A. Place Value of Multi-Digit Whole Numbers

*** Key idea**: Place value
In our number system (base-ten), we only have **ten** symbols (the digits 0 to 9) to represent **zero to nine** ones, tens, hundreds, etc. (a unit) so to represent **ten** ones, tens, hundreds, etc. we use the next "place" to the left and make it 1 of that larger unit (e.g., ten 1s = 10, ten 10s = 100, ten 100s = 1000, ten 1000s = 10000)

- Foundation for multi-digit arithmetic
- Lessons 1 & 2: multiplying and dividing by 10 to observe Lesson 2's objective

Lesson 1: Interpret a multiplication equation as a comparison.

1. Concept Development (CD) Problems 1-3
   - Review place value from ones to thousands
2. CD Problems 4-5
   10 x 40 ("10 times as many as 4 tens")
   = 10 x (4 x 10)
   = (10 x 4) x 10 (10 groups of 4 tens)
   = **(**4 x 10**)** x 10 (4 groups of 10 tens) *Commutative Property of Multiplication
   = 4 x **(**10 x 10**)** *Associative Property of Multiplication
   = 4 x 100 (ten 10s = 100)
   = 400

Lesson 2: Recognize a digit represents 10 times the value of what it represents in the place to its right.

Note: may need to split into two lessons (1-3, 4-5)

1. CD Problem 1
   - Extend place value to millions
2. CD Problem 2
   - Problem 1 + Lesson 1 → 10 x 40000 ("10 times 4 ten thousands")
3. CD Problem 3
   - Problem 2 + definition of division → 4000 ÷ 10 ("4 thousands ÷ 10")
     - want to make 4000 into 10 groups: 10 x __ = 4000
     - 4000 = 4 x 1000 = (4 x **10**) x 100 = **(10** x 4**)** x 100 = **10** x **(**<u>4 x 100</u>**)**
     - so 4000 ÷ 10 = 400
4. CD Problem 4
   10 x 320 ("10 x (3 hundreds 2 tens)")
   = 10 x (3 x 100 + 2 x 10)
   = (10 x 3) x 100 + (10 x 2) x 10 (10 groups of 3 hundreds and 10 groups of 2 tens) *Distributive Property
   = (3 x 10) x 100 + (2 x 10) x 10
   = 3 x (10 x 100) + 2 x (10 x 10)
   = 3200 ("3 thousands 2 hundreds")

5. CD Problem 5
    - Problem 4 + definition of division → 40020 ÷ 10 (“(4 ten thousands) ÷ 10”)
        - want to make 40020 into 10 groups: 10 x __ = 40020
        - 40020
          = (4 x 10000) + (2 x 10)
          = (4 x 10 x 1000) + (2 x 10 x 1)
          = (10 x 4 x 1000) + (10 x 2 x 1)
          = 10 x (<u>4 x 1000 + 2 x 1</u>) *Distributive Property
        - so 40020 ÷ 10 = 4002

Lesson 3: Name numbers within 1 million by building understanding of the place value chart and placement of commas for naming base thousand units.

1. CD Intro
    - Extend place value to billions
2. CD Problem 1
    - Conventions
        i. In English, there are ones, tens, and hundreds for each unit (e.g., thousands, ten thousands, hundred thousands), and every three digits we get a new unit (three digits to the left of thousand is million).
        ii. We use comma (,) to group digits of a common unit.
3. CD Problem 2
   300,000 + 700,000 (“3 hundred thousands + 7 hundred thousands”)
   = (3 x 100,000) + (7 x 100,000)
   = (3 + 7) x 100,000 *Distributive Property
   = 10 x 100,000
   = 1,000,000 (“bundle to make 1 million”)

   (CD Problem 3 = more practice of Lesson 2)

Lesson 4: Read and write multi-digit numbers using base ten numerals, number names, and expanded form.

* Expanded form shows the value of each digit and allows you to see how you are using the properties of operations to do multi-digit arithmetic

1. CD Problem 2
    - Standard form to expanded form and word form (e.g., 27,085 = 20,000 + 7,000 + 80 + 5, “twenty-seven thousand, eighty-five”)
2. Application Problem
    - Apply skills from Lessons 1-4 to solve word problem

B. Comparing Multi-Digit Whole Numbers

Lesson 5: Compare numbers based on meanings of the digits using >, <, or = to record the comparison.

* Key prior knowledge/skill: process of counting and skip-counting by 10s, 100s, etc.
* Definitions: greater than, less than
  - 3 is "greater (or bigger) than" 2 means 3 comes after 2 when counting
  - 2 is "less (or smaller) than" 3 means 2 comes before 3 when counting

1. CD Problem 2
   Compare 43,021 and 45,302.
   Both numbers count up to 40,000 and keep going but because 3,000 comes before 5,000, then 43,021 < 45,302.

   (CD Problem 4 = more practice of Problem 3 and Lesson 4)

Lesson 6: Find 1, 10, and 100 thousand more and less than a given number.

1. CD Problem 1
   19,112 + 1,000 ("What is 1 thousand more than 19,112?")
   = 10,000 + 9,000 + 100 + 10 + 2 + 1,000
   = 10,000 + (9,000 + 1,000) + 100 + 10 + 2 *Commutative & Associative Properties of Addition
   = 10,000 + 10,000 + 100 + 10 + 2
   = 20,000 + 100 + 10 + 2
   = 20,112

C. Rounding Multi-Digit Whole Numbers

* Key prior knowledge/skill:
  - process of counting and skip-counting multiples of 10s, 100s, etc.
  - subtraction: find difference between a number and its nearest multiples of 10s, 100s, etc.
* Definition: rounding

  To round a whole number n to the nearest 10, 100, 1000, etc. means to replace n by the multiple of 10, 100, 1000, etc. which is closest to n. If two multiples of 10, 100, 1000, etc. are equally close to n, the convention is to always choose the bigger number.

Lesson 7: Round multi-digit numbers to the thousands place using the vertical number line.

1. CD Problem 1
   Round 4,100 to the nearest thousand.
   - 4,100 is between 4,000 and 5,000
   - 4,100 is 100 more than 4,000 and 900 less than 5,000 so 4,100 is closer to 4,000
   - 4,100 rounded to the nearest thousand is 4,000
2. CD Problem 2
   Round 14,500 to the nearest thousand.
   - 14,500 is between 14,000 and 15,000
   - 14,500 is 500 more than 14,000 and 500 less than 15,000 so 14,500 is equally close to 14,000 and 15,000
   - 15,000 is the bigger number
   - 14,500 rounded to the nearest thousand is 15,000

Lesson 8: Round multi-digit numbers to any place using the vertical number line.

1. CD Problem 1
   Round 72,744 to the nearest ten thousand.
   - 72,744 is between 70,000 and 80,000
   - 72,744 is between 72,000 and 73,000 (under 75,000)
   - 72,744 is closer to 70,000
   - 72,744 rounded to the nearest ten thousand is 70,000
2. CD Problem 3
   - Problem 1 & Lesson 3 → Estimate the sum of 505,341 and 193,841 by first rounding each addend to the nearest hundred thousand

Lesson 9: Use place value understanding to round multi-digit numbers to any place value.

1. CD Problems 1-3
   - Rounding without using a number line (fluency)

Lesson 10: Use place value understanding to round multi-digit numbers to any place value using real world applications.

1. CD Problems 2-3
   - Applying rounding to real world scenarios
   - The appropriate unit to round to depends on the situation

D. Multi-Digit Whole Number Addition

Lesson 11: Use place value understanding to fluently add multi-digit whole numbers using the standard addition algorithm, and apply the algorithm to solve word problems using tape diagrams.

Definition: variable

A variable is a symbol (usually a letter) used to represent a number (often an unknown number that we are looking for).

1. CD Problem 1
    - Place value and properties of operations (associative, commutative, distributive) → add with composing (renaming) once
      3,134 + 2,493
      = (3 x 1000 + 1 x 100 + 3 x 10 + 4) + (2 x 1000 + 4 x 100 + 9 x 10 + 3)
      = (**4 + 3**) + (**3 + 9**) x 10 + (1 + 4) x 100 + (3 + 2) x 1000 *Properties of Addition, Distributive Property
      = **7** + (**12**) x 10 + (1 + 4) x 100 + (3 + 2) x 1000
      = 7 + **(2 + 10) x 10** + (1 + 4) x 100 + (3 + 2) x 1000
      = 7 + (2 x 10) + **100** + (1 + 4) x 100 + (3 + 2) x 1000 *Distributive Property & compose 10 x 10 = 100
      = 7 + (2 x 10) + (1 + 4 + **1**) **x 100** + (3 + 2) x 1000
      = 7 + (2 x 10) + (6 x 100) + (5 x 1000)
      = 5,627
2. CD Problem 3
    - Place value and properties of operations (associative, commutative, distributive) → add with composing (renaming) multiple units
3. CD Problem 4
    - CD Problems 1-3 & visual representation (number line or tape diagram) → solve one-step word problem

Lesson 12: Solve multi-step word problems using the standard addition algorithm modeled with tape diagrams, and assess the reasonableness of answers using rounding.

1. Prior knowledge
    a. Application Problem: Lesson 11 CD Problem 4
2. CD Problem 2
    - Lesson 11 & Lesson 10 → solve two-step word problem and estimate sum by rounding addends

E. Multi-Digit Whole Number Subtraction

Lesson 13: Use place value understanding to decompose to smaller units once using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams.

1. CD Problem 1
    - Place value and properties of operations (associative, commutative, distributive) → subtract by decomposing (renaming, regrouping) 1 hundred into 10 tens
      4,259 - 2,171
      = (4 x 1000 + 2 x 100 + 5 x 10 + 9) - (2 x 1000 + 1 x 100 + 7 x 10 + 1)
      = (**9 - 1**) + (**5 - 7**) x 10 + (**2** - 1) x 100 + (4 - 2) x 1000 *Properties of Addition, Distributive Property
      = 8 + (5 - 7) x 10 + (**1 + 1** - 1) x 100 + (4 - 2) x 1000
      = 8 + (**10** + 5 - 7) **x 10** + (1 - 1) x 100 + (4 - 2) x 1000 *Distributive Property & decompose 100 = 10 x 10
      = 8 + (15 - 7) x 10 (1 - 1) x 100 + (4 - 2) x 1000
      = 8 + (8 x 10) + (0 x 100) + (2 x 1000)
      = 2,088
2. CD Problem 2
    - Place value and properties of operations (associative, commutative, distributive) → subtract by decomposing (renaming, regrouping) 1 thousand into 10 hundreds

Lesson 14: Use place value understanding to decompose to smaller units up to three times using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams.

1. CD Problem 1 or 2
    - Lesson 13 → subtract by decomposing more than once
2. CD Problem 3
    - CD Problems 1-2 & visual representation (number line or tape diagram) → solve one-step word problem

Lesson 15: Use place value understanding to fluently decompose to smaller units multiple times in any place using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams.

1. CD Problem 2
    - Lesson 14 → subtract numbers from powers of 10
      1,000 - 528

= (1 x 1000 + 0 x 100 + 0 x 10 + 0) - (5 x 100 + 2 x 10 + 8)
= (0 - 8) + (0 - 2) x 10 + (0 - 5) x 100 + (1 x **1000**)
= (0 - 8) + (0 - 2) x 10 + (**10** + 0 - 5) **x 100**
= (0 - 8) + (0 - 2) x 10 + (**1** + 9 - 5) **x 100**
= (0 - 8) + (**10** + 0 - 2) **x 10** + (9 - 5) x 100
= (0 - 8) + (**1** + 9 - 2) x **10** + (9 - 5) x 100
= (**10** + 0 - 8) + (9 - 2) x 10 + (9 - 5) x 100
= 2 + (7 x 10) + (4 x 100)
= 472

Lesson 16: Solve two-step word problems using the standard subtraction algorithm fluently modeled with tape diagrams, and assess the reasonableness of answers using rounding.

1. CD Problem 1 or 2
    - Lessons 14-15, Lesson 10, & visual representation (number line or tape diagram) → solve two-step word problem (given total and numbers of subtotal) and estimate difference by rounding numbers
2. CD Problem 3
    - Lessons 14-15, Lesson 10, & visual representation (number line or tape diagram) → solve two-step word problem (given difference and comparison number) and estimate difference by rounding numbers

F. Addition and Subtraction Word Problems

* Great Minds: “If pacing is a challenge, consider omitting Lesson 17 since multi-step problems are taught in Lesson 18. Instead, embed problems from Lesson 17 into Module 2 or 3 as extensions. Since multi-step problems are taught in Lesson 18, Lesson 19 could also be omitted.”

Lesson 17: Solve *additive compare* word problems modeled with tape diagrams.

1. CD Problem 2-3
    - Solve “how many fewer”, “how much more” word problems

Lesson 18: Solve multi-step problems modeled with tape diagrams, and assess the reasonableness of answers using rounding.

1. CD Problem 3
    - Solve multi-step word problem involving addition and subtraction

Lesson 19: Create and solve multi-step word problems from given tape diagrams and equations.