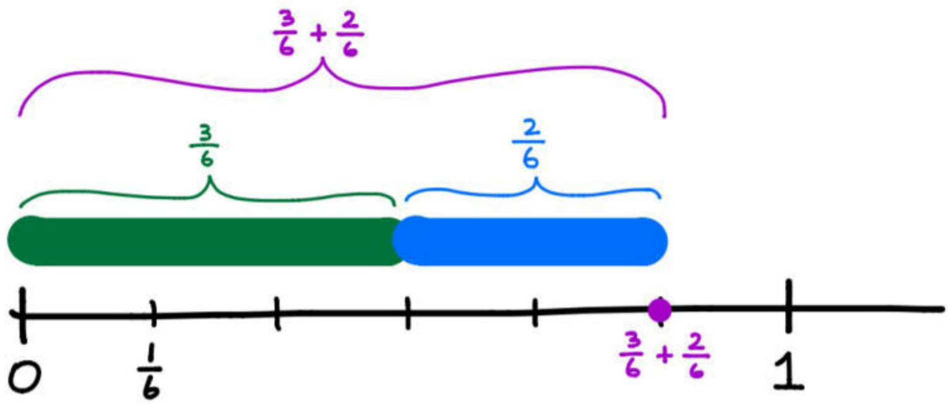
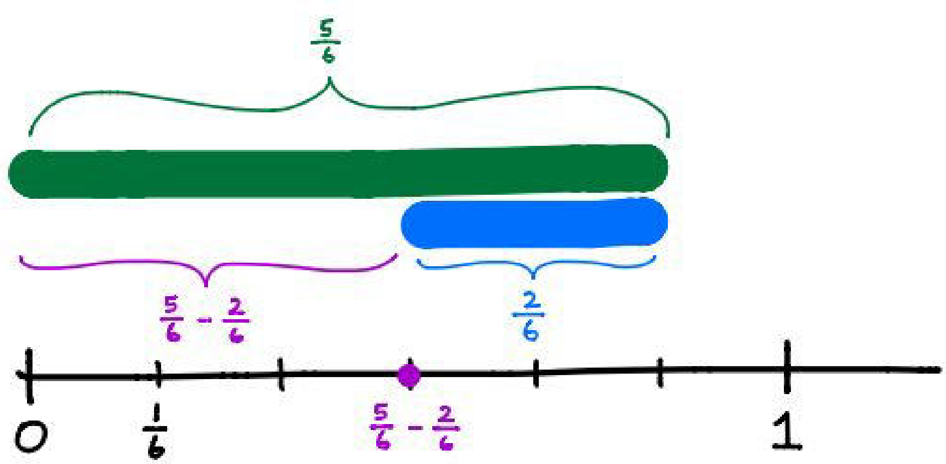
Main ideas: Fractions

- Definitions:
    - Comparison of fractions
        - Equivalent or equal (Grade 3 Standard 3.NF.3a): "$\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$" means "$\frac{3}{6}+\frac{2}{6}$ and $\frac{5}{6}$ are the same point on the number line" or "same area (size)"
        - Greater than: "$\frac{3}{4}>\frac{1}{2}$", "$\frac{3}{4}$ is greater than $\frac{1}{2}$" means "$\frac{3}{4}$ is to the right of $\frac{1}{2}$ on the number line" or "$\frac{3}{4}$ is more area than $\frac{1}{2}$"
        - Less than: "$\frac{4}{10}<\frac{1}{2}$", "$\frac{4}{10}$ is less than $\frac{1}{2}$" means "$\frac{4}{10}$ is to the left of $\frac{1}{2}$ on the number line" or "$\frac{4}{10}$ is less area than $\frac{1}{2}$"
    - Addition of fractions:

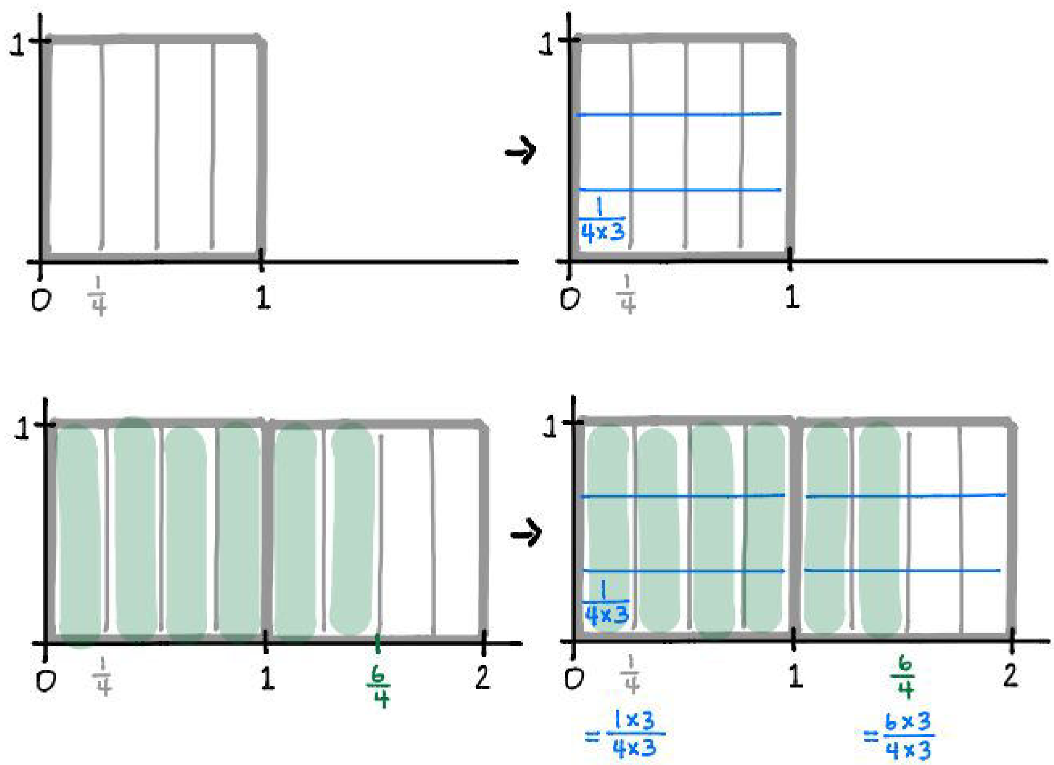


    - Subtraction of fractions:

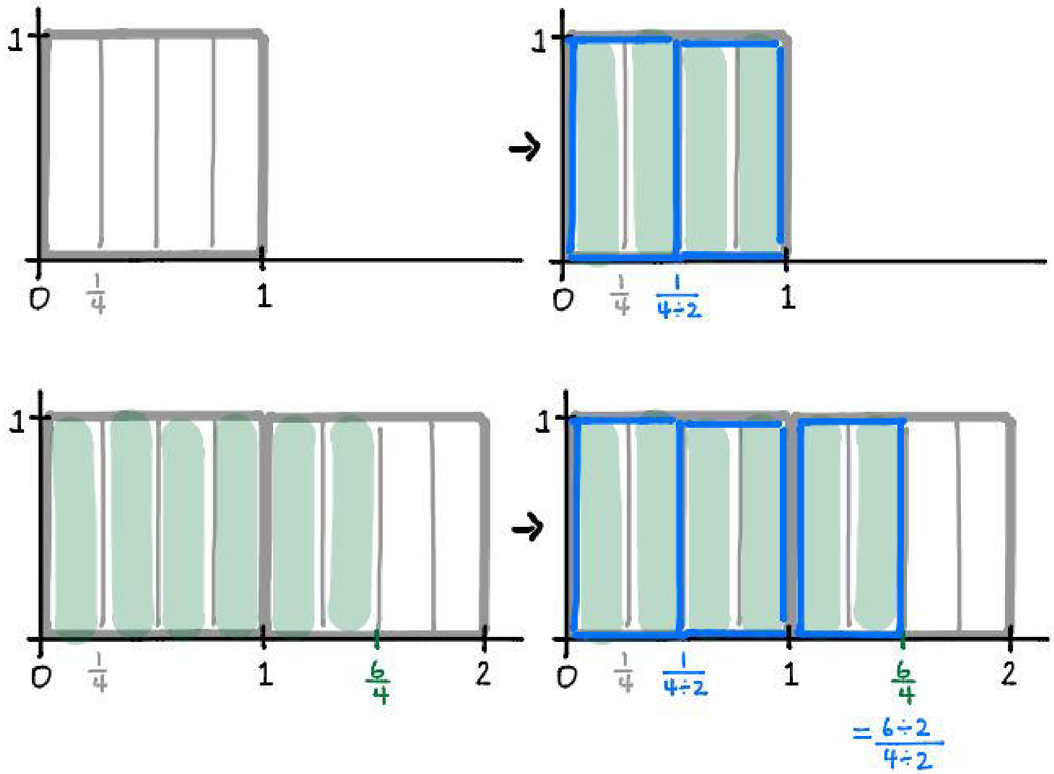


    - Mixed number: "$1\frac{3}{4}$" means "$1+\frac{3}{4}$"
    - Multiplication of whole number and fraction (extension of definition of whole number multiplication): "$2\times\frac{3}{4}$" means "$\frac{3}{4}+\frac{3}{4}$" (2 copies of $\frac{3}{4}$)
    - Rounding to the nearest half: to round a fraction $\frac{a}{b}$ to the nearest half means to replace $\frac{a}{b}$ by the multiple of $\frac{1}{2}$ $(0,\ \frac{1}{2},\ \frac{2}{2}=1,\ \frac{3}{2}=1\frac{1}{2},\ \frac{4}{2}=2,\ etc.)$ which is closest to $\frac{a}{b}$; if two multiples of $\frac{1}{2}$ are equally close to $\frac{a}{b}$, the convention is to always choose the bigger number
- Key ideas:

- Multiplication of whole number and fraction:
  - $\frac{a}{b}$ as a multiple of $\frac{1}{b}$: $\frac{5}{4} = 5 \times \frac{1}{4}$ (5th multiple of $\frac{1}{4}$)
  - $n \times \frac{a}{b} = \frac{n \times a}{b}$
- Equivalent Fractions Theorem:
  - $\frac{a}{b} = \frac{a \times c}{b \times c}$
    - Ex: $\frac{6}{4} = \frac{6 \times 3}{4 \times 3}$
    - Reasoning: When each unit fraction $\frac{1}{4}$ is partitioned into 3 equal parts, it partitions the whole (1) into 4 groups of 3 small parts, or 4×3 small parts (definition of whole number multiplication), and the fraction $\frac{6}{4}$ into 6 groups of 3 small parts, or 6×3 small parts (definition of whole number multiplication). Then each small part is the fraction $\frac{1}{4 \times 3}$ (definition of fraction), and the fraction $\frac{6}{4}$ is 6×3 copies of $\frac{1}{4 \times 3}$, which is the fraction $\frac{6 \times 3}{4 \times 3}$ (i.e., $\frac{6}{4} = \frac{6 \times 3}{4 \times 3}$).



  - $\frac{a}{b} = \frac{a \div c}{b \div c}$ if a and b are multiples of c (or c is a factor of a and b)
    - Ex: $\frac{6}{4} = \frac{6 \div 2}{4 \div 2}$
    - Reasoning: When the unit fractions $\frac{1}{4}$ are formed into groups of 2, it partitions the whole (1) into $4 \div 2$ groups (large parts) and the fraction $\frac{6}{4}$ into $6 \div 2$ groups (definition of whole number division, measurement interpretation). Then each large part is the fraction $\frac{1}{4 \div 2}$ (definition of fraction), and the fraction $\frac{6}{4}$ is $6 \div 2$ copies of $\frac{1}{4 \div 2}$, which is the fraction $\frac{6 \div 2}{4 \div 2}$ (i.e., $\frac{6}{4} = \frac{6 \div 2}{4 \div 2}$).



- Addition and subtraction of fractions: $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$, $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$
- Associative and commutative properties of addition
- Distributive property (for multiplication of whole number and mixed number)

A. Decomposition and Fraction Equivalence

* Great Minds' Suggestions for Consolidation or Omissions: "Study the objectives and the sequence of problems within Lessons 1, 2, and 3, and then consolidate the three lessons. Omit Lesson 4. Instead, in Lesson 5, embed the contrast of the decomposition of a fraction using the tape diagram versus using the area model. Note that the area model's cross hatches are used to transition to multiplying to generate equivalent fractions, add related fractions in Lessons 20 and 21, add decimals in Module 6, add/subtract all fractions in Grade 5's Module 3, and multiply a fraction by a fraction in Grade 5's Module 4. "

Lessons 1-2: Decompose fractions as a sum of unit fractions using tape diagrams.

* Recommendation: include number line with paper strips or tape diagrams

1. Lesson 1 Concept Development (CD) Problem 1; Problem Set
   - Introduce definition of addition of fractions
   - Use visual diagrams (paper strips, number bonds) and definitions of addition of fractions and equivalent (equal) fractions to write a fraction as a sum (decomposition) of smaller fractions

    - Ex: $\frac{5}{6}=\frac{3}{6}+\frac{2}{6}$
2. Lesson 1 CD Problem 2; Problem Set 2g
    - Introduce definition of mixed number
    - Use visual diagrams (paper strips, number bonds) and definitions to write a fraction greater than 1 as a mixed number
        - Ex: $\frac{7}{4}=\frac{4}{4}+\frac{3}{4}=1+\frac{3}{4}=1\frac{3}{4}$
3. Lesson 1 CD Problem 3; Problem Set 1b, 1g
    - Write the addition equation represented by a tape diagram
4. Lesson 1 CD Problem 4; Problem Set 2c-d
    - Draw and label a tape diagram (and/or number line) to represent an addition equation
5. Lesson 2 CD Problem 2 or 3; Problem Set 1b, 2c
    - Represent a fraction as different sums (decompositions)
        - Ex: $\frac{5}{4}=\frac{2}{4}+\frac{2}{4}+\frac{1}{4}$, $\frac{5}{4}=\frac{1}{4}+\frac{3}{4}+\frac{1}{4}$, $\frac{5}{4}=1+\frac{1}{4}$, etc.

Lesson 3: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

* Recommendation: include number line with tape diagrams

1. CD Problem 1 and 2 or 3; Problem Set 1c & e
    - Introduce definition of multiplication of whole number and (unit) fraction
    - Use definitions to conclude Key Idea: $\frac{a}{b}=a\times\frac{1}{b}$

Lesson 4: Decompose fractions into sums of smaller unit fractions using tape diagrams.

* Recommendation: include number line with tape diagrams

1. CD Problem 2; Problem Set 1c, 2b
    - Generate simple equivalent fractions using linear (tape) diagrams (Grade 3 Standard 3.NF.3b)
2. CD Problem 3; Problem Set 3c
    - Use linear (tape) diagrams to show two fractions are equivalent (equal)

Lesson 5: Decompose unit fractions using area models to show equivalence.

* Suggestion: use 1 x 1 (unit) square for area model (instead of rectangle)

- Rationale: area of unit square is 1 (Grade 3 Module 4) so when $\frac{a}{b}$ of the square is shaded, the area of the shaded region will equal the number $\frac{a}{b}$ (instead of only $\frac{a}{b}$ of the rectangle's area)

1. CD Problem 1; Problem Set 1c
    - Generate simple equivalent fractions (less than 1) using area model (Grade 3 Standard 3.NF.3b)
2. CD Problem 3; Problem Set 2a

- Use area model to show two fractions are equivalent (equal)

Lesson 6: Decompose fractions using area models to show equivalence.

* Suggestion: use 1 x 1 (unit) square for area model (instead of rectangle)

1. CD Problem 1 or 2; Problem Set 2a
    - Review/reinforce Lesson 5
2. CD Problem 3
    - Generate equivalent fractions (greater than 1) using area model

B. Fraction Equivalence Using Multiplication and Division

Lessons 7-8: Use the area model and multiplication to show the equivalence of two fractions.

* Suggestion: use 1 x 1 (unit) square for area model (instead of rectangle)

1. Lesson 7 CD Problem 1; Problem Set 1
    - Use area model to show Equivalent Fractions Theorem for unit fraction: $\frac{1}{b} = \frac{1 \times c}{b \times c}$
2. Lesson 8 CD Problems 1-3; Problem Set 1b & d, 3a, 4c, 5a-b
    - Use area model to show Equivalent Fractions Theorem: $\frac{a}{b} = \frac{a \times c}{b \times c}$
    - Use reasoning to verify if two fractions are equivalent (equal)
    - Use Equivalent Fractions Theorem to generate equivalent fractions
    - Note to teacher: include an example of equivalent fractions greater than 1 in preparation for Problem Set and Homework

Lessons 9-10: Use the area model and division to show the equivalence of two fractions.

* Suggestion: use 1 x 1 (unit) square for area model (instead of rectangle)

1. Lesson 9 CD Problem 2; Problem Set 2c
    - Use area model to show Equivalent Fractions Theorem for unit fraction: $\frac{a}{b} = \frac{a \div a}{b \div a}$
2. Lesson 10 CD Problems 1-2 ; Problem Set 1c, 3b
    - Use area model to show Equivalent Fractions Theorem: $\frac{a}{b} = \frac{a \div c}{b \div c}$
3. Lesson 10 CD Problem 4; Problem Set 4b
    - Use Equivalent Fractions Theorem and largest common factor to simplify fraction
        - To simplify a fraction (or write the fraction in simplest form) means to find the equivalent fraction so that the Equivalent Fractions Theorem $\frac{a}{b} = \frac{a \div c}{b \div c}$ cannot be used anymore (i.e., no more common factor in numerator and denominator)

Lesson 11: Explain fraction equivalence using a tape diagram and the number line, and relate that to the use of multiplication and division.

* Suggestion: skip CD Problem 3, Problem Set 5 and Homework problem 5 because decomposing a fraction into *n* equal lengths is not a Grade 4 standard

1. CD Problem 2; Problem Set 1a-b, 2a
    - Use linear (number line and tape) diagrams to show Equivalent Fractions Theorem $\frac{a}{b} = \frac{a \times c}{b \times c}$ and $\frac{a}{b} = \frac{a \div c}{b \div c}$

C. Fraction Comparison

Lessons 12-13: Reason using benchmarks to compare two fractions on the number line.

1. Lesson 12 CD Problems 1-2; Problem Set 1
    - Introduce definitions of greater than, less than
    - Use definitions (greater than, less than, equal) and Equivalent Fractions Theorem to compare fractions less than 1
2. Lesson 13 CD Problem 1; Problem Set 1-2
    - Use definitions (greater than, less than, equal) and Equivalent Fractions Theorem to compare fractions greater than 1

Lessons 14-15: Find common units or number of units to compare two fractions.

1. Lesson 14 CD Problem 1; Problem Set 1c-d, 2c
    - Use reasoning to conclude: if $a < b$, then $\frac{1}{a} > \frac{1}{b}$ and $\frac{n}{a} > \frac{n}{b}$ for any positive whole number n
2. Lesson 14 CD Problems 2-3; Problem Set 2d, 4a
    - Use Equivalent Fractions Theorem to make common numerator or denominator to compare fractions
3. Lesson 15 CD Problems 1-2; Problem Set 1b & d
    - Review/reinforce Lessons 12-13 using area model

D. Fraction Addition and Subtraction

Lesson 16: Use visual models to add and subtract two fractions with the same units.

* Recommendation: Do CD Problem 3 (addition of fractions) before CD Problems 1-2 (subtraction of fractions)

1. CD Problem 3; Problem Set 5a
    - Use definition of addition of fractions (Lesson 1) to find sum of fractions with same denominator
    - Observe that $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
2. CD Problem 1; Problem Set 2a

- Use definition of subtraction of fractions to find difference of fractions with same denominator
- Observe that $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

3. CD Problem 2 or 4; Problem Set 3c, 6d
   - Review/reinforce Lesson 1 (use definition of mixed number to rewrite fraction greater than 1 as mixed number)

Lesson 17: Use visual models to add and subtract two fractions with the same units, including subtracting from one whole.

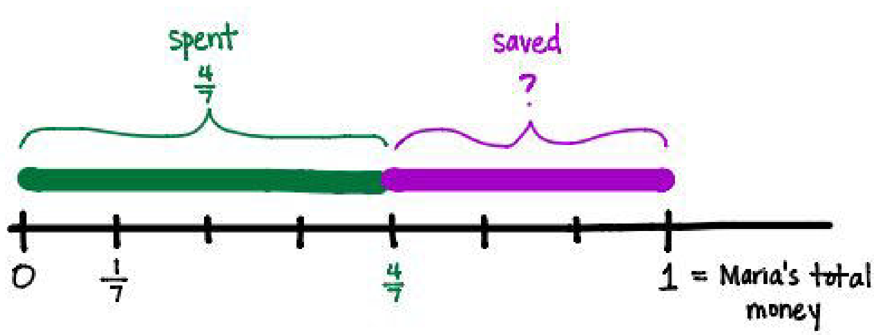
1. CD Problem 1; Problem Set 2d
   - Use definition of subtraction of fractions to subtract fraction from 1
   - Use reasoning to find difference by "counting up"
     - Reasoning: Let $1 - \frac{2}{3} = x$. By definitions of subtraction and addition of fractions, then $x + \frac{2}{3} = 1$. By the commutative property of addition, $\frac{2}{3} + x = 1$.
2. CD Problem 2; Problem Set 3c
   - Use definitions (subtraction of fractions, mixed number) to subtract fraction from number greater than 1
     - $1\frac{1}{5} - \frac{2}{5} = 1 + \frac{1}{5} - \frac{2}{5} = \frac{5}{5} + \frac{1}{5} - \frac{2}{5} = \frac{5+1-2}{5} = \frac{4}{5}$
     - $1\frac{1}{5} - \frac{2}{5} = 1 + \frac{1}{5} - \frac{2}{5} = \left(1 - \frac{2}{5}\right) + \frac{1}{5} = \left(\frac{5}{5} - \frac{2}{5}\right) + \frac{1}{5} = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

Lesson 18: Add and subtract more than two fractions.

1. CD (Practice Sheet) any problem; Problem Set 1c, g-h
   - Build on Lessons 16-17 to add or subtract three or more fractions

Lesson 19: Solve word problems involving addition and subtraction of fractions.

1. CD (Problem Set) Problems 1, 3, 4; Problem Set 5
   - Solve word problems involving addition or subtraction of fractions
   - Recommendation: include number line and label what the whole (1) represents
     - Ex: Problem 3



Lessons 20-21: Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12.

1. Lesson 20 CD Problem 1; Problem Set 1b
    - Use definitions and Equivalent Fractions Theorem to find the sum of unit fractions with related denominators
2. Lesson 20 CD Problem 2 or 3; Problem Set 2d
    - Use definitions and Equivalent Fractions Theorem to find the sum of fractions with related denominators
3. Lesson 21 CD Problem 2; Problem Set 2c
    - Review/reinforce Lesson 20 (and use definition of mixed number to rewrite fraction greater than 1 as mixed number)

E. Extending Fraction Equivalence to Fractions Greater Than 1

Lesson 22: Add a fraction less than 1 to, or subtract a fraction less than 1 from, a whole number using decomposition and visual models.

* Recommendation: include number line

1. CD Problem 1; Problem Set 1b
    - Use definition of mixed number to write a sum that involves a whole number greater than 1
2. CD Problems 2 and 4; Problem Set 3b
    - Use definitions (subtraction of fractions, mixed number) to find a difference that involves a whole number greater than 1
3. CD Problem 3; Problem Set 2
    - Use definitions (addition and subtraction of fractions, mixed number) to write equivalent addition and subtraction equations given three numbers

Lesson 23: Add and multiply unit fractions to build fractions greater than 1 using visual models.

1. CD Problem 1 or 2; Problem Set 3c
    - Build on Lesson 3: use definitions (multiplication of whole number and (unit) fraction, addition of fractions) to find product of whole number and unit fraction
        - Ex: $6 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 3 \times \frac{2}{2} = 3 \times 1 = 3$
2. CD Problem 3; Problem Set 4b
    - Use definitions (multiplication of whole number and (unit) fraction, addition of fractions, mixed number) to write product of whole number and unit fraction as mixed number
        - Ex: $9 \times \frac{1}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = \left(2 \times \frac{4}{4}\right) + \frac{1}{4} = (2 \times 1) + \frac{1}{4} = 2 + \frac{1}{4} = 2\frac{1}{4}$

Lessons 24-25: Decompose and compose fractions greater than 1 to express them in various forms.

1. Lesson 24 CD Problem 1 or 2; Problem Set 2c
    - Use definitions to write fraction greater than 1 as mixed number
        - Ex: $\frac{7}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \left(2 \times \frac{3}{3}\right) + \frac{1}{3} = (2 \times 1) + \frac{1}{3} = 2 + \frac{1}{3} = 2\frac{1}{3}$
2. Lesson 25 CD Problem 1 or 2; Problem Set 1c
    - Use definitions to write mixed number as fraction
        - Ex: $2\frac{1}{6} = 2 + \frac{1}{6} = 1 + 1 + \frac{1}{6} = \frac{6}{6} + \frac{6}{6} + \frac{1}{6} = \frac{(2\times6)+1}{6} = \frac{13}{6}$

Lesson 26: Compare fractions greater than 1 by reasoning using benchmark fractions.

1. CD Problems 1-2; Problem Set 2, 3c
    - Use definitions to compare fractions greater than 1

Lesson 27: Compare fractions greater than 1 by creating common numerators or denominators.

1. CD Problems 1-2; Problem Set 3b, f-g
    - Use definitions and Equivalent Fractions Theorem to compare fractions greater than 1

Lesson 28: Solve word problems with line plots.

1. CD (Problem Set) Problems 1-2
    - Make a line plot to display a data set of measurements with fractions and mixed numbers
    - Solve word problems involving addition and subtraction of fractions by using information presented in line plot

F. Addition and Subtraction of Fractions by Decomposition

Lesson 29: Estimate sums and differences using benchmark numbers.

* Great Minds' Suggestions for Consolidation or Omissions: "Omit Lesson 29, and embed estimation within many problems throughout the module and curriculum."

* Note to teacher: estimating sums and differences using benchmark numbers is not a standard but is helpful for reinforcing understanding of fractions and their location on the number line

1. CD Problems 1-3; Problem Set 1a-c, 2b
    - Use definitions (round to nearest half, addition and subtraction of fractions) to estimate sums and differences of fractions greater than 1

Lesson 30: Add a mixed number and a fraction.

1. CD Problem 1; Problem Set 1c

- Use definition of mixed number and associative property of addition to add mixed number and fraction
  $2\frac{3}{8} + \frac{3}{8}$
  $= \left(2 + \frac{3}{8}\right) + \frac{3}{8}$ by definition of mixed number
  $= 2 + \left(\frac{3}{8} + \frac{3}{8}\right)$ by associative property of addition
  $= 2 + \frac{6}{8}$
  $= 2\frac{6}{8}$ by definition of mixed number

2. CD Problem 2; Problem Set 2b
   - Use number line to find difference between a mixed number and the next whole number
3. CD Problems 3-4; Problem Set 4c
   - Use definition of mixed number and associative property of addition to add mixed number and fraction
     $5\frac{2}{4} + \frac{3}{4}$
     $= 5 + \left(\frac{2}{4} + \frac{3}{4}\right)$ by definition of mixed number, associative property of addition
     $= 5 + \frac{5}{4}$ or $5 + \left(\frac{2}{4} + \frac{2}{4} + \frac{1}{4}\right)$ by applying CD Problem 2
     $= 5 + \left(\frac{4}{4} + \frac{1}{4}\right)$
     $= 5 + \left(1 + \frac{3}{4}\right)$
     $= 6 + \frac{3}{4}$ by associative property of addition
     $= 6\frac{3}{4}$ by definition of mixed number

Lesson 31: Add mixed numbers.

1. CD Problem 1 and 2 or 3; Problem Set 2b, 4a
   - Build on Lesson 30 and use commutative property of addition to add mixed numbers
     $2\frac{1}{8} + 1\frac{5}{8}$
     $= 2 + \frac{1}{8} + 1 + \frac{5}{8}$ by definition of mixed number
     $= (2 + 1) + \left(\frac{1}{8} + \frac{5}{8}\right)$ by commutative and associative properties of addition
     $= 3\frac{6}{8}$

Lesson 32: Subtract a fraction from a mixed number.

1. CD Problems 1-3; Problem Set
   - Key ideas:
     - We can subtract from parts of the total (Problems 1, 3)
       $3\frac{4}{5} - \frac{3}{5} = \left(3 + \frac{4}{5}\right) - \frac{3}{5} = 3 + \left(\frac{4}{5} - \frac{3}{5}\right) =$ etc.
       $4\frac{1}{5} - \frac{3}{5} = \left(1 + 3 + \frac{1}{5}\right) - \frac{3}{5} = 3 + \frac{1}{5} + \left(1 - \frac{3}{5}\right) =$ etc.
     - We can subtract in parts (Problem 2)
       $4\frac{1}{5} - \frac{3}{5} = 4 + \frac{1}{5} - \frac{1}{5} - \frac{2}{5} = 4 - \frac{2}{5} =$ etc.
   - Suggestion: demonstrate Problem 3's strategy with Problem 2

Lesson 33: Subtract a mixed number from a mixed number.

* Suggestion: demonstrate all three strategies with the same problem

1. CD Problem 1; Problem Set
   - Build on Lesson 17 (“counting up” method) to find difference between mixed numbers
2. CD Problems 2-3; Problem Set
   - Key idea: we can subtract in parts from parts of the total
     $11\frac{1}{5} - 2\frac{3}{5} = 11 + \frac{1}{5} - 2 - \frac{3}{5} = (11-2) + \frac{1}{5} - \frac{3}{5} = 9 + \frac{1}{5} - \frac{3}{5} =$ etc.
   - Note: the difference between the strategies in Problems 2 and 3 correspond to Lesson 32’s two Key Ideas

Lesson 34: Subtract mixed numbers.

1. CD Problem 1; Problem Set 1b
   - Rewrite total as sum of whole number and fraction greater than 1 to subtract fraction
     $8\frac{1}{10} - \frac{8}{10} = \left(7 + \frac{10}{10} + \frac{1}{10}\right) - \frac{8}{10} = \left(7 + \frac{11}{10}\right) - \frac{8}{10} = 7 + \left(\frac{11}{10} - \frac{8}{10}\right) =$ etc.
2. CD Problem 2; Problem Set 3a
   - Use Lesson 33 and Problem 1 Key Ideas to subtract mixed numbers

G. Repeated Addition of Fractions as Multiplication

Lessons 35-36: Represent the multiplication of *n* times *a/b* as (*n* x *a*)/*b* using the associative property and visual models.

1. Lesson 35 CD Problems 1-2; Problem Set 3a
   - Build on Lesson 3 by using definition of multiplication of whole number and fraction to conclude Key Idea: $n \times \frac{a}{b} = \frac{n \times a}{b}$
     - Method 1
       $4 \times \frac{3}{5}$
       $= \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$ by definition of $n \times \frac{a}{b}$
       $= \frac{3+3+3+3}{5}$ by Key Idea $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
       $= \frac{4 \times 3}{5}$ by definition of whole number multiplication
     - Method 2
       $4 \times \frac{3}{5}$
       $= 4 \times \left(3 \times \frac{1}{5}\right)$ by Key Idea $\frac{a}{b} = a \times \frac{1}{b}$
       $= (4 \times 3) \times \frac{1}{5}$ by associative property of multiplication
       $= \frac{4 \times 3}{5}$ by Key Idea $\frac{a}{b} = a \times \frac{1}{b}$
2. Lesson 36 CD Problems 1-2; Problem Set 3c
   - Review/reinforce Lesson 35 (definition and Key Idea of $n \times \frac{a}{b}$)
3. Lesson 36 Application Problem or CD Problem 3; Problem Set 5

- Solve word problem involving involving multiplication of whole number and fraction

Lessons 37-38: Find the product of a whole number and a mixed number using the distributive property.

1. Lesson 37 CD Problems 1-2; Problem Set 2g
    - Suggestion: show distributive property (focus of Problem 2) with Problem 1
    - Extend definition of multiplication by whole number to multiply whole number and mixed number and show distributive property
      $2 \times 3\frac{1}{5}$
      $= 2 \times \left(3 + \frac{1}{5}\right)$ by definition of mixed number
      $= 3 + \frac{1}{5} + 3 + \frac{1}{5}$ by extended definition of multiplication by whole number
      $= (3 + 3) + \left(\frac{1}{5} + \frac{1}{5}\right)$ by commutative and associative properties of addition
      $= (2 \times 3) + \left(2 \times \frac{1}{5}\right)$ by definition of $n \times \frac{a}{b}$
      $= 6 + \frac{2}{5}$
      $= 6\frac{2}{5}$ by definition of mixed number
2. Lesson 37 CD Problem 3; Problem Set 3
    - Solve word problem involving multiplication of a whole number and mixed number
3. Lesson 38 CD Problems 2-3; Problem Set 2a, 3
    - Review/reinforce Lesson 37

Lesson 39: Solve multiplicative comparison word problems involving fractions.

1. CD Problem 2-3; Problem Set 5
    - Solve multi-step word problems involving multiplication of a whole number and mixed number ("*n* times as much/tall/long/etc. …")

Lesson 40: Solve word problems involving the multiplication of a whole number and a fraction including those involving line plots.

* Great Minds' Suggestions for Consolidation or Omissions: "Omit Lesson 40, and embed line plot problems in social studies or science. Be aware, however, that there is a line plot question on the End-of-Module Assessment."

1. CD (Problem Set) Problems 1a, 2
    - Build on Lessons 28 and 30-39
        - Make a line plot to display a data set of measurements with mixed numbers
        - Solve word problems using information presented in line plots and involving addition, subtraction of mixed numbers or multiplication by whole number

H. Exploring a Fraction Pattern

Lesson 41: Find and use a pattern to calculate the sum of all fractional parts between 0 and 1. Share and critique peer strategies.

1. CD Problem 1; Problem Set 1e-f
    - Materials: 20 "cards" per student
    - Key idea: $\frac{0}{n}+\frac{1}{n}+...+\frac{n}{n}$
        - If n is odd, then
          $\frac{0}{n}+\frac{1}{n}+...+\frac{n}{n}$
          $=\left(\frac{0}{n}+\frac{n}{n}\right)+\left(\frac{1}{n}+\frac{n-1}{n}\right)+...$ by properties of addition
          $=\frac{n}{n}+\frac{n}{n}+...$
          $=1+1+...$
          $=\left(\frac{n+1}{2}\right)\times 1$
          $=\frac{n+1}{2}$
        - If n is even, then
          $\frac{0}{n}+\frac{1}{n}+...+\frac{n}{n}$
          $=\left(\frac{0}{n}+\frac{n}{n}\right)+\left(\frac{1}{n}+\frac{n-1}{n}\right)+...+\frac{n\div 2}{n}$ by properties of addition
          $=\frac{n}{n}+\frac{n}{n}+...+\frac{n\div 2}{n}$
          $=1+1+...+\frac{1}{2}$
          $=\left(\frac{n}{2}\right)\times 1+\frac{1}{2}$
          $=\frac{n}{2}+\frac{1}{2}$