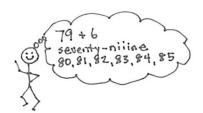
### Sums and Differences to 20

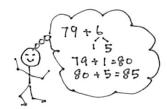
#### **OVERVIEW**

Module 1 sets the foundation for students to master the sums and differences to 20 (2.OA.2) and to subsequently apply these skills to fluently add one-digit to two-digit numbers at least through 100 using place value understandings, properties of operations and the relationship between addition and subtraction (2.NBT.5). In Grade 1, students worked extensively with numbers to 10 and they developed Level 2 and Level 3 mental strategies to add and subtract within 20 (1.OA.1) and 100 (1.NBT.4-6).

Level 2: Count on



Level 3: Decompose an addend to compose ten



For example, to solve 12 + 3 students might make an equivalent but easier problem by decomposing 12 as 10 + 2 and composing 2 with 3 to make 5. Students can use this knowledge to solve related problems such as 92 + 3. They also apply their skill using smaller numbers to subtract problems with larger numbers: 12 - 8 = 10 - 8 + 2 = 2 + 2, just as 72 - 8 = 70 - 8 + 2 = 62 + 2.

Daily fluency activities provide sustained practice to help students attain fluency within 20. This fluency is essential to the work of later modules and future grade levels, where students must fluently recompose place value units to work adeptly with the fouroperations. Activities such as Say Ten counting and Take from 10, and the use of ten-frame cards and Hide Zero cards, solidify student fluency. Because the amount of practice required by each student to achieve mastery will vary, a motivating, differentiated fluency program needs to

be established in these first weeks to set the tone for the rest of the year.

Throughout the module, students will represent and solve one-step word problems through the daily Application Problem (2.OA.1). Application problems can precede a lesson to act as the lead-in to a concept, allowing students to discover through problem-solving the logic and usefulness of a strategy before that strategy is reviewed. Or, they can follow the concept development so that students connect and apply their learning to real-world situations. This latter structure can also serve as a bridge between teacher-directed work and students solving problems independently on activity worksheets and at home. In either case, problem-solving begins as a guided activity, with the goal being to move students to independent problem-solving, wherein they reason through the relationships of the problem and choose an appropriate strategy to solve. In Module 1, application problems follow concept development.

Topic A reactivates students' Kindergarten and Grade 1 learning, as they practice prerequisite skills for Level 3 decomposition and composition methods: partners to 10 and decompositions for all numbers within 10<sup>1</sup>. Students move briskly from concrete to pictorial to abstract as they remember their "make ten" facts. They use ten-frame cards to visualize 10, and they write the number bonds of 10 from memory. They use those facts to see relationships in larger numbers (e.g., 28 needs how many to make 30.) The number bond is also used to represent related facts within 10.

Topic B also moves from concrete to pictorial to abstract, as students use decomposing strategies to add and subtract within 20. By the end of Grade 1, Module 2, students learned to form ten as a unit. Hence, the phrase *make ten* now transitions to *make a ten*. Students use the ten-structure to reason about making a ten to add to the teens, and they use this pattern and math drawings to solve related problem sets (e.g., 9 + 4, 9 + 5, 9 + 6). Students reason about the relationship between problems such as 19 + 5 and 20 + 4 to 9 + 5 and 10 + 4. They use place value understanding to add and subtract within 20 by adding to and subtracting from the ones. The topic ends with a lesson in which students subtract from 10. The goal in making a 10 and taking from 10 is for students to master mental math.

$$13 + 2 = 15$$
  
 $15 - 3 = 12$   
 $10 3$   
 $3+2=6$   
 $13+2=15$   
 $15-3=2$   
 $15-3=12$ 

Add and subtract ones

Take from 10

Topic C calls on students to review foundation for Grade 2's work each student to achieve mastery will established in these first weeks to set strategies to add and subtract within 100 (1.NBT.4–6) to set the student fluency. Because the amount of practice required by vary, a motivating, differentiated fluency program needs to be the tone for the rest of the year.

Throughout the module, students will represent and solve one-step word problems through the daily Application Problem (2.OA.1). Application problems can precede a lesson to act as the lead-in to a concept, allowing students to discover through problem-solving the logic and usefulness of a strategy before that

<sup>&</sup>lt;sup>1</sup> K.OA.4 and K.OA.3

strategy is reviewed. Or, they can follow the concept development so that students connect and apply their learning to real-world situations. This latter structure can also serve as a bridge between teacher-directed work and students solving problems independently on activity worksheets and at home. In either case, problem-solving begins as a guided activity, with the goal being to move students to independent problem-solving, wherein they reason through the relationships of the problem and choose an appropriate strategy to solve. In Module 1, application problems follow concept development.

Topic A reactivates students' Kindergarten and Grade 1 learning, as they practice prerequisite skills for Level 3 decomposition and composition methods: partners to 10 and decompositions for all numbers within  $10^2$ . Students move briskly from concrete to pictorial to abstract as they remember their "make ten" facts. They use ten-frame cards to visualize 10, and they write the number bonds of 10 from memory. They use those facts to see relationships in larger numbers (e.g., 28 needs how many to make 30.) The number bond is also used to represent related facts within 10.

Topic B also moves from concrete to pictorial to abstract, as students use decomposing strategies to add and subtract within 20. By the end of Grade 1, Module 2, students learned to form ten as a unit. Hence, the phrase *make ten* now transitions to *make a ten*. Students use the ten-structure to reason about making a ten to add to the teens, and they use this pattern and math drawings to solve related problem sets (e.g., 9 + 4, 9 + 5, 9 + 6). Students reason about the relationship between problems such as 19 + 5 and 20 + 4 to 9 + 5 and 10 + 4. They use place value understanding to add and subtract within 20 by adding to and subtracting from the ones. The topic ends with a lesson in which students subtract from 10. The goal in making a 10 and taking from 10 is for students to master mental math.

$$13 + 2 = 15$$
 $15 - 3 = 12$ 
 $10 3$ 
 $15 - 3 = 12$ 
 $10 5$ 
 $3 + 2 = 6$ 
 $13 + 2 = 15$ 
 $15 - 3 = 2$ 
 $15 - 3 = 12$ 

Add and subtract ones

Take from 10

Topic C calls on students to review strategies to add and subtract within 100 (1.NBT.4–6) to set the foundation for Grade 2's work towards mastery of fluency with the same set of problems (2.NBT.5). They use basic facts and place value understanding to add and subtract within multiples of 10 without crossing the multiple (e.g., 7-5=2, so 47-5=42.) This segues into the use of basic facts and properties of addition to cross multiples of 10 (e.g., 26+9=20+6+4+5). In the final lesson, students decompose to make a ten, and then to subtract from numbers that have both tens and ones.

<sup>&</sup>lt;sup>2</sup> K.OA.4 and K.OA.3

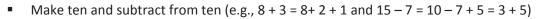
$$87 + 5 = 92$$
 $80 + 32$ 
 $80 + 10 + 2 = 92$ 

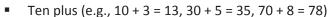
Add basic facts to cross multiples of ten

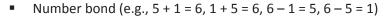
Decompose and subtract from the 10

# **Terminology**

#### **Familiar Terms and Symbols**







Say Ten counting (e.g., 11 is "1 ten 1," 12 is "1 ten 2," twenty is "2 tens," 27 is "2 tens 7," 35 is "3 tens 5," 100 is "1 hundred," 146 is "1 hundred 4 tens 6")

# **Suggested Tools and Representations**

- One set of ten-frame cards per student
  - One each of 1–4 and 6–9
  - Two fives
  - Ten tens
  - Blank frame
- Large set of ten-frame cards for teacher
- A bag of counters for each student (e.g., large white beans spray painted red on one side)
- Ten-strips
- Rekenrek
- Personal boards
- Hide Zero cards
- Linking cubes
- Dice

Regular	Say Ten
fifty-one	5 tens 1
sixty-seven	6 tens 7
seventy-five	7 tens 5
eighty-four	8 tens 4
ninety-five	9 tens 5



Ten-frame cards



•

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# **GRADE 2 • MODULE 2**

# Addition and Subtraction of Length Units

#### **OVERVIEW**

In this 12-day Grade 2 module, students engage in activities designed to deepen their conceptual understanding of measurement and to relate addition and subtraction to length. Their work in Module 2 is exclusively with metric units in order to support place value concepts. Customary units will be introduced in Module 7.

Topic A opens with students exploring concepts about the centimeter ruler. In the first lesson, they are guided to connect measurement with physical units as they find the total number of unit lengths by laying multiple copies of centimeter cubes (physical units) end-to-end along various objects. Through this, the students discover that to get an accurate measurement, there must not be any gaps or overlaps between consecutive length units.

Next, students measure by iterating with one physical unit, using the mark and advance technique. In the following lesson, students repeat the process by laying both multiple copies and a single cube along a centimeter ruler. This helps students create a mental benchmark for the centimeter. It also helps them realize that the distance between 0 and 1 on the ruler indicates the amount of space already covered. Hence 0, not 1, marks the beginning of the total length. Students use this understanding to create their own centimeter rulers using a centimeter cube and the mark and advance technique. Topic A ends with students using their unit rulers to measure lengths (2.MD.1), thereby connecting measurement with a ruler.

Students build skill in measuring using centimeter rulers and meter sticks in Topic B. They learn to see that a length unit is not a cube, or a portion of a ruler (which has width), but is a segment of a line. By measuring a variety of objects, students build a bank of known measurements or benchmark lengths, such as a doorknob being one meter from the floor, or the width of a finger being a centimeter. Then, students learn to estimate length using knowledge of previously measured objects and benchmarks. This enables students to internalize

the mental rulers<sup>3</sup> of a centimeter or meter, which empowers them to mentally iterate units relevant to measuring a given length (**2.MD.3**). The knowledge and experience signal that students are determining which tool is appropriate to make certain measurements (**2.MD.1**).

In Topic C, students measure and compare to determine how much longer one object is than another (2.MD.4). They also measure objects twice using different length units, both standard and nonstandard, thereby developing their understanding of how the total measurement relates to the size of the length unit (2.MD.2). Repeated experience and explicit comparisons will help students

recognize that the smaller the length unit, the larger the number of units, and the larger the length unit, the smaller the number of units.

The module culminates as students relate addition and subtraction to length. They apply their conceptual understanding to choose appropriate tools and strategies, such as the ruler as a number line, benchmarks for estimation, and tape diagrams for comparison, to solve word problems (2.MD.5, 2.MD.6). The problems progress from concrete (i.e., measuring objects and using the ruler as a number line to add and subtract) to abstract (i.e., representing lengths with tape diagrams to solve *start unknown* and two-step problems).

#### Familiar Terms and Symbols<sup>4</sup>

- Length
- Height
- Length Unit
- Combine
- Compare
- Difference
- Tape Diagram

### **Suggested Tools and Representations**

- Manipulatives that are 1 centimeter long (e.g., centimeter cubes)
- Centimeter ruler for each student
- Paper meter strips for each student
- Centimeter ruler, meter stick

<sup>&</sup>lt;sup>3</sup> See the Progression Document "Geometric Measurement." page 14.

## Place Value, Counting, and Comparison of Numbers to 1,000

#### **OVERVIEW**

In Module 2, students added and subtracted measurement units within 100 (2.MD.5, 2.MD.6), a meaningful application of their work from Module 1 (2.NBT.5) and a powerful bridge into the base ten units of Grade 2.

In this 25-day Grade 2 module, students expand their skill with and understanding of units by bundling ones, tens, and hundreds up to a thousand with straws. Unlike the length of 10 centimeters in Module 2, these bundles are discrete sets. One unit can be grabbed and counted just like a banana—1 hundred, 2 hundred, 3 hundred, etc. (2.NBT.1). A number in Grade 1 generally consisted of two different units, tens and ones. Now, in Grade 2, a number generally consists of three units: hundreds, tens, and ones (2.NBT.1). The bundled units are organized by separating them largest to smallest, ordered from left to right. Over the course of the module, instruction moves from physical bundles that show the proportionality of the units to non-proportional place value disks and to numerals on the place value chart (2.NBT.3).

Furthermore, in this module instruction includes a great deal of counting: by ones, tens, and hundreds (2.NBT.2). Counting up using the centimeter tape or a classroom number line shows movement from left to right as the numbers increase. Counting up on the place value chart shows movement from right to left as the numbers increase. For example, as 10 ones are renamed as 1 ten, the larger unit is housed in the place directly to the left. The goal is for students to move back and forth fluidly between these two models, the number line and the place value chart, using either to rename units and compare numbers (2.NBT.4).

In this module, the place value story has advanced. Instead of changing 10 ones to 1 ten, students now are also changing 10 tens for 1 hundred. This changing leads to using counting strategies to solve word problems (2.OA.1). In the next module, this change leads to mental math and the formal algorithms for addition and subtraction. Comparison extends into finding 100 more and 100 less, 10 more and 10 less, etc. Just as in Grade 1, *more* and *less* translate into formal addition and subtraction at the onset of Module 4 (2.NBT.8).

The module includes a sequence of engaging problems in which students are asked to change 1 hundred for 10 units of ten and to change 10 units of ten for 1 hundred. The assessment task following Topic G

culminates this series with variations on the followingproblem: "Mrs. Ortiz has 21 students in her second grade class. All of them have 10 fingers and 10 toes. Write the total number of toes of the students using hundreds, tens and ones. Explain using words, pictures or numbers." In order to explain, students must recognize that each child in the problem represents a group of 10 toes. They then count by tens, changing units of ten for 1 hundred as appropriate to find the solution. This transitions into the coming module where students bring their skill with making and breaking larger and apply it to work with addition and subtraction.

### **Terminology**

#### **New or Recently Introduced Terms**

- Base ten numerals (e.g., a thousand is 10 tens, a hundred is 10 ones, starting in Grade 3 a one is 10 tenths, etc.)
- Expanded form (e.g., 500 + 70 + 6)
- Hundreds place (e.g., the 5 in 576; tells how many hundreds are in a number)
- One thousand (1,000)
- Place value or number disk (pictured to the right)
- Standard form (e.g., 576)
- Word form (e.g., five hundred seventy-six)



Unit form modeled with number disks: 7 hundreds 2 tens 6 ones = 72 tens 6 ones

### Familiar Terms and Symbols<sup>5</sup>

- =, <, > (equal, less than, greater than)
- Altogether (e.g., 59 centimeters and 17 centimeters; altogether there are 76 centimeters)
- Bundling, grouping (putting smaller units together to make a larger one, e.g., putting 10 ones together to make a ten or 10 tens together to make a hundred)
- How many more/less (the difference between quantities)
- How much more/less (the difference between quantities)
- More than/less than (e.g., 576 is more than 76; 76 is less than 576)
- Number sentence
- Ones place (e.g., the 6 in 576; tells how many ones are in a number)
- Place value (the unitary values of the digits in numbers)
- Renaming, changing (instead of "carrying" or "borrowing," e.g., a group of 10 ones is "renamed" a ten when the ones are bundled and moved from the ones to the tens place; if using \$1 bills, they may be "changed" for a \$10 bill when there are enough)
- Tens place (e.g., the 7 in 576; tells how many tens are in a number)

<sup>&</sup>lt;sup>5</sup> These are terms and symbols students have seen previously.

- Unit form counting (unit form counting states the amount of hundreds, tens, and ones in each number, e.g., 11 is stated as 1 ten 1 one, 20 as 2 tens, 27 as 2 tens 7 ones, 100 as 1 hundred, and 146 as 1 hundred 4 tens 6 ones.)
- Units of ones, tens, hundreds, one thousand (a single one and groups of 10s, 100s, and 1,000)

## **Suggested Tools and Representations**

- 2 boxes of 1,000 straws per class of 25
- 12 ones, 10 tens, and 10 hundreds per pair
- 18 ones, 18 tens and 18 hundreds, and 1 one thousand
- Ballpoint pens
- "Clock" number line with corresponding analog template (details in Topic A fluency)
- Dice, 1 per pair
- Dienes blocks or base ten materials: hundreds flats, tens rods, and ones cubes
- Number spelling black line master

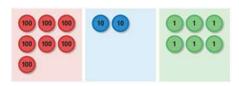
Paper meter	strips	(pictured	in Topic	A fluency)

Personal boards and markers

Place Value Chart with Headings (use with numbers)

hundreds	tens	ones
#	#	#

- Place value "box" (details in Topic C Concept Development)
- Place value charts black line master
- Place value charts and mats: 1 each of the following per student
- Place value cards showing numbers 1–5, 10–50, and 100–500 (1 small set per student)
- Place value cards to 1,000, 1 large teacher set
- Place value disks: suggested minimum of one set per pair
- Play money: \$1, \$5, \$10, and \$100 bills (10 ones, 1 five, 12 tens, and 10 hundreds per pair), and a single set of 16 pennies, 13 dimes
- Rubber bands, 16 per pair
- Small plastic bags (baggies)



Place Value Disks



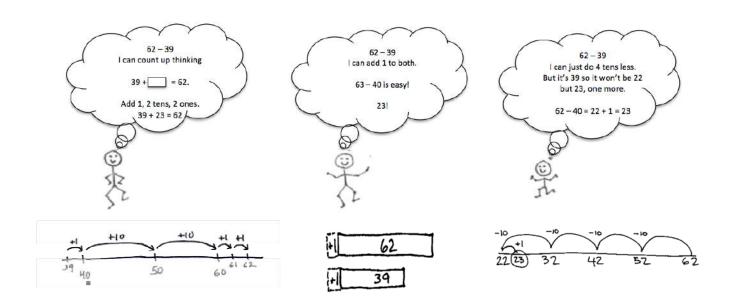
Place Value Chart Without Headings (use with number disks)

# Addition and Subtraction Within 200 with Word Problems to 100

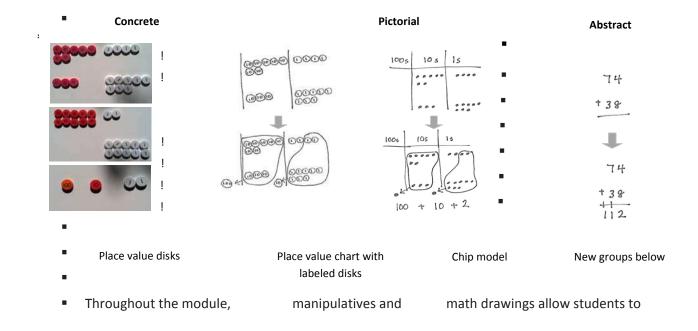
#### OVERVIEW

- In Module 3 students were immersed in the base ten system, as they built a strong foundation through a concrete to pictorial to abstract approach. They bundled groups of 10, and saw that 10 like units could be bundled to produce a new unit that is ten times as large. They progressed from seeing 10 ones as 1 ten (1.NBT.2a) to understanding 10 tens as 1 hundred (2.NBT.2). Module 4 builds on that place value understanding to compose and decompose place value units in addition and subtraction within 200.
- Module 4 is devoted to three major areas of work. The first two are building fluency in two-digit addition and subtraction within 100 (2.NBT.5) and applying that fluency to one- and two-step word problems of varying types within 100 (2.OA.1). Students' increasing fluency with calculations within 100, begun in Grade 1, allows word problems to transition from being mere contexts for calculation into opportunities for students to see and analyze the relationships between quantities (MP1 and 2). Daily application problems and specific lessons in Topics A, C, and F provide students with guided and independent practice as they negotiate a variety of problem types, including the more complex comparison problems. Note that most two-step problems involve single-digit addends, and do not involve the most difficult comparison problem types.<sup>6</sup>
- The third major area of work is developing students' conceptual understanding of addition and subtraction of multi-digit numbers within 200 (2.NBT.7, 2.NBT.9) as a foundation for work with addition and subtraction within 1000 in Module 5.
- In Topic A, students work with place value strategies to fluently add and subtract within 100 (2.NBT.5). The final lessons of Module 3 (finding 1 more, 1 less, 10 more, 10 less) transition into mental addition and subtraction of 1 and 10 (2.NBT.8). Students mentally add and subtract 100 in Topics D and E, as well as during fluency activities throughout the module, as they did in Module 3. This knowledge is then extended and used to solve problems.
- For example, students might count on by ones and tens, e.g.,  $39 + \Box = 62$ , so 40, 50, 60, 61, 62. They might use compensation, adding the same amount to the subtrahend as to the minuend to make a multiple of ten, e.g., 62 39 = 63 40. They might add or subtract a multiple of 10 and adjust the solution as necessary, e.g., 62 39 is 4 tens less than 62 but... one more (2.NBT.5). Students explain why these strategies work using place value language, properties of addition and subtraction, and models, such as the number line (2.NBT.9).

 $<sup>^6</sup>$  See the Progression document "Operations and Algebraic Thinking," p. 18, for the specific types and the rationale.



Topic A's strategies lead naturally to work with the written vertical algorithm for addition (Topic B) and subtraction (Topic C). In these two topics, students represent place value strategies with place value disks and math drawings (see images with strategy names below). Students work with composing 1 ten from 10 ones or decomposing 1 ten as 10 ones (with minuends within 100). After the mid-module assessment, students continue working with manipulatives and math drawings to make sense of problems in which they compose or decompose twice. Topic D focuses on addition, with the new complexity of composing 1 hundred from 10 tens within 200 in problems with up to four addends (2.NBT.6, 2.NBT.7). Subtraction in Topic E involves subtracting when decomposing 1 hundred for 10 tens and 1 ten for 10 ones (2.NBT.7).



- see numbers in terms of place value units and serve as a reminder that they must add like units (e.g., knowing that 74 + 38 is 7 tens + 3 tens and 4 ones + 8 ones).
- The focus is often on computational strategies with bare numbers (i.e., no context) so that total attention is given to understanding the value of each digit within a number, as well as why and how the written method works. Students use the place value chart as an organizer. Simultaneous use of a written method and a place value chart allows students to better recognize both the value of numbers when they are not on the place value chart, and like units. The same is true when students make math drawings and use place value language to relate each step of the drawing to a written method (2.NBT.7). The different representations serve to solidify the understanding of the composition and decomposition of units, moving from concrete to pictorial to abstract. Throughout the work, students are encouraged to explain their actions and analyses, and to use the relationship between addition and subtraction to check their work (2.NBT.9).
- Throughout the module, students are encouraged to be flexible in their thinking and to use multiple strategies in solving problems, including the use of drawings such as tape diagrams, which they relate to equations. In Topic F, students are introduced to the totals below method (pictured below to the far left) and are challenged to explain why both it and the new groups below method (also pictured below to the left) work (2.NBT.9).

# Addition and Subtraction Within 1,000 with Word Problems to 100

#### **OVERVIEW**

In Module 4, students developed addition and subtraction fluency within 100 and began developing conceptual understanding of the standard algorithm via place value strategies. In Module 5, students build upon their mastery of renaming place value units and extend their work with conceptual understanding of the addition and subtraction algorithms to numbers within 1,000, always with the option of modeling with materials or drawings. Throughout the module, students continue to focus on strengthening and deepening conceptual understanding and fluency.

Topic A focuses on place value strategies to add and subtract within 1,000 (**2.NBT.7**). Students relate 100 more and 100 less to addition and subtraction of 100 (2.NBT.8). They add and subtract multiples of 100, including counting on to subtract (e.g., for 650 - 300, they start at 300 and think, "300 more gets me to 600, and 50 more gets me to 650, so... 350"). Students also use simplifying strategies for addition and subtraction: they extend the *make a ten* strategy to make a hundred, mentally decomposing one addend to make a hundred with the other (e.g., 299 + 6 becomes 299 + 1 + 5, or 300 + 5, which equals 305) and use compensation to subtract from three-digit numbers (e.g., for 376 - 59, add 1 to each, 377 - 60 = 317). The topic ends with students sharing and critiquing solution strategies for addition and

subtraction problems. Throughout the topic, students use place value language and properties of operations to explain why their strategies work (2.NBT.9).

In Topics B and C, students continue to build on Module 4's work, now composing and decomposing tens and hundreds within 1,000 (2.NBT.7). As each of these topics begins, students relate manipulative representations to the algorithm, then transition to making math drawings in place of the manipulatives. As always, students use place value reasoning and properties of operations to explain their work.

Throughout Module 5, students maintain addition and subtraction fluency within 100 as they use these skills during their daily application work to solve one- and two-step word problems of all types (2.NBT.5, 2.OA.1). The focus of concept development is reserved for adding and subtracting within 1,000; using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; and relating strategies to a written method (2.NBT.7). Note that a written method can include number bonds, chip models, arrow notation, the algorithm, or tape diagrams. Many students willneed to record these strategies in order to solve correctly. The lessons are designed to provide ample time for discussions that center on student reasoning, explaining why their addition and subtraction strategies work (2.NBT.9). For example, students may use the relationship between addition and subtraction to demonstrate why their subtraction solution is correct.

The module culminates with Topic D, wherein students synthesize their understanding of addition and subtraction strategies and choose which strategy is most efficient for given problems. They defend their choices using place value language and their understanding of the properties of operations (2.NBT.9).

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

### **Terminology**

#### **New or Recently Introduced Terms**

Algorithm (a step-by-step procedure to solve a particular type of problem)

Compensation (simplifying strategy where students add or subtract the same amount to or from both numbers to create an equivalent but easier problem)

Compose (e.g., to make 1 larger unit from 10 smaller units)

Decompose (e.g., to break 1 larger unit into 10 smaller units)

New groups below (show newly composed units on the line below the appropriate place in the addition algorithm)

Simplifying strategy (e.g., to solve 299 + 6, think 299 + 1 + 5 = 300 + 5 = 305.)

#### Familiar Terms and Symbols<sup>7</sup>

Addend

Addition

Bundle

Difference



Place value disks

<sup>&</sup>lt;sup>7</sup> These are terms and symbols students have seen previously.

Equation

Number bond

Place value

Place value chart (pictured right)

Place value or number disk (pictured above right)

Rename

Subtraction

Tape diagram

Total

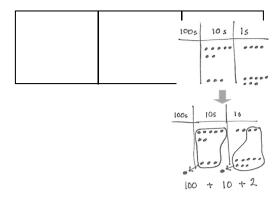
Unbundle

Units of ones, tens, hundreds

#### Place Value Chart with Headings (use with numbers)

hundreds	tens	ones	
7	2	6	
Place Value Chart without Headings			

(use with number disks)



# **Suggested Tools and Representations**

Arrow notation, arrow way

Chip model (pictured right)

Number bond

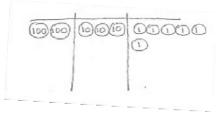
Place value charts and mats (pictured above right)

Place value disk sets (18 ones, 18 tens, 18 hundreds, 1 one thousand per set)

Tape diagram

Note: Students work through a progression of models to represent the addition and subtraction algorithm. Following the use of actual place value disks, students learn to draw the disks to represent numbers. This model provides an added level of support in that students write the value on each disk

(see image below left). Because the value is on the disk, there is no need to label the place value chart. Next, students learn the chip model, drawing dots on a labeled place value chart (see image below right). While still pictorial, this model is more abstract because the value of the chip derives from its placement on the chart.



Chip model

Place value disk drawing