Main topics: Lines and angles; two-dimensional figures

* Great Minds' Suggestions for Consolidation or Omissions: "The placement of Module 4 in A Story of Units was determined based on the New York State Education Department Pre-Post Math Standards document, which placed 4.NF.5–7 outside the testing window and 4.MD.5 inside the testing window. This is not in alignment with PARCC's Content Emphases Clusters, which reverses those priorities, labeling 4.NF.5–7 as Major Clusters and 4.MD.5 as an Additional Cluster, the status of lowest priority. Those from outside New York State may want to **teach Module 4 after Module 6** and truncate the lessons using the Preparing a Lesson protocol (see the Module Overview, just before the Assessment Overview). This would change the order of the modules to the following: Modules 1, 2, 3, 5, 6, 4, and 7. Those from New York State might apply the following suggestions and truncate Module 4's lessons using the Preparing a Lesson protocol. Topic A could be taught simultaneously with Module 3 during an art class. Topics B and C could be taught directly following Module 3, prior to Module 5, since they offer excellent scaffolding for the fraction work of Module 5. Topic D could be taught simultaneously with Module 5, 6, or 7 during an art class when students are served well with hands-on, rigorous experiences. Keep in mind that Topics B and C of this module are foundational to Grade 7's missing angle problems."

Materials:

- Straightedge (or ruler)
- Protractor [Lessons 6-10, 13-16]
- Pattern Blocks [Lessons 9 & 11]
- Ruler [Lessons 13-16]

A. Lines and Angles

Lesson 1: Identify and draw points, lines, line segments, rays, and angles. Recognize them in various contexts and familiar figures.

1. Concept Development (CD) Problem 1; Problem Set 2a-f
    - Definitions:
        - point: a dot to mark a location
            - Name with a letter (e.g., point A)
        - line segment: a straight segment with two endpoints (formed by connecting two points with a straightedge)
            - Name with endpoints (e.g., $\overline{AB}$, "segment AB")
        - line: a straight segment that extends in both directions forever, without end (indicated with arrows at both ends)
            - Name with two points on the line (e.g., $\overleftrightarrow{AB}$., "line AB")
2. CD Problem 2; Problem Set 2a-g
    - Definitions:

    - ray: a straight segment that ends at one point and extends past another point forever (indicated with an arrow)
        - Name with *endpoint first* and then another point on ray (e.g., $\overrightarrow{AB}$, "ray AB")
    - angle: two rays (sides) with the same endpoint (vertex) and the space between the rays
        - Name with the vertex (e.g., $\angle B$, "angle B") OR with one point from one side, *then the vertex*, lastly one point from the other side (e.g., $\angle ABC$, "angle ABC") to be more precise
        - Use small arc to mark angle
3. CD Problem 4; Problem Set 3
    - Use definitions to identify and name point, line segment, line, ray, angle in figures

Lesson 2: Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles.

1. CD Problem 1; Problem Set 1
    - Observe that a right angle is formed when a circle is folded in half twice
        - Reasoning (for teacher): Folding a circle (360°) in half twice means ½ of ½, which results in ¼ of the circle; ¼ of 360° = ¼ x 360° = 360° ÷ 4 = 90° = right angle
        - Symbol to mark right angle: square with vertex and sides of the angle
        - Sample template
    - Definition: equal (angles)
        - "$\angle A$ is equal to $\angle Z$" means, when the vertices are put on top of each other, the sides can line up so that $\angle A$ fits exactly on $\angle Z$, and vice versa (the sides fall on top of each other)
    - Use the folded circle as a right angle reference ("right angle template") to identify other objects with right angle
2. CD Problem 2; Problem Set 1
    - Definitions: less than (angle), greater than (angle), acute angle, obtuse angle
        - "$\angle A$ is less than $\angle Z$" means, when the vertices are put on top of each other, $\angle A$ can fit inside $\angle Z$ (when one side of $\angle A$

is on top of one side of $\angle Z$, the other side of $\angle A$ is in the interior of $\angle Z$)

- "$\angle A$ is greater than $\angle Z$" means, when the vertices are put on top of each other, $\angle A$ can extend <u>outside</u> $\angle Z$ (when one side of $\angle A$ is on top of one side of $\angle Z$, the other side of $\angle A$ is outside of $\angle Z$)
- acute angle: angle that is less than a right angle
- obtuse angle: angle that is greater than a right angle (and less than a straight angle)

3. CD Problem 3; Problem Set 3
   - Definition of straight angle: angle that makes a line
   - Use definitions to draw and label right angle, acute angle, obtuse angle, straight angle

Lesson 3: Identify, define, and draw perpendicular lines.

- Suggestion: Use "Physiometry" fluency activity (pp. 4.A.33-34; and future lessons) to practice Lesson 2 concepts

1. CD Problems 1-4; Problem Set 3-4
   - Definitions: perpendicular, intersecting
     - perpendicular (lines, line segments, rays): form right angle(s)
       - Symbol: $\perp$
       - Ex: $\overline{AB} \perp \overline{CD}$, "segment AB is perpendicular to segment CD"
     - intersecting (lines, line segments, rays): meet at a point
   - Use definition to identify and draw perpendicular lines, line segments, rays

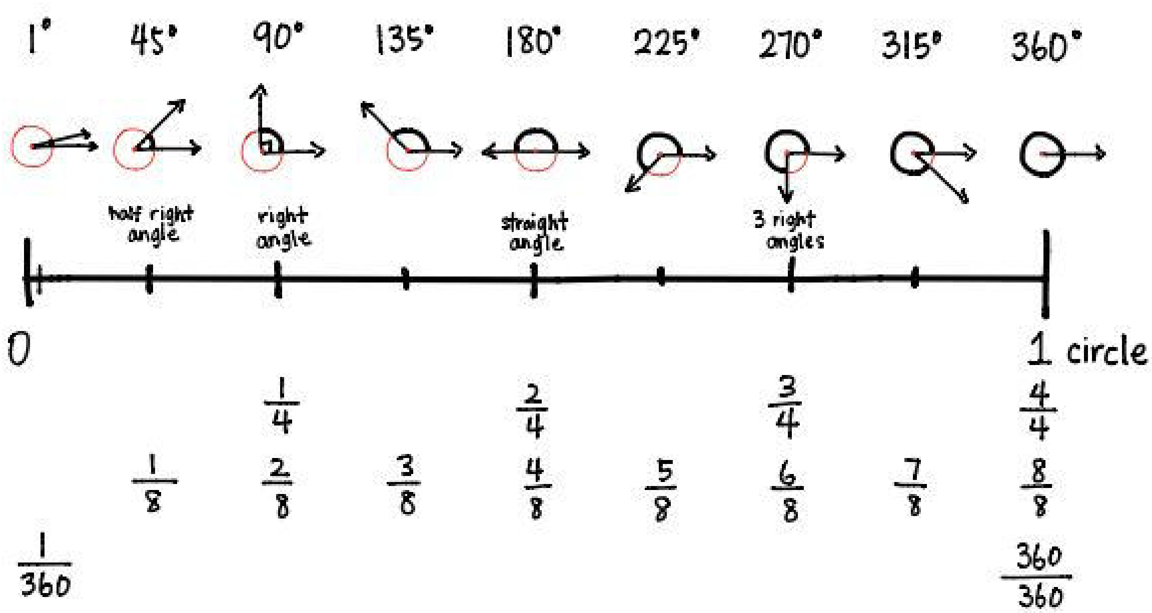
Lesson 4: Identify, define, and draw parallel lines.

1. CD Problems 1-4; Problem Set 3-4
   - Definition of parallel (lines, line segments, rays): never intersect, even if the line segment(s) or ray(s) were extended into lines
     - Symbol: || (or //)
     - Ex: $\overline{AB} || \overline{CD}$ (or $\overline{AB}$ // $\overline{CD}$)
   - Use straightedge and "right angle template" to identify and draw parallel lines (see pp. 4.A.50 & 4.A.51)
     - Reasoning (for teacher): if corresponding angles are equal, then lines/line segments/rays are parallel

B. Angle Measurement

Lesson 5: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of a turn. Explore benchmark angles using the protractor.

1. CD Problems 1-3; Problem Set 1a, 5-6
   - Visualize angles with paper protractor
     - Sample template for paper protractor
   - Measure angles as fractions of the circular arc
     - Definitions: angle measure, degree
       - An angle is measured with reference to a circle with its center at the vertex of the angle, by considering t*he fraction of the circular arc between the points where the two sides of the angle intersect the circle*. (4.MD.5a)
       - "one-degree angle," 1°: angle that turns $\frac{1}{360}$ of a circle
         - 360°: angle that turns full circle
       - angle measure of *n* degrees (*n*°): angle that turns through *n* one-degree angles (4.MD.5b)
     - Right angle = 90°
       - Reasoning: Let *r* = degree measure of right angle. Because 4 right angles make a "full turn," then 4 x *r* = 360°. By definition of division, *r* = 360° ÷ 4 = 90°.
     - Half of right angle = 45°
       - Reasoning: Let *h* = degree measure of half right angle. Because right angle has 2 halves, then 2 x 4 = 8 half right angles make a "full turn" and 8 x *h* = 360°. By definition of division, *h* = 360° ÷ 8 = 45°.
     - Sample template for labeled protractor

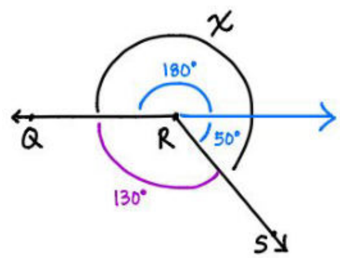


Lesson 6: Use varied protractors to distinguish angle measure from length measurement.

1. CD Problem 2; Problem Set 1
   - Suggestion: omit discussion of arc length (not relevant to standards, Problem Set, Homework, or Assessments)
   - Use protractor to measure angle in degrees
     - Align “center” point/notch of protractor with vertex of angle
     - Align zero/base line of protractor with side of angle so that the angle “opens” counter-clockwise

Lesson 7: Measure and draw angles. Sketch given angle measure and verify with a protractor.

1. CD Problem 1 (Practice Sheet Figures 2-3)
   - Use protractor to measure angle less than 180°
2. CD Problems 2-3 (Practice Sheet Figure 4)
   - Key idea: “When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measure of the parts” (4.MD.7)
     - $x + 130° = 360°$ OR $x = 180° + 50°$



   - Note: can skip these problems because not relevant for Lesson 7 Problem Set or Homework and will be covered in Lessons 9-11
3. CD Problem 4; Problem Set 1-2, 6
   - Use protractor to draw angle less than 180°

Lesson 8: Identify and measure angles as turns and recognize them in various contexts.

1. CD Problem 1; Problem Set 2
   - Identify angle measure formed by hour or minute hand’s movement (e.g., from 12:00 to 3:00 the angle measure is 90°)
   - Use Template on p. 4.B.59 of Teacher Edition
2. CD Problem 2
   - Identify angle measure formed by turning in relation to room (e.g., a complete turn is 360°)
3. CD Problem 3; Problem Set 5

- Identify movement defined by angle measure (e.g., “a skateboarder does a 180” means “a skateboarder spins around to face the other way”)

C. Problem Solving with the Addition of Angle Measures

* Key idea: “When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measure of the parts” (4.MD.7)

Lesson 9: Decompose angles using pattern blocks.

1. CD Problems 1-2 (Problem Set 1)
    - Measure the interior angle of a pattern block by forming 360° around a central point
        - Ex (1c): Three hexagons form 360° around a central point, which means 3 x (interior angle measure) = 360°; by definition of division, 360° ÷ 3 = interior angle measure
    - Note: for 1c-e, interior angle can also be measured by decomposing with 1-2 triangles; for 1f, interior angle can be measured by using two blocks to compose interior angle of a triangle
    - Use protractor to verify interior angle measure
2. CD Problem 3 (Problem Set 2); Problem Set 3
    - Decompose angle with pattern blocks
    - Use protractor to verify angle measure

Lessons 10-11: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.

1. Lesson 10 CD Problems 1-2
    - Use protractor to measure angles formed by folded paper and verify that the angle measure of the whole is the sum of the angle measure of the parts
2. Lesson 10 CD Problem 3; Problem Set 2, 4-5
    - Find an unknown angle measure that is part of a right or straight angle by writing and solving a sum (and/or subtraction) equation
3. Lesson 11 Application Problem & CD Problem 1; Problem Set 3
    - Decompose a 360° angle into smaller angles (using different pattern blocks)
    - Find an unknown angle measure that is part of a 360° angle by writing and solving a sum (and/or subtraction) equation
4. Lesson 11 CD Problem 2; Problem Set 6
    - Find the measurement of three angles given two intersecting lines and the measurement of one angle by focusing on one straight angle at a time
    - Definitions: adjacent angles, vertical angles
        - adjacent angles: angles that share same vertex and one side (“neighboring” or “touching” angles)

- ■ Ex: x and y are adjacent angles
- ○ vertical angles: the two non-adjacent (opposite) angles formed by two intersecting lines
  - ■ Ex: x and z are vertical angles
- Observe that vertical angles have same (equal) measure

5. Lesson 11 CD Problem 3 (slide)
   - Solve word problem involving unknown angle measure(s)
   - Note: part of 4.MD.7 standard ("Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems") though not in Problem Set or Homework
6. Lesson 11 CD Problem 4; Problem Set 7
   - Find the measurements of angles with common vertex

D. Two-Dimensional Figures and Symmetry

Lesson 12: Recognize lines of symmetry for given two-dimensional figures. Identify line-symmetric figures and draw lines of symmetry.

1. Application Problem & CD Problem 1
   - Explore symmetry in figures by folding (sides match perfectly when folded)

Shape	Number of Lines of Symmetry
Pentagon (Template 1)	1
Rectangle	2
Square	4
Parallelogram	0
Rhombus	2
(Isosceles) Trapezoid	1
Circle	Infinite

   - Use Template on p. 4.D.16 of Teacher Edition; doc with other shapes
2. CD Problem 2; Problem Set 1-2
   - Visualize mentally if line of symmetry (folding line to make sides match) exists in figure, then draw line of symmetry
   - Observe that matching points are same distance (equidistant) from line of symmetry (important observation for CD Problem 3)
   - Use Template on p. 4.D.17 of Teacher Edition
3. CD Problem 3; Problem Set 3b-c
   - Suggestion: scaffold by using wax paper to trace and cut out given half of figure, then flip across line of symmetry and trace to draw other half

Lesson 13: Analyze and classify triangles based on side length, angle measure, or both.

- Use Template on pp. 4.D.31-33 of Teacher Edition and Lesson 13 Practice Sheet in Problem Set student booklet

1. CD Problems 1-2 (Practice Sheet); Problem Set 1
    - Classify triangles based on side lengths and angle measures
    - Use dash mark(s) to indicate side lengths of equal length
    - Definitions:

Classification by *side lengths*	Number of sides of *equal* length
equilateral triangle	all (3)
isosceles triangle	at least 2
scalene triangle	none (0) [all 3 of different length]

Classification by *angle measures*	Types of angles
obtuse triangle	1 obtuse (> 90°) angle
right triangle	1 right (90°) angle
acute triangle	3 acute (< 90°) angles
equiangular triangle*	3 equal (60°) angles

* Not in Eureka Math

2. CD Problem 3; Problem Set 2
    - Observe that:
        - When folded on a line of symmetry, the sides and angles that match up are of equal length/measure
        - The two acute angles of a right triangle form a right angle (add up to 90°)
3. CD Problem 4; Problem Set 4-5
    - Observe that a triangle cannot be formed with three collinear points (all three points on the same line)

Lesson 14: Define and construct triangles from given criteria. Explore symmetry in triangles.

1. CD Problems 1-2; Problem Set 1a-b
    - Construct specific triangles (obtuse isosceles triangle, right scalene triangle) with protractor and ruler
2. CD Problem 3; Problem Set 3-5

- Reason about whether certain combinations of triangle classifications can exist

Combination	Possible?	Reason
scalene & equilateral	no	scalene has all sides unequal and equilateral has all sides equal
equilateral & obtuse	no	equilateral will mean equiangular also, so all angles equal and acute, but obtuse has one obtuse angle (and only two acute)
equilateral & right	no	equilateral will mean equiangular also, so all angles equal and acute, but right has one right angle (and only two acute)
scalene & acute	yes	example: 40°-60°-80° triangle
isosceles & equilateral	yes	equilateral has all sides equal, so at least 2 sides are equal

Lesson 15: Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size.

1. CD Problems 1-4 (Problem Set 1-4); Problem Set 5
    - Construct trapezoids, parallelograms, rectangles, and squares based on definitions

Quadrilateral	Definition	Includes...
trapezoid	at least one pair of parallel sides	parallelogram, rhombus, rectangle, square
parallelogram	two pairs of parallel sides	rhombus, rectangle, square
rectangle	four right angles (two pairs of perpendicular sides)	square
square	four right angles and four equal sides (i.e., rectangle & rhombus)	

Lesson 16: Reason about attributes to construct quadrilaterals on square or triangular grid paper.

- Suggestion: omit if pacing is a challenge because content is not directly relevant to standards or Assessments

1. CD Problems 1-2; Problem Set 2b
    - Construct rhombus and rectangle on triangular grid
2. CD Problem 3; Problem Set 1a, c
    - Construct non-rectangular parallelogram on rectangular (square) grid