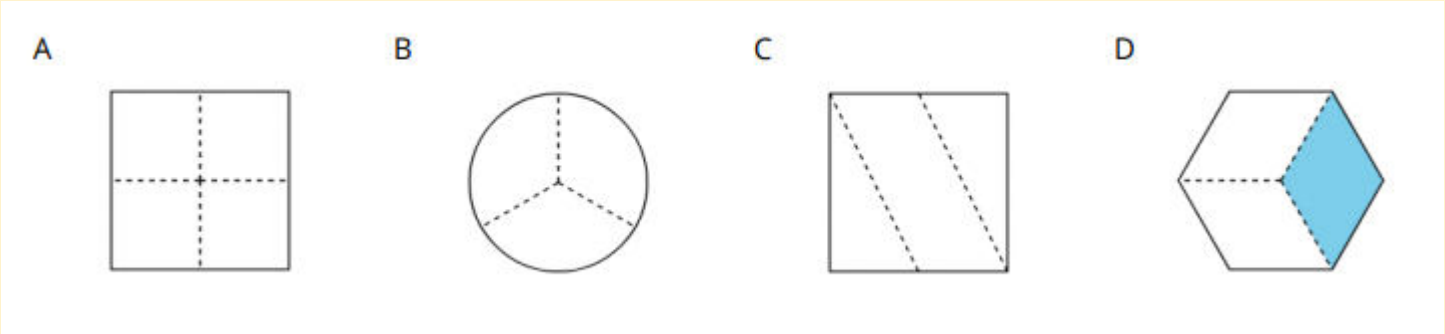
Adaptation
Lesson 2

Representations of Fractions (Part 2)

Let’s name some other fractions and represent them with tape diagrams.

Warm up

Which One Doesn't Belong: All Cut Up

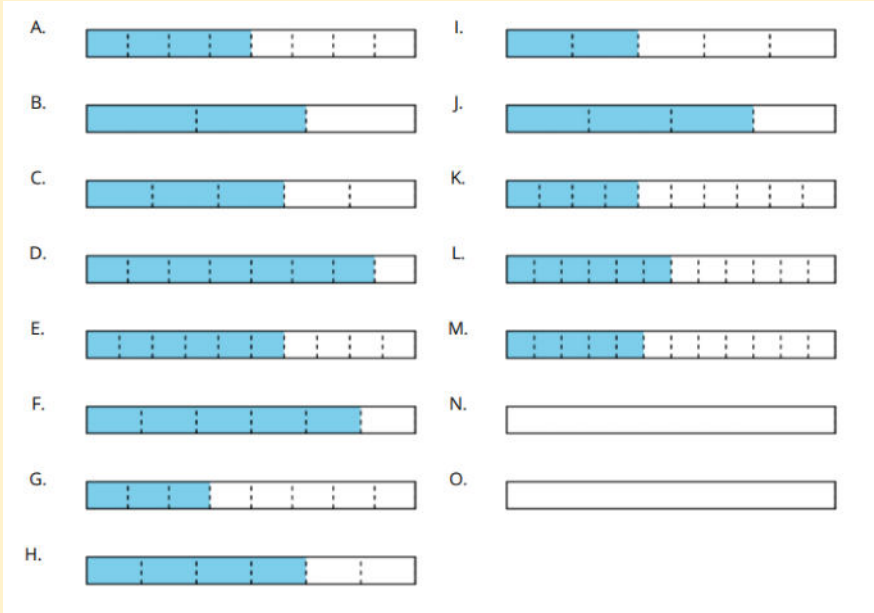


Activity #1

Matching: A Diagram for Each Fraction

Match each fraction with a tape diagram that represents it. Two of the fractions are not represented. Create a representation for each of them.

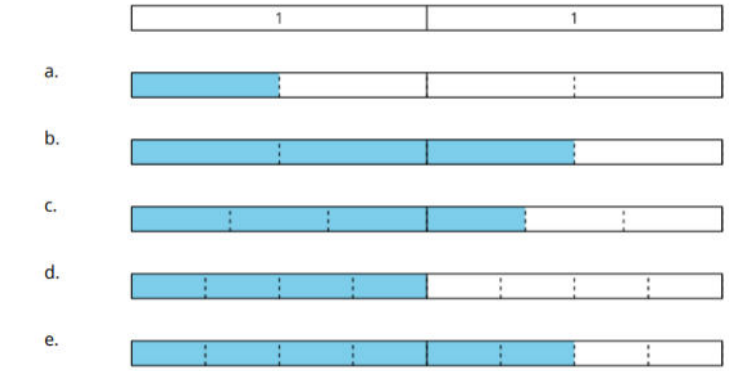
$\frac{2}{3}$	$\frac{3}{8}$	$\frac{4}{6}$	$\frac{4}{10}$	$\frac{6}{6}$
$\frac{3}{5}$	$\frac{4}{8}$	$\frac{6}{12}$	$\frac{6}{10}$	$\frac{3}{4}$
$\frac{5}{12}$	$\frac{7}{10}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{2}{5}$



Activity #2

Diagrams for Some Other Fractions

1. What fraction does each shaded part represent?



2. Here are four fractions and four blank diagrams. Partition each diagram and shade one part so that the shaded part represents the fraction.

a. $\frac{2}{2}$

b. $\frac{4}{2}$

c. $\frac{5}{4}$

d. $\frac{4}{8}$

Lesson Synthesis

Let's Put it All Together

Did you notice anything interesting about the last two diagrams you created and the fractions they represent?

Adaptation
Lesson 3

Representing Equal Groups of Unit Fractions

Let’s identify equal groups of fractions.

Warm up

How Many Do You See: Oranges

How many do you see?

How many do you see them?

Activity #1

Equal Groups of Fractions

1. Lin and Andre brought fruit to share with their class. Draw a diagram to represent each situation.

Lin brought 6 bags with 4 oranges in each.	Andre brought 6 bags with $\frac{1}{4}$ orange in each.

2. Describe the groups you see and how you see them.

3. Which parts of each diagram are the same?

4. Which parts of the diagram changed and how did you change them?

Activity #1

Equal Groups of Fractions

5. What expressions could you write for each situation?

6. Draw a diagram for each situation.
   a. Clare brought 3 bags with 6 strawberries in each.

   b. Diego put $\frac{1}{2}$ apple into each of 8 bags.

   c. Noah had 7 bags with $\frac{1}{3}$ banana in each.

   d. Priya packed $\frac{1}{8}$ of a watermelon into 5 different bags.

Expressions and Diagrams

Activity #2

Expressions and Diagrams

$3 \times \frac{1}{4}$

Expressions and Diagrams

Expressions and Diagrams

5×3

Expressions and Diagrams

Expressions and Diagrams

$4 \times \frac{1}{3}$

Expressions and Diagrams

Expressions and Diagrams

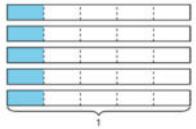
$6 \times \frac{1}{8}$

Expressions and Diagrams

Expressions and Diagrams

$5 \times \frac{1}{5}$

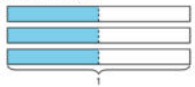
Expressions and Diagrams



Expressions and Diagrams

3×12

Expressions and Diagrams



Expressions and Diagrams

$6 \times \frac{1}{6}$

Expressions and Diagrams

$2 \times \frac{1}{12}$

Expressions and Diagrams

$8 \times \frac{1}{2}$

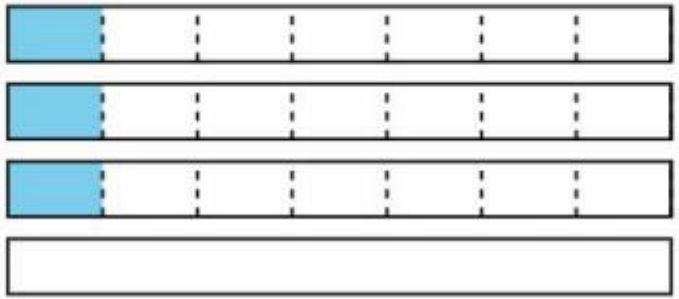
Expressions and Diagrams

3×4

Expressions and Diagrams

Activity #2

3. Han started drawing a diagram to represent $7 \times \frac{1}{8}$ and did not finish. Complete his diagram.

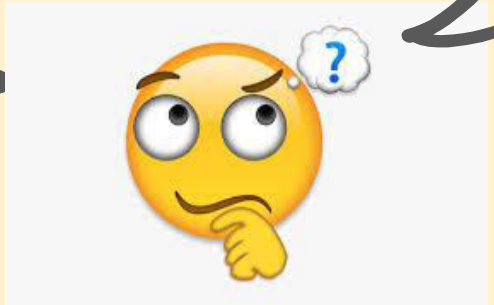


Lesson Synthesis

Let's Put it All Together

What was missing from Han's diagram? How do you know?

If the expression was for 7 groups of $\frac{1}{3}$ instead of $\frac{1}{8}$, how would Han's diagram change?



Adaptation
Lesson 3

Name That Expression

Let’s write multiplication expressions from diagrams and situations.

Warm up

Choral Count

Count by $\frac{1}{4}$ starting at 0.

Count by $\frac{2}{4}$ starting at 0.

Activity #1

Writing Expressions

Five students made slime using this recipe:

3-Ingredient Slime

- 2 bottles of white liquid glue
- 1 tablespoon baking soda
- 2–3 tablespoons of saline solution (contact lens solution)

Optional Ingredients:

- 2 tablespoons of glitter
- 1–3 drops of food coloring

Write an expression to answer each question and identify a matching diagram.

1. Each of the 5 students got $\frac{1}{4}$ pound of slime. How much slime did the students make?

2. Each person added $\frac{1}{3}$ teaspoon of food coloring to their slime. How much food coloring did the students use?

Activity #1

Writing Expressions

3. Next, each person added $\frac{1}{2}$ tablespoon of glitter to their slime. How much glitter did the students use?

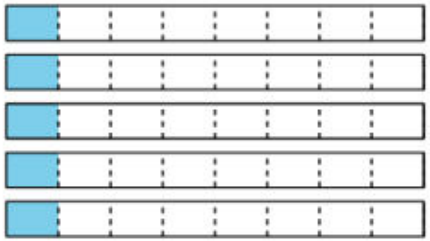
4. The students didn't think there was enough slime, so they made more. This time, each person got $\frac{1}{8}$ pound of the newly made slime. How much slime did they make the second time?

Complete the Story

Activity #2

a. Jada made 7 cups of red slime. She added ____ tablespoon of blue food coloring to each cup to make purple slime. How much food coloring did she use?

b. ____ students each had ____ cup of slime. They combined their slime to make a large slime ball. How many cups of slime is that ball?



Complete the Story

Activity #2

2. A batch of slime takes $\frac{1}{4}$ hour to make. How many hours would it take to make 8 batches of slime?

   Draw a diagram and write an expression to match the problem.

3. Clare gave _____ friends a slime ball. Each slime ball weighed _____ pounds. How much slime did Clare give away?

   Use a whole number and a unit fraction to complete the problem. Draw a diagram and write an expression to answer the question.

Lesson Synthesis

Let's Put it All Together

Describe how you see the groups represented in each student's diagram.

Adaptation
Lesson 3

Multiplication Patterns

Let’s look at patterns in multiplication of a fraction by a whole number.

Warm up

Choral Count

Count by $\frac{1}{8}$ starting at 0.

Activity #1

Describe the Pattern

1. Find the value of each expression. Use a diagram if it is helpful.

Set A

- $1 \times \frac{1}{5}$
- $2 \times \frac{2}{5}$
- $3 \times \frac{3}{5}$
- $4 \times \frac{4}{5}$
- $5 \times \frac{5}{5}$
- $6 \times \frac{6}{5}$

Set B

- $2 \times \frac{1}{4}$
- $2 \times \frac{1}{5}$
- $2 \times \frac{1}{6}$
- $2 \times \frac{1}{7}$
- $2 \times \frac{1}{8}$
- $2 \times \frac{1}{9}$

2. Describe patterns in the products of Sets A and B.

3. Complete each equation to make it true.

a. $4 \times \_\_\_\_ = \frac{4}{5}$

b. $6 \times \_\_\_\_ = \frac{6}{10}$

c. $\_\_\_\_ \times \frac{1}{12} = \frac{7}{12}$

d. $\_\_\_\_ \times \frac{1}{4} = \frac{3}{4}$

Write, Solve, and Draw

Activity #2

1. Complete the table.

diagram	expression	product
		
	$6 \times \frac{1}{3}$	
		$\frac{7}{2}$

Write, Solve, and Draw

Activity #2

2. In your group of 4, complete the following steps. After each step, pass your paper to your right.

   - Step 1: Choose a fraction. Write it down.
   - Step 2: Write the fraction you received as a multiplication expression using a whole number and a unit fraction.
   - Step 3: Find the value of the expression you received.
   - Step 4: Draw a diagram to represent the expression.
   - Step 5: Discuss what's on each paper and make revisions if needed.

Let's Put it All Together

Lesson Synthesis

Today we looked at multiplication expressions in two different ways. In the first activity, the number of groups changed while the unit fraction stayed the same like this:

- $1 \times \frac{1}{5}$
- $2 \times \frac{2}{5}$
- $3 \times \frac{3}{5}$
- $4 \times \frac{4}{5}$
- $5 \times \frac{5}{5}$
- $6 \times \frac{6}{5}$ "

Then we looked at a situation in which the unit fraction changed and the number of groups stayed the same, like this:

- $2 \times \frac{1}{5}$
- $2 \times \frac{1}{3}$
- $2 \times \frac{1}{4}$
- $2 \times \frac{1}{2}$ "

How could we write 7/3 as a product of a whole number 3 7 and a fraction?

Adaptation
Lesson 3

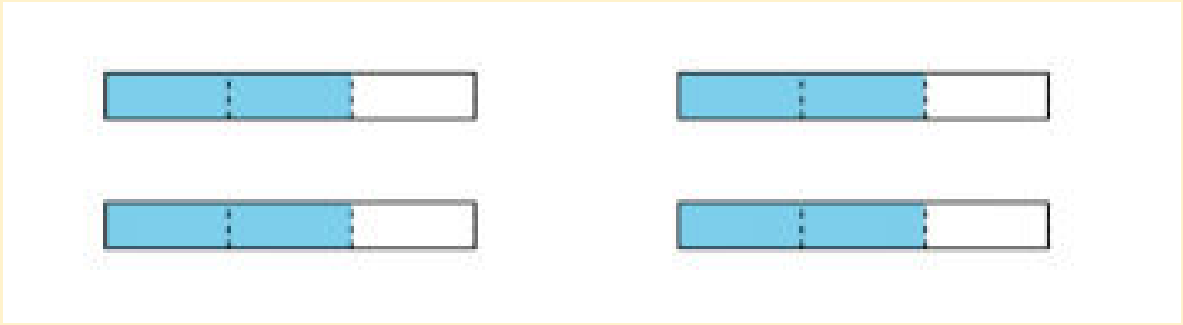
Groups with More Things

Let’s multiply any fraction by a whole number.

Warm up

Notice and Wonder: Thirds

What do you notice?

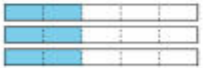


What do you wonder?

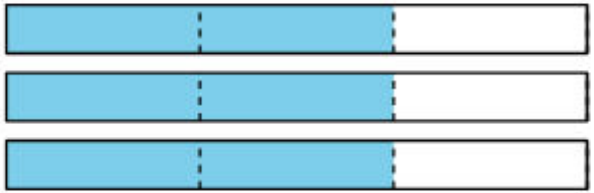
Activity #1

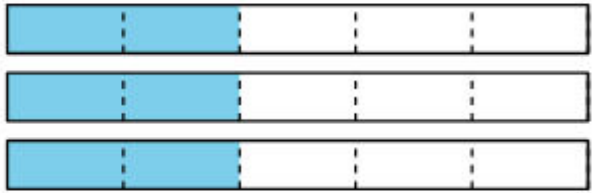
Diagrams for Groups of Many Fractions

1. Write an expression to match each diagram. Evaluate each expression.

diagram	expression	product
		
		

2. Explain the patterns you notice.

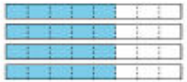
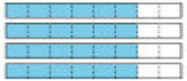




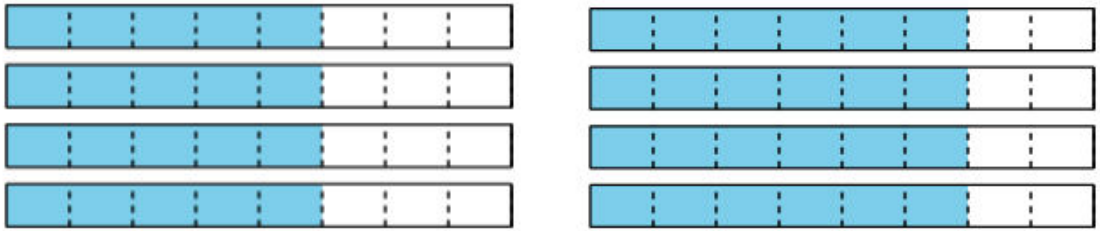
Activity #1

Diagrams for Groups of Many Fractions

3. Write an expression to match each diagram. Find the value of each expression.

diagram	expression	product
		
		

4. Explain how the diagrams in problem 1 are different from the diagrams in problem 2.



Activity #1

Diagrams for Groups of Many Fractions

5. Use the pattern to evaluate each expression. Explain your reasoning. Draw a diagram if it is helpful.

- $2 \times \frac{1}{8}$
- $2 \times \frac{3}{8}$
- $2 \times \frac{5}{8}$
- $5 \times \frac{3}{12}$
- $6 \times \frac{3}{12}$
- $7 \times \frac{3}{12}$

Mai's Big Discovery

Activity #2

1. Evaluate each expression. Use a diagram if it is helpful.

   a. $3 \times \frac{3}{4}$

   b. $8 \times \frac{3}{5}$

   c. $5 \times \frac{6}{8}$

   d. $10 \times \frac{2}{3}$

   e. $9 \times \frac{12}{5}$

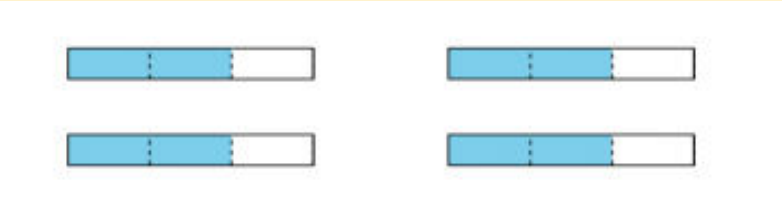
2. Mai said she can multiply any fraction by a whole number by multiplying the whole number by the numerator and keeping the denominator. Do you agree with Mai? Why or why not?

Let's Put it All Together

Lesson Synthesis

Let's discuss Mai's reasoning using the expression 4 x ⅔ and the diagram from today's warm-up.

$$4 \times \frac{2}{3} = \frac{8}{3}$$



- Why can we multiply 4 x 2 to get the numerator of the product?
- Why is the denominator of the product the same as the fraction in the expression?

Adaptation
Lesson 3

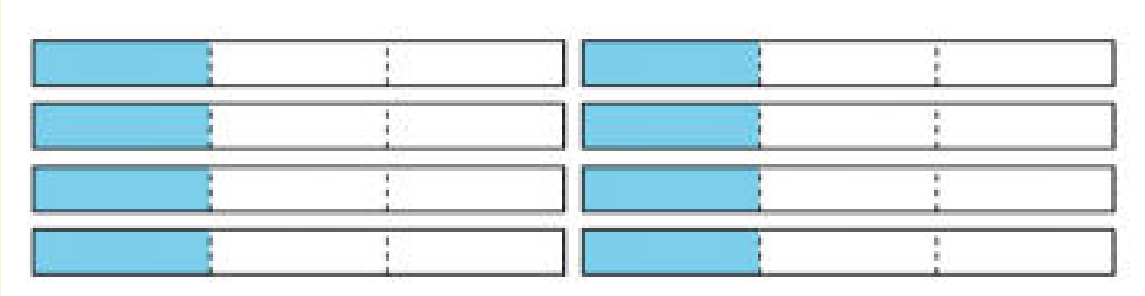
Relate Multiplication Expressions

Let's write multiplication expressions in different ways.

Warm up

How Many Do You See?

How many thirds do you see?



How do you see them?

Activity #1

Complete the Equation

1. Here are two sets of numbers:
   - Set A: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
   - Set B: $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$, $\frac{7}{7}$

   a. Choose a number from Set A and a number from Set B to complete this equation and make it true:

   $\frac{6}{7}$ = ____ x ____

   b. Choose a different number from Set A and a number from Set B to complete the equation to make it true.

   $\frac{6}{7}$ = ____ x ____

Activity #1

Complete the Equation

2. Find the missing number to make each equation true. Draw a diagram if it is helpful.

a. $\frac{12}{5} = 12 \times \_\_\_\_$

b. $\frac{12}{5} = 6 \times \_\_\_\_$

c. $\frac{12}{5} = 4 \times \_\_\_\_$

d. $\frac{12}{5} = 3 \times \_\_\_\_$

e. $\frac{12}{5} = 2 \times \_\_\_\_$

f. $\frac{12}{5} = 1 \times \_\_\_\_$

Activity #1

Complete the Equation

3. Write 3 different expressions that make each equation true:
   - $\frac{8}{10}$ = ____ x ____
   - $\frac{8}{10}$ = ____ x ____
   - $\frac{8}{10}$ = ____ x ____

Equivalent Expressions

Activity #2

Your teacher will give you and your partner a set of cards that have expressions.

Match each card to one of the following fractions.

$\frac{4}{5}$	$\frac{10}{12}$	$\frac{6}{10}$	$\frac{8}{9}$

Complete each equation to make it true. Use the diagrams to support your reasoning.

1. $\frac{4}{5}$ = ____ x ____ = ____ x ____
2. $\frac{10}{12}$ = ____ x ____ = ____ x ____
3. $\frac{6}{10}$ = ____ x ____ = ____ x ____
4. $\frac{8}{9}$ = ____ x ____ = ____ x ____

Equivalent Expressions

Activity #2

Equivalent Expressions $2 \times 2 \times \frac{1}{5}$	Equivalent Expressions $6 \times \frac{1}{10}$
Equivalent Expressions $2 \times \frac{4}{9}$	Equivalent Expressions $5 \times 2 \times \frac{1}{12}$
Equivalent Expressions $4 \times \frac{1}{5}$	Equivalent Expressions $4 \times 4 \times \frac{1}{9}$

Equivalent Expressions $3 \times \frac{2}{10}$	Equivalent Expressions $4 \times \frac{1}{12}$

Let's Put it All Together

Lesson Synthesis

$$\frac{6}{8} = 6 \times \frac{1}{8}$$

$$\frac{6}{8} = 3 \times \frac{2}{8}$$

$$\frac{6}{8} = 2 \times \frac{3}{8}$$

$$\frac{6}{8} = 3 \times 2 \times \frac{1}{8}$$

How does the first equation relate to the second equation, and how does that one relate to the third equation?

1

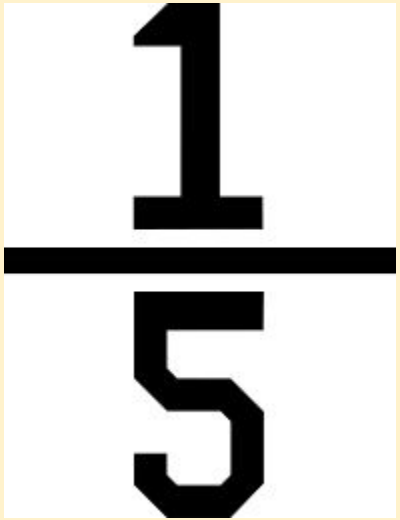
Share Bread

Let’s share one thing.

5.NF.B.3

Warm up

What Do You Know About ⅕ ?



Equal Shares of Bread

Activity #1

1. What kinds of bread do you like to eat?

2. What kinds of bread do you eat on special occasions?

Diego brought in many loaves of bread to share with his class.

For each situation, decide how much bread each person gets if they share the bread equally. Show or explain your reasoning with a diagram or equation.

1. Diego and his three friends share 12 loaves of bread.

2. Priya and her three friends share 8 loaves of bread.

3. Kiran and his three friends share 4 loaves of bread.

4. Jada and her three friends share 1 loaf of bread.

One Loaf

Activity #2

1. Jada baked 1 loaf of bread. Complete the table to show how much bread each person will get if the bread is shared equally and there is none leftover.

number of loaves of bread	number of people sharing bread	amount of bread for each person (loaves)	division expression
1	2		
1	3		
1	4		
1	10		
1	25		

2. Pick one row from the table and draw a diagram to represent your reasoning for that row.

3. What patterns did you notice as the number of people changed?

Lesson Synthesis

Let's Put it All Together

$$1 \div 6$$

What does the expression mean in terms of the problems we were solving about people sharing a loaf of bread?

How much of the loaf of bread will each person get?

2

Share More

Let’s share things equally.

5.NF.B.3

Warm up

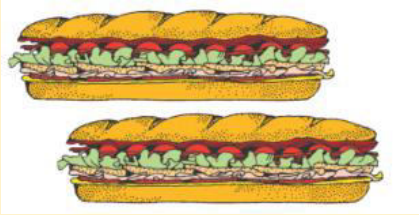
Estimation Exploration: Name that Fraction

The large rectangle represents 1. What fraction of the large rectangle is shaded?

Record an estimate that is:

too low	about right	too high

Share Sandwiches



Activity #1

Tyler, Elena, Mai, Andre, and Lin share some sandwiches.

In each situation decide how much each person will get. Be prepared to explain your reasoning.

1. If they share 2 sandwiches, how much sandwich does each of them get? Make a diagram to support your reasoning.

2. If they share 3 sandwiches, how much sandwich does each of them get? Make a diagram to support your reasoning.

3. If they share 8 sandwiches, how much sandwich does each of them get? Make a diagram to support your reasoning.

Activity #2

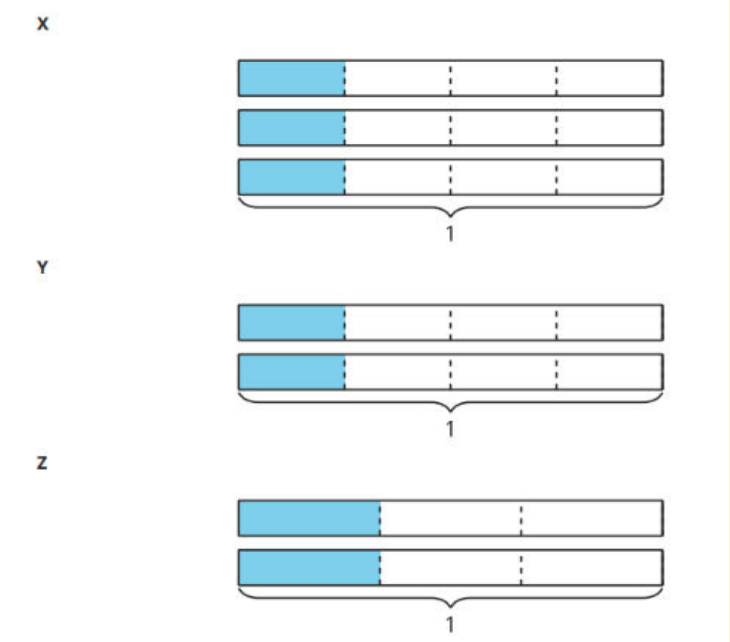
Share More Sandwiches

1. Match each diagram with a situation. Be prepared to explain your reasoning.

a. 4 people equally share 2 sandwiches

b. 4 people equally share 3 sandwiches

c. 3 people equally share 2 sandwiches



Activity #2

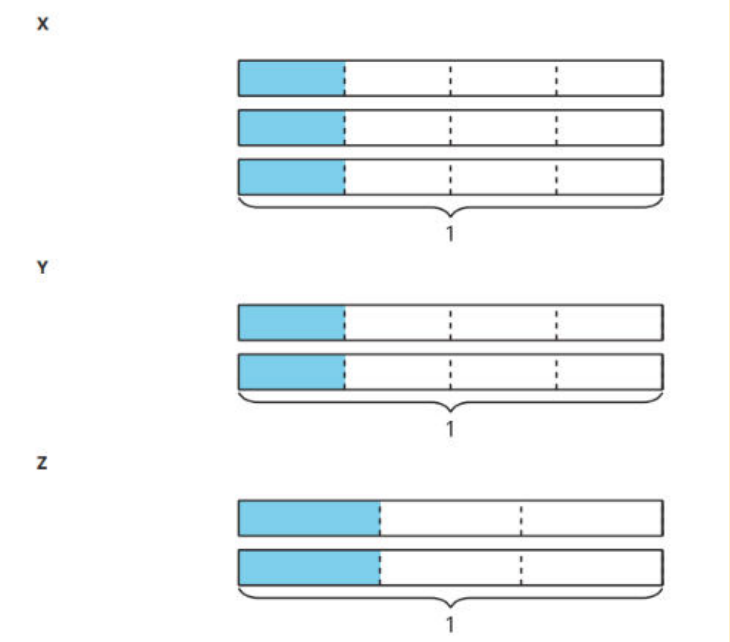
Share More Sandwiches

2. Work with your partner to write a division expression to represent each matched pair.

a. 4 people equally share 2 sandwiches

b. 4 people equally share 3 sandwiches

c. 3 people equally share 2 sandwiches

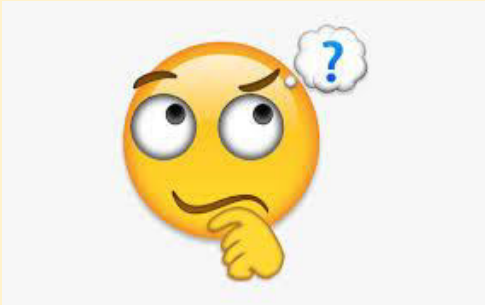


Lesson Synthesis

Let's Put it All Together

"Today we thought about situations where a group of people shares different numbers of things equally. We noticed some patterns in the amount each person gets and in diagrams and expressions to represent the situations."

If 3 people equally share 2 sandwiches, how much sandwich does each person get?



Describe how you would figure out the amount of sandwich each person gets if 3 people share any amount of sandwiches?

3

Interpret Equations

Let’s share more things equally.

5.NF.B.3

What Do You Know About?

Warm up

$$\frac{5}{6}$$

Dehydrated Dancers

Activity #1

1. Some dancers share water.
   a. Partner A: Two dancers share 1 liter of water. $1 \div 2 = \frac{1}{2}$
      Explain how this equation represents the situation.
   b. Partner B: Three dancers share 1 liter of water. $1 \div 3 = \frac{1}{3}$
      Explain how this equation represents the situation.

      Pause for teacher directions.
2. Three dancers share 2 liters of water. How much water does each dancer get?
3. Write a division equation to represent your thinking.
4. Mai said that each dancer gets $\frac{3}{2}$ of a liter of water because it is 3 people sharing 2 liters. Explain why Mai's answer does not make sense.

Interpret Expressions

Activity #2

1. Complete the table. Draw a diagram if it is helpful.

number of dancers	liters of water	division expression	amount of water each dancer drank in liters
4	2		
		$3 \div 4$	
		$3 \div 5$	
4			
	5		

2. What patterns do you notice in the table?

Lesson Synthesis

Let's Put it All Together

What do you know about $\frac{5}{6}$?

Student Responses

Sample responses:

- It is less than 1.
- It is more than $\frac{1}{2}$
- It is close to 1.
- It is 5 groups of $\frac{1}{6}$

What can we add to our answers from the warm-up question based on what we learned today?

If 6 dancers share 5 liters of water, how much water does each get?

4

Situation, Diagrams, and Equations

Let’s solve and represent division situations.

5.NF.B.3

Warm up

Number Talk

Find the value of each expression mentally.

$35 \div 7$

Warm up

Number Talk

Find the value of each expression mentally.

$$1 \div 7$$

Warm up

Number Talk

Find the value of each expression mentally.

$36 \div 7$

Warm up

Number Talk

Find the value of each expression mentally.

$$37 \div 7$$

Card Sort: Division Mix and Match

Activity #1

1. Match each diagram, situation, and equation.

2. Solve each division equation.

Mai, Priya, Tyler, share 5 sandwiches. How much of the sandwich will each person get?

Four friends are sharing 3 pounds of blueberries. How many pounds of blueberries does each friend get?

Five friends are going to share 3 gallons of water so each person gets the same amount of water. How much water will each person get?

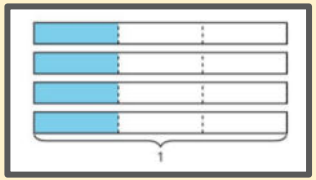
Elena, Lin, and Noah are equally splitting 3 bottles of paint for an art project. How much paint will each person get?

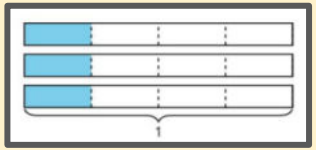
$4 \div 3 =$ _____

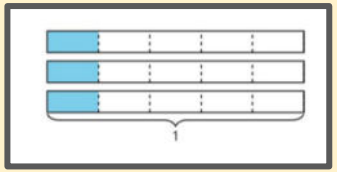
$3 \div 4 =$ _____

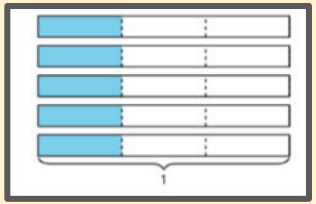
$3 \div 5 =$ _____

$5 \div 3 =$ _____









Activity #2

Creating Division Situations

1. Complete the missing parts of the table. Be prepared to explain your thinking.

2. Discuss both your solutions with your group. What is the same? What is different?

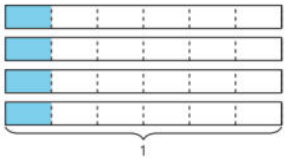
Partner A

Equation	Situation
$4 \div 6 = \frac{4}{6}$	

Diagram

Partner B

Equation	Situation



Diagram

Partner C

Equation	Situation
	Six students were sharing 4 pounds of blueberries. How many pounds of blueberries did each student get?

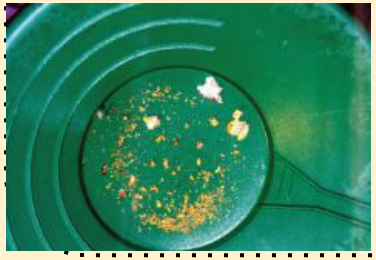
Diagram

Activity #3

Pan for Gold

1. A group of 3 friends spent the afternoon panning for gold. They split the gold equally. Each friend got $\frac{4}{3}$ grams of gold. How much gold did they collect? Explain your reasoning.

2. A group of friends spent the afternoon panning for gold. They split the gold evenly. They collected 5 grams of gold and each friend got $\frac{5}{6}$ grams of gold. How many friends do you think there were? How many grams of gold did they collect altogether? Explain your reasoning.



3. A group of friends spent the afternoon panning for gold. They split the gold equally. Each friend got $\frac{1}{2}$ gram of gold. How much gold did they collect altogether? Explain your reasoning.

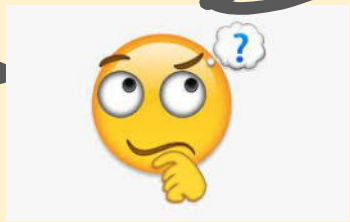
Lesson Synthesis

Let's Put it All Together

$$7 \div ? = \frac{7}{6}$$

What number or numbers make this equation true? How do you know?

What division situation might this equation represent?



5

The Relationship Between Division and Fractions

Let’s explain why fractions can be represented as division expressions.

5.NF.B.3

Warm up

True or False:

Is the statement true or false? Be prepared to explain your reasoning.

$$5 \div 2 = \frac{5}{2}$$

Warm up

True or False:

Is the statement true or false? Be prepared to explain your reasoning.

$$\frac{5}{2} = 5\,\frac{1}{2}$$

Warm up

True or False:

Is the statement true or false? Be prepared to explain your reasoning.

$$\frac{6}{2} = 3$$

Activity #1

Fill in the Blank

Fill in the blank to make the equations true:

1. $9 \div 23 =$ ____

2. ____ $= 4 \div 3$

3. ____ $\div 6 = \frac{14}{6}$

4. $21 \div$ ____ $= 4\frac{1}{5}$

5. $6\frac{4}{5} =$ ____ $\div 5$

Activity #2

Why does it work?

1. What numbers can replace the question marks in each equation? Explain your reasoning.

$$?\div 2 = \frac{?}{2} \qquad 2\div ? = \frac{2}{?}$$

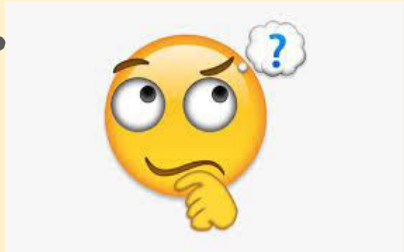
2. Work with your partner to explain why every division expression can be interpreted as a fraction. You can use diagrams, expressions, equations, and words.

Lesson Synthesis

Let's Put it All Together

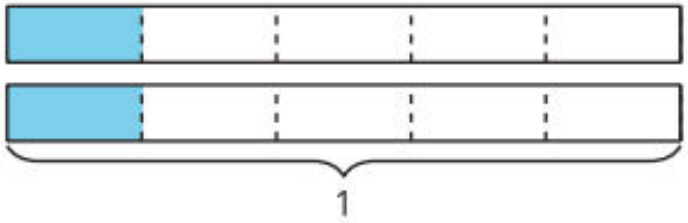
What do you know about the relationship between division and fractions?

How can we represent the relationship between division and fractions?



Section Summary

There is a relationship between division and fractions. We can see this relationship in diagrams, situations, and equations. This diagram represents 2 sandwiches being shared equally by 5 people. Each person will get $\frac{2}{5}$ of a sandwich. The equation, $2 \div 5 = \frac{2}{5}$ also represents the situation.



So far we have:

learned how to represent the relationship between **division** and **unit fractions** and **division** and **non-unit fractions** in a way that makes sense to you.

learned how to represent the relationship between **multiplication** and **division** and **fractions** with **equations.**

figured out how to solve problems involving **division** of whole numbers leading to answers in the form of **mixed numbers** and **fractions**.

are able to explain the relationship between **division** and **fractions.**

6

Divide and Multiply

Let’s explore the relationship between multiplication and division.

5.NF.B.4, 5.NF.B.4.a

Warm up

Number Talk: Multiply & Divide

Find the value of each expression mentally.

$$3 \times \frac{1}{2}$$

Warm up

Number Talk: Multiply & Divide

Find the value of each expression mentally.

$$3 \times \frac{2}{2}$$

Warm up

Number Talk: Multiply & Divide

Find the value of each expression mentally.

$$3 \times \frac{3}{2}$$

Warm up

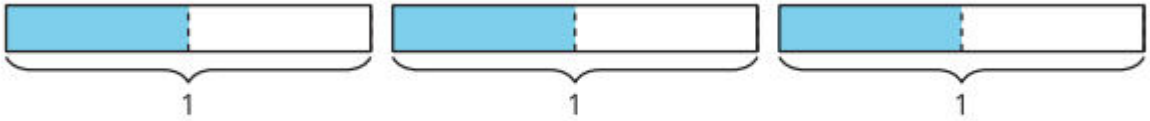
Number Talk: Multiply & Divide

Find the value of each expression mentally.

$$5 \times \frac{3}{2}$$

Activity #1

Where Do You See It?



Explain how each student interprets the diagram.

1. Jada says she sees $\frac{3}{2}$

2. Andre says he sees $3 \times \frac{1}{2}$.

3. Clare says she sees $3 \div 2$.

4. Tyler says he sees $\frac{1}{2}$ of 3.

Activity #2

The Race

1. Andre, Han, and Lin ran a 10 mile relay race as a team. They each ran the same distance. How far did each person run? You can draw a diagram if it helps you.

2. Select all the expressions that represent the situation and explain how each of the expressions you selected represents the situation.
   a. $\frac{1}{3} \times 10$
   b. $3 \div 10$
   c. 10×3
   d. $10 \div 3$

3. Record any other expressions that represent the situation.

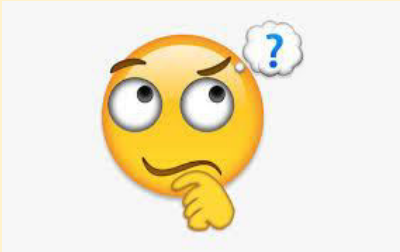
Lesson Synthesis

Let's Put it All Together

When did we use division today?

What did we learn about the relationship between multiplication and division?

When did we use multiplication today?



What do you still wonder about the relationship between multiplication and division?

7

Find a Fraction of a Whole Number

Let’s solve problems about multiplying whole numbers by unit fractions.

5.NF.B.4

Warm up

Estimation Exploration: Number Line

1. What number goes in the box?

2. Record an estimate that is:

too low	about right	too high

How Far Did They Run?

Activity #1

Solve each problem. Draw a diagram if it is helpful

1. Mai ran $\frac{1}{4}$ the length of her road, which is 9 miles long. How far did Mai run?

2. Han ran $\frac{1}{4}$ the length of his road, which is 7 miles long. How far did he run? Represent your thinking with a diagram.

Activity #2

Match the Situation

Han, Lin, Kiran, and Jada together ran a 3 mile relay race. They each ran the same distance.

1. Find the expressions and diagrams that match this situation. Be prepared to explain your reasoning.

2. How far did each person run?

$4 \div 3$

$4 \times \frac{1}{3}$

$\frac{1}{4} \times 3$

$3 \times \frac{1}{4}$

$\frac{4}{3}$

$3 \div 4$

$\frac{3}{4}$

$\frac{1}{3} \times 4$

Lesson Synthesis

Let's Put it All Together

What does it mean to run ¼ of a 10 mile road?

What expressions can you write to represent ¼ of 10?

Which expression helps you calculate how far ¼ of 10 miles is?

8

More Diagrams and Expression

Let’s solve problems about multiplying whole numbers by fractions.

5.NF.B.4

Warm up

True or False:

A Fraction by a Whole Number

Is each statement true or false? Be prepared to explain your reasoning.

- $2 \times \left(\frac{1}{3} \times 6\right) = \frac{2}{3} \times 6$
- $2 \times \left(\frac{1}{3} \times 6\right) = 2 \times (6 \div 3)$
- $\frac{2}{3} \times 6 = 3 \times \left(\frac{1}{2} \times 6\right)$
- $2 \times (6 \div 3) = \frac{12}{3}$

Activity #1

Find the Value

Find the value of each expression. Draw a diagram if it is helpful.

1. $\frac{1}{4} \times 3$

2. $\frac{1}{4} \times 4$

3. $\frac{1}{4} \times 5$

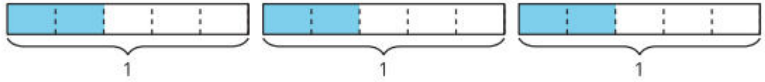
4. $\frac{1}{4} \times 6$

5. $\frac{2}{4} \times 5$

Expressions and Diagrams

Activity #2

1. Explain how each expression represents the shaded region.



a. $2 \times (3 \div 5)$

b. $\frac{6}{5}$

c. $3 \times \frac{2}{5}$

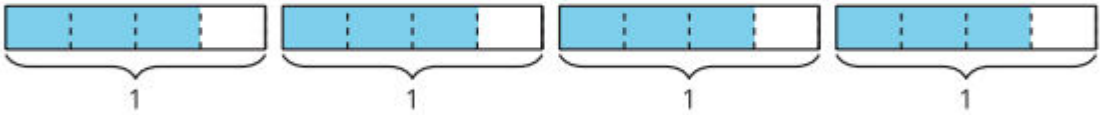
d. $3 \times 2 \times \frac{1}{5}$

e. $\frac{2}{5} \times 3$

Activity #2

Expressions and Diagrams

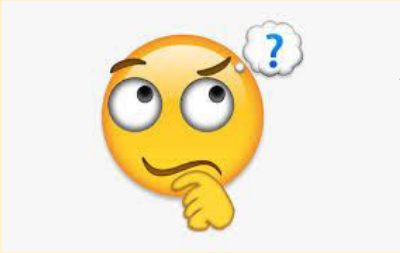
2. Write 4 different expressions for the shaded region. Be prepared to explain your reasoning.



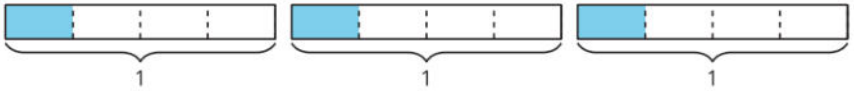
Lesson Synthesis

Let's Put it All Together

What would you add to or revise about the relationship between multiplication and division?



Section Summary



In this section, we explored the relationship between multiplication and division. We learned that 1 diagram can represent a multiplication expression and a division expression. For example, we can see 4 different expressions in this diagram:

- We see $\frac{3}{4}$ because each rectangle is divided into 4 equal pieces and three of them are shaded.
- We see $3 \times \frac{1}{4}$ because there are three pieces shaded and each one is $\frac{1}{4}$ of the rectangle.
- We see $3 \div 4$ because there are three rectangles and each one is divided into 4 equal pieces.
- We see $\frac{1}{4} \times 3$ because there are 3 rectangles and $\frac{1}{4}$ of each one is shaded.

We know that all of these expressions are equal because we can see all of them in the diagram. We can use any of these expressions to represent and solve this problem:

- Mai ate $\frac{1}{4}$ of a 3 pound bag of blueberries. How many pounds of blueberries did Mai eat?

In the past few lessons, we've:

explored the relationships between **multiplication** and **division.**

represented and solved problems involving **multiplication** of a **whole number** by a **unit fraction**.

represented and solved problems involving **multiplication** of a **whole number** by a **non unit fraction.**

9

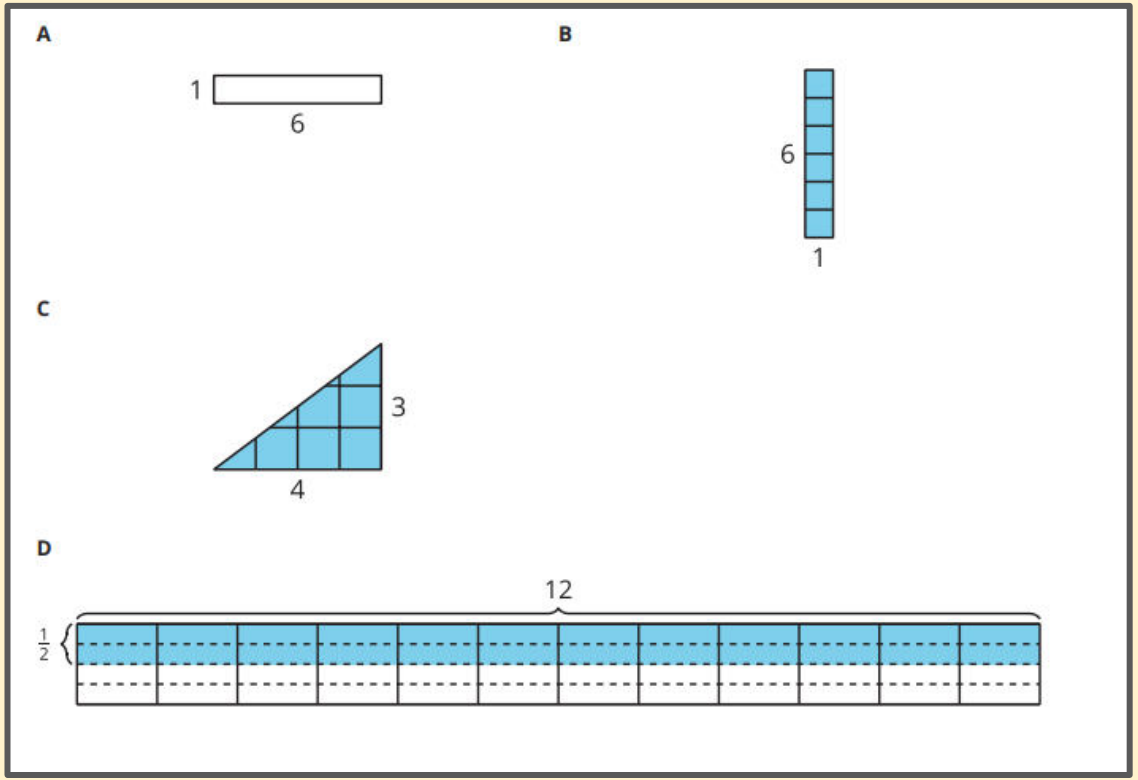
Area With Unit Fraction Side Lengths

Let’s find the area of rectangles with unit fraction side lengths.

5.NF.B.4.b

Warm up

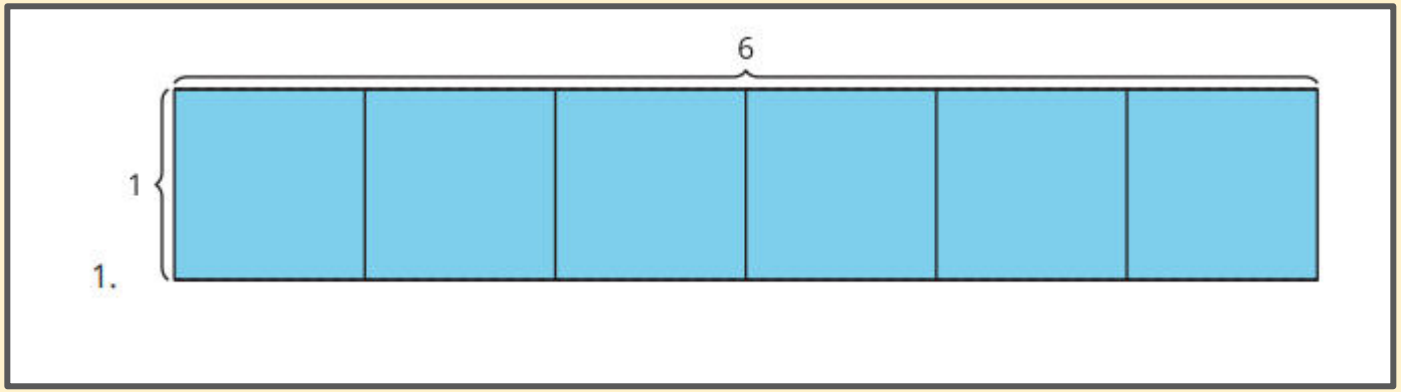
Which One Doesn't Belong: Area



Activity #1

Find the Area

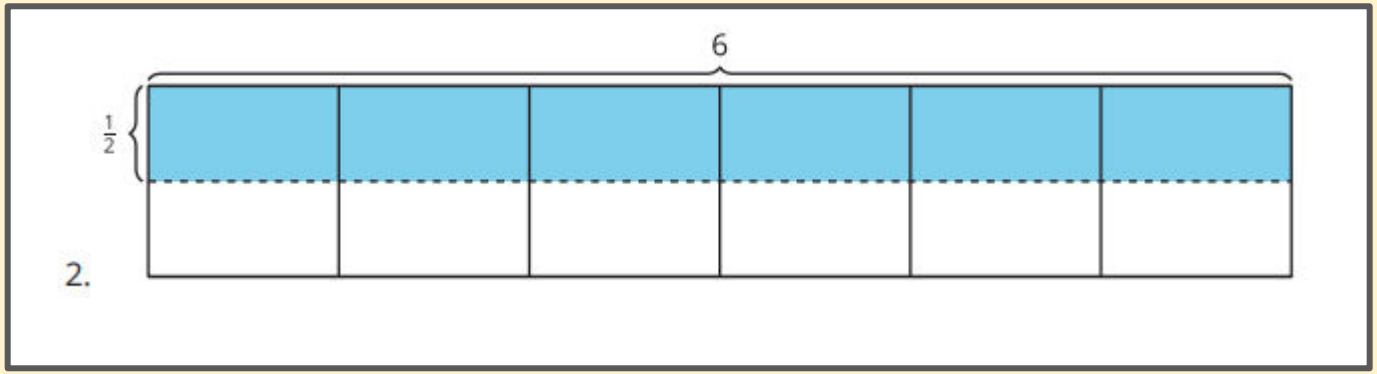
Find the area of the shaded region. Explain your reasoning.



Activity #1

Find the Area

Find the area of the shaded region. Explain your reasoning.

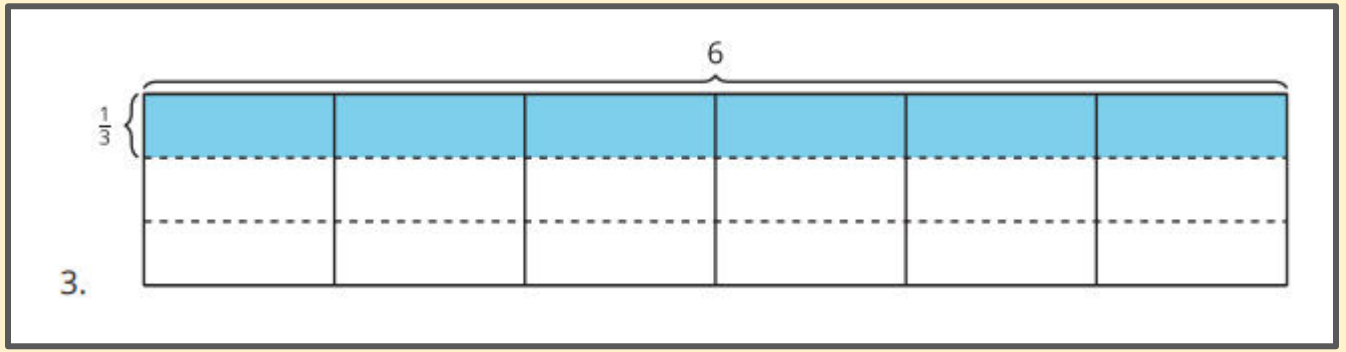


Activity #1

Find the Area

Find the area of the shaded region. Explain your reasoning.

3.

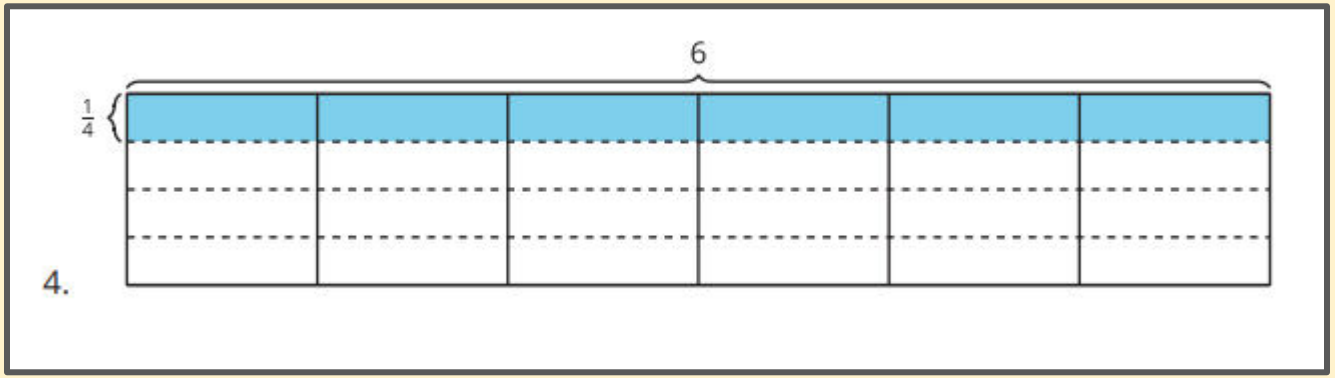


Activity #1

Find the Area

Find the area of the shaded region. Explain your reasoning.

4.



Activity #2

Draw Rectangles

1. Draw each rectangle on grid paper:
   a. $\frac{1}{2}$ unit by 1 unit
   b. $\frac{1}{2}$ unit by 2 units
   c. $\frac{1}{2}$ unit by 3 units
   d. $\frac{1}{2}$ unit by 4 units

2. Use the rectangles you drew to complete the table.

rectangle	multiplication expression	area of the rectangle
$\frac{1}{2}$ unit by 1 unit		
$\frac{1}{2}$ unit by 2 units		
$\frac{1}{2}$ unit by 3 units		
$\frac{1}{2}$ unit by 4 units		

Lesson Synthesis

Let's Put it All Together

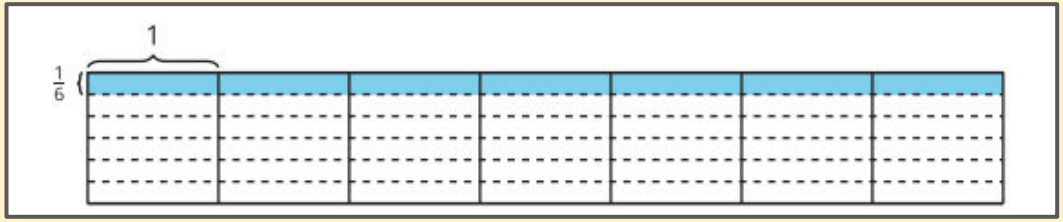
"What do you know about area?"

What information do you need to find the shaded area of this rectangle?

Once you know how long it is, how would you find the shaded area?

Lesson Synthesis

Let's Put it All Together



What is the shaded area?

How do you know?

10

Represent Volume with Expressions

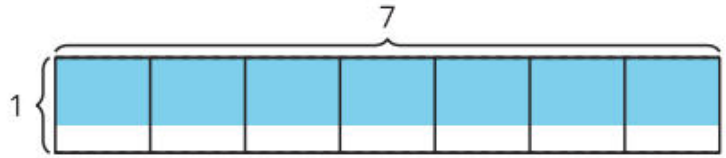
Let’s find the area of rectangles with fractional side lengths.

5.NF.B.4

Warm up

Estimation Exploration: What is the Area?

What might be the area of the shaded region?



Record an estimate that is:

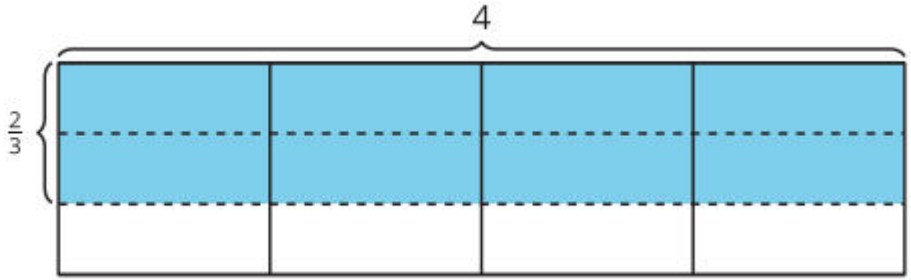
too low	about right	too high

Rectangle With a Fractional Side Length

Activity #1

- Write a multiplication expression to represent the area of each shaded region.
- Then find the area.

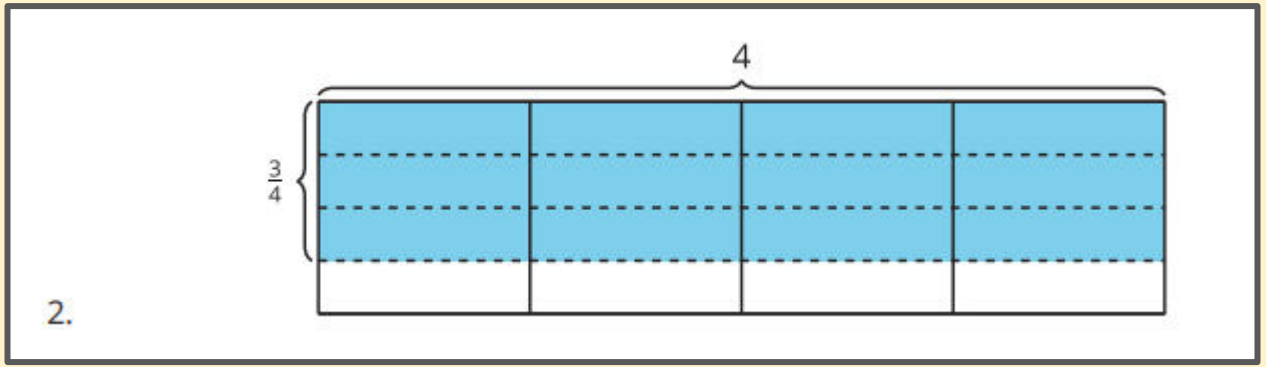
1.



Rectangle With a Fractional Side Length

Activity #1

- Write a multiplication expression to represent the area of each shaded region.
- Then find the area.

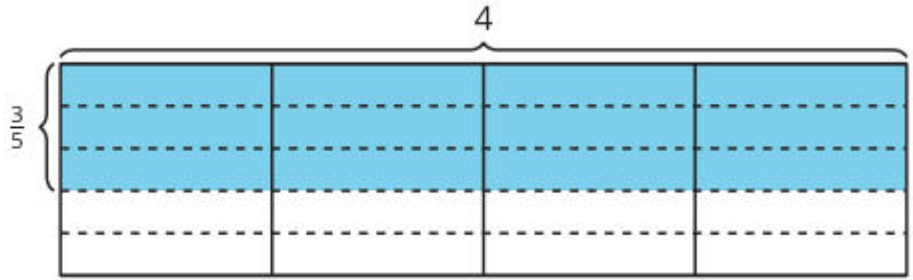


Rectangle With a Fractional Side Length

Activity #1

- Write a multiplication expression to represent the area of each shaded region.
- Then find the area.

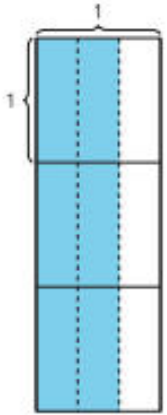
3.



Activity #2

What Are the Side Lengths?

1. Write a multiplication expression to represent the area of the shaded region. What is the area?

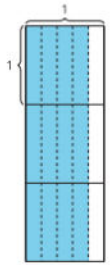


What Are the Side Lengths?

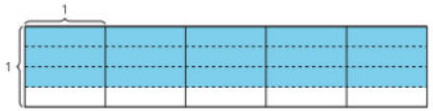
Activity #2

2. Here are two shaded rectangles. Match each expression to a diagram. The diagrams may match more than one expression. Some expressions will not have a match.

X



Y



a. $\frac{3}{4} \times 5$

b. $3 \times \frac{3}{5}$

c. $3 \times 4 \times \frac{1}{5}$

d. $4 \times \frac{3}{4}$

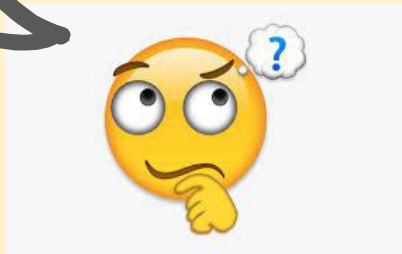
e. $3 \times 3 \times \frac{1}{4}$

3. For each diagram, what is the area?

Lesson Synthesis

Let's Put it All Together

How is finding the area of these rectangles the same as finding the area of rectangles with 2 whole number side lengths?



How is it different?

Fractional Side Lengths Greater Than 1

Let’s find the area of more rectangles.

Warm up

True or False: Thirds

Decide if each statement is true or false. Be prepared to explain your reasoning

$10 \div 3 = 10 \times \frac{1}{3}$

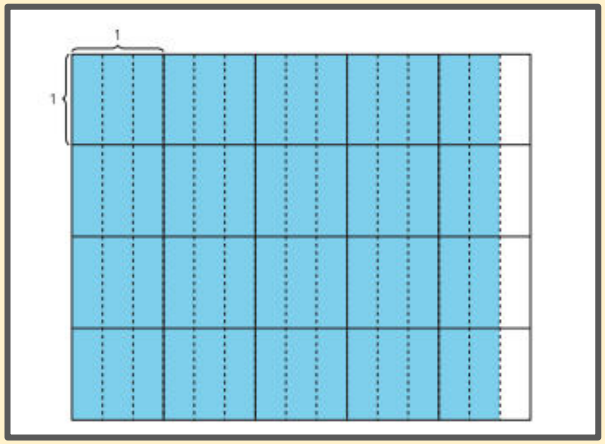
$10 \div 3 = 10\frac{1}{3}$

$\frac{10}{3} = 5 \times \frac{2}{3}$

Greater Than One

Activity #1

1. Find the area of the shaded region in square units. Explain or show your reasoning.



2. Select all the expressions which represent the area in square units. For each correct expression, explain your reasoning.

a. $4\frac{2}{3} \times 4$

b. $16 \times \frac{8}{3}$

c. $\frac{14}{3} \times 4$

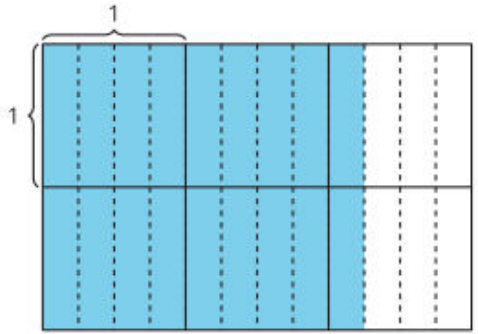
d. $\frac{56}{3}$

e. $4 \times \frac{5}{3}$

Activity #2

Lin's Desk and Garden

1. The diagram represents Lin's desk.



a. Write a multiplication expression to represent the area of Lin's desk.

_____

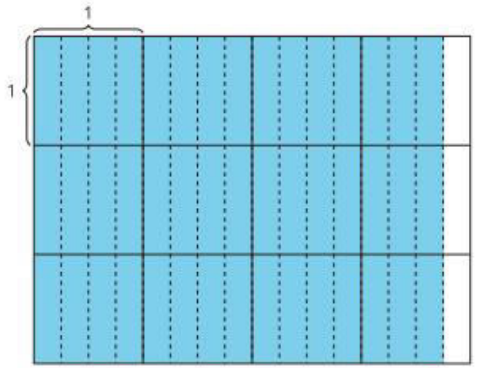
b. What is the area of Lin's desk?

_____

Activity #2

Lin's Desk and Garden

2. The diagram represents Lin's garden in feet.



a. Write a multiplication expression to represent the area of Lin's garden.

_____

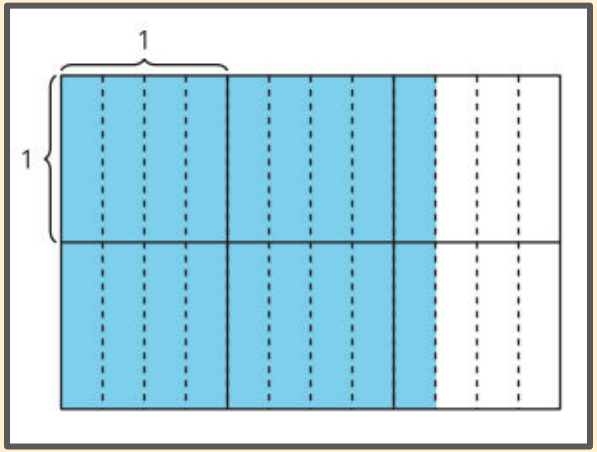
b. What is the area of the garden? Explain your reasoning.

_____

Lesson Synthesis

Let's Put it All Together

Where do you see the expression in the diagram?

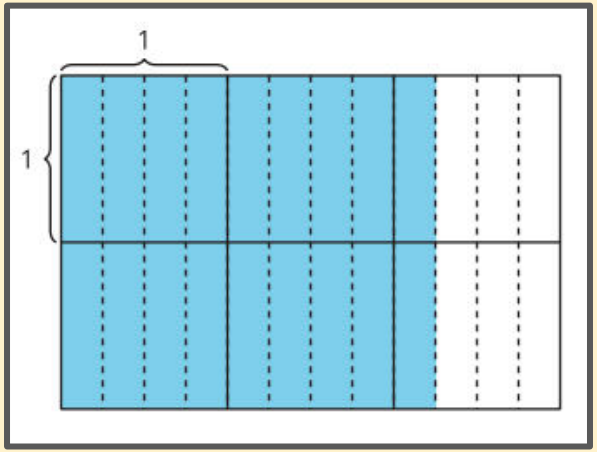


$$2 \times 9 \times \frac{1}{4}$$

Lesson Synthesis

Let's Put it All Together

Where do you see the expression in the diagram?

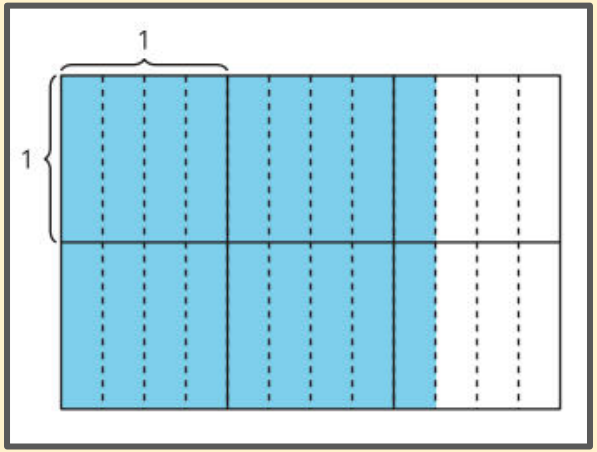


$2 \times \frac{9}{4}$

Lesson Synthesis

Let's Put it All Together

Where do you see the expression in the diagram?



$(2 \times 2) + (2 \times \frac{1}{4})$

Decompose Rectangles

Let's decompose rectangles to find their area

Warm up

Number Talk: Partial Products

Find the value of each product mentally.

$$3 \times 20$$

Warm up

Number Talk: Partial Products

Find the value of each product mentally.

$$3 \times 24$$

Warm up

Number Talk: Partial Products

Find the value of each product mentally.

$$5 \times 2$$

Warm up

Number Talk: Partial Products

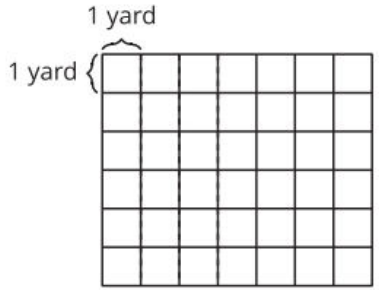
Find the value of each product mentally.

$$5 \times 2\frac{1}{2}$$

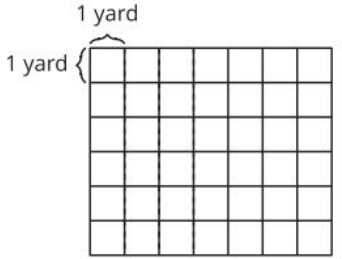
Which Garden Is Larger?

Activity #1

1. Noah's garden is 5 yards by $6\frac{1}{4}$ yards. Draw a diagram of Noah's garden on the grid.



2. Priya's garden is 6 yards by $5\frac{1}{4}$ yards. Draw a diagram of Priya's garden on the grid.

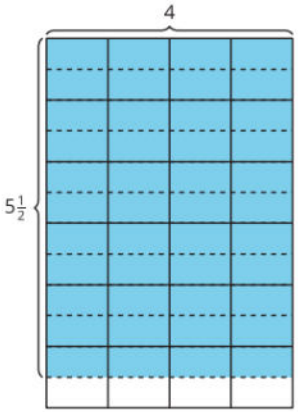


3. Whose garden covers a larger area? Be prepared to explain your reasoning.

Activity #2

Represent Area With Sums and Differences

Partner A

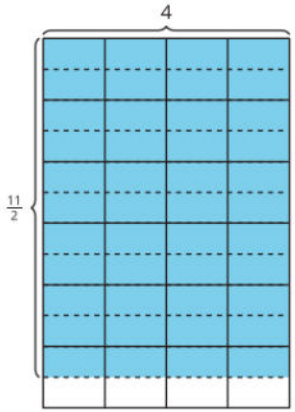


Jada: 4×5

Priya: $4 \times \frac{1}{2}$

Tyler: 6×4

Partner B



Clare: $\frac{10}{2} \times 4$

Diego: 4×6

Elena: $4 \times \frac{1}{2}$

1. Each problem shows the first step a student used to find the area of the shaded region. Explain how each student could finish their process to find the area, in square units, and show your thinking on the diagram.
2. Share your response with your partner. What is the same? What is different?

Lesson Synthesis

Let's Put it All Together

How can we describe the strategies we used today?

13

Diagrams and Expressions

Let’s write expressions to represent the area of rectangles.

5.NF.B.4, 5.NF.B.4.b

Warm up

Number Talk: Parentheses

Find the value of each expression mentally

$$5 \times (7 + 4)$$

Warm up

Number Talk: Partial Products

Find the value of each expression mentally

$$(5 \times 7) + (5 \times 4)$$

Warm up

Number Talk: Partial Products

Find the value of each expression mentally

$$(5 \times 7) + \left(5 \times \frac{1}{4}\right)$$

Warm up

Number Talk: Partial Products

Find the value of each expression mentally

$$(5 \times 7) - \left(5 \times \frac{1}{4}\right)$$

Card Sort: Diagrams and Expressions

Activity #1

Your teacher will give you and your partner a set of cards.

1. Sort the cards in a way that makes sense to you.
2. Match each expression to an appropriate diagram. Some diagrams match more than one expression.
3. Work with your partner to find the area of each shaded region.

Card Sort: Diagrams and Expressions

Activity #1

$5 \times 3\frac{2}{5}$

$(5 \times 3) + (5 \times \frac{2}{5})$

$5 \times (3 + \frac{2}{5})$

$(5 \times 3) - (5 \times \frac{2}{5})$

$5 \times 2\frac{2}{5}$

$(5 \times 2) + (5 \times \frac{2}{5})$

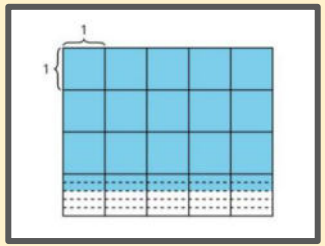
$5 \times \frac{12}{5}$

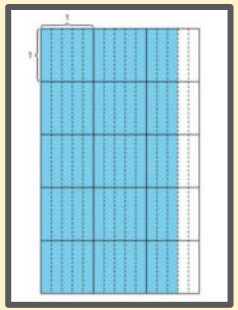
$(3 \times 5) + (3 \times \frac{2}{5})$

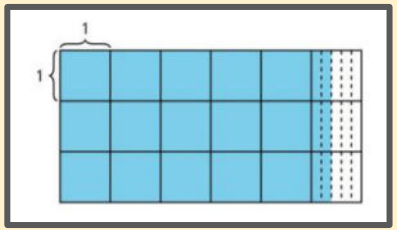
$3 \times 5\frac{2}{5}$

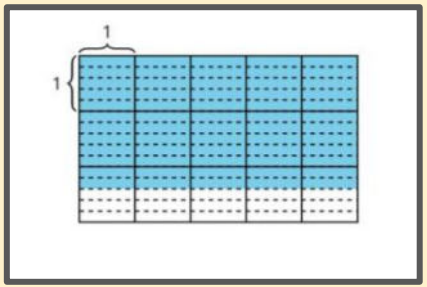
$5 \times (2 + \frac{2}{5})$

$5 \times \frac{17}{5}$





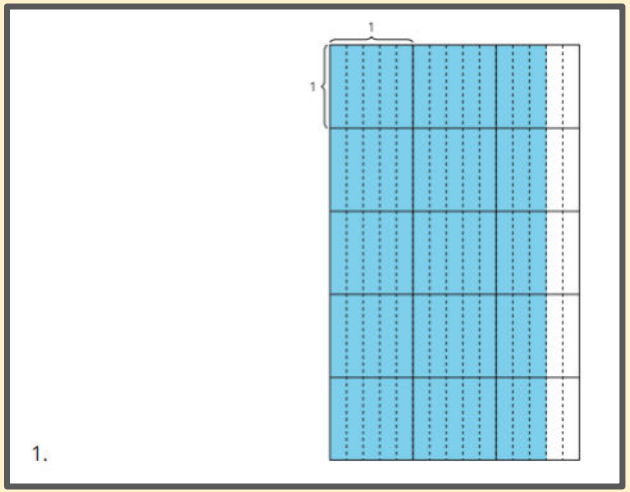


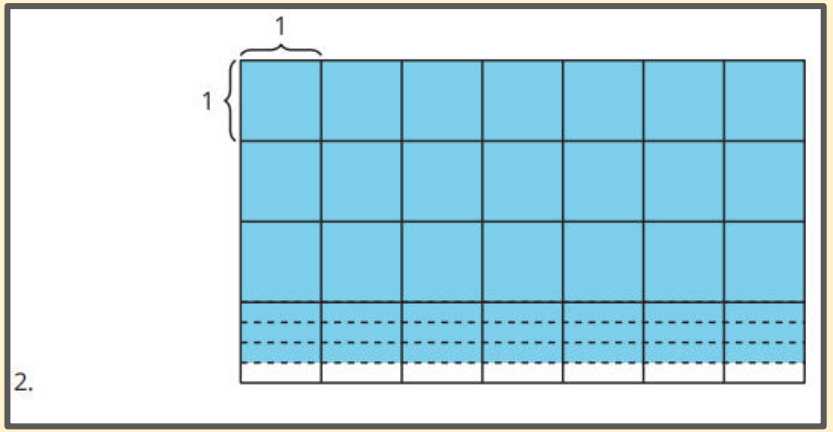


Activity #2

Write Expressions

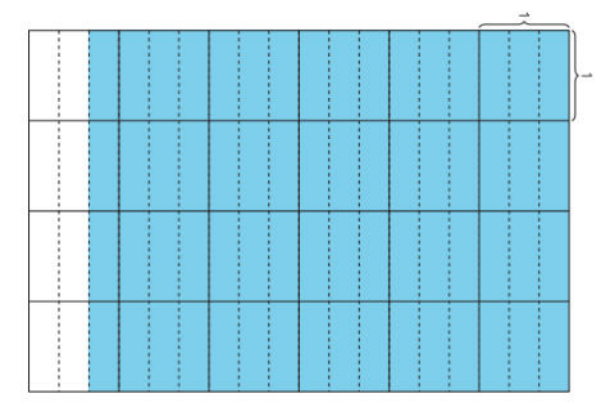
Write as many expressions as you can to match the shaded area in each diagram.





Lesson Synthesis

Let's Put it All Together



$4 \times 5\frac{1}{3}$

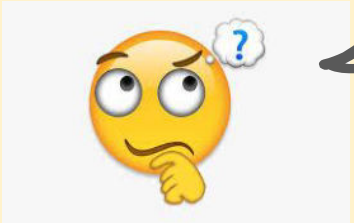
$(4 \times 5) + \left(4 \times \frac{1}{3}\right)$

$(4 \times 6) - \left(4 \times \frac{2}{3}\right)$

$4 \times \frac{16}{3}$

How do we know these expressions are all equivalent?

Where can we see each of these expressions in the diagram?



Reason About Fraction Multiplication

Let’s use the properties of operations to multiply fractions.

Warm up

True or False: Decompose Numbers

Is each statement true or false? Be prepared to explain your reasoning.

$$4 \times 2\frac{1}{3} = 2 \times 2\frac{1}{3} + 2 \times 2\frac{1}{3}$$

Warm up

True or False: Decompose Numbers

Is each statement true or false? Be prepared to explain your reasoning.

$$4 \times 2\frac{1}{3} = (4 \times 2) + \left(4 \times \frac{1}{3}\right)$$

Warm up

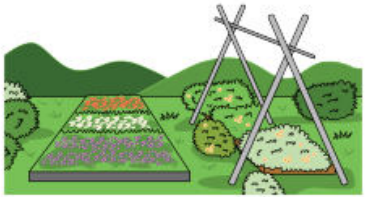
True or False: Decompose Numbers

Is each statement true or false? Be prepared to explain your reasoning.

$$(4 \times 3) - \left(4 \times \frac{1}{3}\right) = 12\,\frac{1}{3}$$

Priya's Garden

Activity #1



Priya has enough materials to build a garden that is 36 square feet.

Choose all the dimensions that are reasonable for her garden. Be prepared to explain your thinking to your partner.

1. 9 feet by $4\frac{2}{3}$ feet

2. 9 feet by $3\frac{8}{9}$ feet

3. 12 feet by $2\frac{11}{12}$ feet

4. 9 feet by $2\frac{2}{3}$ feet

Activity #2

Find the Area

Find the area that each set of dimensions represents.

1. 9 feet by $4\frac{2}{3}$ feet

2. 9 feet by $3\frac{8}{9}$ feet

3. 12 feet by $2\frac{11}{12}$ feet

4. 9 feet by $2\frac{2}{3}$ feet

Lesson Synthesis

Let's Put it All Together

$(5 \times 4) - (5 \times \frac{1}{4})$

- 20
- 19
- 16

What is a reasonable estimate for the value of this expression?

Lesson Synthesis

Let's Put it All Together

$$(5 \times 4) - \left(5 \times \frac{1}{4}\right) = (5 \times 3) + \left(5 \times \frac{3}{4}\right)$$

How do we know this equation is true?

Problem Solve

Let’s apply what we’ve learned about fraction multiplication.

Warm up

Number Talk: Multiply Fractions

Evaluate each expression. Be prepared to explain your reasoning.

$$3 \times (10 \div 2)$$

Warm up

Number Talk: Multiply Fractions

Evaluate each expression. Be prepared to explain your reasoning.

$$\frac{3}{2} \times 10$$

Warm up

Number Talk: Multiply Fractions

Evaluate each expression. Be prepared to explain your reasoning.

$$\left(\frac{14}{7}\right) \times 10$$

Warm up

Number Talk: Multiply Fractions

Evaluate each expression. Be prepared to explain your reasoning.

$$14 \times \frac{10}{7}$$

Info Gap: Area

Activity #1

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.
If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. Solve the problem and explain your reasoning to your partner.

Activity #1

Info Gap: Area

If If your teacher gives you the *data card*:

1. Silently read the information on your card.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask, "Why do you need that information?"
4. After your partner solves the problem, ask them to explain their reasoning and listen to their explanation.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Activity #2

Fill in the Blank

Fill in the blanks to make each equation true. Be prepared to explain your reasoning.

1. $\frac{1}{3} \times 18 = \_\_\_\_$

2. $\frac{7}{9} \times \_\_\_\_ = \frac{21}{9}$

3. $\frac{1}{15} \times \_\_\_\_ = 2$

4. $9 \times 6\frac{2}{3} = \_\_\_\_$

5. $14\frac{99}{100} \times 10 = \_\_\_\_$

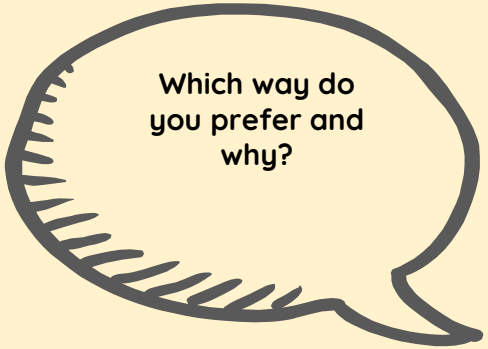
6. $7\frac{3}{5} \times 6 = \_\_\_\_$

7. $4 \times 6\frac{9}{10} = \_\_\_\_$

Lesson Synthesis

Let's Put it All Together

We multiplied whole numbers by fractions that were written as a sum of a whole number and a fraction and we multiplied whole numbers by fractions greater than 1.



16

Multiply More Fractions

Let’s multiply mixed numbers.

5.NF.B.4.b

Warm up

Number Talk: Multiply Mixed Numbers

Find the value of each product mentally.

$$6 \times \frac{3}{8}$$

Warm up

Number Talk: Multiply Mixed Numbers

Find the value of each product mentally.

$$6 \times 2\frac{3}{8}$$

Warm up

Number Talk: Multiply Mixed Numbers

Find the value of each product mentally.

$$7 \times \frac{9}{10}$$

Warm up

Number Talk: Multiply Mixed Numbers

Find the value of each product mentally.

$$7 \times 3\frac{9}{10}$$

Multiply Your Way

Activity #1

Write numbers from the list in the blank spaces so the situations make sense. Each number will be used only one time. Be prepared to explain your thinking.

$$4,\ 5,\ 5\frac{1}{2},\ 3,\ 5\frac{3}{4},\ 2$$

1. The area of the rug is $16\frac{1}{2}$ square feet. The length of the rug is _____ feet. The width of the rug is _____ feet.

2. The puzzle is $2\frac{1}{2}$ feet wide. It is _____ feet long. It has an area of _____ square feet.

3. The area of the whiteboard is 23 square feet. The length of the whiteboard is _____ feet. The width of the whiteboard is _____ feet.

Share your problem and solution with your partner. Explain what choices you made and why.

Activity #2

Equivalent Expressions

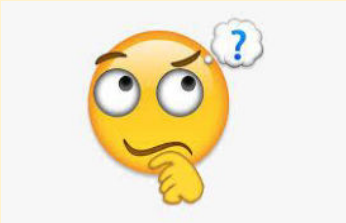
Fill in the missing parts of the table so that each row shows equivalent expressions

product	expression 1	expression 2	value
$5 \times 6\frac{2}{3}$	$(5 \times 6) + (5 \times \frac{2}{3})$	$30 + \frac{10}{3}$	$33\frac{1}{3}$
$3 \times 4\frac{5}{6}$		$15 - \frac{3}{6}$	
$12\frac{3}{4} \times 8$	$(13 \times 8) - (\frac{1}{4} \times 8)$		
$3\frac{7}{12} \times 6$			$21\frac{1}{2}$

Lesson Synthesis

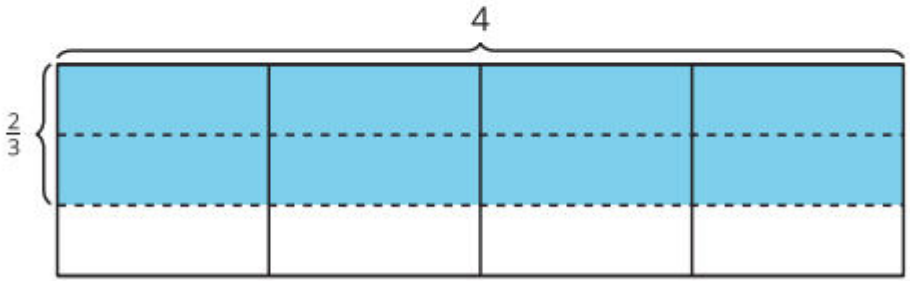
Let's Put it All Together

$$7\frac{3}{5} \times 6$$

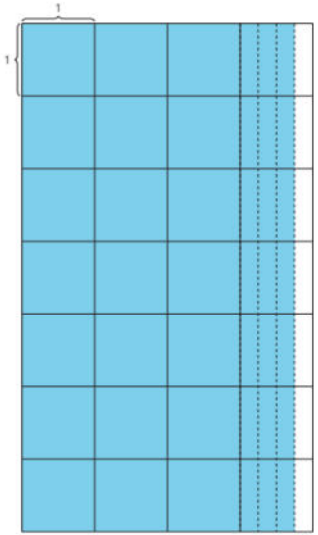


Section Summary

In this section, we learned how to find the area of a rectangle with a fractional side length. The shaded regions has an area of $\frac{8}{3}$ or $2\frac{2}{3}$ because there are 4 unit squares and each one is $\frac{2}{3}$ shaded. The dimensions of the shaded rectangle are 4 by $\frac{2}{3}$. The multiplication expression, $4 \times \frac{2}{3}$ represents the area of the shaded rectangle.



Section Summary



We also learned to multiply a mixed number by a whole number. We used area diagrams and expressions to see why our strategies work. For example, to solve $3\frac{3}{4} \times 7$, we can use the expression $(3 \times 7) + (\frac{3}{4} \times 7)$ We can see both of these expressions in the diagram.

In the past few lessons, we've:

understood that multiplying the **side lengths** of a rectangle to find the **area** still works when the one of the **side lengths** is a **unit fraction.**

found the **area** of a rectangle with one fractional side length in a way that makes sense to you.

found the **area** of a rectangle with one fractional side length greater than one in a way that makes sense to you.

represented the **area** with a multiplication **expression.**

decomposed a rectangle to find its **area.**

represented the **decomposition** of a rectangle with **diagrams** and **expressions.**

solved problems involving multiplication of a whole number by a fraction, including fractions greater than 1.