Main ideas:

- Multiplication (Lessons 1-13, 34-38)
  - Definition of multiplication: multiplication is shorthand for addition of the same number
    - "9 x 4" *means* "4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4"
  - Place value chart/disks or area model → partial products → standard algorithm
    - Key Lessons: Lessons 5, 7, 36
- Division (Lessons 14-33)
  - Definition of division: division is shorthand for multiplication with unknown factor
    - Partitive interpretation: *define* as meaning of division
      - "36 ÷ 9 = N" *means* "9 x N = 36"
      - "Partitive" interpretation: we know how many "parts"
    - Measurement interpretation: *prove* as additional meaning of division
      - If "N x 9 = 36" then "9 x N = 36" because of the commutative property of multiplication
      - "9 x N = 36" means "36 ÷ 9 = N" (definition of division)
      - Conclusion: "36 ÷ 9 = N" *also means* "N x 9 = 36"
      - "Measurement" interpretation: we know how much to "measure"
  - Definition of division with remainder:
    - Partitive interpretation: $a \div b$ is the number Q (quotient) so that $b \times Q + R = a$ where $0 \leq R < b$ (remainder); in other words, when we form b groups from a (partitive interpretation of division), then Q is the most number of items in each group and R is the leftover that is not enough to distribute evenly into b groups
      - Lessons 16-17, 26-30
    - Measurement interpretation: $a \div b$ is the number Q (quotient) so that $Q \times b + R = a$ where $0 \leq R < b$ (remainder); in other words, when we form groups of b from a (measurement interpretation of division), then Q is the most number of groups we can make and R is the leftover that is not enough to form a group of b
      - Lessons 14-15
    - Two interpretations because of the commutative property of multiplication
  - Key Lessons: Lessons 14-15, 17
- Area of rectangle → area model of multiplication → distributive property and area model of division
  - Key Lessons: Lessons 1, 6, 11, 15, 20, 35-36
- Properties of operations (associative and commutative properties of addition and multiplication; distributive property)

Materials: grid paper (Lesson 1)

A. Multiplicative Comparison Word Problems

Lesson 1: Investigate and use the formulas for area and perimeter of rectangles.

* Great Minds' Suggestions for Consolidation or Omissions: "In Lesson 1, omit Problems 1 and 4 of the Concept Development. Problem 1 could have been embedded into Module 2. Problem 4 can be used for a center activity."

Suggestions:

- Use CD Problem 3 in Lesson 14
- Use CD Problem 4 in Lesson 22

Definitions (prior knowledge):

- Perimeter of a figure: total length (distance) around the figure
- Unit square: square with length 1 on each side
- Area of a figure: number of unit squares that fit inside the figure (without overlap or extra space between)

1. Concept Development (CD) Problems 1-3; Problem Set 1-2
   4 units wide, 7 units long rectangle on grid paper
   - Perimeter of rectangle
     = **4 + 7 + 4 + 7** ← definition of perimeter (trace around)
     = (4 + 4) + (7 + 7) ← properties of addition
     = **(2 x 4) + (2 x 7)** ← definition of multiplication
     OR
     = 4 + 7 + 4 + 7
     = (4 + 7) + (4 + 7) ← associative property of addition
     = **2 x (4 + 7)** ← definition of multiplication
   - Area of rectangle
     = 4 + 4 + 4 + 4 + 4 + 4 + 4 ← definition of area (count by rows)
     = **7 x 4** ← definition of multiplication
2. CD Problem 2; Problem Set 4a
   - Given perimeter and one side length → find unknown side length

Lesson 2: Solve multiplicative comparison word problems by applying the area and perimeter formulas.

Definitions: as long/wide as

- "A rectangle is 3 times as long as it is wide" *means* "a rectangle's length (how long it is) is 3 times its width (how wide it is)" or "length = 3 x width"
- "The second rectangle is 3 times as wide (as the first rectangle)" *means* "the second rectangle's width is 3 times the first rectangle's width" or "second rectangle's width = 3 x first rectangle's width"

1. CD Problem 2; Problem Set 2
   - Definition of as long/wide as
2. CD Problem 3; Problem Set 3

- CD Problem 2 & Lesson 1 → solve word problems

(Extension/Optional: CD Problem 4; Problem Set 4)

Lesson 3: Demonstrate understanding of area and perimeter formulas by solving multi-step real world problems.

1. CD Problems 1-4 (Problem Set 1-4)
   - Lessons 1-2 → solve multi-step word problems

B. Multiplication by 10, 100, and 1,000

Lesson 4: Interpret and represent patterns when multiplying by 10, 100, and 1,000 in arrays and numerically.

1. Prior knowledge: Module 1 Lessons 1-2
2. Application Problem
   - 3 x 4 = 12 & 30 x 4 = 120 → previews CD Problem 2
3. CD Problem 1; Problem Set 2, 3g-i
   - Module 1 Lesson 1 & properties of multiplication →
     3,000 = 10 x 300 = 10 x (10 x 10 x 3) = 3 x 10 x 10 x 10 = 3 x 1,000
     OR
     Place value (counting), definitions of addition (count on), multiplication →
     3,000 = 1,000 + 1,000 + 1,000 = 3 x 1,000
4. CD Problem 2; Problem Set 5
   - CD Problem 1 & distributive property →
     15 x 100 = (10 + 5) x 100 = (10 x 100) + (5 x 100) = 1,000 + 500 = 1,500
5. CD Problem 3; Problem Set 8
   - Properties of multiplication & CD Problem 2 →
     6 x 400 = 6 x (4 x 100) = (6 x 4) x 100 = 24 x 100 = 2,400

Lesson 5: Multiply multiples of 10, 100, and 1,000 by single digits, recognizing patterns.

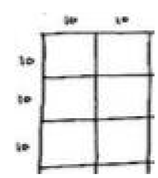
1. CD Problem 1; Problem Set 3, 5 a & k
   - Lesson 4 & properties of multiplication →
     "2 hundreds x 4"
     = 4 x 200 ← commutative property of multiplication
     = 4 x (2 x 100) ← 4 groups of 2 "100" disks
     = (**4 x 2**) x 100
     = 8 x 100
     = 800
   - Common theme: 4 x 2
2. CD Problem 2
   - Properties of multiplication → 8 x 200 = 800 x 2

8 x 200
= 8 x (2 x 100)
= (8 x 100) x 2 ← commutative & associative properties of multiplication
= 800 x 2

3. CD Problems 3-4; Problem Set 6-7
    - CD Problems 1-2 → solve word problems

Lesson 6: Multiply two-digit multiples of 10 by two-digit multiples of 10 with the area model.

1. CD Problem 1; Problem Set 1
    - Using place value chart
    - Lesson 5, properties of multiplication →
      30 x 20
      = (3 x 10) x 20
      = 3 x (10 x 20)
      = 3 x 200
      = 600
2. CD Problem 2; Problem Set 2
    - Using area model
    - Suggestion: use same numbers as in CD Problem 1
    - Formula for area of rectangle, area is additive (3.MD.7d) →
      30 x 20
      = area of rectangle with side lengths 3 x 10 and 2 x 10
      = sum of areas of smaller rectangles
      = (number of small rectangles) x (area of one small rectangle)
      = (3 x 2) x (10 x 10)
      = 6 x 100
      = 600



    - Also note (3 x 10) x (2 x 10) = (3 x 2) x (10 x 10) because of properties of multiplication

C. Multiplication of up to Four Digits by Single-Digit Numbers

Lesson 7: Use place value disks to represent two-digit by one-digit multiplication.

* Show vertical multiplication and partial products alongside place value chart/disks

1. CD Problem 1; Problem Set 1b
    - Using place value chart/disks
      2 x 43
      = 43 + 43 ← definition of multiplication
      = (4 x 10) + 3 + (4 x 10) + 3
      = [(4 x 10) + (4 x 10)] + (3 + 3) ← properties of addition

= 2 x (4 x 10) + (2 x 3) ← definition of multiplication
= (2 x 4) x 10 + (2 x 3)
= 80 + 6
= 86

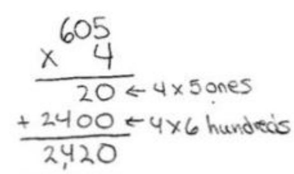
2. CD Problem 2; Problem Set 2b
    - Using place value chart/disks
      5 x 42
      = 42 + 42 + 42 + 42 + 42 ← definition of multiplication
      = 5 x (4 x 10) + (5 x 2) ← properties of addition
      = (2 x 10) x 10 + 10
      = 2 x (10 x 10) + 10 ← associative property of multiplication
      = (2 x 100) + 10
      = 210

Lesson 8: Extend the use of place value disks to represent three- and four-digit by one-digit multiplication.

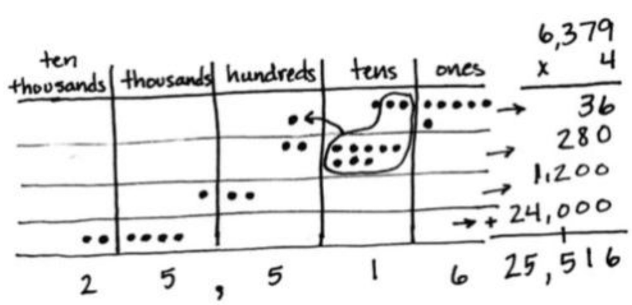
* Great Minds' Suggestions for Consolidation or Omissions: "In Lesson 8, omit the drawing of models in Problems 2 and 4 of the Concept Development and in Problem 2 of the Problem Set. Instead, have students think about and visualize what they would draw."

* Show vertical multiplication and partial products alongside place value chart/disks

1. CD Problem 2; Problem Set 1c
    - Using place value chart/disks
    - Lesson 7 → (one-digit number) x (three-digit number)
    - Note order of partial sums in vertical multiplication:
      4 x 605
      = (4 x 5) + (4 x 600)



2. CD Problem 4; Problem Set 2b
    - CD Problem 2 → (one-digit number) x (four-digit number)
    - Transition to using place value chart/disks to only represent partial products and regrouping/composing
      4 x 6,379
      = 4 x (9 + 70 + 300 + 6000)
      = (4 x 9) + (4 x 70) + (4 x 300) + (4 x 6000)
      = etc.



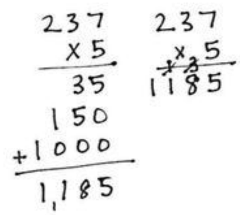
* **Recommendation**: Insert Lesson 11 here to provide alternative approach and explicitly represent the distributive property (3.MD.7c)

Lessons 9-10: Multiply three- and four-digit numbers by one-digit numbers applying the standard algorithm.

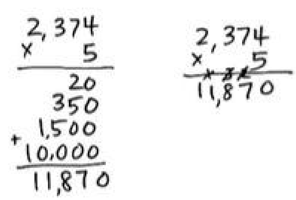
* Great Minds' Suggestions for Consolidation or Omissions: "Omit Lesson 10 because the objective for Lesson 10 is the same as that for Lesson 9."

* Relate partial products approach (from place value chart/disks or area model) to standard algorithm

1. Lesson 9 CD Problem 1 or 2; Problem Set 1b



2. Lesson 10 CD Problem Set 1f



3. Lesson 9 CD Problem 3, Lesson 10 CD Problem 3; Lesson 10 Problem Set 3
    - Lessons 9-10 CD Problems 1-2 → solve word problems

Lesson 11: Connect the area model and partial products method to the standard algorithm.

* **Recommendation**: Move up and insert between Lessons 8 and 9

1. Application Problem: Previews CD
2. CD Problem 1 or 2; Problem Set 1b-c
    - Area model for multiplication (Lesson 6), distributive property →
      8 x 234
      = 8 x (200 + 30 + 4) ← expanded form of number
      = area of rectangle with side lengths 8 and (200 + 30 + 4)
      = sum of areas of smaller rectangles
      = (8 x 200) + (8 x 30) + (8 x 4) ← distributive property
      = (8 x 2) x 100 + (8 x 3) x 10 + (8 x 4) ← Lessons 5-6
      = 1,600 + 240 + 32
      = 1,872
3. CD Problem 3; Problem Set 2
    - CD Problems 1-2 → solve word problems

D. Multiplication Word Problems

Lesson 12: Solve two-step word problems, including multiplicative comparison.

1. CD Problems 2-4 (Problem Set 2-4)
   - Prior lessons → solve two-step word problems

Lesson 13: Use multiplication, addition, or subtraction to solve multi-step word problems.

1. CD Problems 1-4 (Problem Set 1-4)
   - Module 1 & Module 3 Lessons → solve multi-step word problems involving multiplication and addition and/or subtraction

E. Division of Tens and Ones with Successive Remainders

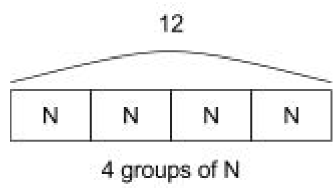
* **Suggestion**: Move Lessons 20 and 21 up and insert between Lessons 17 and 18 to provide alternative approach/explanation for the division algorithm

Lesson 14: Solve division word problems with remainders.

* **Suggestion**: split into two lessons

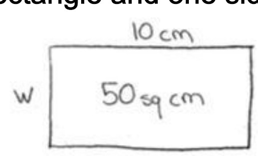
- Lesson 14 Part 1: (1a & b) & (2), no Homework problems in Lesson 14 align with these concepts (although Lesson 1 Homework problem 4 aligns with Lesson 1 CD Problem 3) but Lesson 16+ will use the partitive interpretation of division with remainder
- Lesson 14 Part 2: (1c) & (3), Lesson 15 will use the measurement interpretation of division with remainder

1. Prior knowledge: division
    a. CD Problem 1
    - Given total and number of groups → find number in each group
      "There are 12 students in PE class separated into 4 teams. How many students are on each team?"
        - Let N = number of students on each team



        - 4 x N = 12
    - Definition of division (partitive interpretation): "12 ÷ 4 = N" *means* "4 x N = 12"
        - "Partitive" interpretation: we know how many "parts"
        - 4 x **3** = 12 so 12 ÷ 4 = 3

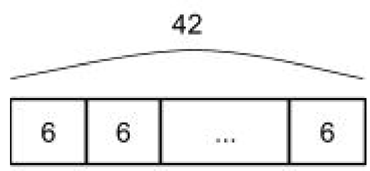
    b. **Lesson 1 CD Problem 3; Lesson 1 Problem Set 5a**
    - **Given area of rectangle and one side length → find unknown side length**



      10 x w = 50 ← length x width = area of rectangle
        - 50 ÷ 10 = w ← partitive interpretation of division

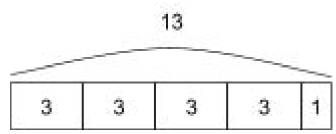
    c. CD Problem 3 modified
    - Given total and number in each group → find number of groups
      "Kristy bought 42 roses. If she puts 6 roses in each vase, how many vases will she use?"
        - Let M = number of vases
        - M x 6 = 42

- 6 x M = 42 ← commutative property of multiplication
- 42 ÷ 6 = M ← definition of division (partitive interpretation)
- "42 ÷ 6 = M" *also means* "M x 6 = 42" (measurement interpretation)
  - "Measurement" interpretation: we know how much to "measure"
- Number line or tape diagram representation:
  M x 6 = 42 *means* 6 + 6 + … + 6 (M times) = 42



M groups of 6

2. CD Problem 2; Problem Set 2
   - Given total and number of groups → find number in each group
     "One more student joined the class described at the beginning of Problem 1. There are now 13 students to be divided into 4 teams."



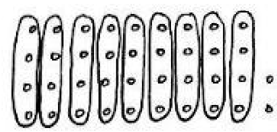
   - 4 x **3** + 1 = 13, which means 4 teams of 3 students and 1 student leftover
   - Definition of division with remainder (partitive interpretation):
     $a \div b$ is the number Q (quotient) so that $b \times Q + R = a$ where $0 \leq R < b$ (remainder); in other words, when we form b groups from a (partitive interpretation of division), then Q is the most number of items in each group and R is the leftover that is not enough to distribute evenly into b groups
3. CD Problem 3; Problem Set 4
   - Given total and number in each group → find number of groups
     "Kristy bought 13 roses. If she puts 6 roses in each vase, how many vases will she use? Will there be any roses left over?"

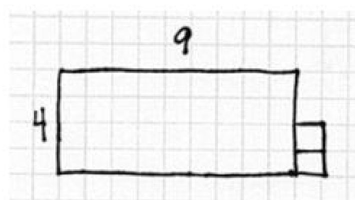
     2 x 6 + 1 = 13, which means 2 vases with 6 roses each and 1 rose leftover
   - Division with remainder (measurement interpretation):
     $a \div b$ is the number Q (quotient) so that $Q \times b + R = a$ where $0 \leq R < b$ (remainder); in other words, when we form groups of b from a (measurement interpretation of division), then Q is the most number of

groups we can make and R is the leftover that is not enough to form a group of b

Lesson 15: Understand and solve division problems with a remainder using the array and area models.

1. CD Problems 1-2; Problem Set 3
    - Use array and area model to represent division with remainder (measurement or partitive interpretation)
      38 ÷ 4
        - 38 = (Q x 4) + R
          Area of figure = Area of rectangle with width 4 + Area of leftover
        - Find the largest rectangle possible with area < 38 and width 4 → length of rectangle = Q, number of leftover unit squares = R

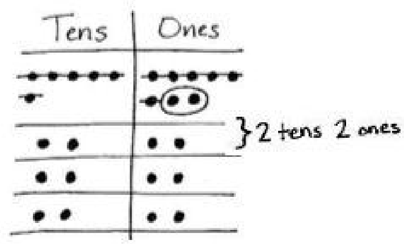


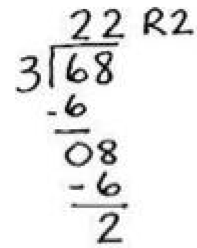


- 38 = (**9** x 4) + **2**, so Q = 9, R = 2

Lesson 16: Understand and solve two-digit dividend division problems with a remainder in the ones place by using place value disks.

1. CD Problem 3; Problem Set 4
    - Use place value chart/disks to represent division with remainder (partitive interpretation)
    - Show division "shorthand" (division algorithm) alongside place value chart/disks and equations
      68 ÷ 3
        - (3 x Q) + R = 68
        - Form 68 into 3 equal groups with as many disks in each group as possible → number of items in each group = Q, leftover disks = R
        - 68
          = 60 + 8
          = (3 x 20) + (3 x 2) + 2 ← form 60 into 3 groups, 8 into 3 groups
          = 3 x (20 + 2) + 2 ← distributive property
          = (3 x **22**) + **2**

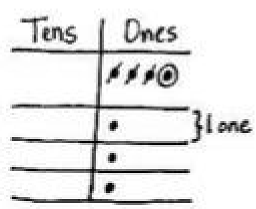


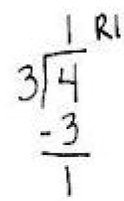


Lesson 17: Represent and solve division problems requiring decomposing a remainder in the tens.

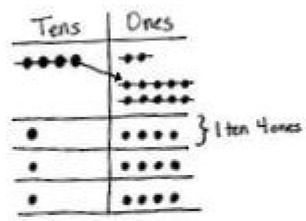
1. CD Problem 2; Problem Set 5-6
   - Use place value chart/disks to represent division with remainder (partitive interpretation)
   - Show division "shorthand" (division algorithm) alongside place value chart/disks and equations

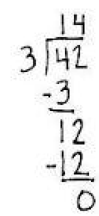
     4 ÷ 3
     - (3 x Q) + R = 4
     - Form 4 into 3 equal groups with as many disks in each group as possible → number of items in each group = Q, leftover disks = R
     - 4 = (3 x 1) + 1





     42 ÷ 3
     - (3 x Q) + R = 42
     - Form 42 into 3 equal groups with as many disks in each group as possible → number of items in each group = Q, leftover disks = R
     - 42
       = 40 + 2
       = (**4** x 10) + 2
       = (**3 x 1 + 1**) x 10 + 2 ← form 4 tens into 3 groups
       = (3 x 1) x 10 + **10** + 2 ← distributive prop (decomposing)
       = (3 x 1) x 10 + 12
       = (3 x 10) + (3 x 4) ← form 12 into 3 groups
       = 3 x (10 + 4) ← distributive property
       = (3 x **14**) + **0**





- Common theme: 4 = (3 x 1) + 1

* **Suggestion**: Insert Lessons 20 and 21 here to provide alternative approach/ explanation for the division algorithm

Lesson 18: Find whole number quotients and remainders.

- Transition to mental model of place value chart/disks
- Review/reinforcement of Lessons 16-17; build fluency with division with remainder

Lesson 19: Explain remainders by using place value understanding and models.

* Great Minds' Suggestions for Consolidation or Omissions: "Omit Lesson 19, and instead, embed discussions of interpreting remainders into other division lessons."

* Suggestion: follow Great Minds' omission suggestion

- Discussion of interpreting remainders naturally arises when using/applying the definition of division with remainder

Lesson 20: Solve division problems without remainders using the area model.

* **Suggestion**: Move up and insert between Lessons 17 and 18

* Show division "shorthand" (division algorithm) alongside area model and equations

1. CD Problem 1
    - Use area model to represent division
      "a rectangle with an area of 48 square units and a width of 4 units"
        - 48 = 4 x Q
        - 48
          = 40 + 8 ← expanded form of number
          = (4 x 10) + (4 x 2) ← form 40 into 4 groups, 8 into 4 groups
          = 4 x (10 + 2) ← distributive property
          = 4 x **12**
2. CD Problem 3; Problem Set 4
    - Use area model to represent division with remainder in tens place
      "The expression 96 ÷ 4 can describe a rectangle with an area of 96 square units. We are trying to find out the length of the unknown side."
        - 96 = 4 x Q

- 96
  = 90 + 6 ← expanded form of number
  = (4 x 20 + 10) + 6 ← form 90 into 4 groups
  = (4 x 20) + 16 ← associative property of addition (decomposing)
  = (**4** x 20) + (**4** x 4) ← form 16 into 4 groups
  = **4** x (20 + 4) ← distributive property
  = 4 x **24**

Lesson 21: Solve division problems with remainders using the area model.

* **Suggestion**: Move up and insert between Lessons 17 and 18
* Show division "shorthand" (division algorithm) alongside area model and equations

1. CD Problem 2; Problem Set 1
   - Lessons 15 & 20 → use area model to represent division with remainder
     76 ÷ 3
     - 76 = (Q x 3) + R
       Area of figure = Area of rectangle with width 3 + Area of leftover
     - Find the largest rectangle possible with area < 76 and width 3 → length of rectangle = Q, number of leftover unit squares = R
     - 76
       = 70 + 6 ← expanded form of number
       = (3 x 20 + 10) + 6 ← form 70 into 3 groups
       = (3 x 20) + 16 ← associative property of addition (decomposing)
       = (**3** x 20) + (**3** x 5 + 1) ← form 16 into 3 groups
       = **3** x (20 + 5) + 1 ← distributive property
       = (3 x **25**) + **1**
     - 76 = (3 x **25**) + **1**, so Q = 25, R = 1

F. Reasoning with Divisibility

* CA Mathematics Framework: "Additional/Supporting Cluster: Gain familiarity with factors and multiples.[1] (4.OA.4) [1]Supports students' work with multi-digit arithmetic as well as their work with fraction equivalence."

- Suggestion: Use Lessons 22-24 to build fluency with multiplication and division.

Lesson 22: Find factor pairs for numbers to 100, and use understanding of factors to define prime and composite.

1. Prior knowledge
   a. CD Problem 1: identify factors and product (Grade 3 Module 1)
2. Lesson 1 CD Problem 4; Lesson 1 Problem Set 6
   - Given the area of a rectangle → find all possible whole number combinations of length and width
     "If a rectangle has an area of 24 square units, what whole numbers could be the length and width of the rectangle?"

        - 1 & 24, 2 & 12, 3 & 8, 4 & 6
        - How to know if all factor pairs listed (see Lesson 22 CD Problems 1-2 for suggestion)
3. CD Problem 2; Problem Set 2
    - Definitions:
        - prime number: a number that has exactly two factors, 1 and the number itself
            - Ex: 3, 5, 7
        - composite number: a number that has at least one factor other than 1 and the number itself
            - Ex: 4, 6, 8, 9

Lesson 23: Use division and the associative property to test for factors and observe patterns.

1. CD Problem 1; Problem Set 1 f & h
    - How to test if a number is a factor
      "How can I find out if 3 is a factor of 54?"
        - Key idea: 3 is a factor of 54 if there is a number N so that 3 x N = 54, which means 3 x N + **0** = 54 (**division with remainder 0**)
        - 3 x **18** + **0** = 54, so 3 is a factor of 54
2. CD Problem 2 or 3; Problem Set 2
    - Use associative property of multiplication to find more factors and factor pairs
      54
      = **6 x 9**
      = (2 x 3) x 9 OR 6 x (3 x 3)
      = 2 x (3 x 9) OR (6 x 3) x 3 ← associative property of multiplication
      = **2 x 27** OR **18 x 3**

Lesson 24: Determine if a whole number is a multiple of another number.

1. CD Problem 1 or 2; Problem Set 2, 3b
    - Definition of multiple: a "multiple of 4" is "N x 4" where N is any whole number
        - Ex: 8 is a multiple of 4 because 8 = **2** x 4
    - How to test if a number is a multiple of x
      "How can we find out if 96 is a multiple of 3?"
        - Key idea: 96 is a multiple of 3 if there is a number N so that N x 3 = 96, which means N x 3 + **0** = 96 (**division with remainder 0**)
            - **"96 is a multiple of 3" ↔ "3 is a factor of 96"**
        - **32** x 3 + **0** = 96, so 96 is a multiple of 3

Lesson 25: Explore properties of prime and composite numbers to 100 by using multiples.

1. CD (Problem Set)
    - 4.OA.4 “a whole number is a multiple of a each of its factors”
        - Ex: 22 is a multiple of 2 (22 = 11 x 2) and a multiple of 11 (22 = 2 x 11)
    - Lesson 24 CD Problem 3 key idea: use the associative property of multiplication to show any multiple of 4 is also a multiple of 2
        - Multiple of 4 = N x 4 = N x (2 x 2) = (N x 2) x 2 = Multiple of 2
    - When the multiples of 2 to 10 (up to 100) have been crossed out, then all the composites have been crossed out

G. Division of Thousands, Hundreds, Tens, and Ones

Lesson 26: Divide multiples of 10, 100, and 1,000 by single-digit numbers.

- Use place value chart/disks to represent division (partitive interpretation)

1. CD Problem 2; Problem Set 3 h & i
   350 ÷ 5
    - 350 = 5 x Q
    - 350
      = **35** x 10
      = (**5 x 7**) x 10 ← form 35 tens into 5 groups
      = 5 x (7 x 10) ← associative property of multiplication
    - = 5 x **<u>70</u>**
2. CD Problem 3; Problem Set 5
    - CD Problem 2 → solve word problems that involve division (partitive interpretation)

Lesson 27: Represent and solve division problems with up to a three-digit dividend numerically and with place value disks requiring decomposing a remainder in the hundreds place.

1. CD Problem 2; Problem Set 2a
    - Use place value chart/disks (and area model) to represent division with remainder (partitive interpretation)
    - Show division “shorthand” (division algorithm) alongside place value chart/disks and equations
    - Lessons 17 & 26 → divide three-digit dividend requiring decomposition
      783 ÷ 3
        - 783 = (3 x Q) + R
        - Form 783 into 3 equal groups with as many disks in each group as possible → number of items in each group = Q, leftover disks = R
        - 783

= 700 + 80 + 3
= (**7** x 100) + (8 x 10) + 3
= (**3 x 2 + 1**) x 100 + (8 x 10) + 3 ⟵ form 7 hundreds into 3 groups
= (3 x 2) x 100 + **100** + (8 x 10) + 3 ⟵ distributive property
= (3 x 2) x 100 + (**10** + 8) **x 10** + 3 ⟵ distributive property (decomposing)
= (3 x 2) x 100 + (**18** x 10) + 3
= (3 x 2) x 100 + (**3 x 6**) x 10 + 3 ⟵ form 18 tens into 3 groups
= (**3** x 2) x 100 + (**3** x 6) x 10 + (**3** x 1)
= 3 x (2 x 100 + 6 x 10 + 1)⟵ distributive property
= (3 x **261**) + **0**

Lesson 28: Represent and solve three-digit dividend division with divisors of 2, 3, 4, and 5 numerically.

1. CD Problem 1; Problem Set 1g
    - Use place value chart/disks (and area model) to represent division with remainder (partitive interpretation)
    - Show division “shorthand” (division algorithm) alongside place value chart/disks and equations
    - Lessons 27 ⟶ divide three-digit dividend with remainder
      297 ÷ 4
        - 297 = (4 x Q) + R
        - Form 297 into 4 equal groups with as many disks in each group as possible ⟶ number of items in each group = Q, leftover disks = R
        - 297
          = (**2** x 100) + (9 x 10) + 7 but 2 hundreds cannot form 4 groups
          = (**2 x 10**) **x 10** + (9 x 10) + 7 ⟵ decomposing
          = (20 + 9) x 10 + 7 ⟵ distributive property
          = (**29** x 10) + 7
          = (**4** x 7 + 1) x 10 + 7 ⟵ form 29 tens into 4 groups
          = (4 x 7) x 10 + 10 + 7 ⟵ distributive property (decomposing)
          = (**4** x 7) x 10 + **(4 x 4) + 1** ⟵ form 17 into 4 groups
          = **4** x (7 x 10 + 4) + 1 ⟵ distributive property
          = (4 x **74**) + **1**

Lesson 29: Represent numerically four-digit dividend division with divisors of 2, 3, 4, and 5, decomposing a remainder up to three times.

- Transition to mental model of place value chart/disks

1. CD Problem 1; Problem Set 1e
    - Lesson 28 ⟶ divide four-digit dividend with remainder
2. CD Problem 2; Exit Ticket Problem 2

- CD Problem 1 → solve word problems that involve division (partitive interpretation)

Lesson 30: Solve division problems with a zero in the dividend or with a zero in the quotient.

* Great Minds' Suggestions for Consolidation or Omissions: "Omit Lesson 33, and embed into Lesson 30 the discussion of the connection between division using the area model and division using the algorithm."

* Suggestion: follow Great Minds' consolidation suggestion

1. CD Problem 1; Problem Set 2
   - Use place value chart/disks and area model to represent division (partitive interpretation) and show what the zero in the dividend means
   - Show division "shorthand" (division algorithm) alongside place value chart/disks and equations
2. CD Problem 2; Problem Set 3
   - Use place value chart/disks (and area model) to represent division (partitive interpretation) and show what the zero in the quotient means
   - Show division "shorthand" (division algorithm) alongside place value chart/disks and equations

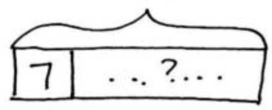
Lesson 31: Interpret division word problems as either *number of groups unknown* or *group size unknown*.

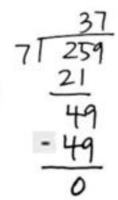
* Great Minds' Suggestions for Consolidation or Omissions: "Omit Lesson 31, and instead, embed analysis of division situations throughout later lessons."

* Suggestion: follow Great Minds' omission suggestion and also embed analysis of division situations in earlier lessons (e.g., Lessons 14, 26, 29)

Lesson 32: Interpret and find whole number quotients and remainders to solve one-step division word problems with larger divisors of 6, 7, 8, and 9.

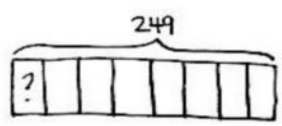
1. CD Problem 1; Problem Set
   - Solve word problems that involve division (measurement interpretation) [note: word problems in earlier lessons focus on the partitive interpretation]
     "How many weeks are in 259 days?"
     - Form 259 into groups of 7 → find number of groups
     - ? x 7 = 259

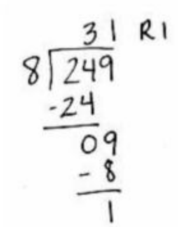


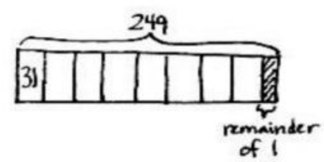


There are 37 weeks in 259 days.

2. CD Problem 2; Problem Set
    - Solve word problems that involve division with remainder (partitive interpretation)
    "Everyone is given the same number of colored pencils in art class. If there are 249 colored pencils and 8 students, how many pencils does each student receive?"
        - Form 249 into 8 equal groups with as many pencils in each group as possible → number of pencils in each group (student) = Q, leftover pencils = R
        - 249 = (8 x Q) + R

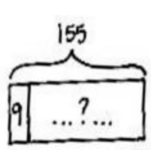


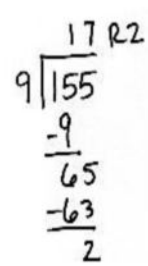


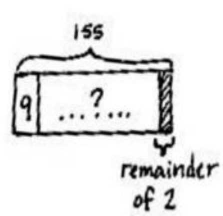


Each student receives 31 pencils. There is 1 pencil remaining.

3. CD Problem 3; Problem Set
    - Solve word problems that involve division with remainder (measurement interpretation)
    "Mr. Hughes has 155 meters of volleyball netting. How many nets can he make if each court requires 9 meters of netting?"
        - Form 155 into as many groups of 9 as possible → number of groups (nets) = Q, leftover netting = R
        - 155 = (Q x 9) + R







Mr. Hughes can make 17 nets.

Lesson 33: Explain the connection of the area model of division to the long division algorithm for three- and four-digit dividends.

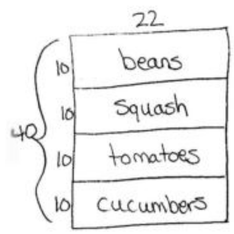
* Great Minds' Suggestions for Consolidation or Omissions: "Omit Lesson 33, and embed into Lesson 30 the discussion of the connection between division using the area model and division using the algorithm."

* Suggestion: follow Great Minds' omission suggestion

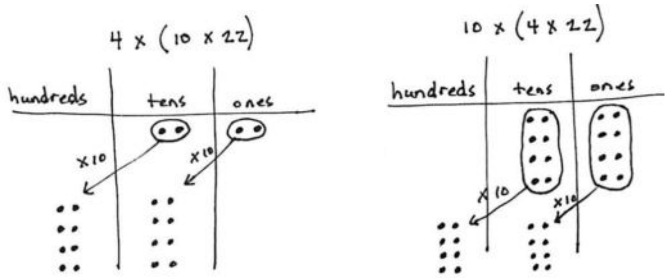
H. Multiplication of Two-Digit by Two-Digit Numbers

Lesson 34: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart.

1. Application Problem, CD Problems 1-2; Problem Set 2a
    - Definition of multiplication, area model, Lessons 4 & 7 (multiplication with place value chart/disks), properties of operations → multiplication of two-digit multiples of 10 and two-digit numbers



Total number of plants
= (10 + 10 + 10 + 10) x 22
= (4 x 10) x 22 (= 40 x 22)
= **4 x (10 x 22)**
= **10 x (4 x 22)** ← commutative property of multiplication

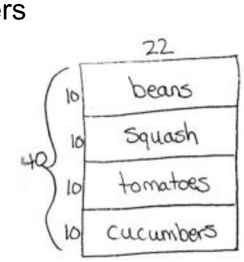


= 880

Lesson 35: Multiply two-digit multiples of 10 by two-digit numbers using the area model.

* Show vertical multiplication and partial products alongside area model

1. Lesson 34 Application Problem continued
    - Lessons 6 & 11 (area model of multiplication), expanded form of number, distributive property → multiplication of two-digit multiples of 10 and two-digit numbers

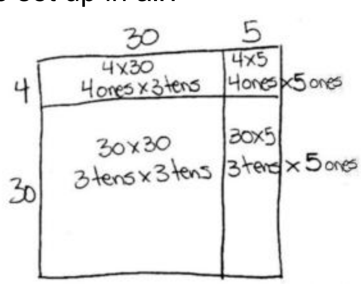


- Method 1 (based on picture)
  40 x 22
  = (10 + 10 + 10 + 10) x 22
  = (10 x 22) + (10 x 22) + (10 x 22) + (10 x 22) ← distributive property
  = 220 + 220 + 220 + 220
  = 880
- Method 2 (based on expanded form of number)
  40 x 22
  = 4 x 10 x (20 + 2)
  = 10 x (4 x (20 + 2)) ← commutative & associative property
  = 10 x ((4 x 20) + (4 x 2)) ← distributive property
  = 10 x (80 + 8)
  = 880
- Method 3 (based on expanded form of number)
  40 x 22
  = 40 x (20 + 2)
  = (40 x 20) + (40 x 2) ← distributive property
  = 800 + 80
  = 880

2. CD Problem 2 or 3; Problem Set 2
   - Method 3 above

Lesson 36: Multiply two-digit by two-digit numbers using four partial products.

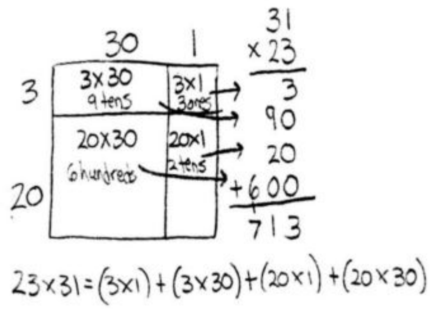
1. (Application Problem), CD Problem 1
   - Lesson 35, expanded form of number, distributive property → two-digit multiplication
     Mr. Goggins set up 30 rows of chairs in the gym. If each row had 35 chairs, how many chairs did Mr. Goggins set up? Mr. Goggins then set up an additional 4 rows of chairs with 35 chairs in each row. How many chairs did he set up in all?



34 x 35
= (4 + 30) x (30 + 5)
= 4 x (30 + 5) + 30 x (30 + 5) ← distributive property

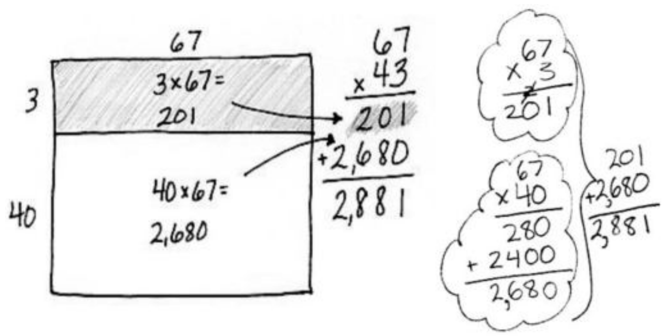
= (4 x 30) + (4 x 5) + (30 x 30) + (30 x 5) ← distributive property
= 120 + 20 + 900 + 150
= 1,190

2. CD Problem 2 or 3; Problem Set 2
    - List partial products in order of standard algorithm; show vertical multiplication and partial products alongside area model



Lessons 37-38: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.

1. Lesson 37 CD Problem 2; Problem Set 3
    - Transition to two partial product area model; show vertical multiplication and partial products alongside area model



2. Lesson 38 CD Problem 2 or 3; Problem Set 5
    - Multiplication with regrouping in partial product