Grade 4

Eureka Essentials

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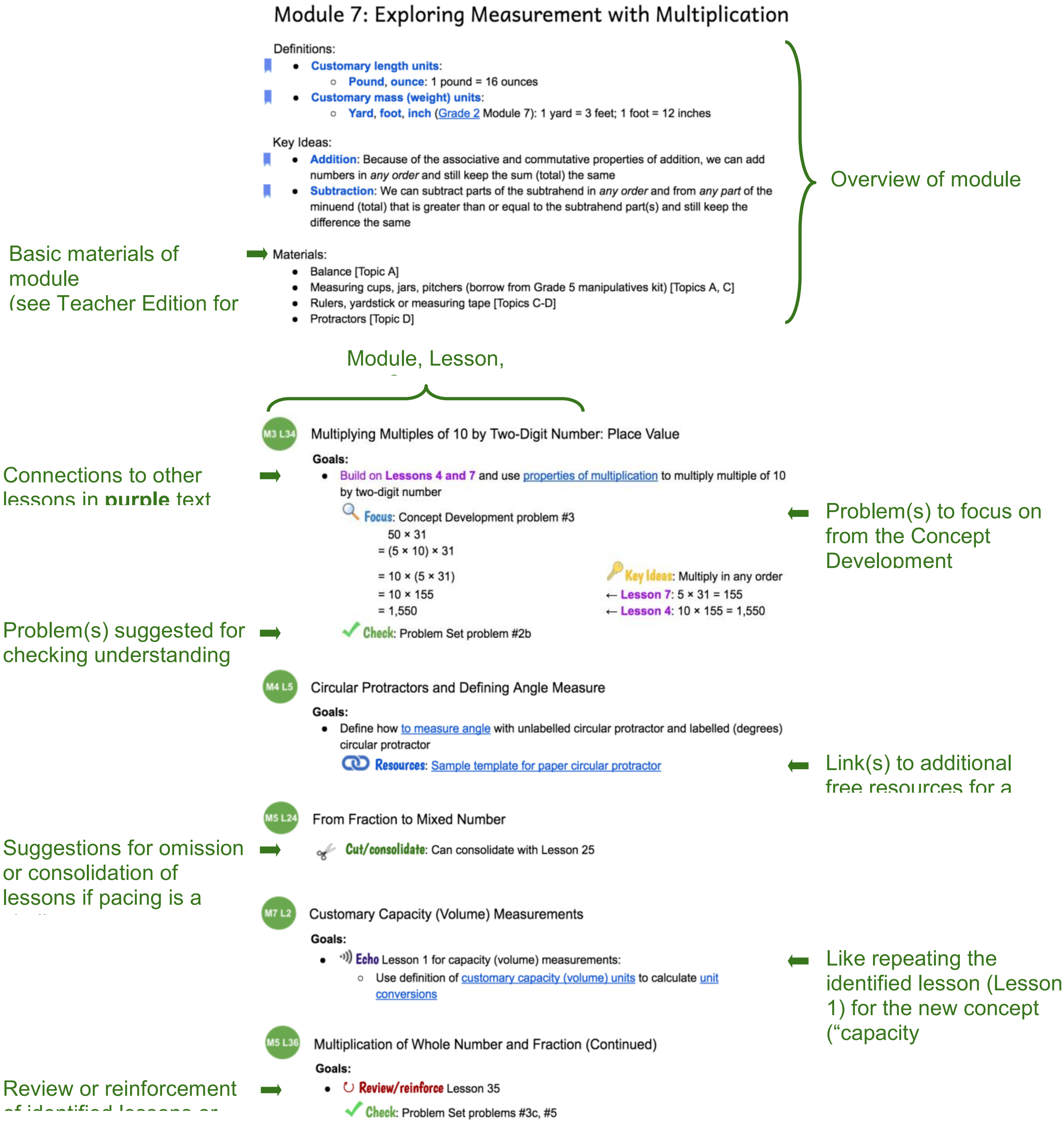
Overview

How to Read **Eureka Essentials**


Module 7: Exploring Measurement with Multiplication

Definitions:
- **Customary length units**:
  - **Pound, ounce**: 1 pound = 16 ounces
- **Customary mass (weight) units**:
  - **Yard, foot, inch** (Grade 2 Module 7): 1 yard = 3 feet; 1 foot = 12 inches

Key Ideas:
- **Addition**: Because of the associative and commutative properties of addition, we can add numbers in *any order* and still keep the sum (total) the same
- **Subtraction**: We can subtract parts of the subtrahend in *any order* and from *any part* of the minuend (total) that is greater than or equal to the subtrahend part(s) and still keep the difference the same

Materials:
- Balance [Topic A]
- Measuring cups, jars, pitchers (borrow from Grade 5 manipulatives kit) [Topics A, C]
- Rulers, yardstick or measuring tape [Topics C-D]
- Protractors [Topic D]

M3 L34 Multiplying Multiples of 10 by Two-Digit Number: Place Value

Goals:
- Build on **Lessons 4 and 7** and use properties of multiplication to multiply multiple of 10 by two-digit number

Focus: Concept Development problem #3

50×31
$= (5 \times 10) \times 31$
$= 10 \times (5 \times 31)$
$= 10 \times 155$
$= 1{,}550$

Key Ideas: Multiply in any order
← **Lesson 7**: 5 × 31 = 155
← **Lesson 4**: 10 × 155 = 1,550

Check: Problem Set problem #2b

M4 L5 Circular Protractors and Defining Angle Measure

Goals:
- Define how to measure angle with unlabelled circular protractor and labelled (degrees) circular protractor

Resources: Sample template for paper circular protractor

M5 L24 From Fraction to Mixed Number

Cut/consolidate: Can consolidate with Lesson 25

M7 L2 Customary Capacity (Volume) Measurements

Goals:
- Echo Lesson 1 for capacity (volume) measurements:
  - Use definition of customary capacity (volume) units to calculate unit conversions

M5 L36 Multiplication of Whole Number and Fraction (Continued)

Goals:
- Review/reinforce Lesson 35

Check: Problem Set problems #3c, #5

How to Approach Pacing

- ➢ Cover Major Cluster Standards more thoroughly and move through other Standards more quickly
  - **Major Clusters**: Modules 1, 3, 5-6
  - **Additional/Supporting Clusters**: Modules 2, 3 Topic F, 4, 7
    - *Spread Module 4 across the year or cover after Module 6*
- ➢ Use lesson connections (in purple text) to foresee when and how certain concepts will be revisited and further developed in later lessons
- ➢ Embed "reteaching" into the next lesson's activities (Fluency Practice, Application Problems, Concept Development, etc.) rather than repeat a prior lesson when formative assessments indicate lack of student understanding
- ➢ Omit or differentiate lessons labelled ↻ **Review/reinforce** based on students' strengths and needs
- ➢ Follow suggestions for omission or consolidation of lessons labelled **Cut/consolidate** based on students' strengths and needs

References

- Great Minds Eureka Math Teacher Edition version 3.0 (2015)
- California Common Core State Standards (2013)
- Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2016)
- Wu, H. (2011). *Understanding numbers in elementary school mathematics*. Providence: American Mathematical Society.

Module 1: Place Value, Rounding, and Algorithms for Addition and Subtraction

Definitions:

- **Multiplication** (Grade 3 Modules 1, 3): "10 × 100" means "100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100"
- **Division** (Grade 3 Modules 1, 3): two interpretations
  - Partitive interpretation
    - The unknown size of each group when given the total and the number of equal groups (number of *parts* is known)
    - "4,000 ÷ 10 = __" means "10 × __ = 4,000"
  - Measurement interpretation
    - The unknown number of equal groups when given the total and the size of each group (how much to *measure* is known)
    - "4,000 ÷ 10 = __" means "__ × 10 = 4,000"
- **Standard form of a number (base-ten numeral*)**: "standard form (or base-ten numeral) of 12,345" is "12,345"
  * State standard's terminology
- **Word form of a number (number name*)**: "word form (or number name) of 12,345" is "twelve-thousand three hundred forty-five"
  * State standard's terminology
- **Expanded form of a number**: "expanded form of 12,345" is "10,000 + 2,000 + 300 + 40 + 5"
- **Greater than, less than, equals** (Grade 1 Modules 4 & 6; Grade 2 Module 3):

Word form	*Symbolic form*	*Meaning (Definition)*
3 is greater (bigger, more) than 2	3 > 2	3 comes after 2 when counting
2 is less (smaller, fewer) than 3	2 < 3	2 comes before 3 when counting
3 + 0 equals 2 + 1	3 + 0 = 2 + 1	3 + 0 counts to the same number as 2 + 1

- **Rounding to the nearest 1,000**: to round a whole number n to the nearest 1,000 means to replace n by the multiple of 1,000 which is closest to n; if two multiples of 1,000 (0, 1000, 2000, etc.) are equally close to n, the convention is to always choose the bigger number
- **Variable**: a symbol (usually a letter) used to represent a number (often an unknown number that we are looking for)

Key Ideas:

- **Place value**: In our number system (base-ten), we only have **ten** symbols (the digits 0 to 9) to represent **zero to nine** ones, tens, hundreds, etc. (a unit) so to represent **ten** ones, tens, hundreds, etc. we use the next “place” to the left and make it 1 of that larger unit (e.g., ten 1s = 10, ten 10s = 100, ten 100s = 1000, ten 1000s = 10000)

- **Addition**: Because of the associative and commutative properties of addition, we can add numbers in *any order* and still keep the sum (total) the same
- **Subtraction**: We can subtract parts of the subtrahend in *any order* and from *any part* of the minuend (total) that is greater than or equal to the subtrahend part(s) and still keep the difference the same
- **Distributive property** (Grade 3 Modules 1, 3): 10 × (100 + 20) = (10 × 100) + (10 × 20)
- **Conventions**:
    1. In English, there are ones, tens, and hundreds for each unit (e.g., thousands, ten thousands, hundred thousands), and every three digits we get a new unit (three digits to the left of thousand is million).
    2. We use comma (,) to group digits of a common unit.

Topic A: Place Value of Multi-Digit Whole Numbers

Multiplying by 10 Within 10,000

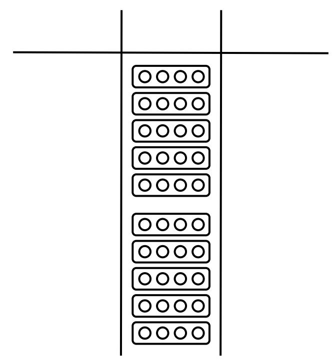
Goals:

- ↻ **Review/reinforce** place value from Grade 2 Module 3:
  - 10 × 1 (“10 times 1 one”) = 10
  - 10 × 10 (“10 times 1 ten”) = 100
  - 10 × 100 (“10 times 1 hundred”) = 1,000

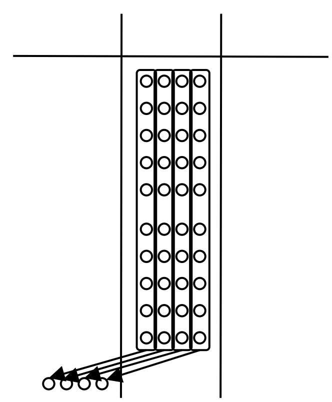
  Focus: Concept Development problems #1-3
- Use place value to multiply a number by 10
  - Example:

    10 × 40 (“10 times as many as 4 tens”)

    = 10 × 4 × 10 (“10 groups of 4 tens”)



    = 4 × 10 × 10



    = 4 × 100 (ten 10s = 100)

    = 400

  Focus: Concept Development problem #4

  Check: Problem Set problem #1b

Multiplying and Dividing by 10 Within 1,000,000

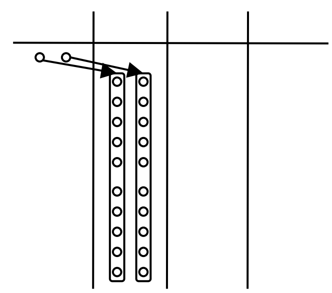
Suggestions: Split into two lessons (Concept Development Problems 1-3, 4-5)

Goals:

- Extend place value to millions
  - **Focus**: Concept Development problem #1
- Build on **Lesson 1** to multiply by 10 within 1,000,000
  - **Focus**: Concept Development problem #2
    - 10 × 40,000 (“10 times 4 ten thousands”)
    - = 10 × 4 × 10,000
    - = 4 × 10 × 10,000
    - = 4 × 100,000 (ten 10,000s = 100,000)
    - = 400,000
  - **Check**: Problem Set problem #1b
- Use definition of division to divide by 10 within 1,000,000
  - **Focus**: Concept Development problem #3
    - 2,000 ÷ 10 (“2 thousands ÷ 10”) = 200
      - Reasoning 1:
        2,000 ÷ 10 (“2 thousands ÷ 10”)

        = (2 × 1,000) ÷ 10

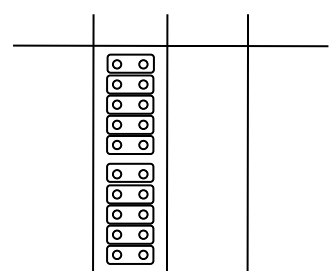
        = (2 × 10 × 100) ÷ 10

        = (10 × 200) ÷ 10

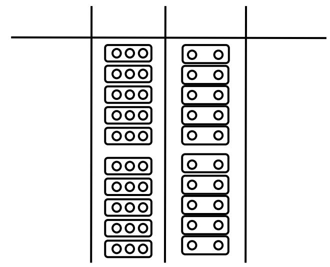
        = how many in each group when (10 × 200) is divided into 10 groups

        = 200

        
      - Reasoning 2:
        - “2,000 ÷ 10 = __” means “__ × 10 = 2,000”
        - 200 × 10 = 2 × 100 × 10 = 2 × 1,000 = 2,000
  - **Check**: Problem Set problem #1c
- Use the distributive property to multiply by 10
  - **Focus**: Concept Development problem #4
    - 10 × 320

= 10 × (3 × 100 + 2 × 10)

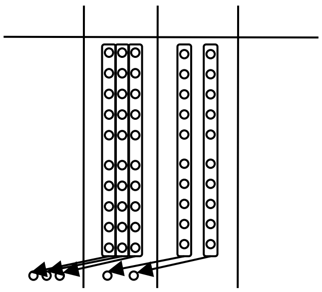
= (10 × 3 × 100) + (10 × 2 × 10)
"10 groups of 3 hundreds + 10 groups of 2 tens"



= (3 × 10 × 100) + (2 × 10 × 10)
"3 groups of ten 100s + 2 groups of ten 10s"

= 3 × 1,000 + 2 × 100

= 3,200



Check: Problem Set problem #6

- Use definition of division (and the distributive property) to divide by 10

Focus: Concept Development problem #4

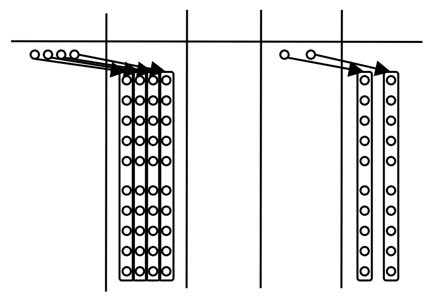
40,020 ÷ 10 = 4,002

- Reasoning 1:

  40,020 ÷ 10

  = (4 × 10,000 + 2 × 10) ÷ 10

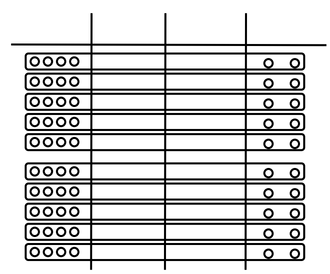
  = (4 × 10 × 1,000 + 2 × 10 × 1) ÷ 10



  = (10 × 4,000 + 10 × 2) ÷ 10

  = how many in each group when (10 × 4,000 + 10 × 2) is divided into 10 groups

  = 4,000 + 2
  = 4,002



- Reasoning 2:
  - "40,020 ÷ 10 = __" means "__ × 10 = 40,020"
  - 4002 × 10 = (4000 + 2) × 10 = (4000 × 10) + (2 × 10) = 40,020

Check: Problem Set problem #5

Adding Within 1,000,000

Goals:

- Extend place value to billions
  - Focus: Concept Development Introduction
- Use conventions to write numbers
  - Focus: Concept Development problem #1
  - Check: Problem Set problem #1b & d
- Use place value and properties of addition to add within 1,000,000
  - Focus: Concept Development problem #2
  - Check: Problem Set problem #3a

Notes to teacher:

- Will revisit/reinforce multi-digit addition in **Lessons 11-12** with the standard algorithm for addition

Different Forms of a Number

Goals:

- Write a number in standard form, word form, and expanded form
  - Focus: Concept Development problem #2
  - Check: Problem Set problem #2

Topic B: Comparing Multi-Digit Whole Numbers

Comparing Numbers

Goals:

- Review definitions of greater than, less than, equals (Grade 1 Modules 4 & 6; Grade 2 Module 3)

- Use above definitions to compare and order numbers
  - Example: Compare 43,021 and 45,302
    - Both numbers count up to 40,000 and keep going
    - 3,000 comes before 5,000
    - Therefore, 43,021 < 45,302

  Focus: Concept Development problems #2, #3

  Check: Problem Set problems #2a-b, #4

1,000, 10,000, and 100,000 More or Less

Goals:

- Use place value and properties of addition and subtraction to find 1,000, 10,000, or 100,000 more or less than a number

  Check: Problem Set problem #1

Topic C: Rounding Multi-Digit Whole Numbers

Rounding to the Nearest Thousand

Goals:

- Generalize definition of rounding to the nearest 10, 100 (Grade 3 Module 2) to define rounding to the nearest 1,000
- Round multi-digit numbers to the nearest 1,000
  - Example: Round 4,100 to the nearest thousand
    - 4,100 is between 4,000 and 5,000
    - 4,100 is 100 more than 4,000 and 900 less than 5,000 so 4,100 is closer to 4,000
    - 4,100 rounded to the nearest thousand is 4,000

  Check: Problem Set problem #1c-d

Suggestions: Give students choice to show rounding with horizontal number line or vertical number line (see Suggestion for Grade 3 Module 2 Lesson 12)

Rounding to the Nearest Ten Thousand, Hundred Thousand

Goals:

- Build on **Lesson 7** to round to the nearest 10,000 or 100,000

  Focus: Concept Development problems #1, #2

  Check: Problem Set problems #1b, #2b
- Build on Grade 3 **Module 2 Lessons 17 and 20** to estimate a sum or difference by first rounding each number to a given place value and then adding or subtracting the rounded numbers

  Focus: Concept Development problem #3

  Check: Problem Set problem #5a

Rounding (Continued)

Goals:

- Round to the nearest 1,000, 10,000, or 100,000 without using a number line by attending to place value
  - Example: Round *18*,753 to the nearest thousand
    - Between *18*,000 and the next thousand, *19*,000
    - 700 is closer to the upper 1,000
    - 18,753 rounded to the nearest thousand is 19,000

  Check: Problem Set problems #1c, #2c

Applications of Rounding

Goals:

- Apply rounding to real world scenarios and observe that the appropriate unit to round to depends on the situation

  Focus: Concept Development problems #2, #3

  Check: Problem Set problem #3

Topic D: Multi-Digit Whole Number Addition

Multi-Digit Addition

Goals:

- Use variable to represent unknown number
- Build on **Lesson 3** and Grade 3 **Module 2 Lesson 15** to add with composing once

  Focus: Concept Development problem #1

  Check: Problem Set problem #1c
- Build on Grade 3 **Module 2 Lesson 16** to add with composing more than once

  Focus: Concept Development problem #3

  Check: Problem Set problem #1f
- Solve one-step word problems that involve multi-digit addition

  Focus: Concept Development problem #4

  Check: Problem Set problem #2

Word Problems

Goals:

- Build on **Lessons 8-11** by solving two-step word problems that involve multi-digit addition and estimating sums by rounding addends

  Focus: Concept Development problem #2

  Check: Problem Set problem #2

Topic E: Multi-Digit Whole Number Subtraction

Subtraction with One Decomposition

Goals:

- Build on Grade 3 **Module 2 Lesson 18** to subtract by decomposing once (1 hundred into 10 tens, 1 thousand into 10 hundreds, etc.)
  - Focus: Concept Development problem #2
  - Check: Problem Set problem #1g

Subtraction with Multiple Decompositions

Goals:

- Build on Grade 3 **Module 2 Lesson 19** to subtract by decomposing more than once
  - Focus: Concept Development problem #2
  - Check: Problem Set problems #1d-e
- Solve one-step word problems that involve multi-digit subtraction
  - Focus: Concept Development problem #1
  - Check: Problem Set problem #3

Subtraction from Powers of 10

Goals:

- Build on **Lesson 14** to subtract numbers from powers of 10 (100, 1000, etc.)
  - Focus: Concept Development problem #2
  - Check: Problem Set problem #1e
- ↻ **Review/reinforce** solving one-step word problems that involve multi-digit subtraction
  - Focus: Concept Development problem #3
  - Check: Problem Set problem #3

Word Problems

Goals:

- Solve two-step word problems (given total and numbers of subtotal) and estimate answer by rounding numbers

  Focus: Concept Development problem #1

  Check: Problem Set problem #1
- Solve two-step word problem (given difference and comparison number) and estimate answer by rounding numbers

  Focus: Concept Development problem #3

  Check: Problem Set problem #2a

Topic F: Addition and Subtraction Word Problems

Comparison Word Problems

Cut/consolidate: Can consolidate with Lesson 18

Goals:

- Solve “how many fewer”, “how much more” word problems

  Focus: Concept Development (Problem Set) problems #2, #3

Multi-Step Word Problems

Goals:

- Solve multi-step word problems involving addition and subtraction

  Focus: Concept Development (Problem Set) problem #3

Creating Word Problems

Cut/consolidate: Can omit lesson or use as “challenge” task

Goals:
- Create a story to match a given tape diagram or equation

Module 2: Unit Conversions and Problem Solving with Metric Measurement

Definitions:

- **Metric length units**:
  - **Meter**, **centimeter** (Grade 2 Module 2): 1 meter = 100 centimeter
  - **Kilometer**: 1,000 meters = 1 kilometer
- **Metric mass (weight) units**:
  - **Gram**, **kilogram** (Grade 3 Module 2): 1,000 grams = 1 kilogram
- **Metric capacity (volume) units**:
  - **Liter**, **milliliter** (Grade 3 Module 2): 1 liter = 1,000 milliliters
- **Mixed unit**:
  - "2 km 5 m" means "2 km + 5 m"
  - Mixed unit is shorthand for writing a sum of two units
    - Preview of mixed numbers (Module 5): "$2\frac{5}{8}$" means "$2+\frac{5}{8}$"
  - We make as many of the larger unit as possible
    - Example: 2005 m as a mixed unit is 2 km 5 m and not 1 km 1005 m
- **Unit conversion**: to convert 1 km 500 m to meters means to find how long 1 km 500 m is in meters (1 km 500 m = ? m)
- **Equal**, **greater than**, **less than** (Grade 2 Module 3; Grade 3 Module 5)

Word	*Symbolic*	*Meaning (Definition)*
123 cm is equal to 1 m 23 cm	123 cm = 1 m 23 cm	• Counting: 123 cm counts to the SAME number as 1 m 23 cm • Number line: 123 cm and 1 m 23 cm are the SAME point
1 m is greater than 10 cm	1 m > 10 cm	• Counting: 1 m comes AFTER 10 cm • Number line: 1 m is to the RIGHT of 10 cm
1 m is less than 1 km	1 m < 1 km	• Counting: 1 m comes BEFORE 1 km • Number line: 1 m is to the LEFT of 1 km

Key Ideas:

- **Addition**: Because of the associative and commutative properties of addition, we can add numbers in *any order* and still keep the sum (total) the same
- **Subtraction**: We can subtract parts of the subtrahend in *any order* and from *any part* of the minuend (total) that is greater than or equal to the subtrahend part(s) and still keep the difference the same

Topic A: Metric Unit Conversions

Metric Length Measurements

Goals:

- ↻ **Review/reinforce** definitions of meter, centimeter and define kilometer
- Visualize relative sizes of 1 cm, 1 m, 1 km with concrete objects and places

  Focus: Concept Development problem #1
- Use definitions of mixed unit and metric length units to calculate unit conversions

  Focus: Concept Development problem #2

  ✓ **Check**: Problem Set problems #1c & g, #2b & e
- Use definitions of mixed unit and metric length units and properties of addition and subtraction to add and subtract mixed units
  - Example:

    1 km 734 m + 4 km 396 m
    = 1 km + 734 m + 4 km + 396 m ← definition of mixed unit
    = (1 km + 4 km) + (734 m + 396 m) **Key Ideas**: Add in any order
    ...
    = 5 km + 1 km + 130 m
    = 6 km + 130 m
    = 6 km 130 m ← definition of mixed unit

  Focus: Concept Development problems #3, #4

  ✓ **Check**: Problem Set problems #3c & d

Metric Mass (Weight) Measurements

Goals:

- **Echo** Lesson 1 for mass (weight) measurements:
  - ↻ **Review/reinforce** definitions of gram, kilogram
  - Visualize relative sizes of 1 g, 1 kg with concrete objects
  - Use definitions of mixed unit and metric mass units to calculate unit conversions

    Focus: Concept Development problem #1

Check: Problem Set problems #2d & e

- Use definitions of mixed unit and metric mass units and properties of addition and subtraction to add and subtract mixed units

  Focus: Concept Development problems #2, #3

  Check: Problem Set problems #3c & d

Metric Capacity (Volume) Measurements

Goals:

- **Echo** Lesson 1 for capacity (volume) measurements:
  - ↻ **Review/reinforce** definitions of liter, milliliter
  - Visualize relative sizes of 1 mL, 1 L with concrete objects
  - Use definitions of mixed unit and metric capacity units to calculate unit conversions

    Focus: Concept Development problem #1

    Check: Problem Set problems #2d & e
  - Use definitions of mixed unit and metric capacity units and properties of addition and subtraction to add and subtract mixed units

    Focus: Concept Development problems #2, #3

    Check: Problem Set problems #3c & d

Topic B: Application of Metric Unit Conversions

Place Value and Metric Units

Goals:

- Relate metric units to place value units for unit conversion

  Focus: Concept Development problem #2

Thousands 1,000 mL (= 1 L)	*Hundreds* 100 mL	*Tens* 10 mL	*Ones* 1 mL

1	2	0	0

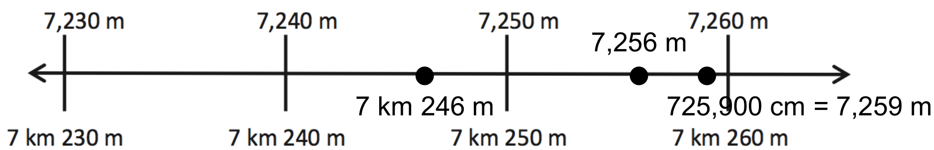
1,200 mL
= 1 x 1000 mL + 2 x 100 mL ← expanded form of number
= 1 L + 200 mL ← definition of metric units
= 1 L 200 mL ← definition of mixed unit

✓ **Check**: Problem Set problems #2b & f

- Use definitions of equal, greater than, less than to compare metric units with place value or number line

  Focus: Concept Development problem #3

	1,000 m = 1 km	100 m	10 m	100 cm = 1 m	10 cm	1 cm
7,256 m =	7	2	5	6		
7 km 246 m =	7	2	4	6		
725,900 cm =	7	2	5	9	0	0



Answer: 7 km 246 m < 7,256 m < 725,900 cm

✓ **Check**: Problem Set problems #5b, #6

Word Problems

Goals:

- Build on **Lessons 1-3** by solving two-step word problems that involve metric unit measurements

  Focus: Concept Development (Problem Set) problem #1

  ✓ **Check**: Problem Set problem #6

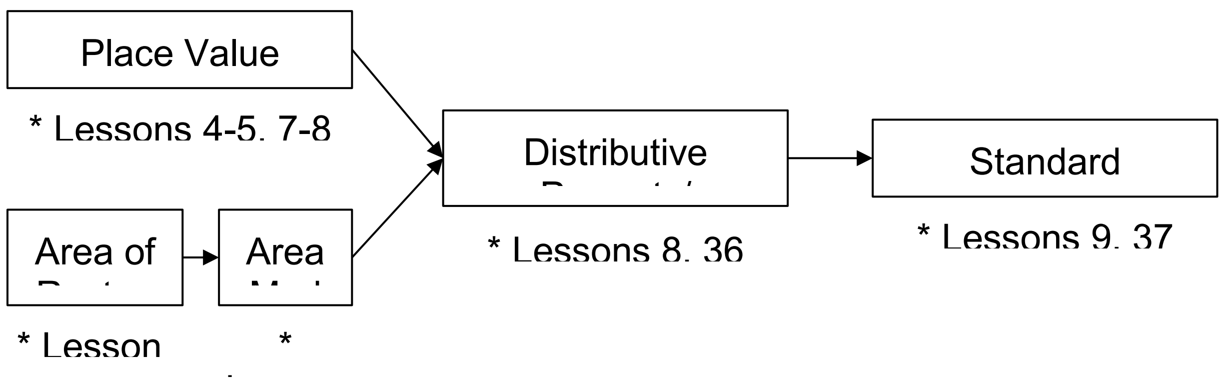
- Build on **Lessons 1-3** by solving three-step word problems that involve metric unit measurements

  Focus: Concept Development (Problem Set) problem #3

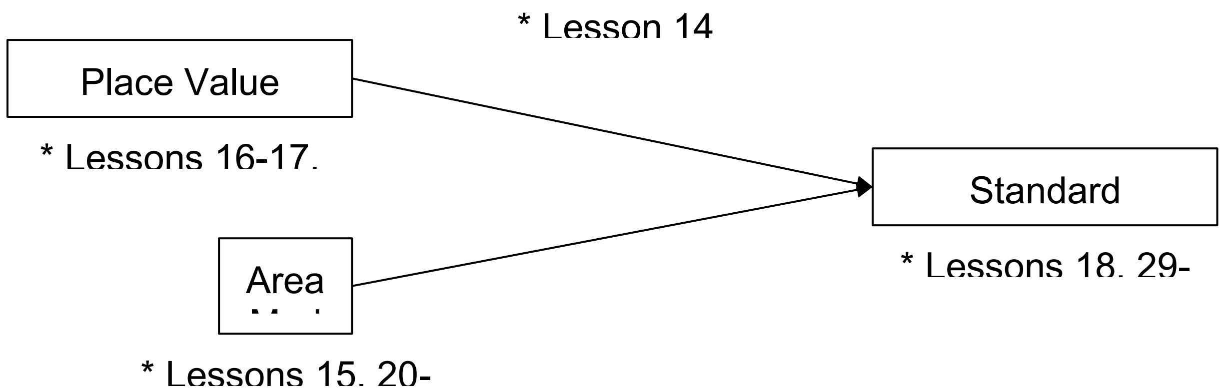
Module 3: Multi-Digit Multiplication and Division

Summary:

MULTIPLICATION



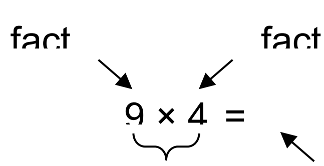
DIVISION



* Key Lessons

Definitions:

- **Perimeter** (Grade 3 Module 7): total length (number of unit lengths) around a figure
- **Area** (Grade 3 Module 4): total number of unit squares that fit inside a figure (without overlap or extra space between)
- **Multiplication** (Grade 3 Modules 1, 3): multiplication is shorthand for addition of the same number
  - “9 × 4” means “4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4”



- **Division** (Grade 3 Modules 1, 3): division is shorthand for multiplication with unknown factor
  - Partitive interpretation:
    - The unknown size of each group when given the total and the number of equal groups (number of *parts* is known)
    - “36 ÷ 9 = N” means “9 × N = 36”
  - Measurement interpretation:
    - The unknown number of equal groups when given the total and the size of each group (how much to *measure* is known)
    - “36 ÷ 9 = N” means “N × 9 = 36”
  - *Choose one interpretation as the meaning (definition) of division and justify the other interpretation with the commutative property of multiplication (reasoning)*

    Resources: Video introduction to division

    Example: starting with the partitive interpretation as the definition of division
    - “24 ÷ 8 = __” means “8 × __ = 24”
    - “__ × 8 = 24” by commutative property of multiplication (8 × __ = __ × 8)
    - so “24 ÷ 8 = __” also means “__ × 8 = 24” (measurement interpretation)
- **Division with remainder**:
  - Partitive interpretation: $a \div b$ is the number Q(quotient) so that $b \times Q + R = a$ where $0 \leq R < b$ (remainder)
    - When we form b groups from a (partitive interpretation of division), then Q is the most number of items in each group and R is the leftover that is not enough to distribute evenly into b groups
  - Measurement interpretation: $a \div b$ is the number Q(quotient) so that $Q \times b + R = a$ where $0 \leq R < b$ (remainder)
    - When we form groups of b from a (measurement interpretation of division), then Q is the most number of groups we can make and R is the leftover that is not enough to form a group of b
- **“As long/wide as”**:
  - “A rectangle is 3 times as long as it is wide” means “a rectangle’s length (how long it is) is 3 times its width (how wide it is)” or “length = 3 × width”
  - “The second rectangle is 3 times as wide (as the first rectangle)” means “the second rectangle’s width is 3 times the first rectangle’s width” or “second rectangle’s width = 3 × first rectangle’s width”
- **Prime number**: a number that has exactly two factors, 1 and the number itself
  - Examples: 3, 5, 7
- **Composite number**: a number that has at least one factor other than 1 and the number itself
  - Examples: 4, 6, 8, 9

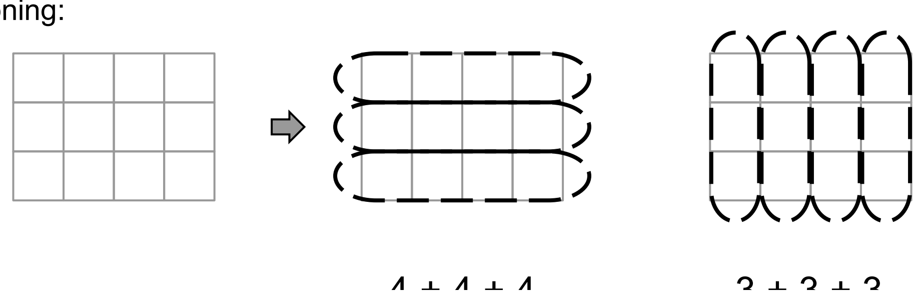
Key Ideas:

- **Perimeter of rectangle formula** (Grade 3 Module 7):

  Perimeter of rectangle = 2 × (length + width) OR (2 × length) + (2 × width)
  - Reasoning:

- Perimeter of rectangle
  = length + width + length + width ← definition of perimeter
  = (length + width) + (length + width) ← associative property of addition
  = 2 × (length + width) ← definition of multiplication
- Perimeter of rectangle
  = length + width + length + width ← definition of perimeter
  = (length + length) + (width + width) ← commutative, associative properties of addition
  = (2 × length) + (2 × width) ← definition of multiplication

- **Area of rectangle formula** (Grade 3 Module 4):
  Area of rectangle = (length of side 1) × (length of side 2)
  - Reasoning:



Area of 3 inch by 4 inch rectangle
= "total number of square inch tiles" ← definition of measure area
= 4 + 4 + 4 or 3 + 3 + 3 + 3 ← definition of addition
= 3 × 4 or 4 × 3 ← definition of multiplication
= 12 square inches

- **Distributive property** (Grade 3 Modules 1, 3): 10 × (100 + 20) = (10 × 100) + (10 × 20)
- **Multiplication**: Because of the associative and commutative properties of multiplication, we can multiply numbers in *any order* and still keep the product (total) the same

Topic A: Multiplicative Comparison Word Problems

Area and Perimeter of Rectangles

Suggestions: Save Concept Development problem #4 for Lesson 22 (factor pairs)

Goals:

- ↻ **Review/reinforce** definitions of perimeter and area of rectangle

  Focus: Concept Development problem #1

- ↻ **Review/reinforce** formulas for perimeter and area of rectangle

  Focus: Concept Development problem #2 (part 1), #3 (part 1)

  Check: Problem Set problem #2a

- Find unknown side length of a rectangle when given perimeter or area and one side length

  Focus: Concept Development problem #2 (part 2), #3 (part 2)

  Check: Problem Set problem #4a, #5a

Multiplicative Comparisons

Goals:

- Define "as long/wide as"

  Focus: Concept Development problem #2

  Check: Problem Set problem #2

Word Problems

Goals:

- Build on **Lessons 1-2** by solving multi-step word problems that involve area or perimeter of rectangles, multiplicative comparisons

  Focus: Concept Development (Problem Set) problem #2

Topic B: Multiplication by 10, 100, and 1,000

Multiplying by 10, 100, and 1,000

Goals:

- Build on Module 1 **Lessons 1-2** to multiply single-digit number by 10, 100 (= 10 × 10), or 1,000 (= 10 × 10 × 10)
    - Example: 3 × 10 × 10 × 10 = 3 × 1,000

      ↙ ↘

      = 30 × 10 × 10
      = 300 × 10
      = 3,000

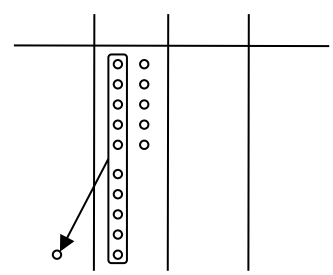
      = 1,000 + 1,000 + 1,000
      = 3,000

  Focus: Concept Development problem #1

  Check: Problem Set problem #2
- Use distributive property to multiply two-digit number by 10, 100, or 1,000

  Focus: Concept Development problem #2

  15 × 100
  = (10 + 5) × 100
  = (10 × 100) + (5 × 100)
  = 1,000 + 500
  = 1,500



  Check: Problem Set problem #5

Suggestions: Omit or use as “challenge” tasks Concept Development problem #3, Problem Set problem #8, Homework problems #7-10 because the concept will be covered in Lesson 5

Multiplying One-Digit Numbers by Multiples of 10, 100, and 1,000

Goals:

- Build on **Lesson 4** to multiply single-digit number by multiple of 10, 100, or 1,000
  - Focus: Concept Development problem #1

    200×4
    $= 4 \times 200$
    $= 200 + 200 + 200 + 200$ OR $= 4 \times 2 \times 100$
    $= 800$ $\quad$ $= 800$ ← **Lesson 4**: $8 \times 100 = 800$

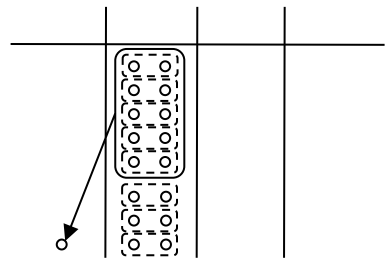
  - Check: Problem Set problem #3
- Multiply single-digit number by multiple of 10, 100, or 1,000 with composition (regrouping)
  - Focus: Concept Development problem #2

    8×200

    $= 8 \times 2 \times 100$

    $= 16 \times 100$

    $= 1,600$



← **Lesson 4**: $16 \times 100 = 1,600$

  - Check: Problem Set problem #5a & c

Multiplying Multiples of 10, 100, and 1,000

Goals:

- Build on **Lessons 4-5** (place value) and use [properties of multiplication]() to multiply two-digit multiples of 10
  - Focus: Concept Development problem #1

    30×20
    $= (3 \times 10) \times 20$ OR $= 30 \times (2 \times 10)$
    $= 3 \times (10 \times 20)$ $\quad$ $= (30 \times 2) \times 10$ Key Ideas: Multiply in any order
    $= 3 \times 200$ $\quad$ $= 60 \times 10$
    $= 600$ $\quad$ $= 600$

Check: Problem Set problem #1

- Use area model to multiply two-digit multiples of 10
    - Example:

      30 × 20

      = (3 × 10) × (2 × 10)

      = area of rectangle with side lengths (3 × 10) and (2 × 10)

      = sum of areas of small congruent squares

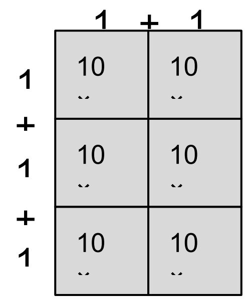
      = (number of small squares) × (area of one small square)

      = (3 × 2) × (10 × 10)

      Multiply in any order

      = 6 × 100

      = 600



Key Ideas:

Focus: Concept Development problem #2

Check: Problem Set problem #2

Suggestions: Use the same multiplication expression (e.g., 30 × 20) to demonstrate and connect the two methods (place value and area model)

Topic C: Multiplication of up to Four Digits by Single-Digit Numbers

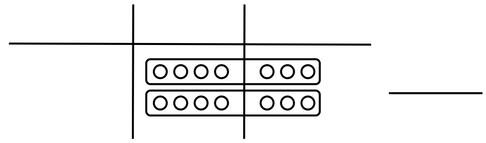
Suggestions:

- Show vertical multiplication and partial products alongside place value disks in Lessons 7-8
- Insert Lesson 11 between Lessons 8 and 9 to show area model of the multiplication algorithm and represent the distributive property (3.MD.7c) before focusing exclusively on practicing the multiplication algorithm (Lessons 9-10)
- Relate partial products (from place value or area model) to standard algorithm (Lessons 9-10)

Multiplying Two-Digit Number by One-Digit Number

Goals:

- Use distributive property to multiply
  - Example:

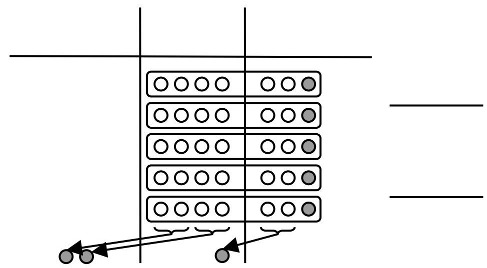


$$
\begin{aligned}
&2 \times 43 \\
&= 2 \times (4 \times 10 + 3) \\
&= (2 \times 4 \times 10) + (2 \times 3) \\
&= 80 + 6 \\
&= 86
\end{aligned}
$$

Focus: Concept Development problem #1

Check: Problem Set problem #1b

- Use distributive property to multiply with composition (regrouping)
  - Example:



$$
\begin{aligned}
&5 \times 43 \\
&= 5 \times (4 \times 10 + 3) \\
&= (5 \times 4 \times 10) + (5 \times 3) \\
&= 200 + 15 \\
&= 215
\end{aligned}
$$

Focus: Concept Development problem #2

Check: Problem Set problem #2a

Multiplying Three- and Four-Digit Numbers by One-Digit Number: Place Value

Goals:

- Build on **Lesson 7** and use distributive property to multiply three-digit number by one-digit number

  Focus: Concept Development problem #2

  Check: Problem Set problem #1c

- Use distributive property to multiply four-digit number by one-digit number

  Focus: Concept Development problem #4

  Check: Problem Set problem #2b

M3 [illegible]

Standard Multiplication Algorithm

Cut/consolidate: Can consolidate with Lesson 10

Goals:

- Use standard algorithm to multiply three-digit number by one-digit number

Focus: Concept Development problem #2

Partial Products	Standard Algorithm
237 × 5 35 = 5 [illegible] 150 = 5 × 3 [illegible] + [illegible] = 5 × 2 × [illegible] [illegible]	237 × 5 [illegible]

Check: Problem Set problem #1b

M3 [illegible]

Standard Multiplication Algorithm (Continued)

Goals:

- Use standard algorithm to multiply four-digit number by one-digit number

Focus: Concept Development problem #1

Check: Problem Set problem #1f

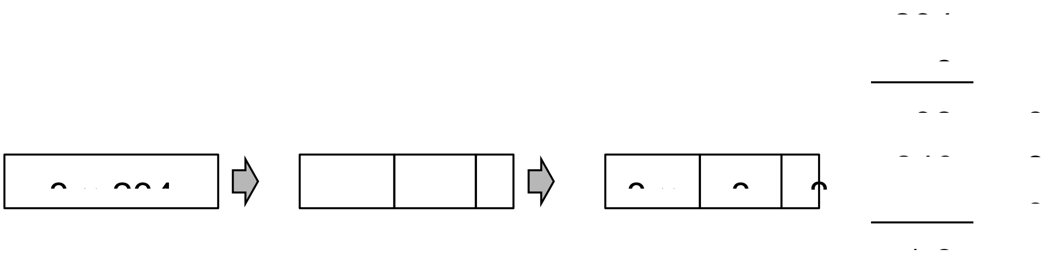
M3 [illegible]

Multiplying Three- and Four-Digit Numbers by One-Digit Number: Area Model

Goals:

- Build on **Lesson 6** and use distributive property to multiply three-digit number by one-digit number

  Focus: Concept Development problem #1



  Check: Problem Set problem #1b-c

Topic D: Multiplication Word Problems

Two-Step Word Problems

Goals:

- Build on **Lessons 2-11** to solve two-step word problems that involve multiplication

Multi-Step Word Problems

Goals:

- Build on Module 1 and Module 3 lessons to solve multi-step word problems that involve multiplication and addition and/or subtraction

Topic E: Division of Tens and Ones with Successive Remainders

Suggestions:

- Insert Lessons 20 and 21 between Lessons 17 and 18 show area model of division algorithm before focusing exclusively on practicing the division algorithm (Lesson 18)
- Omit Lesson 19 and embed discussion of interpreting remainders when using/applying the definition of division with remainder
- Show division “shorthand” (division algorithm) alongside place value chart/disks or area model

Division and Division with Remainder

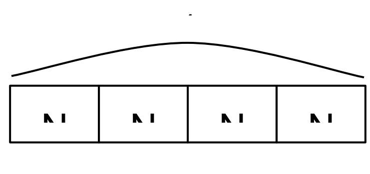
Suggestions: Split into two lessons

- Part 1 Division: partitive interpretation (Concept Development problem #1) and measurement interpretation (Concept Development problem #3 modified)
- Part 2 Division with Remainder: partitive interpretation (Concept Development problem #2) and measurement interpretation (Concept Development problem #3)
    - Assign Homework after Part 2

Goals:

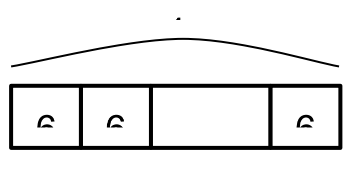
- ↻ **Review/reinforce** partitive interpretation of division (given the total and number of groups, find the number in each group)

  Focus: Concept Development problem #1

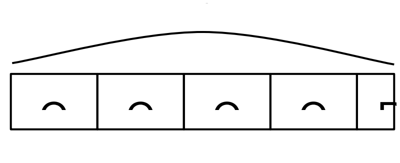
    - $4 \times N = 12$
    - $12 \div 4 = N$ ← partitive interpretation (we know how many "parts")
- ↻ **Review/reinforce** measurement interpretation of division (given the total and number in each group, find the number of groups)

  Focus: Concept Development problem #3 modified

    - $M \times 6 = 42$
    - $42 \div 6 = M$ ← measurement interpretation (we know how much to "measure")
- Define partitive interpretation of division with remainder

  Focus: Concept Development problem #2

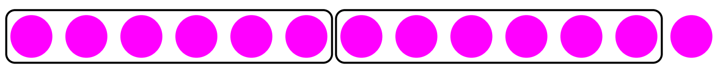


- $4 \times \mathbf{Q} + \underline{R} = 13$
- $4 \times \mathbf{3} + \underline{1} = 13$, which means 4 teams of **3** students each and <u>1</u> student leftover

Check: Problem Set problem #2

- Define measurement interpretation of division with remainder

  Focus: Concept Development problem #3

  - $\mathbf{Q} \times 6 + \underline{R} = 13$
  - $\mathbf{2} \times 6 + \underline{1} = 13$, which means **2** vases with 6 roses each and <u>1</u> rose leftover

  Check: Problem Set problem #4

Notes to teacher:

- **Lessons 16 and on** will use the partitive interpretation of division with remainder

Division with Remainder: Area Model

Goals:

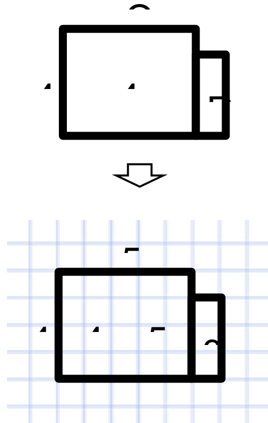
- Use array and area model to represent division with remainder

  Focus: Concept Development problem #1

  - 23 ÷ 4 means

    23 = (4 × **Q**) + <u>R</u>

    Area of figure = Area of rectangle with width 4 + Area of leftover



- ○ $23 = 4 \times \mathbf{5} + \underline{3}$

Check: Problem Set problem #3

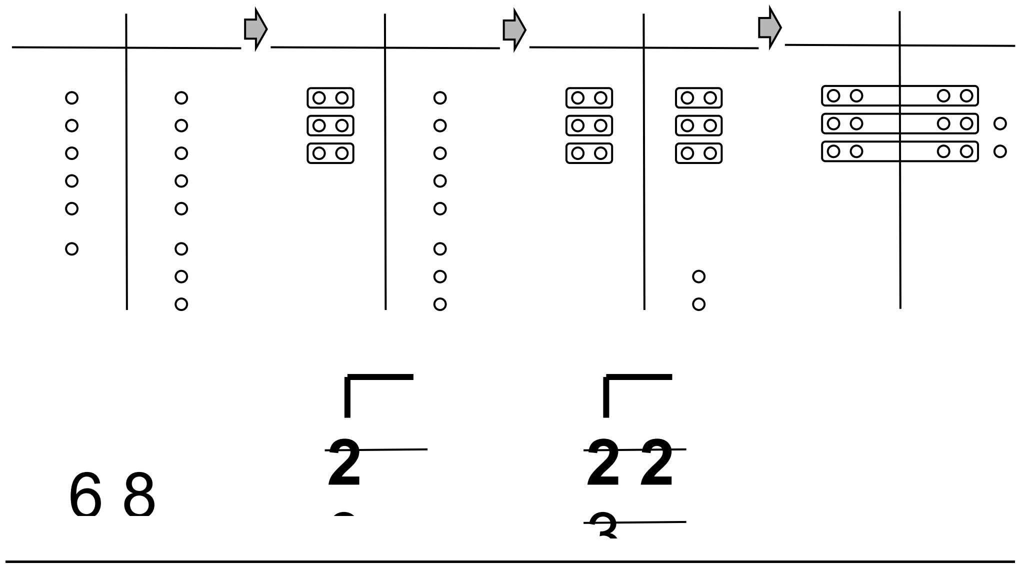
Division with Remainder: Place Value

Goals:

- Use place value chart/disks to represent division with remainder ([partitive interpretation]())

  Focus: Concept Development problem #3

  - ○ $68 \div 3$ means $68 = 3 \times \mathbf{Q} + \underline{R}$
  - ○ Goal: Divide 68 into 3 equal groups
  - ○ Strategy:
    - ■ Divide tens into equal groups
    - ■ Divide ones into equal groups



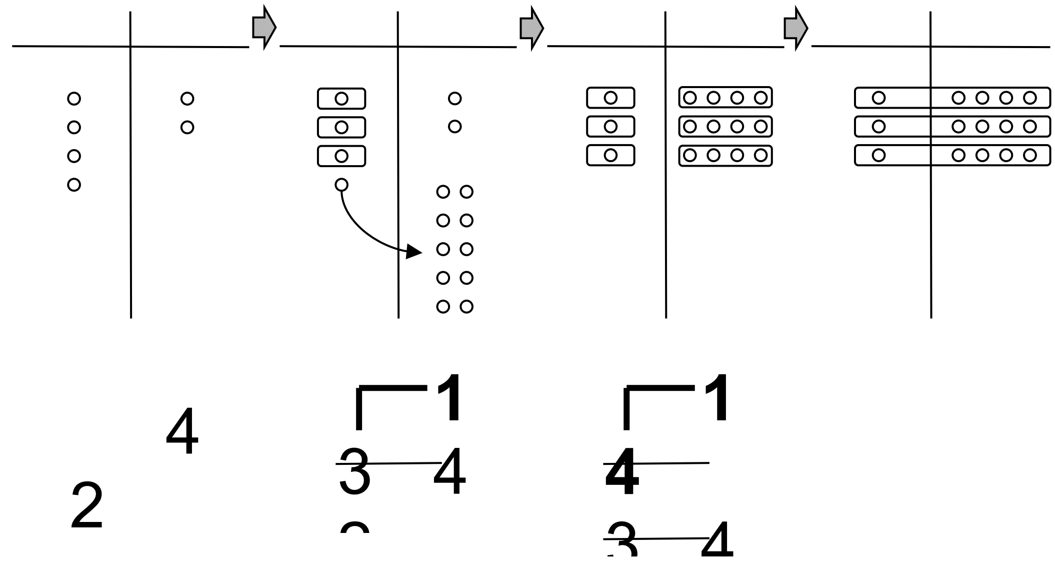
- ○ 68 = 3 × **22** + 2, Quotient = **22**, Remainder = 2

Check: Problem Set problem #4

Division with Remainder of Tens: Place Value

Goals:

- Use place value chart/disks to represent division with remainder of tens (partitive interpretation)
  - **Focus**: Concept Development problem #2
    - 42 ÷ 3 means 42 = 3 × **Q** + R
    - Goal: Divide 42 into 3 equal groups
    - Strategy:
      - Divide tens into equal groups
      - Decompose remainder of tens into ones
      - Divide ones into equal groups



- 42 = 3 × **14**, Quotient = **14**, Remainder = 0

✓ Check: Problem Set problem #4

Division with Remainder: Standard Division Algorithm

Goals:

- Transition to mental model of place value chart/disks and practice standard division algorithm (with remainder of tens and ones)

  Focus: Concept Development problem #2

  ✓ Check: Problem Set problem #6

Interpreting Remainders

Cut/consolidate: Can omit lesson by embedding discussion of interpreting remainders in earlier lessons

Goals:

- Interpret the meaning of the remainder in division with remainder problems

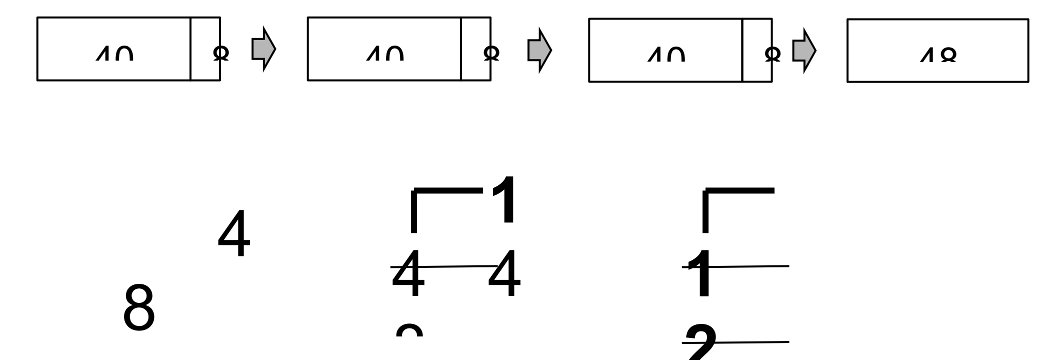
  Focus: Concept Development problem #1

  ✓ Check: Problem Set problem #1

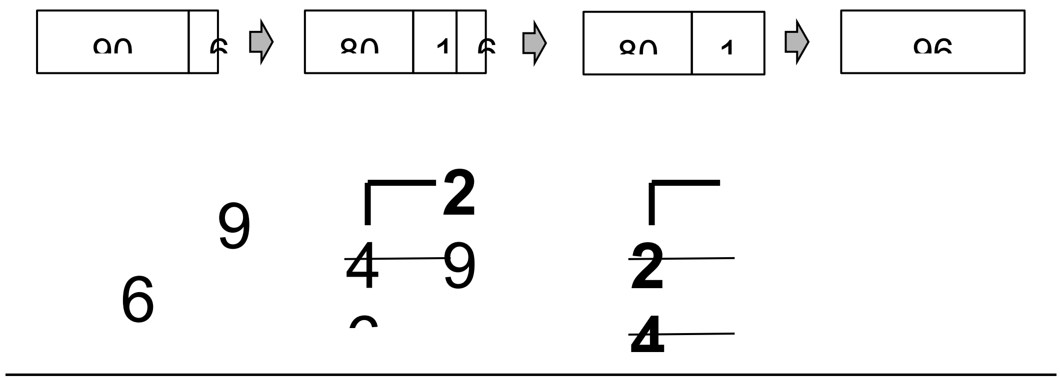
Division with Remainder of Tens: Area Model

Goals:

- Use area model and place value to represent division
  - **Focus**: Concept Development problem #1
    - 48 ÷ 4 means 48 = 4 × **Q**
    - Goal: Find length of rectangle with area 48 and width 4
    - Strategy:
      - Find length of rectangle with area 40 (tens) and width 4
      - Find length of rectangle with area 8 (ones) and width 4



    - 48 = 4 × **12**, Quotient = **12**, Remainder = 0
- Use area model to represent division with remainder of tens
  - **Focus**: Concept Development problem #3
    - 96 ÷ 4 means 96 = 4 × **Q**
    - Goal: Find length of rectangle with area 96 and width 4
    - Strategy:
      - Find length of rectangle with area 90 (tens) and width 4
      - Merge remainder of tens with ones into new rectangle
      - Find length of rectangle with area 16 (ones) and width 4



- 96 = 4 × **24**, Quotient = **24**, Remainder = 0

✔ Check: Problem Set problem #4

Division with Remainder: Area Model

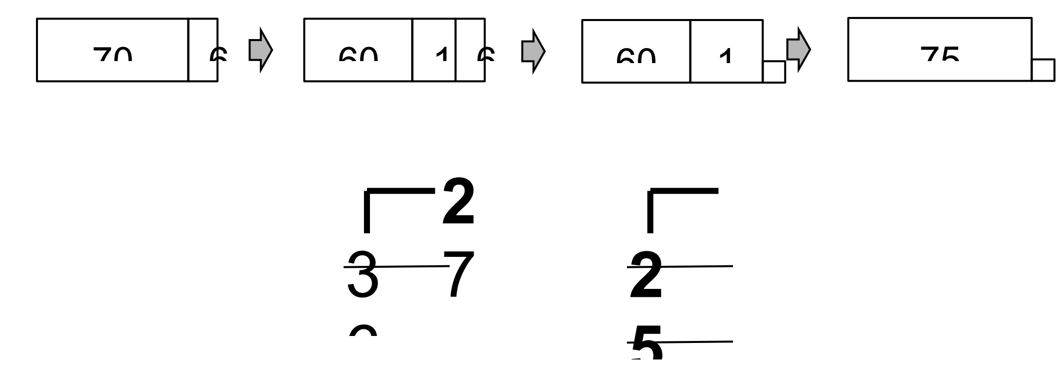
Goals:

- Build on **Lessons 15 and 20** to use area model for representing division with remainder

🔍 Focus: Concept Development problem #2

- 76 ÷ 3 means

76	=	(3 × **Q**)	+	R
Area of figure	=	Area of rectangle with width 3	+	Area of leftover



- 76 = 3 × **25** + 1, Quotient = **25**, Remainder = 1

✔ Check: Problem Set problem #1

Topic F: Reasoning with Divisibility

Cut/consolidate: Can revisit Lessons 22-25 later in the year to build fluency with multiplication, division, and division with remainder because Standard 4.OA.4 “Gain familiarity with factors and multiples” is a Supporting Cluster, not a Major Cluster Standard

Factor Pairs, Prime and Composite Numbers

Goals:

- ↻ **Review/reinforce** factors and product

  Focus: Concept Development problem #1

  Suggestions: Revisit Lesson 1 Concept Development problem #4 as an application of identifying factor pairs
- Define prime number, composite number

  Focus: Concept Development problem #2

  Check: Problem Set problem #2

Testing and Finding Factors

Goals:

- Use division with remainder to test if a number is a factor
  - Example: Is 3 a factor of 54?
    - Key idea: 3 is a factor of 54 if there is a number N so that 3 × N = 54, which means 3 × **N** + 0 = 54 (division with remainder 0)
    - 3 × **18** + 0 = 54, so 3 is a factor of 54

  Focus: Concept Development problem #1

  Check: Problem Set problem #1e & g
- Use the associative property of multiplication to find more factors or factor pairs

  Focus: Concept Development problem #2

  54
  = 6 × 9
  = (2 × 3) × 9 OR 6 × (3 × 3)
  = 2 × (3 × 9) OR (6 × 3) × 3 ← associative property

= 2 × 27 OR 18 × 3

Check: Problem Set problem #2

Testing Multiples

Goals:

- Define multiple

  Focus: Concept Development problem #1

  Check: Problem Set problem #2

- Use division with remainder to test if a number is a multiple

  Focus: Concept Development problem #2

  - Is 96 a multiple of 3?
    - Key idea: 96 is a multiple of 3 if there is a number N so that N × 3 = 96, which means **N** × 3 + <u>0</u> = 96 (division with remainder <u>0</u>)
    - **32** × 3 + <u>0</u> = 96, so 96 is a multiple of 3

  Check: Problem Set problem #3b

Exploring Prime and Composite Numbers

Goals:

- Use definitions of factor, multiple, prime number, composite number to observe:
  - Standard 4.OA.4 “A whole number is a multiple of each of its factors”
    - Example: 22 is a multiple of 2 (22 = 11 × 2) and 11 (22 = 2 × 11)
  - Key idea of Lesson 24 Concept Development problem #3: Any multiple of a whole number is also a multiple of its factors
    - Example: Multiple of 4 = N × 4 = N × (2 × 2) = (N × 2) × 2 = Multiple of 2
  - When the multiples of 2 to 10 have been crossed out in a 1-100 chart, then all the composites (up to 100) have been crossed out

Topic G: Division of Thousands, Hundreds, Tens, and Ones

Suggestions:

- If student learning of Topic E is solid, move through Topic G more quickly because it applies the same ideas to larger numbers

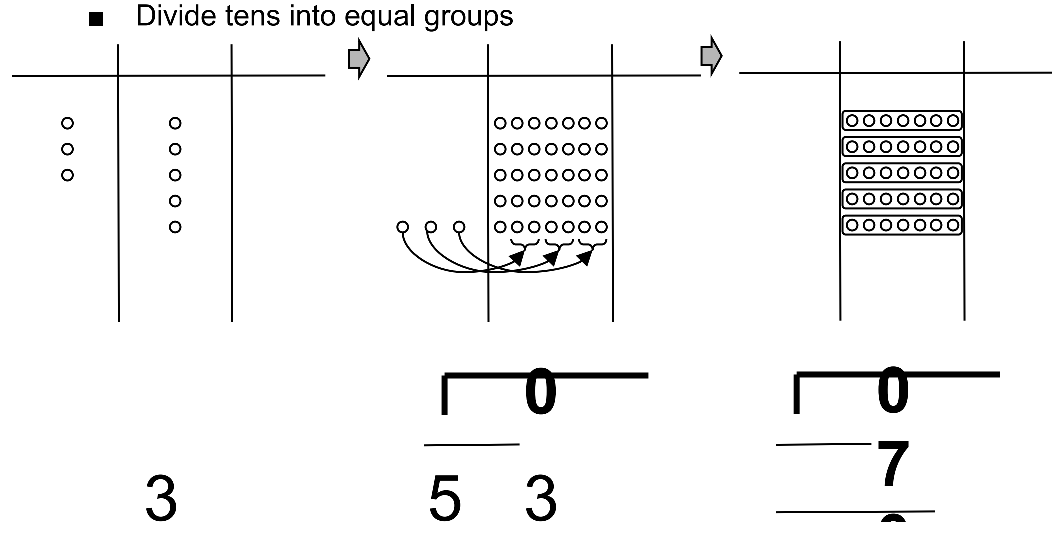
- Insert Lesson 33 after Lesson 27 or 28 to show area model of division algorithm before focusing exclusively on practicing the division algorithm (Lesson 29-30)

Division with Remainder of Thousands or Hundreds: Place Value

Goals:

- Build on **Lesson 17** to use place value chart/disks for representing division with remainder of thousands or hundreds (partitive interpretation)

  Focus: Concept Development problem #2
  - 350 ÷ 5 means 350 = 5 × **Q**
  - Goal: Divide 350 into 5 equal groups
  - Strategy:
    - Divide hundreds into equal groups
    - Decompose remainder of hundreds into tens
    - Divide tens into equal groups



  - 350 = 5 × **70**, Quotient = **70**, Remainder = 0

  Check: Problem Set problem #3h & k

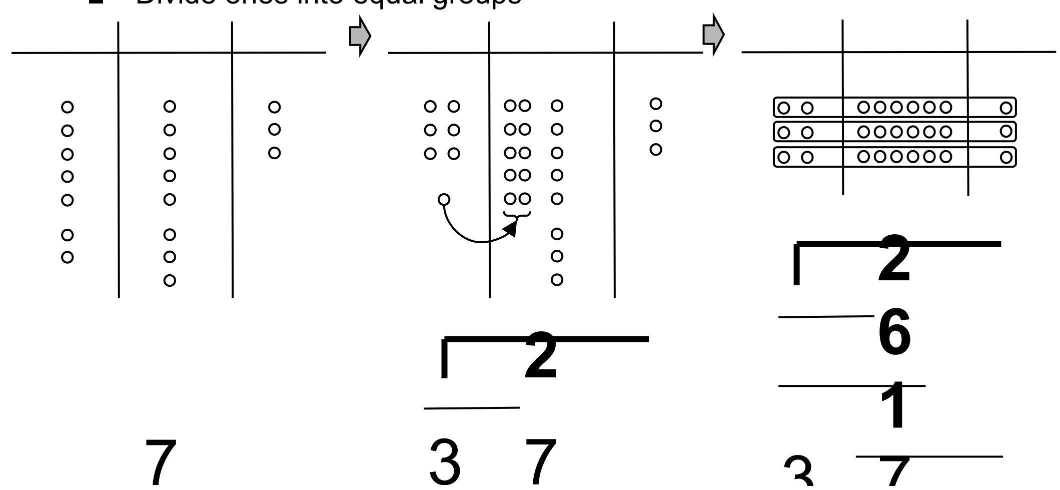
Division with Remainder of Hundreds: Place Value (Continued)

Goals:

- Build on **Lessons 17 and 26** to use place value chart/disks for representing division with remainder of hundreds (partitive interpretation)

Focus: Concept Development problem #2

- 783 ÷ 3 means 783 = 3 × **Q**
- Strategy:
  - Divide hundreds into equal groups
  - Decompose remainder of hundreds into tens
  - Divide tens into equal groups
  - Divide ones into equal groups



- 783 = 3 × **261**, Quotient = **261**, Remainder = <u>0</u>

Check: Problem Set problem #2a

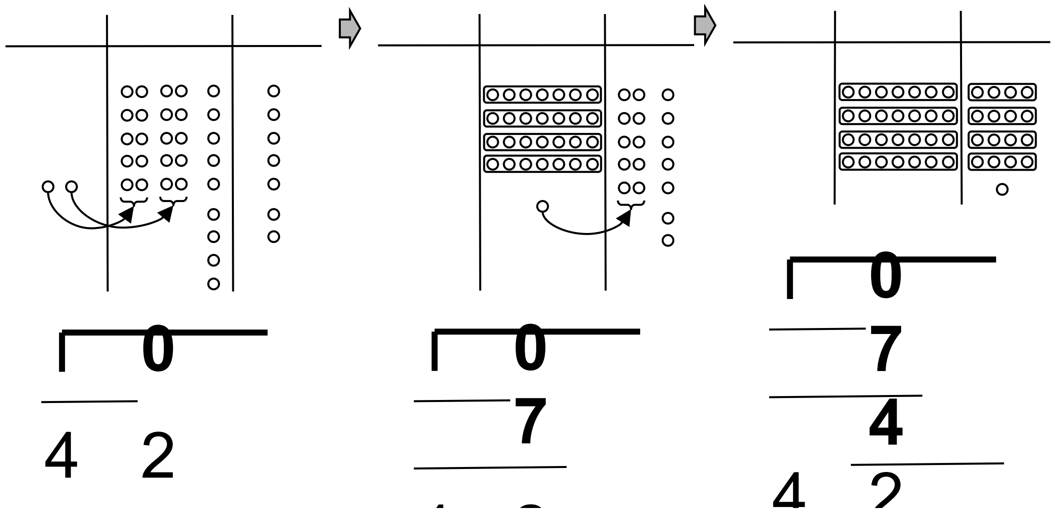
Division with Remainder: Place Value

Goals:

- Build on **Lessons 16 and 26-27** to use place value chart/disks for representing division with remainder ([partitive interpretation]())

Focus: Concept Development problem #1

- 297 ÷ 4 means 297 = 4 × **Q** + <u>R</u>
- Strategy:
  - Divide hundreds into equal groups
  - Decompose remainder of hundreds into tens
  - Divide tens into equal groups
  - Decompose remainder of tens into ones
  - Divide ones into equal groups



- 297 = 4 × **74** + 1, Quotient = **74**, Remainder = 1

✓ **Check**: Problem Set problem #1g

Division with Remainder: Standard Division Algorithm

Goals:

- Transition to mental model of place value chart/disks and practice standard division algorithm with four-digit dividend

  🔍 **Focus**: Concept Development problem #1

  ✓ **Check**: Problem Set problem #1e

Division with Zero in the Dividend or Quotient

Goals:

- Use place value chart/disks and area model to represent [division](#) ([partitive interpretation](#)) and show what the zero in the dividend means

  🔍 **Focus**: Concept Development problem #1

  ✓ **Check**: Problem Set problem #2

- Use place value chart/disks (and area model) to represent [division](#) ([partitive interpretation](#)) and show what the zero in the quotient means

  🔍 **Focus**: Concept Development problem #2

Check: Problem Set problem #3

Word Problems: Division

Goals:

- Solve word problems that involve the partitive interpretation of division: number of groups ("parts") known, number in each group unknown
  - Focus: Concept Development problem #1
  - Check: Problem Set problem #4
- Solve word problems that involve the measurement interpretation of division: number in each group (how much to "measure") known, number of groups unknown
  - Focus: Concept Development problem #3
  - Check: Problem Set problem #3

Word Problems: Division with Remainder

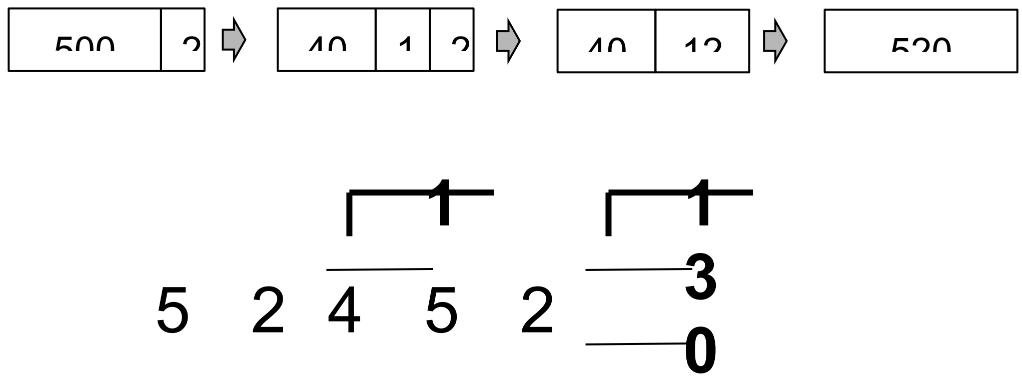
Goals:

- Solve word problems that involve the partitive interpretation of division with remainder
  - Focus: Concept Development problem #2
  - Check: Problem Set problem #2
- Solve word problems that involve the measurement interpretation of division with remainder
  - Focus: Concept Development problem #3
  - Check: Problem Set problem #3

Division with Remainder of Thousands or Hundreds: Area Model

Goals:

- Build on **Lesson 20** to use area model for representing division with remainder of thousands or hundreds
  - Example: 520 ÷ 4



520 = 4 × **130**, Quotient = **130**, Remainder = <u>0</u>

Focus: Concept Development problem #2

Check: Problem Set problem #2

Topic H: Multiplication of Two-Digit by Two-Digit Numbers

Multiplying Multiples of 10 by Two-Digit Number: Place Value

Goals:

- Build on **Lessons 4 and 7** and use [properties of multiplication]() to multiply multiple of 10 by two-digit number

Focus: Concept Development problem #3

50 × 31
= (5 × 10) × 31
= 10 × (5 × 31)
= 10 × 155
= 1,550

Key Ideas: Multiply in any order
← **Lesson 7**: 5 × 31 = 155
← **Lesson 4**: 10 × 155 = 1,550

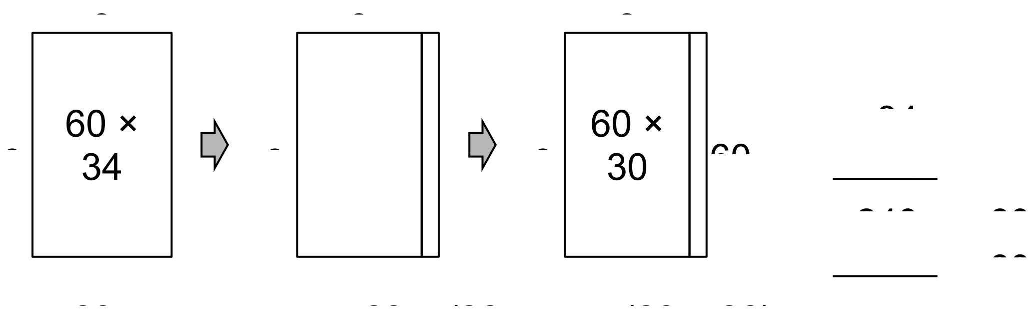
Check: Problem Set problem #2b

Multiplying Multiples of 10 by Two-Digit Number: Area Model

Goals:

- Build on **Lessons 6 and 11** and use distributive property to multiply multiple of 10 by two-digit number

Focus: Concept Development problem #3



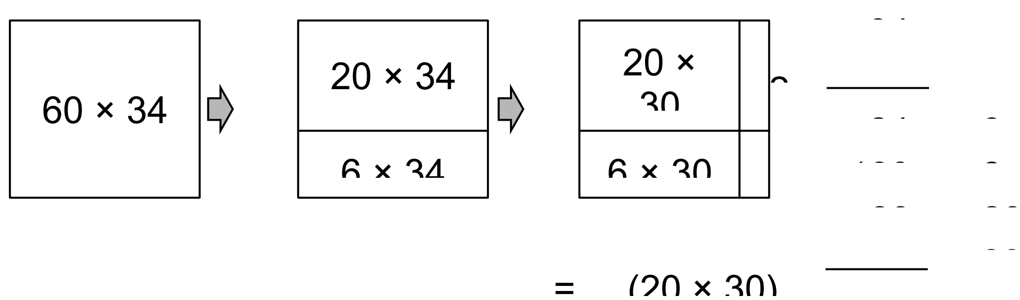
Check: Problem Set problem #2

Multiplying Two-Digit Numbers: Area Model

Goals:

- Build on **Lesson 35** and use distributive property to multiply two-digit numbers

Focus: Concept Development problem #3



Check: Problem Set problem #3

Multiplying Two-Digit Numbers: Standard Multiplication Algorithm

Goals:

- Use standard algorithm to multiply two-digit numbers

Focus: Concept Development problem #2

Check: Problem Set problem #3

Multiplying Two-Digit Numbers: Standard Multiplication Algorithm (Continued)

Goals:

- ↻ Review/reinforce Lesson 37

  Focus: Concept Development problem #2

  Check: Problem Set problem #4

Module 4: Angle Measure and Plane Figures

Definitions:

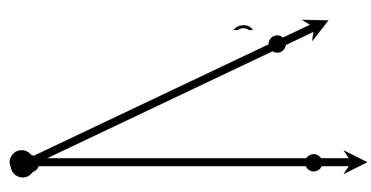
- **Point**: a dot to mark a location
    - Name with a letter
    - Example: point A

- **Line segment**: a straight segment with two endpoints (formed by connecting two points with a straightedge)
    - Name with endpoints
    - Example: segment AB, $\overline{AB}$

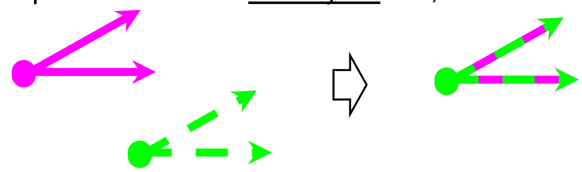
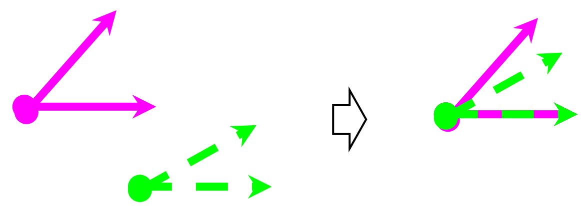
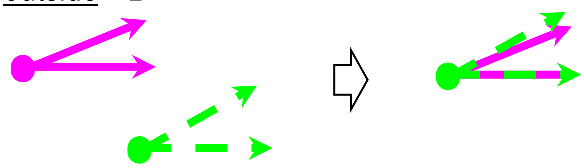
- **Line**: a straight segment that extends in both directions forever, without end (indicated with arrows at both ends)
    - Name with two points on the line
    - Example: line AB, $\overleftrightarrow{AB}$

- **Ray**: a straight segment that ends at one point and extends past another point forever (indicated with an arrow)
    - Name with *endpoint first* and then another point on ray
    - Example: ray AB, $\overrightarrow{AB}$

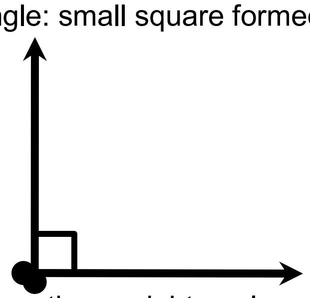
- **Angle**: two rays (sides) with the same endpoint (vertex) and the space between the rays
    - Name with the vertex OR with one point from one side, *then the vertex*, lastly one point from the other side to be more precise
    - Use small arc to mark angle
    - Example: angle CAB, $\angle CAB$



- **Equal**, **greater than**, **less than (angle)**

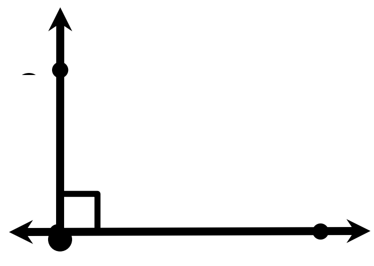
Word	*Symbolic*	*Meaning (Definition)*
$\angle A$ is equal to $\angle B$	$\angle A = \angle B$	When the vertices are put on top of each other, the sides can line up so that $\angle A$ fits exactly on $\angle B$, and vice versa 
$\angle A$ is greater than $\angle B$	$\angle A > \angle B$	When the vertices are put on top of each other, $\angle A$ can fit inside $\angle B$ 
$\angle A$ is less than $\angle B$	$\angle A < \angle B$	When the vertices are put on top of each other, $\angle A$ can extend outside $\angle B$ 

- **Right angle**:
  - Grades K-2: "L-shaped" corner; angle formed by sides of index card
  - Grade 4: 90° angle
  - Symbol to mark right angle: small square formed with vertex and sides of the angle

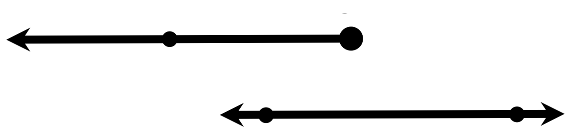


- **Acute angle**: an angle that is less than a right angle
- **Obtuse angle**: an angle that is greater than a right angle (and less than a straight angle)
- **Straight angle**: an angle that makes a line; 180° angle
- **Intersecting** (lines, line segments, rays): lines, line segments, rays that meet at a point
- **Perpendicular** (lines, line segments, rays): lines, line segments, rays that form right angle(s) at their point of intersection

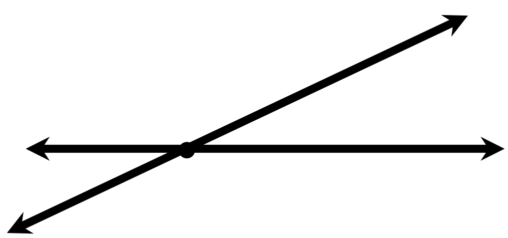
- Example: $\overleftrightarrow{AB} \perp \overrightarrow{AC}$



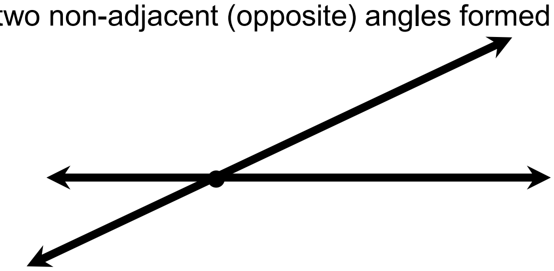
- **Parallel** (lines, line segments, rays): lines, line segments, rays that never intersect, even if the line segment(s) or ray(s) were extended into lines
  - Example: $\overleftrightarrow{AB} // \overrightarrow{CD}$



- **To measure angle**: an angle is measured with reference to a circle with its center at the vertex of the angle, "by considering the fraction of the circular arc between the points where the two sides of the angle intersect the circle" (4.MD.5a)
  - One-degree angle, 1°: an angle that turns $\frac{1}{360}$ of a circle
    - Corollary: 360° is an angle that turns full circle
  - Angle measure of *n* degrees (*n*°): an angle that turns through *n* one-degree angles (4.MD.5b)
- **Adjacent angles**: angles that share the same vertex and one side ("neighboring" or "touching" angles)



- **Vertical angles**: the two non-adjacent (opposite) angles formed by two intersecting lines



- **Line of symmetry**: the line to fold across to make sides of figure match and show symmetry
- **Triangles**:

Classification by *side lengths*	Number of sides of *equal* length

equilateral triangle	all (3)
isosceles triangle	at least 2
scalene triangle	none (0) [all 3 of different length]

Classification by *angle measures*	Types of angles
obtuse triangle	1 obtuse (> 90°) angle
right triangle	1 right (90°) angle
acute triangle	3 acute (< 90°) angles
equiangular triangle*	3 equal (60°) angles

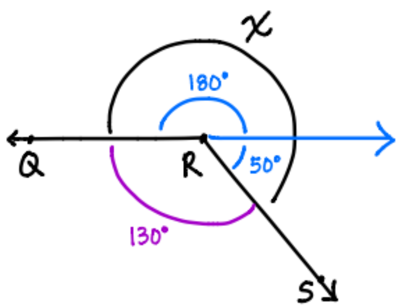
* Not in Eureka Math

- **Quadrilaterals** (Grade 3 Module 7):

Quadrilateral	Definition	Includes...
trapezoid	at least one pair of parallel sides	parallelogram, rhombus, rectangle, square
parallelogram	two pairs of parallel sides	rhombus, rectangle, square
rectangle	four right angles (two pairs of perpendicular sides)	square
square	four right angles and four equal sides (i.e., rectangle & rhombus)	

Key Ideas:

- **Angles as additive**: "When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measure of the parts" (4.MD.7)



$x + 130° = 360°$ OR $x = 180° + 50°$

Materials:

- Straightedge (or ruler)
- Protractor [Topics B-D]

- Pattern Blocks [Topic C]
- Ruler [Topic D]

Suggestions: Spread Module 4 across the year or cover after Module 6 because the concepts are Additional/Supporting Cluster Standards

Topic A: Lines and Angles

Lines and Angles

Goals:

- Define point, line segment, line

  Focus: Concept Development problem #1

  Check: Problem Set problem #2a-f
- Define ray, angle

  Focus: Concept Development problem #2

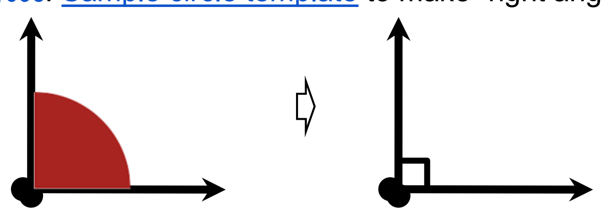
  Check: Problem Set problem #2a-g

Right, Obtuse, and Acute Angles

Goals:

- Observe that a right angle is formed when a circle is folded in half two times

  Resources: Sample circle template to make “right angle template”

  Focus: Concept Development problem #1
- Define equal, greater than, less than for angles and acute, obtuse, and straight angles

  Focus: Concept Development problems #2, #3
- Use “right angle template” to identify and draw right, acute, obtuse, and straight angles

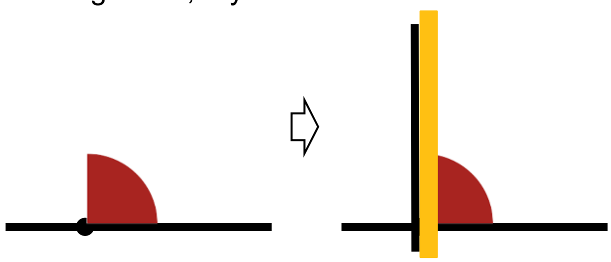
  Check: Problem Set problems #1, #3

Perpendicular Lines

Goals:

- Define intersecting, perpendicular lines/line segments/rays

  Focus: Concept Development problems #1, #3
- Use “right angle template” and straightedge to identify and draw perpendicular lines, line segments, rays



  Focus: Concept Development problems #2, #4

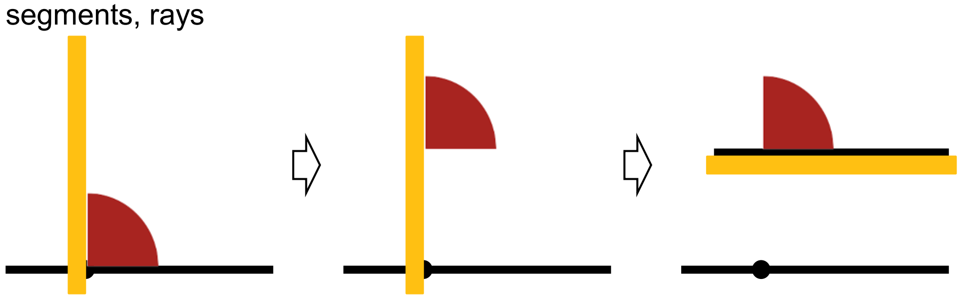
  Check: Problem Set problems #3, #4

Parallel Lines

Goals:

- Define parallel lines/line segments/rays

  Focus: Concept Development problems #1, #3
- Use “right angle template” and straightedge to identify and draw parallel lines, line segments, rays



  Focus: Concept Development problems #2, #4

  Check: Problem Set problems #3, #4

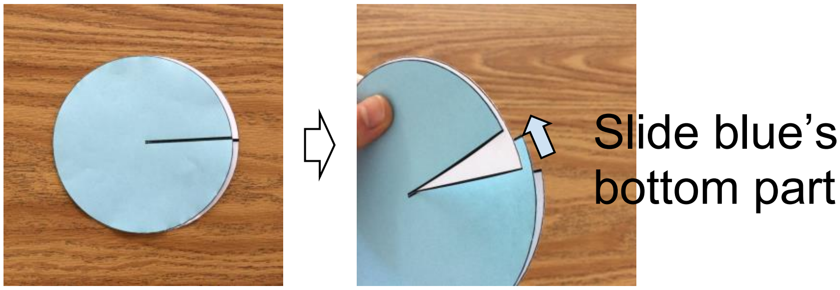
Topic B: Angle Measurement

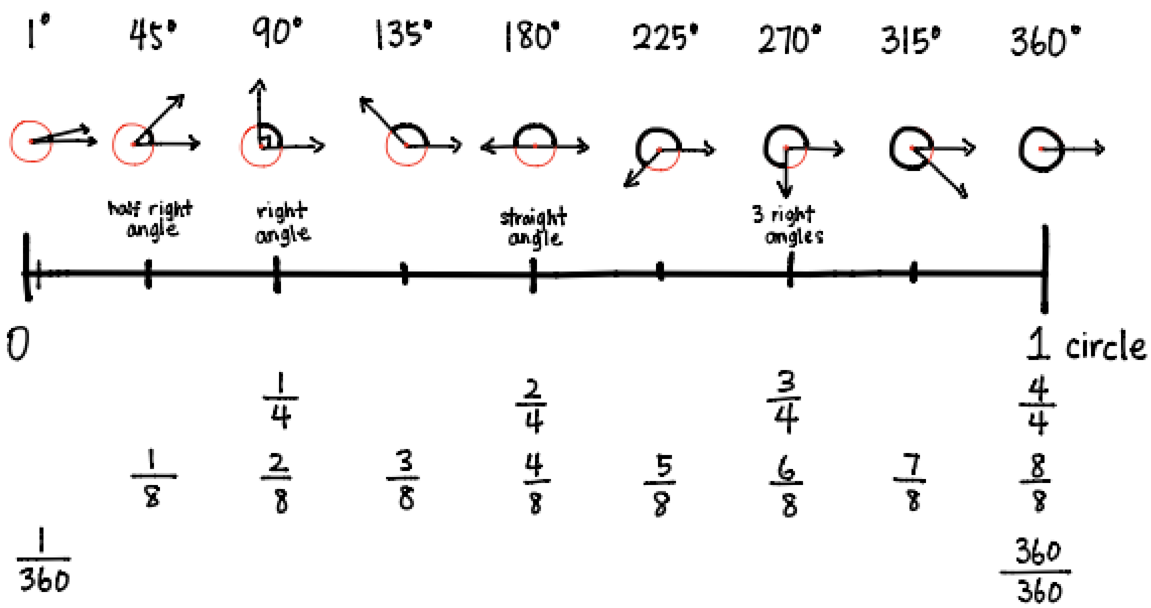
Circular Protractors and Defining Angle Measure

Goals:

- Define how [to measure angle](#) with unlabelled circular protractor and labelled (degrees) circular protractor

  Resources: Sample template for paper circular protractor





Check: Problem Set problem #6

Other Protractors

Goals:

- Use protractor (for example, semi-circular protractor) to measure angle in degrees:
  - Align "center" point/notch of protractor with vertex of angle
  - Align zero/base line of protractor with side of angle so that the angle "opens" counter-clockwise

Focus: Concept Development problem #2

Check: Problem Set problem #1

Measuring and Drawing Angles

Goals:

- Review/reinforce Lesson 6: Use protractor to measure angle less than 180°

  Focus: Concept Development problem #1
- Use protractor to draw angle less than 180°

  Focus: Concept Development problem #4

  Check: Problem Set problems #1, #2, #6

Suggestions: Skip Concept Development problems #2, #3 (Practice Sheet Figure 4) because they are not relevant for Lesson 7 Problem Set or Homework and will be covered in Lessons 9-11

Angles as Movement

Cut/consolidate: Can omit lesson because not directly relevant to Grade 4 Standards

Goals:

- Identify angle measure formed by movement:
  - Example: the angle measure of an hour or minute hand's movement from 12:00 to 3:00 is 90°

  Focus: Concept Development problem #1

  Check: Problem Set problem #2
- Identify movement defined by angle measure
  - Example: "a skateboarder does a 180" means "a skateboarder spins around to face the other way"

  Focus: Concept Development problem #3

  Check: Problem Set problem #5

Topic C: Problem Solving with the Addition of Angle Measures

Decomposing Angles with Pattern Blocks

Goals:

- Measure the interior angle of a pattern block by forming 360° (circle) around a central point (or by decomposing the interior angle with smaller blocks or composing a larger block) and use protractor to verify angle measure

  Focus: Concept Development problems #1, #2 (Problem Set problem #1)
  - Example: Hexagon
    - Three hexagons form 360° around a central point, which means 3 × (interior angle measure) = 360°
    - By definition of division, 360° ÷ 3 = interior angle measure
- Decompose angle with pattern blocks and use protractor to verify angle measure

  Focus: Concept Development problem #3 (Problem Set problem #2)

  Check: Problem Set problem #3

Addition of Adjacent Angles

Goals:

- Use angles as additive to find an unknown angle measure that is part of a right or straight angle

  Focus: Concept Development problem #3

  Check: Problem Set problem #2, #5

Addition of Adjacent Angles (Continued)

Goals:

- Use angles as additive to find an unknown angle measure that is part of a 360° angle

  Focus: Concept Development problem #1

  Check: Problem Set problem #3

- Define adjacent angles, vertical angles
- Use angles as additive to find the measurement of unknown angles given two intersecting lines and the measurement of one angle
  - Focus on one straight angle at a time
  - Observe that vertical angles have same (equal) measure

  Focus: Concept Development problem #2

  Check: Problem Set problem #6
- Solve word problems that involve finding unknown angle measure(s)

  Focus: Concept Development problem #3

  Resources: Sample slide
  - Note: Though not in the Problem Set or Homework, this is part of Standard 4.MD.7 (“Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems”)

Topic D: Two-Dimensional Figures and Symmetry

Symmetry

Goals:

- Explore symmetry in figures by folding (sides match perfectly when folded) and observe that **matching points are the same distance (equidistant) from the line of symmetry**

  Focus: Concept Development problem #1

  Resources: Doc with additional shapes that are not included in Templates

Shape	Number of Lines of Symmetry
Pentagon (Template 1)	1
Rectangle	2
Square	4
Parallelogram	0
Rhombus	2
Isosceles Trapezoid	1

Circle	Infinite

- Mentally visualize whether a line of symmetry exists in a figure, then draw line of symmetry
  - Verify existence or accuracy of line of symmetry by checking that matching points are equidistant from the line of symmetry

  Focus: Concept Development problem #2

  Check: Problem Set problem #2
- Draw the other half of a symmetrical figure given one half and the line of symmetry
  - Verify accuracy of drawing by checking that matching points are equidistant from the line of symmetry

  Focus: Concept Development problem #3

  Check: Problem Set problem #3b

  Suggestions: Scaffold by using wax paper to trace and cut out given half of figure, then flip across line of symmetry and trace to draw other half

Triangles

Goals:

- Classify triangles based on *side lengths* and *angle measures*
  - Use dash mark(s) to indicate side lengths of equal length

  Focus: Concept Development problems #1-2 (Practice Sheet)

  Check: Problem Set problem #1
- Observe that:
  - When folded on a line of symmetry, the sides and angles that match up have equal length/measure
  - The two acute angles of a right triangle form a right angle (add up to 90°)

  Focus: Concept Development problem #3

  Check: Problem Set problem #2
- Observe that a triangle cannot be formed with three collinear points (all three points on the same line)

  Focus: Concept Development problem #4

  Check: Problem Set problem #5

Triangles (Continued)

Goals:

- Construct specific triangles (obtuse isosceles triangle, right scalene triangle, etc.) with protractor and ruler
  - Focus: Concept Development problems #1, #2
  - Check: Problem Set problems #1a-b
- Reason about whether certain combinations of triangle classifications can exist
  - Focus: Concept Development problem #3

Combination	Possible?	Reason
scalene & equilateral	no	scalene has all sides unequal and equilateral has all sides equal
equilateral & obtuse	no	equilateral will mean equiangular also, so all angles equal and acute, but obtuse has one obtuse angle (and only two acute)
equilateral & right	no	equilateral will mean equiangular also, so all angles equal and acute, but right has one right angle (and only two acute)
scalene & acute	yes	example: 40°-60°-80° triangle
isosceles & equilateral	yes	equilateral has all sides equal, so at least 2 sides are equal

  - Check: Problem Set problem #5

Quadrilaterals

Goals:

- ↻ **Review/reinforce** definitions of trapezoids, parallelograms, rectangles, squares (Grade 3 Module 7)
- Construct trapezoids, parallelograms, rectangles, and squares with protractor and ruler
  - Check: Problem Set problem #5

Quadrilaterals (Continued)

Cut/consolidate: Can omit lesson because not directly relevant to Grade 4 Standards

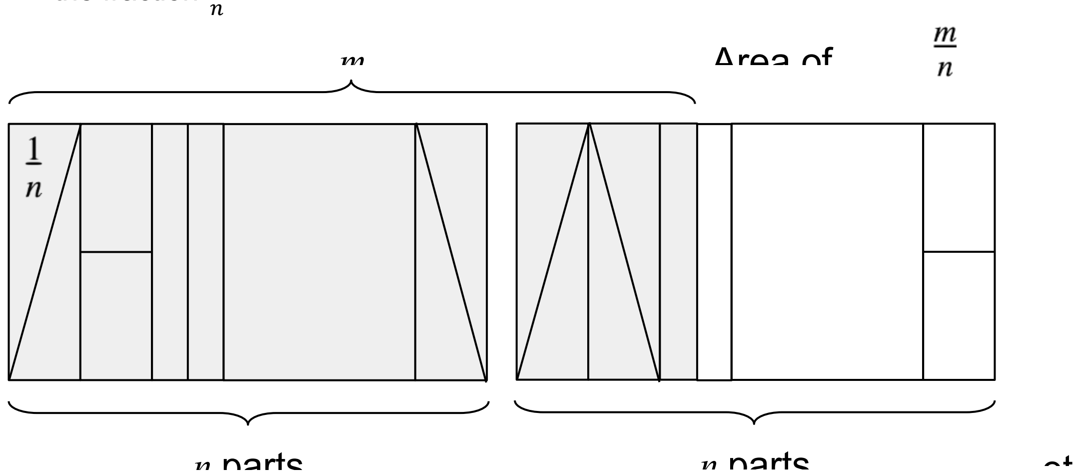
Goals:

- Construct rhombus and rectangle on triangular grid
  - **Focus**: Concept Development problems #1, #2
  - **Check**: Problem Set problem #2b
- Construct non-rectangular parallelogram on rectangular (square) grid
  - **Focus**: Concept Development problem #3
  - **Check**: Problem Set problems #1a & c

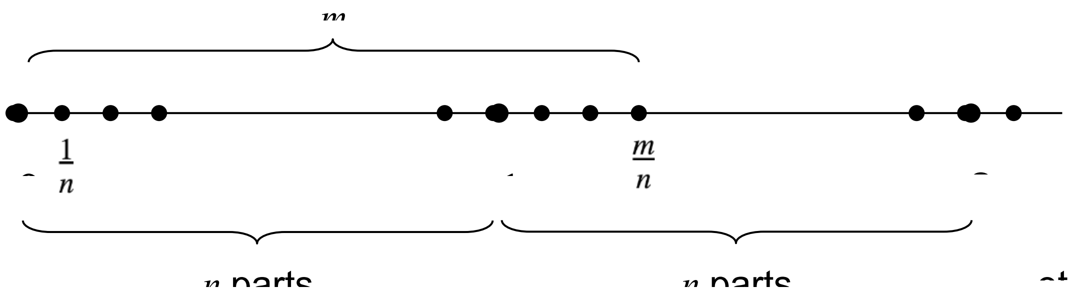
Module 5: Fraction Equivalence, Ordering, and Operations

Definitions:

- **Fraction** (Grade 3 Module 5):
  - **Geometric**: we partition each unit segment (shape, object that represents the number 1) into n parts of equal length (area, volume); then
    - **Fractional unit**: n-th
      - Example: If $n = 6$, then the fractional unit is sixth
    - **Unit fraction**: the length (area, volume) of1 copy of the parts is the unit fraction $\frac{1}{n}$
    - **Fraction**: the length (area, volume) of m copies of the parts (unit fraction $\frac{1}{n}$) is the fraction $\frac{m}{n}$



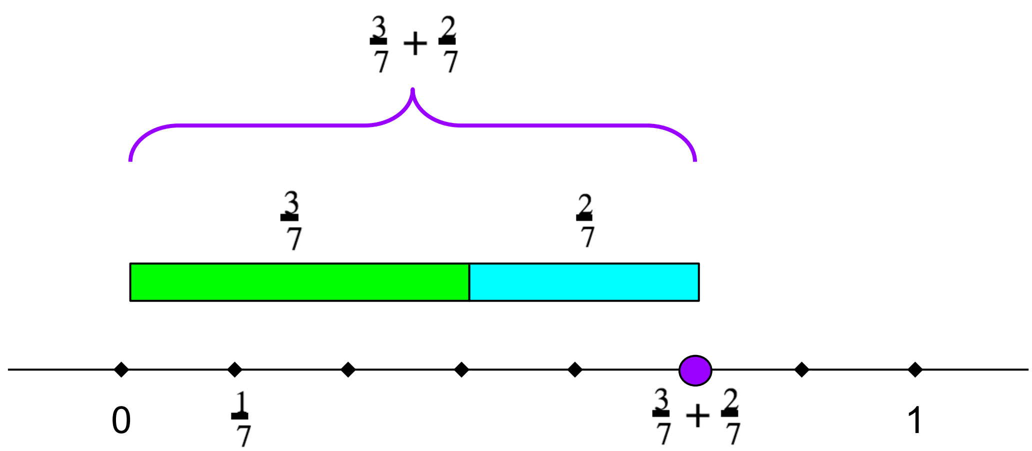
  - **Number line**: we partition each unit segment $[0,1], [1,2], [2,3], \ldots$ into n parts of equal length; then
    - **Unit fraction**: first partition point to the right of 0 is the unit fraction $\frac{1}{n}$
    - **Fraction**: the point which is the m-th copy (multiple) of the parts (unit fraction $\frac{1}{n}$) to the right of 0 is the fraction $\frac{m}{n}$



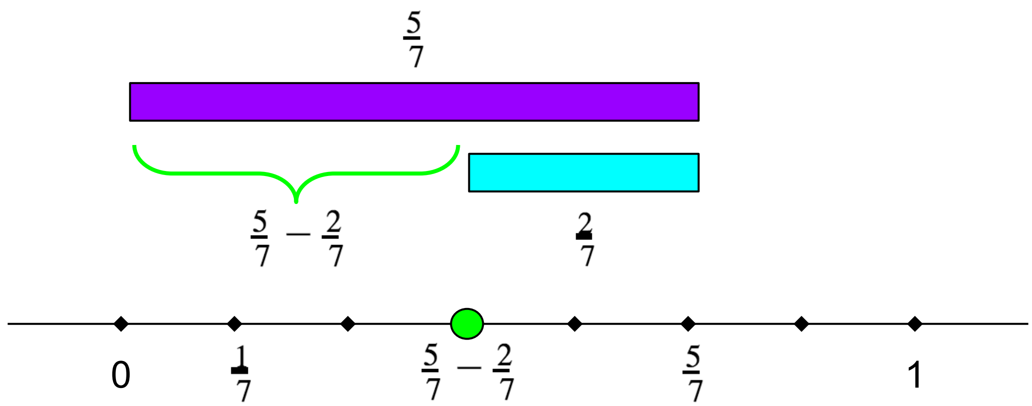
- **Equivalent** or **equal**, **greater than**, **less than** (Grade 3 Module 5):

Word	*Symbolic*	*Meaning (Definition)*
1 is equivalent or equal to $\frac{3}{3}$	$1 = \frac{3}{3}$	• Geometric: 1 and $\frac{3}{3}$ have SAME length (area, volume) • Number line:1 and $\frac{3}{3}$ are the SAME point
$\frac{3}{4}$ is greater than $\frac{1}{2}$	$\frac{3}{4} > \frac{1}{2}$	• Geometric: $\frac{3}{4}$ has MORE length (area, volume) than $\frac{1}{2}$ • Number line: $\frac{3}{4}$ is to the RIGHT of $\frac{1}{2}$
$\frac{4}{10}$ is less than $\frac{1}{2}$	$\frac{4}{10} < \frac{1}{2}$	• Geometric:$\frac{4}{10}$ has LESS length (area, volume) than $\frac{1}{2}$ • Number line: $\frac{4}{10}$ is to the LEFT of $\frac{1}{2}$

- **Addition of fractions**:



- **Subtraction of fractions**:



- **Mixed number**: "$1\frac{3}{4}$" means "$1+\frac{3}{4}$"
- **Multiplication of whole number and fraction**: "$2\times\frac{3}{4}$" means "$\frac{3}{4}+\frac{3}{4}$" (2 copies of $\frac{3}{4}$)
- **Rounding to the nearest half**: to round a fraction $\frac{□}{□}$ to the nearest half means to replace $\frac{□}{□}$ by the multiple of $\frac{1}{2}$ ($0, \frac{1}{2}, \frac{2}{2}=1, \frac{3}{2}=1\frac{1}{2}, \frac{4}{2}=2$, □□□.) which is <u>closest</u> to $\frac{□}{□}$; if two multiples of $\frac{1}{2}$ are equally close to $\frac{□}{□}$, the convention is to always choose the bigger number

Key Ideas:

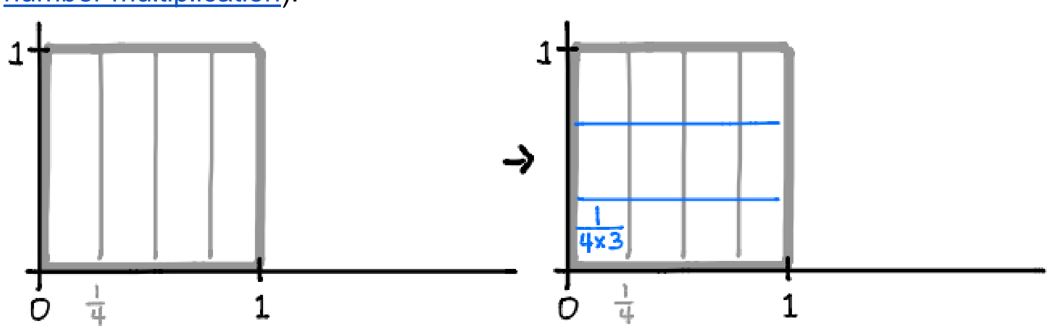
- **Fraction as multiple of unit fraction**: $\frac{a}{b}=a\times\frac{1}{b}$
  - Reasoning:

    $\frac{5}{4}$

    $=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ ← <u>definition of addition of fractions</u>

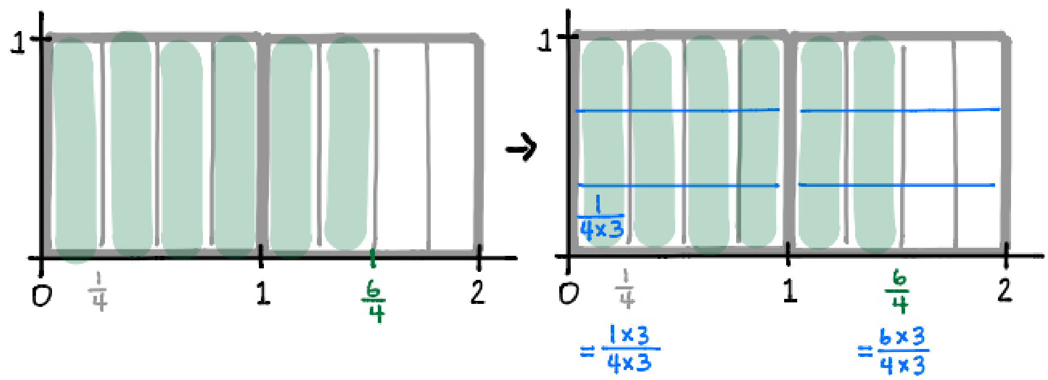
    $=5\times\frac{1}{4}$ (5 copies of $\frac{1}{4}$) ← <u>definition of multiplication of whole number & fraction</u>
- **Equivalent Fractions Theorem (multiplication)**: $\frac{a}{b}=\frac{a\times c}{b\times c}$
  - Reasoning for $\frac{6}{4}=\frac{6\times3}{4\times3}$:

    When each unit fraction $\frac{1}{4}$ is partitioned into 3 equal parts, it partitions the whole (the number 1) into 4 groups of 3 small parts, or4×3 small parts (<u>definition of whole number multiplication</u>):



And the fraction $\frac{6}{4}$ is partitioned into 6 groups of 3 small parts, or 6×3 small parts (<u>definition of whole number multiplication</u>). Then each small part is the fraction $\frac{1}{4\times3}$

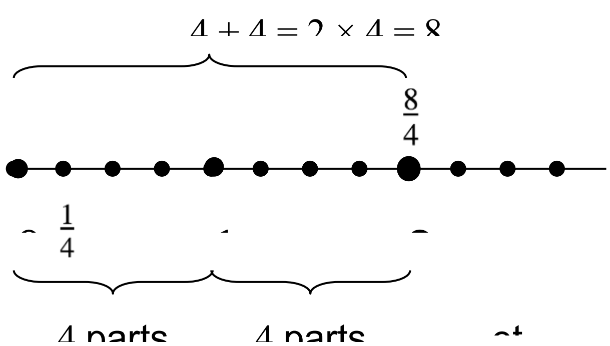
(definition of fraction), and the fraction $\frac{6}{4}$ is 6×3 copies of $\frac{1}{4\times3}$, which is the fraction $\frac{6\times3}{4\times3}$ (i.e., $\frac{6}{4} = \frac{6\times3}{4\times3}$):



- **Equivalent Fractions Theorem (whole number)**: $w = \frac{w\times n}{n}$ for whole numbers w, n
  - Example: $2 = \frac{2\times4}{4}$
    - Reasoning 1 (Grade 3 Module 5): When the fractional unit is 4ths, every unit segment is a group of 4parts (of the unit fraction $\frac{1}{4}$). Because 2 is made of 2 unit segments, then 2 is made of 2 groups of 4 parts (unit fraction $\frac{1}{4}$):

$$\begin{aligned} & 2 \\ &= (4+4) \text{ copies of } \tfrac{1}{4} && \leftarrow \text{definition of fraction, equivalent} \\ &= (2\times4) \text{ copies of } \tfrac{1}{4} && \leftarrow \text{definition of multiplication} \\ &= \tfrac{2\times4}{4} && \leftarrow \text{definition of fraction} \end{aligned}$$

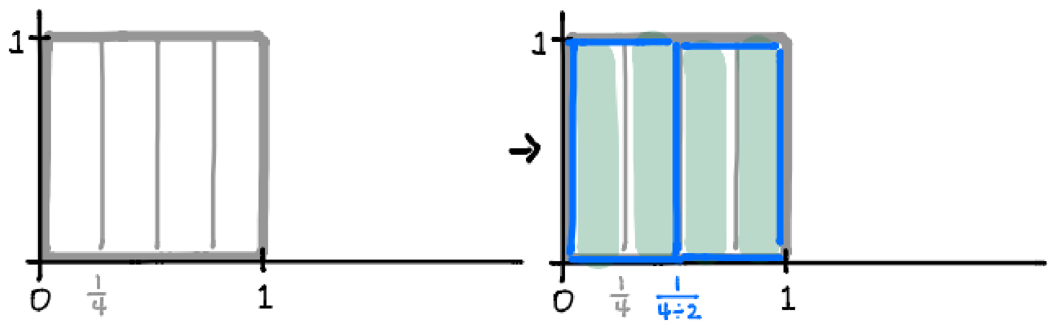
$$= \frac{8}{4}$$



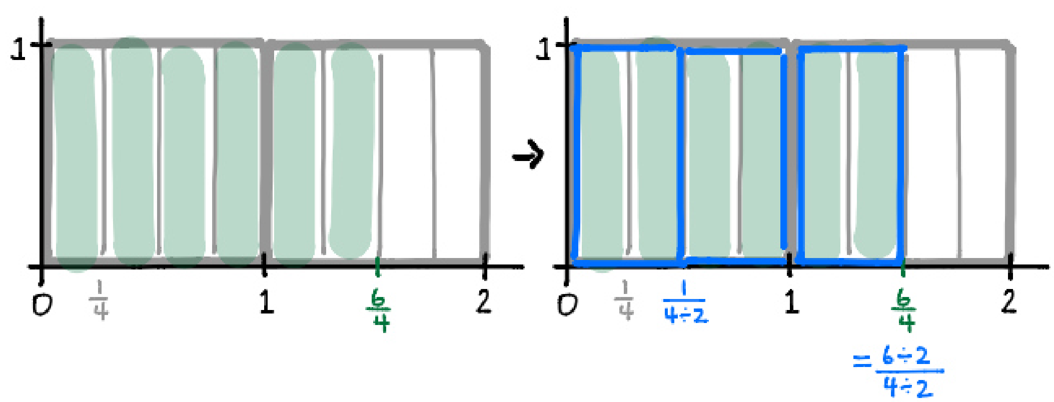
    - Reasoning 2 (Grade 4 Module 5):

$$\begin{aligned} & 2 \\ &= \tfrac{2}{1} && \leftarrow \text{definition of fraction, equivalent} \\ &= \tfrac{2\times4}{1\times4} && \leftarrow \text{Equivalent Fractions Theorem} \\ &= \tfrac{2\times4}{4} && \leftarrow 1\times4=4 \end{aligned}$$

- **Equivalent Fractions Theorem (division)**: $\frac{a}{b} = \frac{a \div c}{b \div c}$ if a and b are multiples of c (or c is a factor of a and b)
  - Reasoning for $\frac{6}{4} = \frac{6 \div 2}{4 \div 2}$:
    When the unit fractions $\frac{1}{4}$ are formed into groups of 2, it partitions the whole (the number 1) into $4 \div 2$ groups (definition of whole number division, measurement interpretation):



And the fraction $\frac{6}{4}$ is partitioned into $6 \div 2$ groups (definition of whole number division, measurement interpretation). Then each large part is the fraction $\frac{1}{4 \div 2}$ (definition of fraction), and the fraction $\frac{6}{4}$ is $6 \div 2$ copies of $\frac{1}{4 \div 2}$, which is the fraction $\frac{6 \div 2}{4 \div 2}$ (i.e., $\frac{6}{4} = \frac{6 \div 2}{4 \div 2}$):



- **Addition and subtraction of fractions**: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$, $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$
- **Addition**: Because of the associative and commutative properties of addition, we can add numbers in *any order* and still keep the sum (total) the same
- **Subtraction**: We can subtract parts of the subtrahend in *any order* and from *any part* of the minuend (total) that is greater than or equal to the subtrahend part(s) and still keep the difference the same
- **Multiplication of whole number and fraction**: $n \times \frac{a}{b} = \frac{n \times a}{b}$
  - Reasoning 1:
    $7 \times \frac{5}{4}$
    $= \frac{5}{4} + \frac{5}{4} + \frac{5}{4} + \frac{5}{4} + \frac{5}{4} + \frac{5}{4} + \frac{5}{4}$ ← definition of multiplication of whole number & fraction

$= 7 \times 5$ copies of $\frac{1}{4}$ ← definition of multiplication

$= \frac{7\times5}{4}$ ← definition of fraction

- Reasoning 2:

  $7 \times \frac{5}{4}$

  $= 7 \times \left(5 \times \frac{1}{4}\right)$ ← fraction as multiple of unit fraction: $\frac{a}{b} = a \times \frac{1}{b}$

  $= (7 \times 5) \times \frac{1}{4}$ ← multiply in any order

  $= \frac{7\times5}{4}$ ← fraction as multiple of unit fraction: $a \times \frac{1}{b} = \frac{a}{b}$

Topic A: Decomposition and Fraction Equivalence

Suggestions:

- Include number line with paper strips or tape diagrams
- Use 1 × 1 (unit) square for area model, instead of rectangle
  - Rationale: The area of a unit square is 1 (Grade 3 Module 4) so when $\frac{a}{b}$ of the square is shaded, the area of the shaded region will equal the number $\frac{a}{b}$ (instead of only $\frac{a}{b}$ of the rectangle's area).

Fraction as Sum of Fractions

Goals:

- ↻ **Review/reinforce** definitions of fraction, equivalent fractions (Grade 3 Module 5) and define addition of fractions, mixed number
- Use visual diagrams (paper strips, number bonds) and definition of addition of fractions to write a fraction as a sum (decomposition) of smaller fractions

  Focus: Concept Development problems #1, #3

  Check: Problem Set problems #1b & g
- Use visual diagrams (paper strips, number bonds) and definitions of equivalent fractions and mixed number to write a fraction greater than 1 as a mixed number

  Focus: Concept Development problems #2, #3

  Check: Problem Set problem #1g
  - Add instruction: "If a fraction is greater than 1, also express it as a mixed number."
- Draw and label a tape diagram (and/or number line) to represent an addition equation

  Focus: Concept Development problem #4

  Check: Problem Set problems #2c-d & g

Fraction as Sum of Fractions (Continued)

Goals:

- Build on **Lesson 1** to represent a fraction as different sums (decompositions)
  - Example: $\frac{5}{4} = \frac{2}{4} + \frac{2}{4} + \frac{1}{4}, \frac{5}{4} = \frac{1}{4} + \frac{3}{4} + \frac{1}{4}, \frac{5}{4} = 1 + \frac{1}{4}$, etc.

Focus: Concept Development problem #3

Check: Problem Set problems #1b, #2c

Fraction as Multiple of Unit Fraction

Goals:

- Define multiplication of whole number and fraction and conclude fraction as multiple of unit fraction: $\frac{\square}{\square} = \square \times \frac{1}{\square}$

  Focus: Concept Development problems #1, #2

  Check: Problem Set problems #1c & e

Notes to teacher:

- Will revisit/reinforce multiplication of whole number and unit fraction in **Lesson 23**

Equivalent Fractions: Linear Model (Tape Diagram)

Goals:

- Generate equivalent fractions by decomposing a fraction into a smaller unit fraction using linear model (tape diagram or number line)

  Focus: Concept Development problem #2

  Check: Problem Set problems #1c, #2b
- Use linear model (tape diagram or number line) to show two fractions are equivalent (equal)

  Focus: Concept Development problem #3

  Check: Problem Set problem #3c

Equivalent Fractions: Area Model

Goals:

- Generate equivalent fractions by decomposing a fraction into a smaller unit fraction using area model
  - Focus: Concept Development problem #1
  - Check: Problem Set problem #1c
- Use area model to show two fractions are equivalent (equal)
  - Focus: Concept Development problem #3
  - Check: Problem Set problem #2a

Equivalent Fractions: Area Model (Continued)

Goals:

- ↻ Review/reinforce Lesson 5
  - Focus: Concept Development problem #2
  - Check: Problem Set problem #2a

Topic B: Fraction Equivalence Using Multiplication and Division

Equivalent Fractions Theorem (Multiplication): Unit Fraction

Goals:

- Use area model to show Equivalent Fractions Theorem (multiplication) for unit fraction: $\frac{1}{b} = \frac{1 \times c}{b \times c}$
  - Focus: Concept Development problem #1
  - Check: Problem Set problem #1

Equivalent Fractions Theorem (Multiplication)

Goals:

- Use area model to show Equivalent Fractions Theorem (multiplication): $\frac{a}{b} = \frac{a \times c}{b \times c}$

  Focus: Concept Development problem #1

  Check: Problem Set problems #1b & d
- Use area model and/or Equivalent Fractions Theorem (multiplication) to check if two fractions are equivalent (equal)

  Focus: Concept Development problem #2

  Check: Problem Set problems #5a-b
- Use Equivalent Fractions Theorem (multiplication) to generate equivalent fractions

  Focus: Concept Development problem #3

  Check: Problem Set problem #4c

Suggestions: Include an example of equivalent fractions greater than 1 in preparation for Problem Set and Homework

Equivalent Fractions Theorem (Division): Unit Fraction

Goals:

- Use area model to show Equivalent Fractions Theorem (division) for unit fraction: $\frac{a}{b} = \frac{a \div a}{b \div a}$

  Focus: Concept Development problem #2

  Check: Problem Set problem #2c

Equivalent Fractions Theorem (Division)

Goals:

- Use area model to show Equivalent Fractions Theorem (division): $\frac{a}{b} = \frac{a \div c}{b \div c}$

  Focus: Concept Development problem #1

  Check: Problem Set problems #1b & d

- Use Equivalent Fractions Theorem (division) and largest common factor to "simplify a fraction"
    - To "simplify a fraction" (or write the fraction in simplest form) means to find the equivalent fraction so that the Equivalent Fractions Theorem (division) $\frac{a}{b} = \frac{a \div c}{b \div c}$ cannot be used anymore (i.e., there is no more common factor in numerator and denominator)

  Focus: Concept Development problem #4

  Check: Problem Set problem #4b

Equivalent Fractions Theorem: Linear Model

Goals:

- Use linear model (number line or tape diagram) to show Equivalent Fractions Theorem (multiplication and division)

  Focus: Concept Development problem #2

  Check: Problem Set problems #1a-b, #2a

Suggestions: Skip Concept Development problem #3, Problem Set problem #5, and Homework problem #5 because decomposing a fraction into *n* equal lengths is not a Grade 4 standard

Topic C: Fraction Comparison

Comparing Fractions Less Than 1

Goals:

- ↻ **Review/reinforce** definitions of greater than, less than (Grade 3 Module 5)
- Use definitions of equivalent, greater than, less than and the Equivalent Fractions Theorem to compare fractions less than 1

  Focus: Concept Development problem #2

  $\frac{5}{12} < \frac{1}{2}$ because $\frac{1}{2} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12}$ (Equivalent Fractions Theorem) and $\frac{5}{12}$ is to the left of $\frac{6}{12}$ on the number line so $\frac{5}{12} < \frac{6}{12}$ (definition of less than) and $\frac{5}{12} < \frac{1}{2}$

Check: Problem Set problem #1

Comparing Fractions Between 1 and 2

Goals:

- Use definitions of equivalent, greater than, less than and the Equivalent Fractions Theorem to compare fractions between 1 and 2

  Focus: Concept Development problem #1

  Check: Problem Set problems #1, #2

Comparing Fractions with Common Numerator or Denominator

Goals:

- Use linear model (number line or tape diagram) to show:
  If $a < b$, then $\frac{1}{a} > \frac{1}{b}$ and $\frac{n}{a} > \frac{n}{b}$ for any positive whole number n.

  Focus: Concept Development problem #1

  Check: Problem Set problems #1c-d
- Use Equivalent Fractions Theorem to make common numerator or denominator to compare fractions

  Focus: Concept Development problems #1, #2

  Check: Problem Set problems #2d, #4a

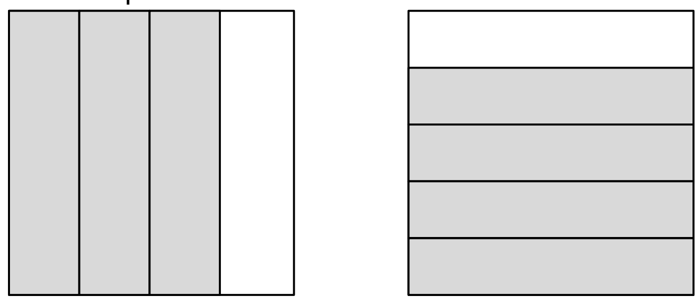
Comparing Fractions: Area Model

Goals:

- Use definitions of equivalent, greater than, less than and the Equivalent Fractions Theorem to compare fractions using the area model

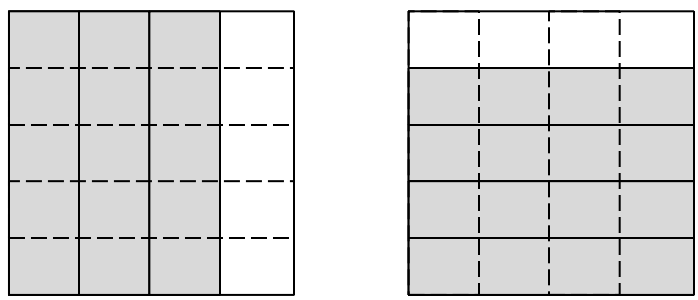
Focus: Concept Development problem #1

Strategy: Represent one fraction with vertical partitions and the other fraction with horizontal partitions:



$\frac{3}{4}$ $\frac{4}{5}$

Then partition in the other direction to form common unit fractions for comparison:



$\frac{3\times5}{4\times5} < \frac{4\times4}{5\times4}$

Check: Problem Set problems #1b & d

Notes to teacher:

- Will revisit/reinforce fraction comparison in **Lesson 27**

Topic D: Fraction Addition and Subtraction

Adding and Subtracting Fractions with Same Denominator

Suggestions: Do Concept Development problem #3 (addition of fractions) before problems #1-2 (subtraction of fractions)

Goals:

- Use definition of addition of fractions (Lesson 1) to add fractions with the same denominator and observe that $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

Focus: Concept Development problem #3

Check: Problem Set problem #5a

- Use definition of subtraction of fractions to subtract fractions with the same denominator and observe that $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$

  Focus: Concept Development problem #1

  Check: Problem Set problem #2a

- ↻ Review/reinforce Lesson 1: Use definition of mixed number to rewrite a sum or difference greater than 1 as a mixed number

  Focus: Concept Development problem #2 or #4

  Check: Problem Set problems #3c & #6d

Subtracting Fractions from Numbers Between 1 and 2

Goals:

- Use definition of subtraction of fractions to subtract fractions from 1

  Focus: Concept Development problem #1

  Check: Problem Set problem #2d

- Use definitions of subtraction of fractions and mixed number to subtract a fraction from a number between 1 and 2

  Focus: Concept Development problem #2

  $1\frac{1}{5}-\frac{2}{5}$

  $=1+\frac{1}{5}-\frac{2}{5}$ ← definition of mixed number

  $=\frac{5}{5}+\frac{1}{5}-\frac{2}{5}$ OR $\left(1-\frac{2}{5}\right)+\frac{1}{5}$ Key Ideas: Subtract from any part of minuend

  $=\frac{5+1-2}{5}$ OR $\frac{3}{5}+\frac{1}{5}$ Key Ideas: $\frac{a}{c}\pm\frac{b}{c}=\frac{a\pm b}{c}$

  $=\frac{4}{5}$

  Check: Problem Set problem #3c

Adding and Subtracting Three or More Fractions

Goals:

- Build on **Lessons 16-17** to add or subtract three or more fractions

  Focus: Concept Development (Practice Sheet) problems B, C

  Check: Problem Set problems #1c-d

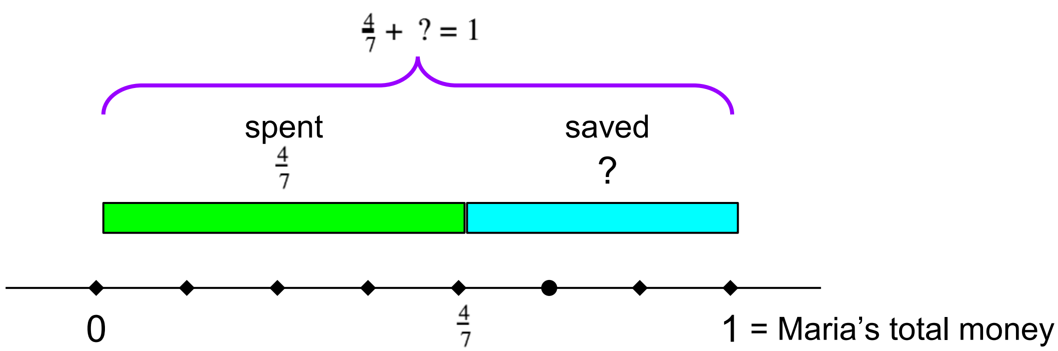
Word Problems

Goals:

- Solve word problems involving addition or subtraction of fractions

  Suggestions: Include number line and label what the whole (1) represents

  Focus: Concept Development (Problem Set) problem #3



  Check: Problem Set problem #5

Adding Fractions with Related Denominators

Goals:

- Use the [Equivalent Fractions Theorem (multiplication)]() and [addition of fractions]() to add unit fractions with related denominators

  Focus: Concept Development problem #1

$$\frac{1}{2} + \frac{1}{8}$$
$$= \frac{1\times4}{2\times4} + \frac{1}{8}$$
$$= \frac{4+1}{8}$$

Key Ideas: $\frac{1}{b} = \frac{1\times c}{b\times c}$

Key Ideas: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

$$= \frac{5}{8}$$

Check: Problem Set problem #1b

- Use the Equivalent Fractions Theorem (multiplication) and addition of fractions to add fractions with related denominators

  Focus: Concept Development problem #2

  $\frac{3}{4} + \frac{5}{8}$

  $= \frac{3 \times 2}{4 \times 2} + \frac{5}{8}$ **Key Ideas**: $\frac{a}{b} = \frac{a \times c}{b \times c}$

  $= \frac{6+5}{8}$ **Key Ideas**: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

  $$= \frac{11}{8}$$

  Check: Problem Set problem #2c

Adding Fractions with Related Denominators (Continued)

Goals:

- **Review/reinforce** Lesson 20
  - Use definition of mixed number to rewrite a sum or difference greater than 1 as a mixed number

  Focus: Concept Development problem #2

  Check: Problem Set problems #2c

Topic E: Extending Fraction Equivalence to Fractions Greater Than 1

Adding to or Subtracting from Whole Numbers Greater Than 1

Suggestions: Include number line to represent and solve problems

Goals:

- **Review/reinforce** Lesson 1: Use definition of mixed number to write a sum that involves a whole number greater than 1

  Focus: Concept Development problem #1

Check: Problem Set problem #1b

- Build on **Lesson 17** to subtract a fraction from a whole number greater than 1

  Focus: Concept Development problem #2

  $3-\frac{1}{4}$

  $=2+1-\frac{1}{4}$

  $=2+\left(\frac{4}{4}-\frac{1}{4}\right)$ **Key Ideas**: Subtract from any part of minuend

  $=2+\frac{4-1}{4}$ **Key Ideas**: $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$

  $=2+\frac{3}{4}$

  $=2\frac{3}{4}$ ← definition of mixed number

  OR $3-\frac{1}{4}$

  $=\frac{3\times4}{4}-\frac{1}{4}$ **Key Ideas**: $w=\frac{w\times n}{n}$

  $=\frac{12-1}{4}$ **Key Ideas**: $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$

  $=\frac{11}{4}$

  Check: Problem Set problem #3b

- Write equivalent addition and subtraction equations with three given numbers

  Focus: Concept Development problem #3

  $4\frac{1}{3}+\frac{2}{3}=5,\ \frac{2}{3}=5-4\frac{1}{3}$, etc.

  Check: Problem Set problem #2

Multiplying Whole Number and Unit Fraction

Goals:

- Build on **Lesson 3** to multiply whole number and unit fraction

  Focus: Concept Development problems #2, #3

  $9\times\frac{1}{4}$

  $=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ ← definition of multiplication of whole number and fraction

  $=\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)+\frac{1}{4}$ **Key Ideas**: Add in any order

$$= 2 + \frac{1}{4}$$

$= 2\frac{1}{4}$ ← definition of mixed number

Check: Problem Set problems #3c, #4b

From Fraction to Mixed Number

Cut/consolidate: Can consolidate with Lesson 25

Goals:

- Write a fraction greater than 1 as a mixed number

Focus: Concept Development problem #1

$\frac{7}{3}$

$$= \frac{3+3+1}{3}$$

$= \frac{3}{3} + \frac{3}{3} + \frac{1}{3}$ **Key Ideas**: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$$= 2 + \frac{1}{3}$$

$= 2\frac{1}{3}$ ← definition of mixed number

Check: Problem Set problem #2c

From Mixed Number to Fraction

Goals:

- Write a mixed number as a fraction

Focus: Concept Development problem #1

$2\frac{1}{6}$

$= 2 + \frac{1}{6}$ ← definition of mixed number

$= \frac{2\times 6}{6} + \frac{1}{6}$ **Key Ideas**: $w = \frac{w \times n}{n}$

$$= \frac{12}{6} + \frac{1}{6}$$

$= \frac{13}{6}$ **Key Ideas**: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

Check: Problem Set problem #1c

Comparing Numbers Greater Than 1: Number Line

Cut/consolidate: Can consolidate with Lesson 27

Goals:

- Build on **Lessons 13-14** to compare fractions and mixed numbers greater than 1 using the number line

  Focus: Concept Development problem #1

  Check: Problem Set problem #2a-b

Comparing Numbers Greater Than 1: Tape Diagram, Area Model

Goals:

- Build on **Lesson 15** to compare fractions and mixed numbers greater than 1 using a tape diagram or the area model

  Focus: Concept Development problem #2

  Check: Problem Set problems #3f-g

Line Plots

Cut/consolidate: Can omit lesson or consolidate with Lesson 40

Goals:

- Make a line plot to display a data set of measurements with fractions and mixed numbers
- Solve word problems involving addition and subtraction of fractions by using information presented in line plot

Topic F: Addition and Subtraction of Fractions by Decomposition

Estimating Sums and Differences

Cut/consolidate: Can omit lesson and embed estimation in Lessons 30-34

Goals:

- Use definitions of addition and subtraction of fractions and rounding to the nearest half to estimate a sum or difference of fractions greater than 1

  Focus: Concept Development problems #2, #3

  Check: Problem Set problems #1b-c

Adding Mixed Number and Fraction

Goals:

- Add mixed number and fraction *without* composing another whole (1)

  Focus: Concept Development problem #1

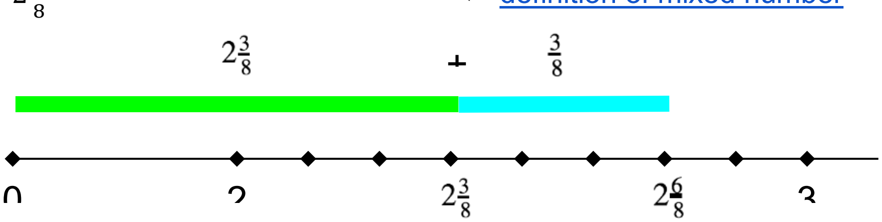
  $2\frac{3}{8} + \frac{3}{8}$

  $= 2 + \frac{3}{8} + \frac{3}{8}$ ← definition of mixed number

  $= 2 + \left(\frac{3}{8} + \frac{3}{8}\right)$ **Key Ideas**: Add in any order

  $= 2 + \frac{6}{8}$

  $= 2\frac{6}{8}$ ← definition of mixed number

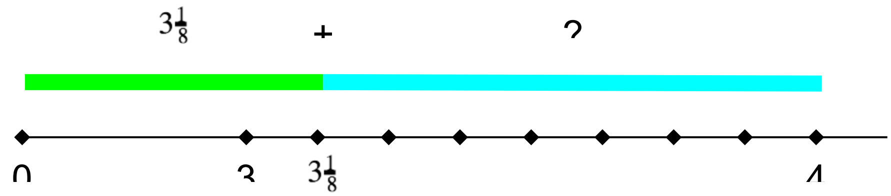


  Check: Problem Set problem #1c

- Find fraction to add to mixed number to make next whole number (the difference between a mixed number and the next whole number)

  Focus: Concept Development problem #2

$$3\frac{1}{8} + ? = 4$$



Check: Problem Set problem #2b

- Add mixed number and fraction *with* composing another whole (1)

  Focus: Concept Development problems #3, #4

  $5\frac{2}{4} + \frac{3}{4}$

  $= 5 + \frac{2}{4} + \frac{3}{4}$ ← definition of mixed number

  $= 5 + \frac{5}{4}$ OR $5 + \left(\frac{2}{4} + \frac{2}{4}\right) + \frac{1}{4}$ **Key Ideas**: Add in any order

  $= 5 + \frac{4}{4} + \frac{1}{4}$ **Key Ideas**: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

  $= 5 + 1 + \frac{1}{4}$

  $= 6\frac{1}{4}$ ← definition of mixed number

  Check: Problem Set problem #4c

Adding Mixed Numbers

Goals:

- Build on **Lesson 30** to add mixed numbers

  Focus: Concept Development problems #1, #2

  $2\frac{1}{8} + 1\frac{5}{8}$

  $= 2 + \frac{1}{8} + 1 + \frac{5}{8}$ ← definition of mixed number

  $= (2 + 1) + \left(\frac{1}{8} + \frac{5}{8}\right)$ **Key Ideas**: Add in any order

  $= 3 + \frac{6}{8}$ **Key Ideas**: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

  $= 3\frac{6}{8}$ ← definition of mixed number

  Check: Problem Set problems #1b, #4b

Subtracting Fraction from Mixed Number

Goals:

- Subtract fraction from mixed number *without* decomposing a whole (1)

  Focus: Concept Development problem #1

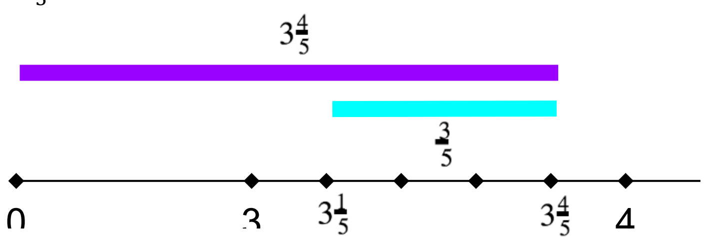
  $3\frac{4}{5} - \frac{3}{5}$

  $= 3 + \frac{4}{5} - \frac{3}{5}$ ← definition of mixed number

  $= 3 + \left(\frac{4}{5} - \frac{3}{5}\right)$ **Key Ideas**: Subtract from any part of the minuend

  $$= 3 + \frac{1}{5}$$

  $= 3\frac{1}{5}$ ← definition of mixed number



  Check: Problem Set problem #1b

- Subtract fraction from mixed number *with* decomposing a whole (1)

  Focus: Concept Development problems #2, #3

  - Method 1: Subtracting subtrahend parts in any order

  $4\frac{1}{5} - \frac{3}{5}$

  $= 4 + \frac{1}{5} - \frac{3}{5}$ ← definition of mixed number

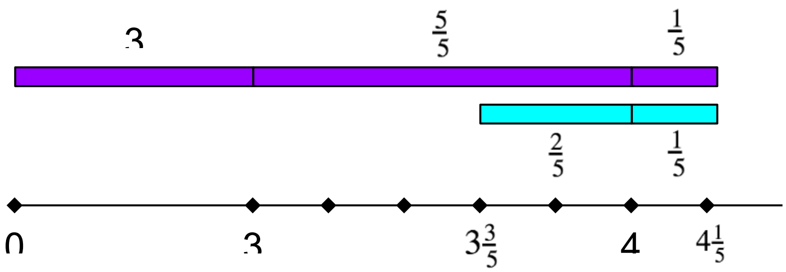
  $$= 4 + \frac{1}{5} - \frac{1}{5} - \frac{2}{5}$$

  $= 4 - \frac{2}{5}$ **Key Ideas**: Subtract subtrahend parts in any order

  $$= 3 + \frac{5}{5} - \frac{2}{5}$$

  $$= 3 + \frac{3}{5}$$

  $= 3\frac{3}{5}$ ← definition of mixed number



- Method 2: Subtracting from any part of minuend (total)

$4\frac{1}{5}-\frac{3}{5}$

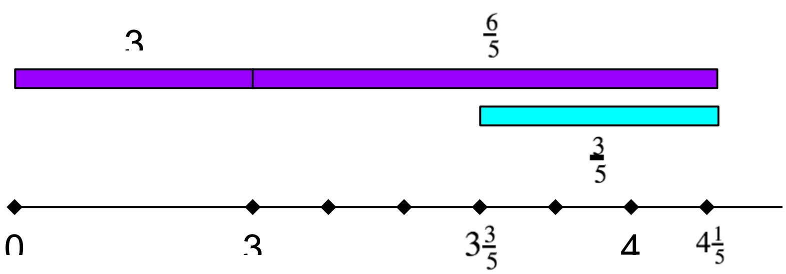
$=4+\frac{1}{5}-\frac{3}{5}$ ← definition of mixed number

$$=3+\frac{5}{5}+\frac{1}{5}-\frac{3}{5}$$

$=3+\frac{6-3}{5}$ 🔑 Key Ideas: Subtract from any part of the minuend

$$=3+\frac{3}{5}$$

$=3\frac{3}{5}$



✔ Check: Problem Set problem #3c

Subtracting Mixed Numbers

Goals:

- Build on **Lesson 32** to subtract mixed numbers
  - Method 1: Subtracting subtrahend parts in any order

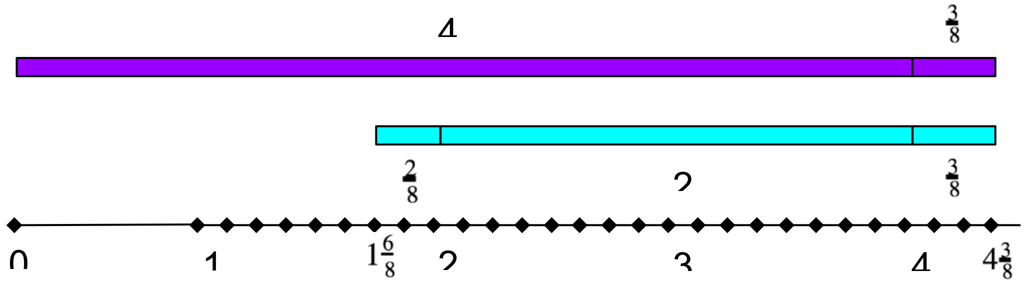
$4\frac{3}{8}-2\frac{5}{8}$

$$=4+\frac{3}{8}-2-\frac{5}{8}$$

$$=(4-2)+\left(\frac{3}{8}-\frac{3}{8}-\frac{2}{8}\right)$$

$=2-\frac{2}{8}$ 🔑 Key Ideas: Subtract subtrahend parts in any order

$$= 1 + \frac{8}{8} - \frac{2}{8}$$
$$= 1\frac{6}{8}$$



- Method 2: Subtracting from any part of minuend (total)

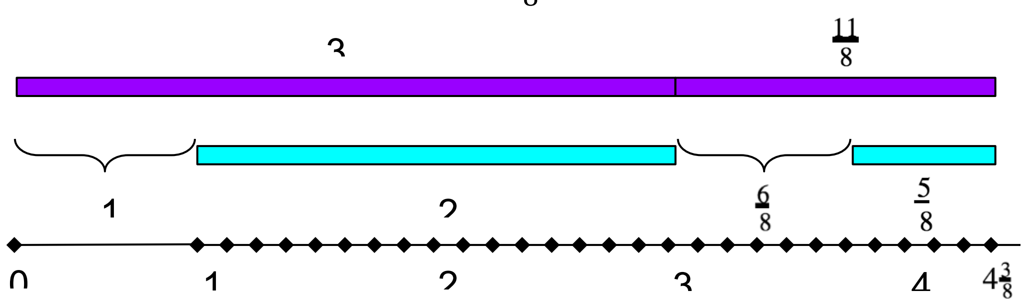
  $4\frac{3}{8} - 2\frac{5}{8}$

$$= 4 + \frac{3}{8} - 2 - \frac{5}{8}$$
$$= 3 + \frac{8}{8} + \frac{3}{8} - 2 - \frac{5}{8}$$
$$= (3-2) + \left(\frac{8}{8} + \frac{3}{8} - \frac{5}{8}\right)$$

$= 1 + \frac{11-5}{8}$

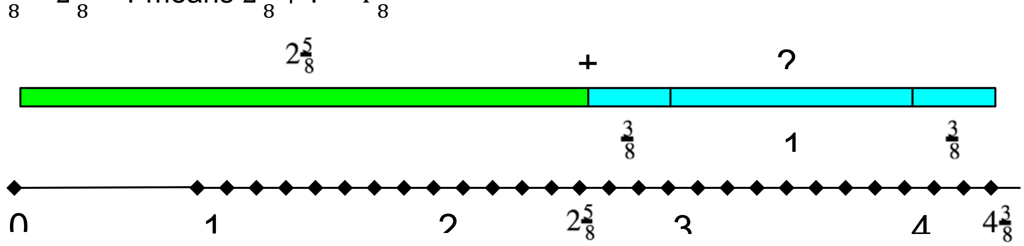
Key Ideas: [Subtract from any part of the minuend]

$$= 1\frac{6}{8}$$



- Method 3: "Counting up"

  $4\frac{3}{8} - 2\frac{5}{8} = ?$ means $2\frac{5}{8} + ? = 4\frac{3}{8}$



Check: Problem Set problem #4a

Subtracting Mixed Numbers (Continued)

Goals:

- ↻ **Review/reinforce** Lessons 32-33
  - **Focus**: Concept Development problems #1, #2
  - **Check**: Problem Set problems #1b, #3a

Topic G: Repeated Addition of Fractions as Multiplication

Multiplication of Whole Number and Fraction

Goals:

- Build on **Lesson 3** to conclude multiplication of whole number and fraction: $n \times \frac{a}{b} = \frac{n \times a}{b}$
  - **Focus**: Concept Development problem #2
  - **Check**: Problem Set problem #3a

Multiplication of Whole Number and Fraction (Continued)

Goals:

- ↻ **Review/reinforce** Lesson 35
  - **Check**: Problem Set problems #3c, #5

Multiplication of Whole Number and Mixed Number

Goals:

- Build on **Lessons 35-36** to multiply whole number with mixed number and show distributive property:
  - **Focus**: Concept Development problem #1

$2 \times 3\frac{1}{5}$

$= 2 \times \left(3 + \frac{1}{5}\right)$

$= 3 + \frac{1}{5} + 3 + \frac{1}{5}$ ← definition of multiplication of whole number & fraction

$= (3 + 3) + \left(\frac{1}{5} + \frac{1}{5}\right)$ Key Ideas: Add in any order

$= (2 \times 3) + \left(2 \times \frac{1}{5}\right)$ ← definition of multiplication of whole number & fraction

$= 6 + \frac{2}{5}$ ← fraction as multiple of unit fraction: $a \times \frac{1}{b} = \frac{a}{b}$

$= 6\frac{2}{5}$ ← definition of mixed number

✓ Check: Problem Set problem #2c

Multiplication of Whole Number and Mixed Number (Continued)

Goals:

- ↻ **Review/reinforce** Lesson 37

  ✓ Check: Problem Set problems #2a, #3

Word Problems: Multiplicative Comparisons

Goals:

- Build on **Lessons 37-38** and Module 3 Topic A to solve multi-step word problems involving multiplication of a whole number and mixed number ("*n* times as much/tall/long/etc. …")

Line Plots (Continued)

Goals:

- Build on **Lessons 28 and 37-39** to solve word problems using information presented in line plots and involving addition, subtraction of mixed numbers or multiplication by whole number

Topic H: Exploring a Fraction Pattern

Sum of Fractions Between 0 and 1

Cut/consolidate: Can revisit later in the year because Standard 4.OA.5 “Generate and analyze patterns” is an Additional Cluster, not a Major Cluster Standard

Goals:

- Observe and justify that $\frac{0}{n}+\frac{1}{n}+\ldots+\frac{n}{n}$ equals $(n+1)\div 2$ if n is odd and $(n\div 2)+\frac{1}{2}$ if n is even

Focus: Concept Development problem #1

- For $n=5$ (odd):

$$\frac{0}{5}+\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}+\frac{5}{5}$$

$$=\left(\frac{0}{5}+\frac{5}{5}\right)+\left(\frac{1}{5}+\frac{4}{5}\right)+\left(\frac{2}{5}+\frac{3}{5}\right)$$

Key Ideas: Add in any order

$$=\frac{5}{5}+\frac{5}{5}+\frac{5}{5}$$

Key Ideas: $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

$$=1+1+1$$

$$=3\times 1$$

$$=\big((5+1)\div 2\big)\times 1$$

$$=(5+1)\div 2$$

$$=3$$

- For $n=6$ (even):

$$\frac{0}{6}+\frac{1}{6}+\frac{2}{6}+\frac{3}{6}+\frac{4}{6}+\frac{5}{6}+\frac{6}{6}$$

$$=\left(\frac{0}{6}+\frac{6}{6}\right)+\left(\frac{1}{6}+\frac{5}{6}\right)+\left(\frac{2}{6}+\frac{4}{6}\right)+\frac{3}{6}$$

Key Ideas: Add in any order

$$=\frac{6}{6}+\frac{6}{6}+\frac{6}{6}+\frac{1}{2}$$

Key Ideas: $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$, $\frac{a}{b}=\frac{a\div c}{b\div c}$

$$=1+1+1+\frac{1}{2}$$

$$=(3\times 1)+\frac{1}{2}$$

$$=\big((6\div 2)\times 1\big)+\frac{1}{2}$$

$$=(6\div 2)+\frac{1}{2}$$

$$=3\frac{1}{2}$$

Module 6: Decimal Fractions

Definitions:

- **Decimal fraction**: a fraction with a denominator that is a product of 10’s (positive power of 10)
  - Examples:
    - $\frac{3}{10}$
    - $\frac{487}{100} = \frac{487}{10\times10}$
    - $\frac{181}{10000} = \frac{181}{10\times10\times10\times10}$
- **Decimal number**: shorthand for expressing a decimal fraction
  - The number of zeros in the denominator of the decimal fraction corresponds to the number of digits to the right of the decimal point (**decimal digits**)
  - “It has been recognized since 1593 by the German Jesuit astronomer C. Clavius that a decimal fraction is easier to write if we abandon the fraction symbol: just use the numerator and then keep track of the number of zeros in the denominator by the use of a so-called decimal point…” (Wu, 2011, p. 187)
  - Examples:
    - 3/10 = 0.3 (1 zero in denominator → 1 decimal digit)
    - 487/100 = 4.87 (2 zeros in denominator → 2 decimal digits)
    - 181/10000 = 0.0181 (4 zeros in denominator → 4 decimal digits)
  - Conventions:
    - The zero in front (to the left) of the decimal point is only for the purpose of clarity and is optional
      - Example: 3/10 = 0.3 or .3
    - When the number of digits in the numerator is *fewer* than the number of zeros in the denominator, zeros are inserted to the right of the decimal point to make the decimal digits clear and obvious
      - Example: $\frac{181}{10000}$= 0.0181 (3 digits in numerator, 4 zeros in denominator → need 1 zero inserted to the right of the decimal point)
- **Equivalent** or **equal**, **greater than**, **less than**:

Word	*Symbolic*	*Meaning (Definition)*
$1\frac{3}{10}$ is equivalent or equal to $\frac{13}{10}$	$1\frac{3}{10} = \frac{13}{10}$	• Geometric: $1\frac{3}{10}$ and $\frac{13}{10}$ have SAME length (area, volume) • Number line:$1\frac{3}{10}$ and $\frac{13}{10}$ are the SAME point
$\frac{13}{10}$ is greater than 0.31	$\frac{13}{10} > 0.31$	• Geometric: $\frac{13}{10}$ has MORE length (area, volume) than 0.31 • Number line: $\frac{13}{10}$ is to the RIGHT of 0.31
0.31 is less than $\frac{13}{10}$	$0.31 < \frac{13}{10}$	• Geometric:0.31 has LESS length (area, volume) than $\frac{13}{10}$ • Number line: 0.31 is to the LEFT of $\frac{13}{10}$

Key Ideas:

- Decimal Table [Lessons 1-8]: 1.2 = one and 2 tenths, 0.23 = 2 tenths and 3 hundredths, 1.23 = one and 2 tenths and 3 hundredths, etc.

Suggestions:

- Begin module with Lesson 0 Introduction to Decimals
- Use Decimal Table to organize and summarize key ideas for Lessons 1-8

Topic A: Exploration of Tenth

Tenths Less Than 1: Linear Model

Goals:

- Define decimal fraction, decimal number for tenths
- Represent tenths less than 1 as a decimal number with a linear model and express a decimal fraction as a decimal number (and vice versa)

  Focus: Concept Development Activity 1

  Check: Problem Set problems #2, #3
- Express 1 as a sum of tenths

  Focus: Concept Development Activity 3

  $\frac{9}{10}+\frac{1}{10}=\frac{10}{10}$ ← $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

  $\frac{9}{10}+\frac{1}{10}=1$ ← definition of equivalent fractions: $\frac{10}{10}=1$

  $0.9+0.1=1$ ← definition of decimal number: $\frac{9}{10}=0.9$, $\frac{1}{10}=0.1$

  * Note: $1.0=1$ because

  1.0

  $=\frac{10}{10}$ ← definition of decimal number

  $=1$ ← definition of equivalent fractions

  Check: Problem Set problem #5a

Tenths Greater Than 1: Linear and Area Model

Goals:

- Represent tenths greater than 1 as a mixed number and as a decimal number with:
  - Linear model

    Focus: Concept Development problem #1

    Check: Problem Set problem #1b

- Area model
  - Focus: Concept Development problem #2
  - Check: Problem Set problems #2c & e
- Observe and justify: the whole number in a mixed number corresponds to the digits to the *left* of the decimal point and the numerator of the fraction in the mixed number corresponds to the digits to the *right* of the decimal point
  - Example: $1\frac{2}{10} = 1.2$

    Reasoning:

    $1\frac{2}{10}$

    $= 1 + \frac{2}{10}$ ← definition of mixed number

    $= \frac{10}{10} + \frac{2}{10}$ ← definition of equivalent fractions: $\frac{10}{10} = 1$

    $= \frac{12}{10}$ ← $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

    $= 1.2$ ← definition of decimal number

Tenths Greater Than 1: Linear Model, Place Value Disks

Goals:

- Represent tenths greater than 1 as a mixed number, decimal number, and expanded forms with
  - Place value disks
    - Focus: Concept Development problem #2
    - Check: Problem Set problem #1a
  - Number line
    - Focus: Concept Development problem #3
    - Check: Problem Set problems #3a-b

Topic B: Tenths and Hundredths

Hundredths Less Than 1: Linear Model

Goals:

- Build on **Lesson 1** to define decimal fraction, decimal number for hundredths
- Use equivalent fractions to represent tenths as hundredths in fraction or decimal form with linear model
    - Example: $0.1 = 0.10$

$$
\begin{aligned}
& 0.1 \\
&= \frac{1}{10} && \leftarrow \text{definition of decimal number} \\
&= \frac{1\times 10}{10\times 10} && \leftarrow \frac{a}{b}=\frac{a\times c}{b\times c} \\
&= \frac{10}{100} && \text{* 10 hundredths in every tenth} \\
&= 0.10 && \leftarrow \text{definition of decimal number}
\end{aligned}
$$

Focus: Concept Development problem #1

Check: Problem Set problem #2c

- Represent sum of tenths and hundredths less than 1 in fraction and decimal form with linear model

Focus: Concept Development problem #2

$$
\begin{aligned}
& 0.2 + 0.05 \\
&= \frac{2}{10}+\frac{5}{100} && \leftarrow \text{definition of decimal number} \\
&= \frac{20}{100}+\frac{5}{100} && \leftarrow \frac{2}{10}=\frac{2\times 10}{10\times 10} \\
&= \frac{25}{100} && \leftarrow \frac{a}{c}+\frac{b}{c}=\frac{a+b}{c} \\
&= 0.25 && \leftarrow \text{definition of decimal number}
\end{aligned}
$$

Check: Problem Set problem #3c

Hundredths Less Than 1: Area Model, Place Value Disks

Goals:

- ↻ **Review/reinforce** Lesson 4: Use equivalent fractions to represent tenths as hundredths (or hundredths as tenths) in fraction or decimal form with area model

Focus: Concept Development problem #1

Check: Problem Set problem #1

- Represent hundredths less than 1 in fraction and decimal form and as sum of tenths and hundredths with:
    - Area model

Focus: Concept Development problem #2

0.25

$= \frac{25}{100}$ ← definition of decimal number

$= \frac{20}{100} + \frac{5}{100}$ $\leftarrow \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$= \frac{2}{10} + \frac{5}{100}$ $\leftarrow \frac{20}{100} = \frac{20 \div 10}{100 \div 10}$

$= 0.2 + 0.05$ ← definition of decimal number

Check: Problem Set problem #2a

- Place value disks

  Focus: Concept Development problem #3

  Check: Problem Set problems #3b, #4c

Hundredths Greater Than 1: Linear and Area Models

Goals:

- Represent hundredths greater than 1 as mixed number and decimal number and in "unit form" with:
  - Area model

    Focus: Concept Development problem #1

    Check: Problem Set problem #1b
  - Number line

    Focus: Concept Development problem #2

    Check: Problem Set problem #1b

Hundredths Greater Than 1: Place Value; Expanded Forms

Goals:

- Identify the value of each digit in a decimal (hundredths)

  Focus: Concept Development problem #2

  Check: Problem Set problems #2a-d
- Express decimal number in decimal and fraction expanded forms

Focus: Concept Development problem #3

Check: Problem Set problems #3b-c

Hundredths Greater Than 1: Expressing in Different Units

Goals:

- Use an area model or place value chart to express a mixed number or decimal number in tenths or hundredths
    - Example 1: $2\ ones\ 4\ tenths\ =\ 240\ hundredths$

      $2\ ones\ 4\ tenths$

      $= 2 + \frac{4}{10}$

      $= \frac{20}{10} + \frac{4}{10}$ $\leftarrow 2 = \frac{2\times10}{10}$

      $= \frac{24}{10}$ $\leftarrow \frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

      $= \frac{240}{100}$ $\leftarrow \frac{24}{10} = \frac{24\times10}{10\times10}$

      $= 240\ hundredths$
    - Example 2: $2.4 = 240\ hundredths$

      2.4

      $= \frac{24}{10}$ $\leftarrow$ definition of decimal number

      $= \frac{240}{100}$ $\leftarrow \frac{24}{10} = \frac{24\times10}{10\times10}$

      $= 240\ hundredths$

Focus: Concept Development problems #1, #2

Check: Problem Set problems #2, #5

Topic C: Decimal Comparison

Comparing Decimal Numbers with Linear Model

Goals:

- Use linear models (tape measurement, graduated cylinder, weight scale) and definitions of equal, greater than, less than to compare decimal numbers

Check: Problem Set problems #1a, #2a, #3b

Comparing Decimal Numbers with Area and Linear Models

Goals:

- Use an area model and definitions of equal, greater than, less than to compare decimal numbers

  Focus: Concept Development problem #1

  Check: Problem Set problem #1b

- Use a number line and definitions of equal, greater than, less than to compare decimal numbers

  Focus: Concept Development problem #2

  Check: Problem Set problem #2b

Ordering Numbers with Number Line

Goals:

- Order numbers by locating them on the number line

  Focus: Concept Development problem #1

  Check: Problem Set problem #1b

Topic D: Addition with Tenths and Hundredths

Adding Tenths and Hundredths Less Than 1

Goals:

- Use the Equivalent Fractions Theorem to add tenths and hundredths

  Focus: Concept Development problems #1, #2

$$\frac{3}{10}+\frac{13}{100}$$

$$=\frac{30}{100}+\frac{13}{100} \qquad \leftarrow \frac{3}{10}=\frac{3\times 10}{10\times 10}$$

$= \frac{43}{100}$ ← $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
$= 0.43$ ← definition of decimal number

Check: Problem Set problems #2b, #3b

Adding Mixed Numbers and Decimal Numbers Greater Than 1

Goals:

- Build on **Lessons 2-8 and 12** to add mixed numbers and decimal numbers

Focus: Concept Development problem #3

- Method 1:

  $3.5 + 2.49$
  $= \frac{35}{10} + \frac{249}{100}$ ← definition of decimal number
  $= \frac{350}{100} + \frac{249}{100}$ ← $\frac{35}{10} = \frac{35 \times 10}{10 \times 10}$
  $= \frac{599}{100}$ ← $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
  $= 5.99$ ← definition of decimal number

- Method 2:

  $3.5 + 2.49$
  $= 3\frac{5}{10} + 2\frac{49}{100}$ ← Lessons 2 & 6
  $= 3\frac{50}{100} + 2\frac{49}{100}$ ← $\frac{5}{10} = \frac{5 \times 10}{10 \times 10}$
  $= (3 + 2) + \left(\frac{50}{100} + \frac{49}{100}\right)$ Key Ideas: Add in any order
  $= 5\frac{99}{100}$ ← $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
  $= 5.99$ ← Lesson 6

Check: Problem Set problems #2c, #3c

Word Problems

Goals:

- Solve word problems involving addition of decimal measurements
  - Focus: Concept Development problems #2, #3

Topic E: Money Amounts as Decimal Numbers

Money Value as Decimal Fraction or Number

Goals:

- Express the value of coins as decimal fraction or decimal number
  - Focus: Concept Development problem #1
  - Check: Problem Set problems #3, #8, #12
- Express the total value of a combination of coins or sets of bills and coins as a decimal fraction or decimal number
  - Focus: Concept Development problem #3
  - Check: Problem Set problem #20

Word Problems

Goals:

- Solve word problems involving addition, subtraction, multiplication, and/or division of the values of bills and coins
  - Focus: Concept Development problems #1, #3

Module 7: Exploring Measurement with Multiplication

Definitions:

- **Customary length units**:
    - **Pound**, **ounce**: 1 pound = 16 ounces
- **Customary mass (weight) units**:
    - **Yard**, **foot**, **inch** (Grade 2 Module 7): 1 yard = 3 feet; 1 foot = 12 inches
- **Customary capacity (volume) units**:
    - **Gallon**, **quart**: 1 gallon = 4 quarts
    - **Pint**: 1 quart = 2 pints
    - **Cup**: 1 pint = 2 cups
- **Time units**:
    - **Day**, **Hour**, **Minute**, **Second**: 1 day = 24 hours; 1 hour = 60 minutes; 1 minute = 60 seconds
- **Mixed unit** (Module 2):
    - "2 yd 5 ft" means "2 yd + 5 ft"
    - Mixed unit is shorthand for writing a sum of two units
        - Review of mixed numbers (Module 5): "$2\frac{5}{8}$" means "$2+\frac{5}{8}$"
    - We make as many of the larger unit as possible
        - Example: 36 oz as a mixed unit is 2 lb 4 oz and not 1 lb 20 oz
- **Unit conversion**: to convert 12 lb 10 oz to ounces means to find how heavy 12 lb 10 oz is in ounces (12 lb 10 oz = ? oz)
- **Rectilinear figure**: a figure composed of rectangles

Key Ideas:

- **Addition**: Because of the associative and commutative properties of addition, we can add numbers in *any order* and still keep the sum (total) the same
- **Subtraction**: We can subtract parts of the subtrahend in *any order* and from *any part* of the minuend (total) that is greater than or equal to the subtrahend part(s) and still keep the difference the same

Materials:

- Balance [Topic A]
- Measuring cups, jars, pitchers (borrow from Grade 5 manipulatives kit) [Topics A, C]
- Rulers, yardstick or measuring tape [Topics C-D]
- Protractors [Topic D]

Topic A: Measurement Conversion Tables

Customary Mass (Weight) and Length Measurements

Goals:

- Use definitions of customary mass (weight) units and customary length units to calculate unit conversions

  Suggestions: Include products in the conversion chart to make explicit how conversion values are calculated

Pounds	**Ounces**
1	16
2	2 × 16 = 32
3	3 × 16 = 48

  Check: Problem Set problem #1

- Use definition of mixed unit to express a mixed unit measurement in terms of a smaller unit
  - Example:

$$12\ pounds\ 10\ ounces$$
$$= 12\ pounds + 10\ ounces \quad \leftarrow \text{definition of mixed unit}$$
$$= (12 \times 1\ pound) + 10\ ounces$$
$$= (12 \times 16\ ounces) + 10\ ounces \quad \leftarrow \text{1 pound = 16 ounces}$$
$$= 192\ ounces + 10\ ounces$$
$$= 202\ ounces$$

  Check: Problem Set problems #3, #5a

Customary Capacity (Volume) Measurements

Goals:

- **Echo** Lesson 1 for capacity (volume) measurements:
  - Use definition of customary capacity (volume) units to calculate unit conversions

  Suggestions:

- Include products in the conversion chart to make explicit how conversion values are calculated
- Provide table (example below) or linear model (p. 30 of Teacher Edition) that shows relationship between gallon, quart, pint, cup

Gallons	Quarts	Pints	Cups
		1	2
	1	2	(2 × 2 = 4)
1	4	(4 × 2 = 8)	(8 × 2 = 16)

Check: Problem Set problem #1

- Use definition of mixed unit to express a mixed unit measurement in terms of a smaller unit

Check: Problem Set problems #5b & d

Time

Goals:

- Echo Lessons 1-2 for time:
  - Use definition of time units to calculate unit conversions

Suggestions:

- Include products in the conversion chart to make explicit how conversion values are calculated
- Provide table (example below) that shows relationship between day, hours, minutes, seconds

Days	Hours	Minutes	Seconds
		1	60
	1	60	(60 × 60)
1	24	(24 × 60)	(24 × 60 × 60)

Check: Problem Set problem #2

- Use definition of mixed unit to express a mixed unit measurement in terms of a smaller unit

✓ Check: Problem Set problems #4b-c

Word Problems

Goals:

- Solve word problems that involve finding “*n* times as much” and measurement conversion from larger unit to smaller unit

  Focus: Concept Development (Problem Set) problem #2

Creating Word Problems

Cut/consolidate: Can omit lesson or use as “challenge” task

Goals:

- Create a story to match a given set of tape diagrams

Topic B: Problem Solving with Measurement

Adding and Subtracting Mixed Units of Capacity (Volume)

Goals:

- Build on **Lesson 2** (unit conversion for capacity) to add or subtract mixed units of capacity

  ✓ Check: Problem Set problems #1c & g, #2b

Adding and Subtracting Mixed Units of Length

Goals:

- Build on **Lesson 1** (unit conversion for length) to add or subtract mixed units of length

  Check: Problem Set problems #1c-d, #2f

Adding and Subtracting Mixed Units of Mass (Weight)

Goals:

- Build on **Lesson 1** (unit conversion for weight) to add or subtract mixed units of weight

  Check: Problem Set problems #1c-f

Adding and Subtracting Mixed Units of Time

Goals:

- Build on **Lesson 3** (unit conversion for time) to add or subtract mixed units of time

  Check: Problem Set problems #1c & f, #2b & d

Multi-Step Word Problems

Goals:

- Solve multi-step word problems involving addition and/or subtraction of mixed units of measurement

  Focus: Concept Development (Problem Set) problem #3

Multi-Step Word Problems (Continued)

Goals:

- Solve multi-step word problems involving addition, subtraction, multiplication, and/or division of mixed units of measurement

Focus: Concept Development (Problem Set) problem #3

Topic C: Investigation of Measurements Expressed as Mixed Numbers

Customary Capacity and Length Measurements (Continued)

Goals:

- Use definitions of customary capacity (volume) units and customary length units to calculate unit conversions with fractions and mixed numbers
    - Example:

$4\frac{1}{2}\ feet$

$= \left(4 + \frac{1}{2}\right) \times 1\ foot$ ← definition of mixed unit

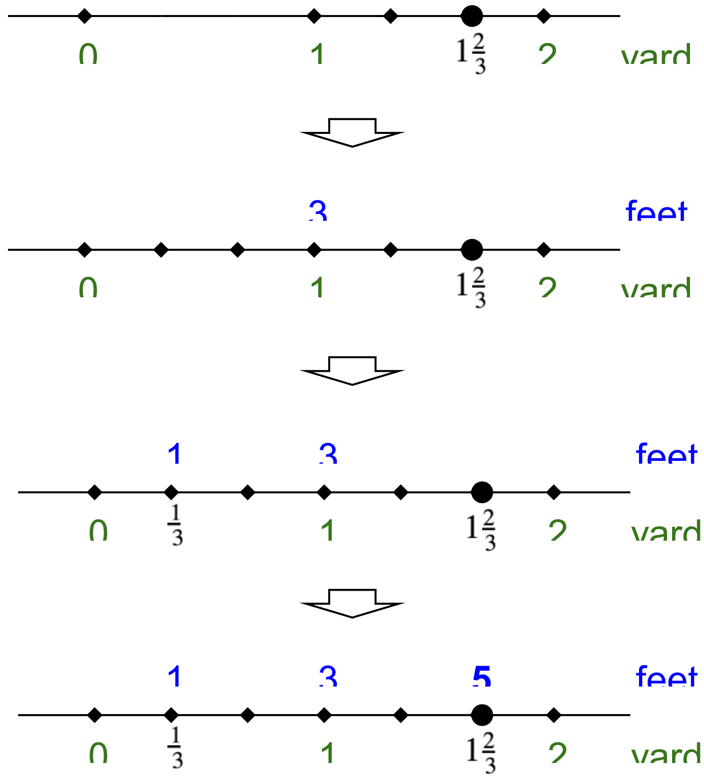
$= \left(4 + \frac{1}{2}\right) \times 12\ inches$ ← 1 foot = 12 inches

$= (4 \times 12) + \left(12 \times \frac{1}{2}\right)$ ← distributive property

$= 48 + 6$ ← Module 5: $12 \times \frac{1}{2} = \frac{12}{2} = \frac{12 \div 2}{2 \div 2} = \frac{6}{1} = 6$

$= 54\ inches$

Suggestions: Use “double unit” number line and label “1” for each unit



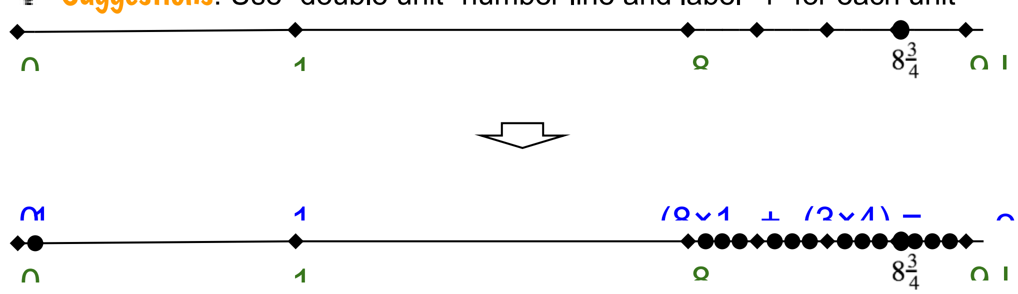
Check: Problem Set problems #1b, #6d-e

Converting Customary Mass or Time Measurement to Smaller Unit

Goals:

- **Echo** Lesson 12 for mass and time:
    - Use definitions of customary mass (weight) units and time to calculate unit conversions with fractions and mixed numbers

      Suggestions: Use “double unit” number line and label “1” for each unit



✔ **Check**: Problem Set problems #1c, #5c & g

Multi-Step Word Problems (Continued)

Goals:

- Solve multi-step word problems involving addition, subtraction, and/or multiplication of mixed units of measurement (and conversion to a single unit of measurement)

  Focus: Concept Development (Problem Set) problem #1

Topic D: Year in Review

Area of Rectilinear Figures

Goals:

- ↻ **Review/reinforce** Grade 3 Module 4 Lessons 13-14 (area of rectilinear figures) with customary units (feet, square feet)

  Focus: Concept Development (Problem Set) problems #1, #3

Area of Floor Plan

Goals:

- Build on Module 4 **Lesson 15** (construction of rectangles) and Grade 3 **Module 4 Lessons 15-16** (areas of rooms in floor plan) to draw floor plan and find areas given dimensions of rooms and furniture

Fluency Activities

Goals:

- ↻ **Review/reinforce** Modules 1-7

Vocabulary Review

Goals:

- ↻ **Review/reinforce** Modules 1-7