Main ideas: Decimal numbers (and fractions)

- Decimal Table [Lessons 1-8]
- Purpose of decimal numbers: to express decimal fractions in easier way
  - "It has been recognized since 1593 by the German Jesuit astronomer C. Clavius that a decimal fraction is easier to write if we abandon the fraction symbol: just use the numerator and then keep track of the number of zeros in the denominator by the use of a so-called decimal point…" (Wu, 2011, p. 187)
- Definitions:
  - decimal fraction: fraction with denominator that is product of 10's (positive power of 10)
    - Examples:
      - $\frac{3}{10}$
      - $\frac{487}{100} = \frac{487}{10\times10}$
      - width of hair: between $\frac{17}{10{,}000} = \frac{17}{10\times10\times10\times10}$ & $\frac{181}{10{,}000} = \frac{181}{10\times10\times10\times10}$ cm
  - decimal number: alternative notation for expressing decimal fraction
    - The number of zeros in the denominator of the decimal fraction corresponds to the number of digits to the right of the decimal point (decimal digits)
    - Examples:
      - 3/10 = 0.3 (1 zero in denominator → 1 decimal digit)
      - 487/100 = 4.87 (2 zeros in denominator → 2 decimal digits)
      - 181/10000 = 0.0181 (4 zeros in denominator → 4 decimal digits)
    - Conventions:
      - The zero in front (to left) of the decimal point is only for the purpose of clarity and is optional
        - Ex: 3/10 = 0.3 or .3
      - When the number of digits in numerator is *fewer* than number of zeros in denominator, zeros are inserted to the right of the decimal point to make the decimal digits clear and obvious
        - Ex: 181/10000 = 0.0181 (3 digits in numerator, 4 zeros in denominator → need 1 zero inserted to right of decimal point)
  - Comparison of numbers (fractions, decimals)
    - Equivalent or equal (Grade 3 Standard 3.NF.3a): "$1\frac{3}{10} = \frac{13}{10}$" means "$1\frac{3}{10}$ and $\frac{13}{10}$ are the same point on the number line" or "same length (area, volume)"
    - Greater than: "$\frac{13}{10} > 0.31$", "$\frac{13}{10}$ is greater than 0.31" means "$\frac{13}{10}$ is to the right of 0.31 on the number line" or "$\frac{13}{10}$ has more length (area, volume) than 0.31"
    - Less than: "$0.31 < \frac{13}{10}$", "0.31 is less than $\frac{13}{10}$" means "0.31 is to the left of $\frac{13}{10}$ on the number line" or "0.31 has less length (area, volume) than $\frac{13}{10}$

Reference: Wu, H. (2011). *Understanding numbers in elementary school mathematics*. Providence: American Mathematical Society.

* Great Minds' Suggestions for Consolidation or Omissions: "In Module 6, students explore decimal numbers for the first time by means of the decimal numbers' relationship to decimal fractions. Module 6 builds directly from Module 5 and is foundational to students' Grade 5 work with decimal operations. Therefore, **it is not recommended to omit any lessons from Module 6."**

A. Exploration of Tenths

Lesson 1: Use metric measurement to model the decomposition of one whole into tenths.

1. CD Activities 1 & 3; Problem Set 2-3, 5a
    - Introduce definition of decimal number, decimal fraction (see Main Ideas at top of this page)
    - Use definition of decimal number to represent a measurement as a decimal number and express a decimal fraction (tenths) as a decimal number (and vice versa)
    - Use definitions of decimal number and fraction addition to express 1 cm as sum of tenths of cm
        - Example:
          $\frac{9}{10} + \frac{1}{10} = \frac{10}{10} = 1$ by fraction addition and definition of fraction
          $\frac{9}{10} = 0.9,\ \frac{1}{10} = 0.1,\ \frac{10}{10} = 1.0$ by definition of decimal number
          $\Rightarrow\ 0.9 + 0.1 = 1.0 = 1$
        - Note: $1.0 = 1$ *because* $1.0 = \frac{10}{10}$ (definition of decimal number) and $1 = \frac{10}{10}$ (definition of fraction)
    - See Materials on p. 14 of Teacher Edition; if not available, modify lesson to convey the key ideas

Lesson 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

1. CD Problems 1-2; Problem Set 1b, 2c & e
    - Represent tenths greater than 1:
        - with linear (measurement) and area models
        - as mixed number and decimal number
    - **Key idea**: the whole number in a mixed number corresponds to the digits to the *left* of the decimal point and the numerator of the fraction in the mixed number corresponds to the digits to the *right* of the decimal point
        - Ex: $2\frac{6}{10}$ = 2.6

- **Reasoning**:

$2\frac{6}{10}$
$= 2 + \frac{6}{10}$ by definition of mixed number
$= \frac{20}{10} + \frac{6}{10}$ by equivalent fractions (Module 5): $2 = \frac{2}{1} = \frac{2\times10}{10}$
$= \frac{26}{10}$ by fraction addition (Module 5)
$= 2.6$ by definition of decimal number

Lesson 3: Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

1. CD Problems 1-3; Problem Set 1a, 3a-b
   - Represent tenths greater than 1:
     - with place value disks and on number line
     - as mixed number and decimal number
     - in fraction expanded form and decimal expanded form

B. Tenths and Hundredths

Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

1. CD Problem 1; Problem Set 1b, 2b-c
   - Build on Lesson 1 by introducing decimal form of hundredths (definition of decimal number)
   - Use definitions to express 1 cm as $\frac{1}{100}$ m and 0.01 m
   - Use equivalent fractions to express tenths meter as hundredths meter in fraction and decimal form
     - **Key fact**: there are 10 hundredths in every tenth
2. CD Problem 2; Problem Set 3c, 4b-c
   - Represent sum of tenths and hundredths less than 1:
     - with linear (measurement) model
     - in fraction and decimal form
   - **Key idea**: $0.2 + 0.05 = 0.25$

     Reasoning:

$0.2 + 0.05$
$= \frac{2}{10} + \frac{5}{100}$ by definition of decimal number
$= \frac{20}{100} + \frac{5}{100}$ by equivalent fractions (Module 5): $\frac{2}{10} = \frac{2\times10}{10\times10}$
$= \frac{25}{100}$ by fraction addition (Module 5)
$= 0.25$ by definition of decimal number

Lesson 5: Model the equivalence of tenths and hundredths using the area model and place value disks.

1. CD Problems 1 & 3; Problem Set 2, 3b, 4b-c

- Represent equivalence of tenths and hundredths with area model and place value disks
- Reverse direction of Lesson 4 CD Problem 2:
  - Represent decimal (hundredths) in fraction form and as sum of hundredths and tenths ("unit form")

    0.16
    $= \frac{16}{100}$ by definition of decimal number
    $= \frac{10}{100} + \frac{6}{100}$ by fraction addition (Module 5)
    $= \frac{1}{10} + \frac{6}{100}$ by equivalent fractions (Module 5): $\frac{10}{100} = \frac{10 \div 10}{100 \div 10}$
    $= 1 \textit{ tenth } + \textit{ } 6 \textit{ hundredths}$
- **Key idea**: $0.10 = 0.1$

  Reasoning:

  0.10
  $= \frac{10}{100}$ by definition of decimal number
  $= \frac{10 \div 10}{100 \div 10}$ by equivalent fractions (Module 5)
  $= \frac{1}{10}$
  $= 0.1$ by definition of decimal number

Lesson 6: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.

1. CD Problems 1-2; Problem Set 2, 3a-b
   - Represent hundredths greater than 1:
     - with area model and on number line
     - as mixed number and decimal number
     - in "unit form"

Lesson 7: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

1. CD Problem 2; Problem Set 2
   - Identify the value of each digit in a decimal (hundredths)
2. CD Problem 3; Problem Set 3 b-c
   - Express decimal number in decimal and fraction expanded form

Lesson 8: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

1. CD Problems 1-2; Problem Set 2 & 5
   - Use area model or place value chart to express a mixed number or decimal number in tenths or hundredths
     - $2 \textit{ ones } 4 \textit{ tenths}$

$= 2 + \frac{4}{10}$
$= \frac{20}{10} + \frac{4}{10}$ by equivalent fractions: $2 = \frac{2}{1} = \frac{2 \times 10}{10}$
$= \frac{24}{10}$ by fraction addition $\rightarrow$ 24 *tenths*
$= \frac{240}{100}$ by equivalent fractions: $\frac{24}{10} = \frac{24 \times 10}{10 \times 10} \rightarrow$ 240 *hundredths*

- 2.4
  $= \frac{24}{10}$ by definition of decimal number $\rightarrow$ 24 *tenths*
  $= \frac{240}{100}$ by equivalent fractions: $\frac{24}{10} = \frac{24 \times 10}{10 \times 10} \rightarrow$ 240 *hundredths*

C. Decimal Comparison

Lesson 9: Use the place value chart and metric measurement to compare decimals and answer comparison questions.

1. CD Problems 1-3; Problem Set 1a, 2a, 3b
   - Use linear models (tape measurement, graduated cylinder, weight scale) and definitions of equal, greater than, less than to compare decimal numbers
   - See Materials on p. 145 of Teacher Edition; if not available, modify lesson to convey the key ideas
   - **Recommendation**: introduce comparison on number line and area model (Lesson 10) before comparison with place value chart

Lesson 10: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

1. CD Problems 1-2; Problem Set 1a, 2a
   - Use linear (number line) and area models and definitions of equal, greater than, less than to compare decimal numbers

Lesson 11: Compare and order mixed numbers in various forms.

1. CD Problem 1; Problem Set 1b
   - Use definitions of fractions, mixed numbers, and decimals to locate numbers on number line
2. CD Problem 2; Problem Set 2a
   - Use definitions of equal, greater than, less than to order numbers

D. Addition with Tenths and Hundredths

Lesson 12: Apply understanding of fraction equivalence to add tenths and hundredths.

1. CD Problems 1-3; Problem Set 2b, 3b, 4a
   - Use equivalent fractions to add decimal fractions (tenths and hundredths)

Lesson 13: Add decimal numbers by converting to fraction form.

1. CD Problem 3; Problem Set 2b, 3a & d
    - Build on Lessons 5-8 to add mixed numbers and decimal numbers

Lesson 14: Solve word problems involving the addition of measurements in decimal form.

1. CD Problems 2-3 (Problem Set); Exit Ticket
    - Solve word problems involving addition of decimal measurements

E. Money Amounts as Decimal Numbers

Lesson 15: Express money amounts given in various forms as decimal numbers.

1. CD Problem 1; Problem Set 3, 8, 12
    - Express value of coins as decimal fraction or decimal number
2. CD Problems 2-3; Problem Set 16 & 20
    - Express total value of combination of coins or sets of bills and coins as decimal fraction or decimal number

Lesson 16: Solve word problems involving money.

1. CD Problems 1, 3 (Problem Set); Problem Set 2
    - Solve word problems involving addition, subtraction, multiplication, and/or division of values of bills and coins