GLENCOE MATHEMATICS

Interactive Chalkboard

Algebra 1

CHAPTER 14
Probability

EXIT

Lesson 14-1 Contents

Example 1

At football games, a student concession stand sells sandwiches on either wheat or rye bread. The sandwiches come with salami, turkey, or ham, and either chips, a brownie, or fruit. Use a tree diagram to determine the number of possible sandwich combinations.

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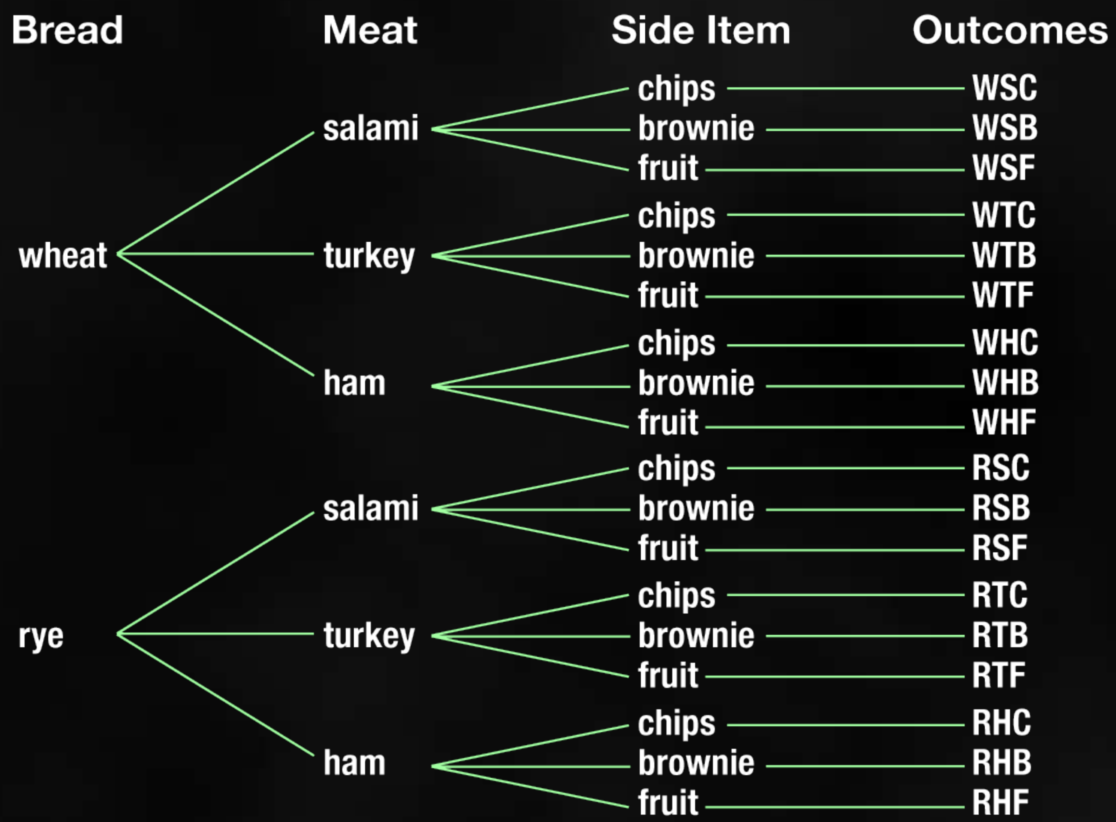
Example 1



Answer: The tree diagram shows that there are 18 possible combinations.

Your Turn

A lunch buffet offers a combination of a meat, a vegetable, and a drink for $5.99. The choices of meat are chicken or pork; the choices of vegetable are carrots, broccoli, green beans, or potatoes; and the choices of drink are milk, lemonade, or a soft drink. Use a tree diagram to determine the number of possible lunch combinations.

Answer: 24 different lunches

STOP! End of slide

Example 2

The Too Cheap computer company sells custom made personal computers. Customers have a choice of 11 different hard drives, 6 different keyboards, 4 different mice, and 4 different monitors. How many different custom computers can you order?

Multiply to find the number of custom computers.

hard drive choices		keyboard choices		mice choices		monitor choices		number of custom computers
11	$\cdot$	6	$\cdot$	4	$\cdot$	4	$=$	1056

Answer: The number of different custom computers is 1056.

Your Turn

A major league team is trying to organize their draft. In their first five rounds, they want to pick a pitcher, a catcher, a first baseman, a third basemen, and an outfielder. They are considering 7 pitchers, 9 catchers, 3 first baseman, 4 third baseman, and 12 outfielders. How many ways can they draft players for these five positions?

Answer: 9072

Example 3

There are 8 students in the Algebra Club at Central High School. The students want to stand in a line for their yearbook picture. How many different ways could the 8 students stand for their picture?

The number of ways to arrange the students can be found by multiplying the number of choices for each position.

- There are eight people from which to choose for the first position.
- After choosing a person for the first position, there are seven people left from which to choose for the second position.

End of slide—continued on the next slide

Example 3

- There are now six choices for the third position.
- This process continues until there is only one choice left for the last position.

Let n represent the number of arrangements.

$$n = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \text{ or } 40,320$$

Answer: There are 40,320 different ways they could stand.

Your Turn

There are 11 people performing in a talent show. The program coordinator is trying to arrange the order in which each participant will perform. How many different ways can the order of performances be arranged?

Answer: 39,916,800 ways

Example 4

Find the value of $9!$.

$$9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$ Definition of factorial

Answer: $= 362{,}880$ Simplify.

End of slide

Your Turn

Find the value of $7!$.

Answer: 5040

Example 5a

Jill and Miranda are going to a national park for their vacation. Near the campground where they are staying, there are 8 hiking trails.

How many different ways can they hike all of the trails if they hike each trail only once?

Use a factorial.

$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$	Definition of factorial
$= 40,320$	Simplify.

Answer: There are 40,320 ways in which Jill and Miranda can hike all 8 trails.

Example 5b

Jill and Miranda are going to a national park for their vacation. Near the campground where they are staying, there are 8 hiking trails.

If they only have time to hike on 5 of the trails, how many ways can they do this?

Use the Fundamental Counting Principle to find the sample space.

$s = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$ Fundamental Counting Principle

$= 6720$ Simplify.

Answer: There are 6720 ways that Jill and Miranda can hike 5 of the trails.

Your Turn

Jack and Renee want to take a cross-country trip over the summer to 10 different cities. They are trying to decide the order in which they should travel.

a. How many different orders can they travel to the 10 cities if they go to each city once?

Answer: 3,628,800

b. Suppose they only have time to go to 8 of the cities. How many ways can they do this?

Answer: 1,814,400

End of

Lesson 14-1

Click the mouse button to return to the Contents screen.

Lesson 14-2 Contents

Example 1

Ms. Baraza asks pairs of students to go in front of her Spanish class to read statements in Spanish, and then to translate the statement into English. One student is the Spanish speaker and one is the English speaker. If Ms. Baraza has to choose between Jeff, Kathy, Guillermo, Ana, and Patrice, how many different ways can Ms. Baraza pair the students?

Use a tree diagram to show the possible arrangements.

End of slide—continued on the next slide

Example 1

Answer: There are 20 different ways for the 5 students to be paired.

Your Turn

There are five finalists in the student art contest: Cal, Jeanette, Emily, Elizabeth, and Ron. The winner and the runner-up of the contest will receive prizes. How many possible ways are there for the winners to be chosen?

Answer: 20

End of slide

Example 2

Find $_8P_4$.

$$_nP_r = \frac{n!}{(n-r)!}$$ Definition of $_nP_r$

$$_8P_4 = \frac{8!}{(8-4)!}$$ $n = 8,\ r = 4$

$$_8P_4 = \frac{8!}{4!}$$ Subtract.

End of slide—continued on the next slide

Example 2

$${}_8P_4 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}^{\,1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}_{\,1}}$$ Definition of factorial

$${}_8P_4 = 8 \cdot 7 \cdot 6 \cdot 5 \text{ or } 1680$$ Simplify.

Answer: There are 1680 permutations of 8 objects taken 4 at a time.

Your Turn

Find ${}_9P_5$.

Answer: 15,120

Example 3a

Shaquille has a 5-digit pass code to access his e-mail account. The code is made up of the even digits 2, 4, 6, 8, and 0. Each digit can be used only once.

How many different pass codes could Shaquille have?

Since the order of the numbers in the code is important, this situation is a permutation of 5 digits taken 5 at a time.

$${}_nP_r = \frac{n!}{(n-r)!}$$ Definition of permutation

$${}_5P_5 = \frac{5!}{(5-5)!}$$ $n = 5, r = 5$

End of slide—continued on the next slide

Example 3a

$${}_5P_5 = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} \text{ or } 120$$ Definition of factorial

Answer: There are 120 possible pass codes with the digits 2, 4, 6, 8, and 0.

End of slide

Example 3b

Shaquille has a 5-digit pass code to access his e-mail account. The code is made up of the even digits 2, 4, 6, 8, and 0. Each digit can be used only once.

What is the probability that the first two digits of his code are both greater than 5?

Use the Fundamental Counting Principle to determine the number of ways for the first two digits to be greater than 5.

- There are 2 digits greater than 5 and 3 digits less than 5.
- The number of choices for the first two digits, if they are greater than 5, is 2 • 1.
- The number of choices for the remaining digits is 3 • 2 • 1.

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Example 3b

- The number of favorable outcomes is 2 • 1 • 3 • 2 • 1 or 12. There are 12 ways for this event to occur out of the 120 possible permutations.

$$P(\text{first 2 digits} > 5) = \frac{12}{120} \leftarrow \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$= \frac{1}{10} \quad \text{Simplify.}$$

Answer: The probability that the first two digits of the pass code are greater than 5 is $\frac{1}{10}$ or 10%.

End of slide

Your Turn

Bridget and Brittany are trying to find a house, but they cannot remember the address. They can remember only that the digits used are 1, 2, 5, and 8, and that no digit is used twice.

a. How many possible addresses are there?

Answer: 24 addresses

b. What is the probability that the first two numbers are odd?

Answer: $\frac{1}{6}$ or about 17%

End of slide

Example 4

Multiple-Choice Test Item

Customers at Tony's Pizzeria can choose 4 out of 12 toppings for each pizza for no extra charge. How many different combinations of pizza toppings can be chosen?

A 495 **B** 792
C 11,880 **D** 95,040

Read the Test Item

The order in which the toppings are chosen does not matter, so this situation represents a combination of 12 toppings taken 4 at a time.

End of slide—continued on the next slide

Example 4

Solve the Test Item

$${}_nC_r = \frac{n!}{(n-r)!r!}$$ Definition of combination

$${}_{12}C_4 = \frac{12!}{(12-4)!4!}$$ $n = 12, r = 4$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot \overset{1}{\cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}}{\underset{1}{\cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$ Definition of factorial

End of slide—continued on the next slide

Example 4

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} \text{ or } 495$$

Simplify.

Answer: There are 495 different ways to select toppings. Choice A is correct.

Your Turn

Multiple-Choice Test Item

A cable company is having a sale on their premium channels. Out of 8 possible premium channels, they are allowing customers to pick 5 channels at no extra charge. How many channel packages are there?

A 6720 B 56
C 336 D 120

Answer: B

Example 5a

Diane has a bag full of coins. There are 10 pennies, 6 nickels, 4 dimes, and 2 quarters in the bag.

How many different ways can Diane pull four coins out of the bag?

The order in which the coins are chosen does not matter, so we must find the number of combinations of 22 coins taken 4 at a time.

$${}_{n}C_{r} = \frac{n!}{(n-r)!r!}$$ Definition of combination

$${}_{22}C_{4} = \frac{22!}{(22-4)!4!}$$ $n = 22, r = 4$

End of slide—continued on the next slide

Example 5a

$$= \frac{22!}{18!4!}$$

$22 - 4 = 18$

$$= \frac{22 \cdot 21 \cdot 20 \cdot 19 \cdot \overset{1}{\cancel{18!}}}{\underset{1}{\cancel{18!}}\,4!}$$

Divide by the GCF, 18!.

$$= \frac{175,560}{24} \text{ or } 7315$$

Simplify.

Answer: There are 7315 ways to pull 4 coins out of a bag of 22.

Example 5b

Diane has a bag full of coins. There are 10 pennies, 6 nickels, 4 dimes, and 2 quarters in the bag.

What is the probability that she will pull two pennies and two nickels out of the bag?

There are two questions to consider.

- How many ways can 2 pennies be pulled from 10?
- How many ways can 2 nickels be pulled from 6?

Using the Fundamental Counting Principle, the answer can be determined with the product of the two combinations.

STOP! End of slide—continued on the next slide

Example 5b

ways to choose 2 pennies out of 10 · ways to choose 2 nickels out of 6

$$\left({}_{10}C_2\right) \cdot \left({}_{6}C_2\right)$$

$$\left({}_{10}C_2\right)\left({}_{6}C_2\right) = \frac{10!}{(10-2)!2!} \cdot \frac{6!}{(6-2)!2!}$$ Definition of combination

$$= \frac{10!}{8!2!} \cdot \frac{6!}{4!2!}$$ Simplify.

End of slide—continued on the next slide

Example 5b

$$= \frac{10 \cdot 9}{2!} \cdot \frac{6 \cdot 5}{2!}$$

Divide the first term by its GCF, 8!, and the second term by its GCF, 4!.

$$= 675$$

Simplify.

There are 675 ways to choose this particular combination out of 7315 possible combinations.

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Example 5b

$$P(2 \text{ pennies}, 2 \text{ nickels}) = \frac{675}{7315} \quad \frac{\leftarrow \text{number of favorable outcomes}}{\leftarrow \text{number of possible outcomes}}$$

$$= \frac{135}{1463} \quad \text{Simplify.}$$

Answer: The probability that Diane will select two pennies and two nickels is $\frac{135}{1463}$ or about 9%.

End of slide

Your Turn

At a factory, there are 10 union workers, 12 engineers, and 5 foremen. The company needs 6 of these workers to attend a national conference.

a. How many ways could the company choose the 6 workers?

Answer: 296,010 ways

b. If the workers are chosen randomly, what is the probability that 3 union workers, 2 engineers, and 1 foreman are selected?

Answer: $\frac{40}{299}$ or about 13%

End of slide

End of

Lesson 14-2

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Lesson 14-3 Contents

Example 1

Roberta is flying from Birmingham to Chicago to visit her grandmother. She has to fly from Birmingham to Houston on the first leg of her trip. In Houston she changes planes and heads on to Chicago. The airline reports that the flight from Birmingham to Houston has a 90% on time record, and the flight from Houston to Chicago has a 50% on time record. What is the probability that both flights will be on time?

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Example 1

$P(A \text{ and } B) = P(A) \cdot P(B)$ Definition of independent events

$P(\text{B-H on time and H-C on time}) = P(\text{B-H on time}) \cdot P(\text{H-C on time})$

$= 0.9 \cdot 0.5$ $90\% = 0.9$ and $50\% = 0.5$

$= 0.45$ Multiply.

Answer: The probability that both flights will be on time is 45%.

Your Turn

Two cities, Fairfield and Madison, lie on different faults. There is a 60% chance that Fairfield will experience an earthquake by the year 2010 and a 40% chance that Madison will experience an earthquake by 2010. Find the probability that both cities will experience an earthquake by 2010.

Answer: 24%

Example 2a

At the school carnival, winners in the ring-toss game are randomly given a prize from a bag that contains 4 sunglasses, 6 hairbrushes, and 5 key chains. Three prizes are randomly drawn from the bag and not replaced. Find $P($sunglasses, hairbrush, key chain$)$**.**

The selection of the first prize affects the selection of the next prize since there is one less prize from which to choose. So, the events are dependent.

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Example 2a

First prize: $P(\text{sunglasses}) = \frac{4}{15}$ ← number of sunglasses / total number of prizes

Second prize: $P(\text{hairbrush}) = \frac{6}{14}$ or $\frac{3}{7}$ ← number of hairbrushes / total number of prizes

Third prize: $P(\text{key chain}) = \frac{5}{13}$ ← number of key chains / total number of prizes

STOP! End of slide—continued on the next slide

Example 2a

$$P(\text{sunglasses, hairbrush, key chain}) = P(\text{sunglasses}) \cdot P(\text{hairbrush}) \cdot P(\text{key chain})$$

$$= \frac{4}{15} \cdot \frac{3}{7} \cdot \frac{5}{13}$$ Substitution

$$= \frac{60}{1365} \text{ or } \frac{4}{91}$$ Multiply.

Answer: The probability of drawing sunglasses, a hairbrush, and a key chain is $\frac{4}{91}$.

End of slide

Example 2b

At the school carnival, winners in the ring-toss game are randomly given a prize from a bag that contains 4 sunglasses, 6 hairbrushes, and 5 key chains. Three prizes are randomly drawn from the bag and not replaced. Find P(hairbrush, hairbrush, key chain).

Notice that after selecting a hairbrush, not only is there one fewer prize from which to choose, there is also one fewer hairbrush.

End of slide—continued on the next slide

Example 2b

$$P(\text{hairbrush, hairbrush, key chain}) = P(\text{hairbrush}) \bullet P(\text{hairbrush}) \bullet P(\text{key chain})$$

$$= \frac{6}{15} \bullet \frac{5}{14} \bullet \frac{5}{13}$$

Substitution

$$= \frac{150}{2730} \text{ or } \frac{5}{91}$$

Multiply.

Answer: The probability of drawing two hairbrushes and then a key chain is $\frac{5}{91}$.

End of slide

Example 2c

At the school carnival, winners in the ring-toss game are randomly given a prize from a bag that contains 4 sunglasses, 6 hairbrushes, and 5 key chains. Three prizes are randomly drawn from the bag and not replaced. Find P(sunglasses, hairbrush, *not* key chain).

Since the prize that is not a key chain is selected after the first two prizes, there are 10 – 2 or 8 prizes that are not key chains.

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Example 2c

$$P(\text{sunglasses, hairbrush, } \textit{not} \text{ key chain}) = P(\text{sunglasses}) \cdot P(\text{hairbrush}) \cdot P(\textit{not} \text{ key chain})$$

$$= \frac{4}{15} \cdot \frac{6}{14} \cdot \frac{8}{13}$$

Substitution

$$= \frac{192}{2730} \text{ or } \frac{32}{455}$$

Multiply.

Answer: The probability of drawing sunglasses, a hairbrush, and *not* a key chain is $\frac{32}{455}$.

End of slide

Your Turn

A gumball machine contains 16 red gumballs, 10 blue gumballs, and 18 green gumballs. Once a gumball is removed from the machine, it is not replaced. Find each probability if the gumballs are removed in the order indicated.

a. P(red, green, blue)

Answer: $\frac{120}{3311}$

b. P(blue, green, green)

Answer: $\frac{255}{6622}$

c. P(green, blue, *not* red)

Answer: $\frac{195}{3311}$

Example 3

Alfred is going to the Lakeshore Animal Shelter to pick a new pet. Today, the shelter has 8 dogs, 7 cats, and 5 rabbits available for adoption. If Alfred randomly picks an animal to adopt, what is the probability that the animal would be a cat or a dog?

Since a pet cannot be both a dog and a cat, the events are mutually exclusive.

End of slide—continued on the next slide

Example 3

$$P(\text{cat}) = \frac{7}{20}$$ ← number of cats / ← total number of pets

$$P(\text{dog}) = \frac{8}{20}$$ ← number of dogs / ← total number of pets

$$P(\text{cat or dog}) = P(\text{cat}) + P(\text{dog})$$ Definition of mutually exclusive events

$$= \frac{7}{20} + \frac{8}{20}$$ Substitution

STOP! End of slide—continued on the next slide

Example 3

$$= \frac{15}{20} \text{ or } \frac{3}{4}$$ Add.

Answer: The probability of randomly picking a cat or a dog is $\frac{3}{4}$.

End of slide

Your Turn

The French Club has 16 seniors, 12 juniors, 15 sophomores, and 21 freshmen as members. What is the probability that a member chosen at random is a junior or a senior?

Answer: $\frac{7}{16}$

Example 4

A dog has just given birth to a litter of 9 puppies. There are 3 brown females, 2 brown males, 1 mixed-color female, and 3 mixed-color males. If you choose a puppy at random from the litter, what is the probability that the puppy will be male or mixed-color?

Since three of the puppies are both mixed-colored and males, these events are inclusive.

End of slide—continued on the next slide

Example 4

$P(A \text{ or } B) = P(A) + P(A) - P(A \text{ and } B)$

Definition of inclusive events

$P(\text{male or mixed-color})$

$= \underbrace{P(\text{male})}_{} + \underbrace{P(\text{mixed-color})}_{} - \underbrace{P(\text{male and mixed-color})}_{}$

$= \frac{5}{9} + \frac{4}{9} - \frac{3}{9}$

Substitution

$= \frac{5+4-3}{9}$

LCD is 9.

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Example 4

$$= \frac{6}{9} \text{ or } \frac{2}{3}$$ Simplify.

Answer: The probability of a puppy being a male or mixed-color is $\frac{2}{3}$ or about 67%.

Your Turn

In Mrs. Kline's class, 7 boys have brown eyes and 5 boys have blue eyes. Out of the girls, 6 have brown eyes and 8 have blue eyes. If a student is chosen at random from the class, what is the probability that the student will be a boy or have brown eyes?

Answer: $\frac{9}{13}$

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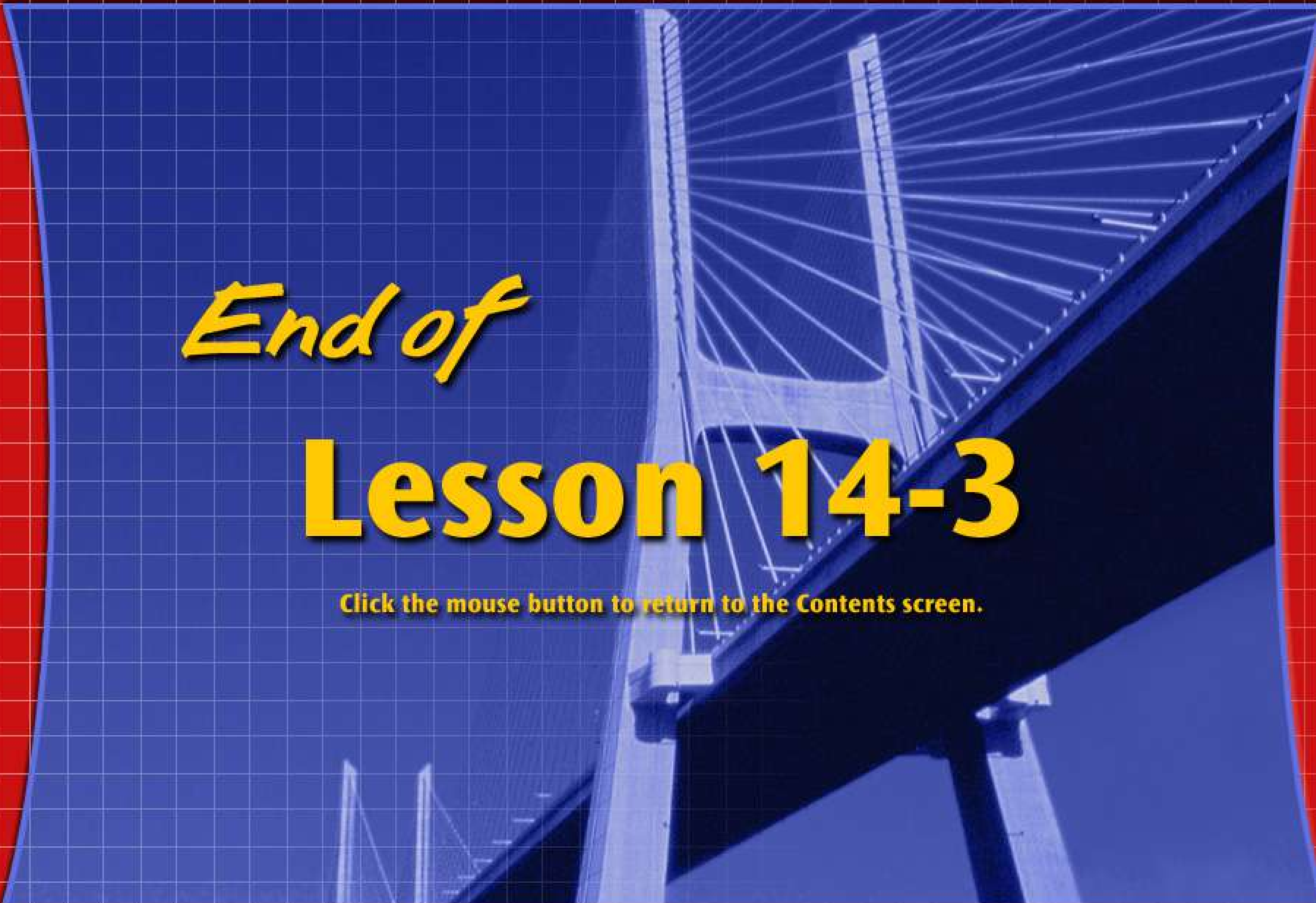
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Lesson 14-3
Click the mouse button to return to the Contents screen.

Lesson 14-4 Contents

Example 1a

The owner of a pet store asked customers how many pets they owned. The results of this survey are shown in the table.

Number of Pets	Number of Customers
0	3
1	37
2	33
3	18
4	9

Find the probability that a randomly chosen customer has at most 2 pets.

There are $3 + 37 + 33$ or 73 outcomes in which a customer owns at most 2 pets, and there are 100 survey results.

STOP! End of slide—continued on the next slide

Example 1a

$$P(X \le 2) = \frac{73}{100}$$

Answer: The probability that a randomly chosen customer owns at most 2 pets is $\frac{73}{100}$ or 73%.

End of slide

Example 1b

The owner of a pet store asked customers how many pets they owned. The results of this survey are shown in the table.

Find the probability that a randomly chosen customer has 2 or 3 pets.

Number of Pets	Number of Customers
0	3
1	37
2	33
3	18
4	9

There are $33 + 18$ or 51 outcomes in which a customer owns 2 or 3 pets.

End of slide—continued on the next slide

Example 1b

$$P(X = 2 \text{ or } 3) = \frac{51}{100}$$

Answer: The probability that a randomly chosen customer owns 2 or 3 pets is $\frac{51}{100}$ or 51%.

Your Turn

A survey was conducted concerning the number of movies people watch at the theater per month. The results of this survey are shown in the table.

Number of movies (per month)	Number of people
0	7
1	23
2	30
3	29
4	11

a. Find the probability that a randomly chosen person watches at most 1 movie per month.

Answer: $P(X \leq 1) = \frac{3}{10}$ or 30%

STOP! End of slide—continued on the next slide

Your Turn

A survey was conducted concerning the number of movies people watch at the theater per month. The results of this survey are shown in the table.

Number of movies (per month)	Number of people
0	7
1	23
2	30
3	29
4	11

b. Find the probability that a randomly chosen person watches 0 or 4 movies per month.

Answer: $P(X = 0 \text{ or } 4) = \frac{9}{50} \text{ or } 18\%$

End of slide

Example 2a

The table shows the probability distribution of the number of students in each grade at Sunnybrook High School.

X = Grade	$P(X)$
9	0.29
10	0.26
11	0.25
12	0.2

If a student is chosen at random, what is the probability that he or she is in grade 11 or above?

Recall that the probability of a compound event is the sum of the probabilities of each individual event.

The probability of a student being in grade 11 or above is the sum of the probability of grade 11 and the probability of grade 12.

End of slide—continued on the next slide

Example 2a

$$P(X \geq 11) = P(X = 11) + P(X = 12)$$

Sum of individual probabilities

$$= 0.25 + 0.2 \text{ or } 0.45$$

$P(X = 11) = 0.25$,
$P(X = 12) = 0.2$

Answer: The probability of a student being in grade 11 or above is 0.45.

Example 2b

The table shows the probability distribution of the number of students in each grade at Sunnybrook High School.

Make a probability histogram of the data.

X = Grade	$P(X)$
9	0.29
10	0.26
11	0.25
12	0.2

Draw and label the vertical and horizontal axes. Remember to use equal intervals on each axis. Include a title.

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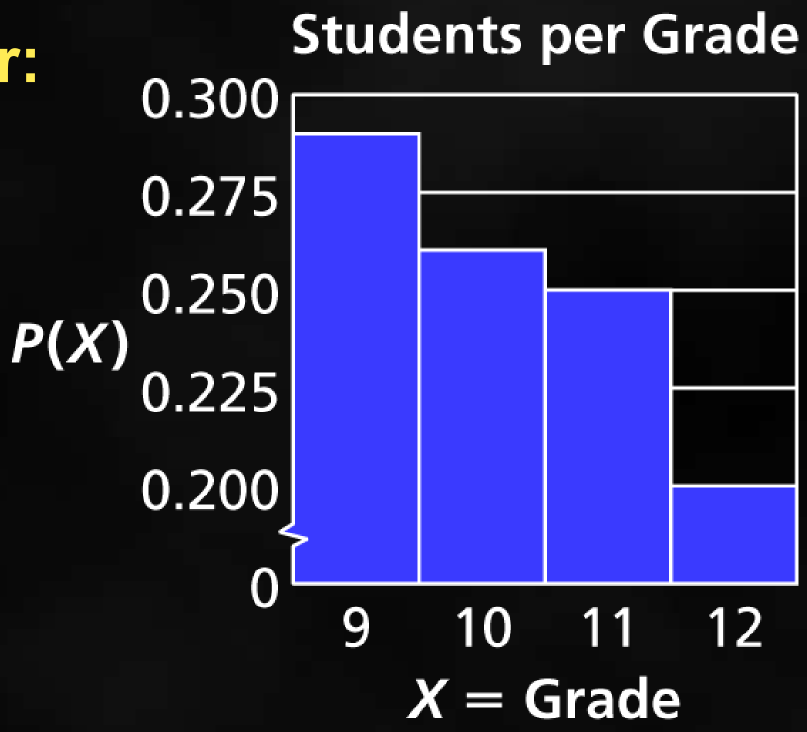
Example 2b

Answer:



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Your Turn

The table shows the probability distribution of the number of children per family in the city of Maplewood.

X = Number of Children	$P(X)$
0	0.11
1	0.23
2	0.32
3	0.26
4	0.08

a. If a family was chosen at random, what is the probability that they have at least 2 children?

Answer: 0.66

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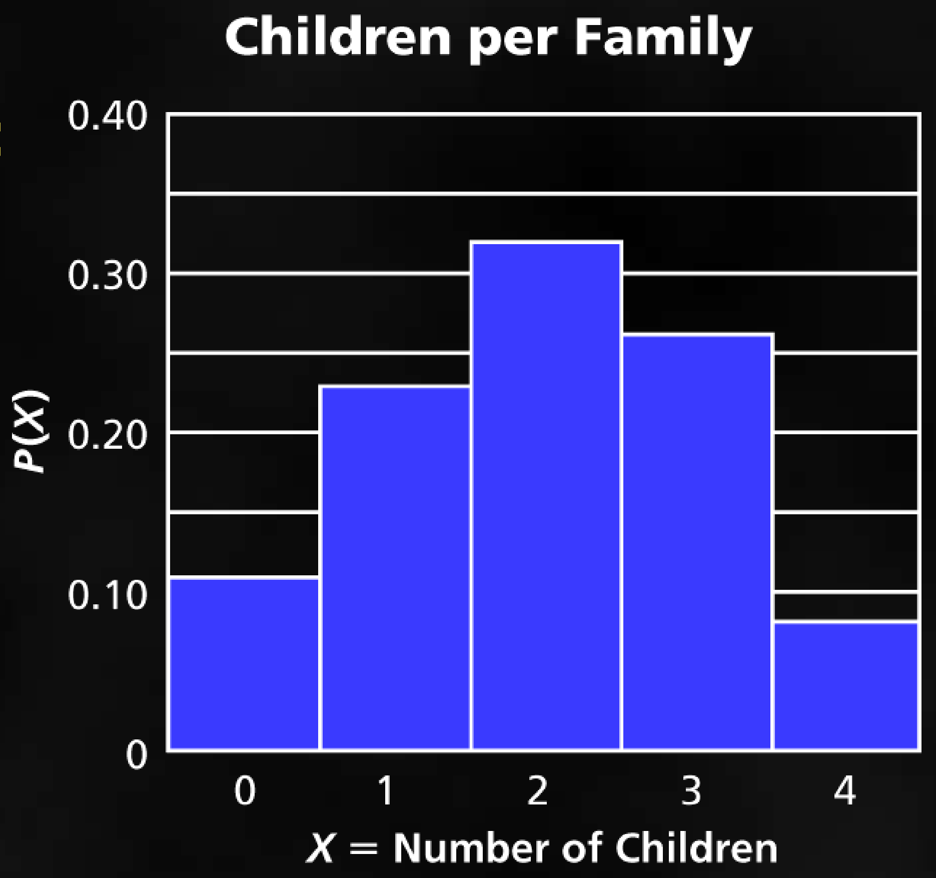
Your Turn

b. Make a probability histogram of the data.

Answer:



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End of

Lesson 14-4

Click the mouse button to return to the Contents screen.

Lesson 14-5 Contents

Example 1

Miguel shot 50 free throws in the gym and found that his experimental probability of making a free throw was 40%. How many free throws did Miguel make?

Miguel's experimental probability of making a free throw was 40%. The number of successes can be written as 40 out of every 100 free throws.

$$\text{experimental probability} = 40\% \text{ or } \frac{40}{100}$$

40 ← number of success

100 ← total number of free throws

End of slide—continued on the next slide

Example 1

Since Miguel only shot 50 free throws, write and solve a proportion.

experimental successes →	$$\frac{40}{100} = \frac{x}{50}$$	← Miguel's successes
experimental total free throws →		← Miguel's total free throws

$$50(40) = 100(x)$$ Find the cross products.

$$2000 = 100x$$ Simplify.

$$x = 20$$ Divide each side by 100.

Answer: Miguel made 20 free throws.

STOP! End of slide—continued on the next slide

Your Turn

Nancy was testing her serving accuracy in volleyball. She served 80 balls and found that her experimental probability of keeping it in bounds was 60%. How many serves did she keep in bounds?

Answer: 48

Example 2

A pharmaceutical company performs three clinical studies to test the effectiveness of a new medication. Each study involves 100 volunteers. The results of the studies are shown in the table.

Study of New Medication			
Result	**Study 1**	**Study 2**	**Study 3**
Expected Success Rate	70%	70%	70%
Condition Improved	61%	74%	67%
No Improvement	39%	25%	33%
Condition Worsened	0%	1%	0%

What is the experimental probability that the drug showed no improvement in patients for all three studies?

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Example 2

The number of outcomes with no improvement for the three studies was $39 + 25 + 33$ or 97 out of the 300 total patients.

$$\text{experimental probability} = \frac{97}{300}$$

Answer: The experimental probability of the three studies was $\frac{97}{300}$ or about 32%.

Your Turn

A new study is being developed to analyze the relationship between heart rate and watching scary movies. A researcher performs three studies, each with 100 volunteers. Based on similar studies, the researcher expects that 80% of the subjects will experience a significant increase in heart rate. The table shows the results of the study.

Study of Heart Rate			
Result	**Study 1**	**Study 2**	**Study 3**
Expected Success Rate	80%	80%	80%
Rate increased significantly	83%	75%	78%
Little or no increase	16%	24%	19%
Rate decreased	1%	1%	3%

End of slide—continued on the next slide

Your Turn

What is the experimental probability that the movie would cause a significant increase in heart rate for all three studies?

Answer: $\frac{59}{75}$ or about 79%

Example 3a

In the last 30 school days, Bobbie's older brother has given her a ride to school 5 times.

What could be used to simulate whether Bobbie's brother will give her a ride to school?

Bobbie got a ride to school on $\frac{5}{30}$ days. or $\frac{1}{6}$

Answer: Since a die has 6 sides, you could use one side of a die to represent a ride to school.

Example 3b

In the last 30 school days, Bobbie's older brother has given her a ride to school 5 times.

Describe a way to simulate whether Bobbie's brother will give her a ride to school in the next 20 school days.

Choose the side of the die that will be used to represent a ride to school.

Answer: Let the 1-side of the die equal a ride to school. Toss the die 20 times and record each result.

Your Turn

In the last 52 days, it has rained 4 times.

a. What could be used to simulate whether it will rain on a given day?

Answer: It rained on $\frac{4}{52}$ or $\frac{1}{13}$ of the days. You could use a deck of cards to simulate the situation.

b. Describe a way to simulate whether it will rain in the next 15 days.

Answer: Let the aces equal a rainy day. Draw cards 15 times and record the results.

Example 4a

Dogs Ali raises purebred dogs. One of her dogs is expecting a litter of four puppies, and Ali would like to figure out the most likely mix of male and female puppies.

Assume that $P(\text{male}) = P(\text{female}) = \frac{1}{2}$.

One possible simulation would be to toss four coins, one for each puppy, with heads representing female and tails representing male. What is an alternative to using 4 coins that could model the possible combinations of the puppies?

End of slide—continued on the next slide

Example 4a

Each puppy can be male or female, so there are $2 \cdot 2 \cdot 2 \cdot 2$ or 16 possible outcomes for the litter

Sample answer: a spinner with 16 equal divisions

End of slide

Example 4b

Dogs Ali raises purebred dogs. One of her dogs is expecting a litter of four puppies, and Ali would like to figure out the most likely mix of male and female puppies.

Assume that $P(\text{male}) = P(\text{female}) = \frac{1}{2}$.

Find the theoretical probability that there will be 4 female puppies in a litter.

End of slide—continued on the next slide

Example 4b

There are 16 possible outcomes, and the number of combinations that have 4 female puppies is ${}_4C_4$ or 1.

Answer: So the theoretical probability is $\frac{1}{16}$.

Example 4c

Dogs Ali raises purebred dogs. One of her dogs is expecting a litter of four puppies, and Ali would like to figure out the most likely mix of male and female puppies.

Assume that $P(\text{male}) = P(\text{female}) = \frac{1}{2}$.

The results of a simulation Ali performed are shown in the table on the following slide. How does the theoretical probability that there will be 4 females compare with Ali's results?

End of slide—continued on the next slide

Example 4c

Outcomes	Frequency
4 female, 0 male	3
3 female, 1 male	13
2 female, 2 male	18
1 female, 3 male	12
0 female, 4 male	4

Theoretical probability

$$P(\text{4 females}) = \frac{{}_4C_4}{16}$$

${}_4C_4$ ← combinations with 4 female puppies

16 ← possible outcomes

End of slide—continued on the next slide

Example 4c

$$= \frac{1}{16} \text{ or } 6.25\%$$

Experimental probability

Ali performed 50 trials and 3 of those resulted in 4 females. So, the experimental probability is $\frac{3}{50}$ or 6%.

Answer: The theoretical probability is a little more than 6% and the experimental probability is 6%, so they are very close.

Your Turn

In baseball, the Cleveland Indians and Chicago White Sox play each other five times in the next week. The manager would like to figure out the most likely mix of wins and losses.

Assume that $P(\text{Indians win}) = P(\text{White Sox win}) = \frac{1}{2}$.

a. What objects can be used to model the possible outcomes of the games?

Sample Answer: Flip five coins, one for each game, with heads representing an Indians win, and tails representing a White Sox win.

End of slide—continued on the next slide

Your Turn

b. Find the theoretical probability that the Indians will win three games.

Answer: $\frac{10}{32}$ or $\frac{5}{16}$

End of slide—continued on the next slide

Your Turn

c. Below are the results of the last thirty 5-game series between the two teams. How does the theoretical probability that the Indians will win three games compare with the results?

Outcomes	Frequency
Indians win every game	2
Indians win four, White Sox win one	6
Indians win three, White Sox win two	10
Indians win two, White Sox win three	7
Indians win one, White Sox win four	4
White Sox win every game	1

End of slide—continued on the next slide

Your Turn

Answer: The theoretical probability is a little more than 31% and the experimental probability is a little more than 33%, so they are moderately close.

End of

Lesson 14-5

Click the mouse button to return to the Contents screen.

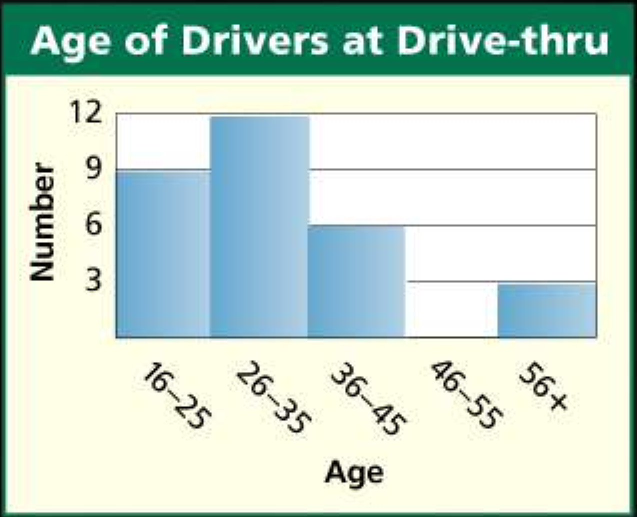
Extra Examples

Explore online information about the information introduced in this chapter.

Click on the **Connect** button to launch your browser and go to the *Algebra 1* Web site. At this site, you will find extra examples for each lesson in the Student Edition of your textbook. When you finish exploring, exit the browser program to return to this presentation. If you experience difficulty connecting to the Web site, manually launch your Web browser and go to **www.algebra1.com/extra_examples**.

1. Simplify $\begin{bmatrix} 4 & 0 & 2 \\ 5 & 3 & 1 \\ -2 & -1 & 6 \end{bmatrix} - \begin{bmatrix} -2 & -4 & -1 \\ 11 & 0 & 6 \\ 1 & -8 & 5 \end{bmatrix}$.

For Questions 2 and 3, use the histogram.



2. How many drivers are in this data set?

3. Describe the distribution of these data.

4. What are the quartiles of the average monthly rainfall data shown in the box-and-whisker plot?

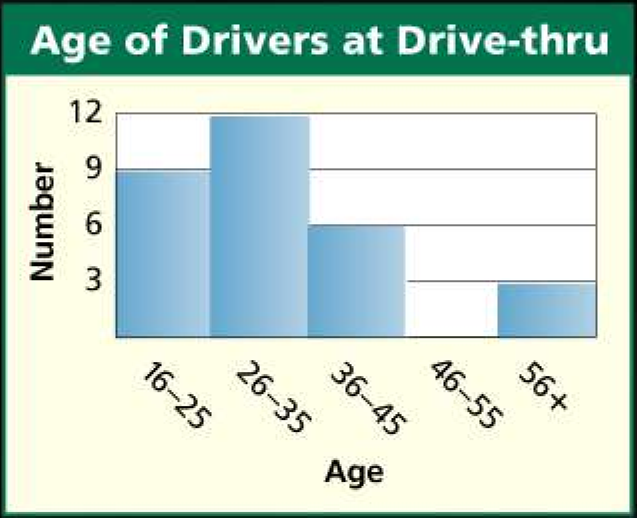
5. **Standardized Test Practice** Which term describes the sampling of two random students from each of five classrooms?

- (A) simple
- (B) convenient
- (C) stratified
- (D) systematic

5-Minute Check *(over Chapter 13)* Transparency 14-1

1. Simplify $\begin{bmatrix} 4 & 0 & 2 \\ 5 & 3 & 1 \\ -2 & -1 & 6 \end{bmatrix} - \begin{bmatrix} -2 & -4 & -1 \\ 11 & 0 & 6 \\ 1 & -8 & 5 \end{bmatrix}$. $\begin{bmatrix} 6 & 4 & 3 \\ -6 & 3 & -5 \\ -3 & 7 & 1 \end{bmatrix}$

For Questions 2 and 3, use the histogram.



2. How many drivers are in this data set? 30

3. Describe the distribution of these data.
The data is skewed to the left.

4. What are the quartiles of the average monthly rainfall data shown in the box-and-whisker plot? 2.4, 3.0, 3.6

5. **Standardized Test Practice** Which term describes the sampling of two random students from each of five classrooms?

Ⓐ simple
Ⓑ convenient
Ⓒ stratified
Ⓓ systematic

1. Draw a tree diagram to show the sample space for reading the national, local, arts, or sports section of the *Times* or the *Reporter*. Determine the number of possible outcomes.

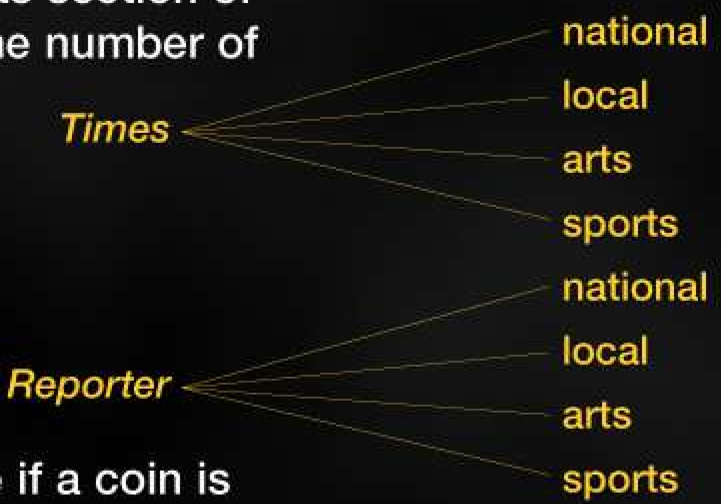
2. Find the value of 6!.

3. How many possible outcomes are there if a coin is flipped, and then one card is chosen from a deck of 52 cards?

4. **Standardized Test Practice** A music club offers 3 free CDs from a list of 12 possible choices. How many possible ways are there to list a choice of 3 free CDs?

Ⓐ 12! Ⓑ 3×12 Ⓒ $12 \times 11 \times 10$ Ⓓ 3!

1. Draw a tree diagram to show the sample space for reading the national, local, arts, or sports section of the *Times* or the *Reporter*. Determine the number of possible outcomes. 8



2. Find the value of 6!. 720

3. How many possible outcomes are there if a coin is flipped, and then one card is chosen from a deck of 52 cards? 104

4. **Standardized Test Practice** A music club offers 3 free CDs from a list of 12 possible choices. How many possible ways are there to list a choice of 3 free CDs?

(A) 12!   (B) 3×12 (C) $12 \times 11 \times 10$ (D) 3!

Determine whether each situation involves a *permutation* or *combination*.

1. voting for three of the five candidates running for the city council
2. the announcement of 10 different awards given to graduates
3. Evaluate ${}_9P_4$.
4. Gina needs to pick 9 songs from a list of 12 songs to play during her radio show. How many ways can she include 9 songs in her show in any order?
5. From a 10-member basketball team, a coach needs to pick 5 starters. His lineup must include only one of two centers. How many starting lineups can the coach make?
6. **Standardized Test Practice** Twelve people apply for 5 different job openings. Which expression shows the number of different ways in which the openings could be filled?

   A $\frac{12!}{5!}$ B $\frac{5!}{12!}$ C $\frac{12!}{(12-5)!}$ D $\frac{(12-5)!}{12!}$

Determine whether each situation involves a *permutation* or *combination*.

1. voting for three of the five candidates running for the city council combination
2. the announcement of 10 different awards given to graduates permutation
3. Evaluate ${}_9P_4$. 3024
4. Gina needs to pick 9 songs from a list of 12 songs to play during her radio show. How many ways can she include 9 songs in her show in any order? 220
5. From a 10-member basketball team, a coach needs to pick 5 starters. His lineup must include only one of two centers. How many starting lineups can the coach make? 140
6. **Standardized Test Practice** Twelve people apply for 5 different job openings. Which expression shows the number of different ways in which the openings could be filled?

   A $\frac{12!}{5!}$ B $\frac{5!}{12!}$ C $\frac{12!}{(12-5)!}$ D $\frac{(12-5)!}{12!}$

For Questions 1–3, the table shows the number of students who are eligible for the Athlete of the Year award. Find each probability if the student will be picked at random. Assume no student is eligible for more than one sport.

Sport	Male	Female
hockey	5	3
soccer	1	6
basketball	4	2
swimming	2	5

1. picking a girl who plays hockey
2. picking a boy or a soccer player
3. picking a swimmer or a hockey player
4. From a deck of playing cards, what is the probability of drawing a king, and then drawing a queen without replacement?
5. What is the difference between events that are *inclusive* and those that are *mutually exclusive*?
6. **Standardized Test Practice** What is the probability of drawing a heart or a face card from a deck of 52 cards?

For Questions 1–3, the table shows the number of students who are eligible for the Athlete of the Year award. Find each probability if the student will be picked at random. Assume no student is eligible for more than one sport.

Sport	Male	Female
hockey	5	3
soccer	1	6
basketball	4	2
swimming	2	5

1. picking a girl who plays hockey $\frac{3}{28}$
2. picking a boy or a soccer player $\frac{9}{14}$
3. picking a swimmer or a hockey player $\frac{15}{28}$
4. From a deck of playing cards, what is the probability of drawing a king, and then drawing a queen without replacement? $\frac{4}{663}$
5. What is the difference between events that are *inclusive* and those that are *mutually exclusive*? Sample answer: Inclusive events can happen at the same time; but mutually exclusive events cannot happen at the same time.
6. **Standardized Test Practice** What is the probability of drawing a heart or a face card from a deck of 52 cards? $\frac{11}{26}$

For Questions 1–4, the spinner shown is spun two times.

1. Write the sample space with all possible outcomes.

2. Find the probability distribution of X, where X represents the number of times the spinner lands on green for $X = 0$, $X = 1$, and $X = 2$.

3. Make a probability histogram for X.

4. Do all possible outcomes have the same chance of occurring? Explain.

5. **Standardized Test Practice** On a random roll of two dice, what is the probability that the sum of the two numbers showing is greater than 10?

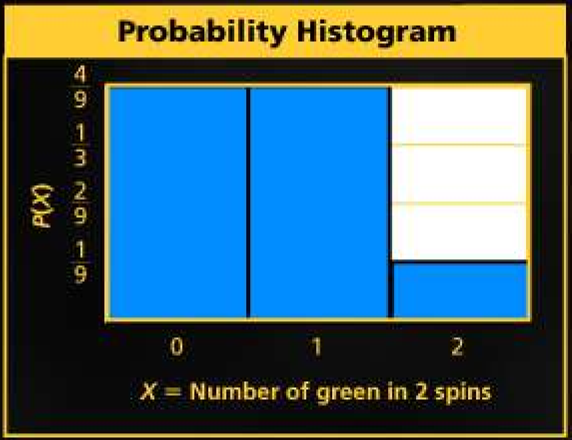
(A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{5}{36}$ (D) $\frac{1}{36}$

For Questions 1–4, the spinner shown is spun two times.

1. Write the sample space with all possible outcomes. GG, GY, GR, YY, YG, YR, RR, RY, RG

2. Find the probability distribution of X, where X represents the number of times the spinner lands on green for $X = 0$, $X = 1$, and $X = 2$. $\frac{4}{9}, \frac{4}{9}, \frac{1}{9}$

3. Make a probability histogram for X.



4. Do all possible outcomes have the same chance of occurring? Explain. Yes; each choice is equally likely to occur.

5. **Standardized Test Practice** On a random roll of two dice, what is the probability that the sum of the two numbers showing is greater than 10?

(A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{5}{36}$ (D) $\frac{1}{36}$

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Click the **Previous** button to return to the previous slide.

Click the **Section Back** button to return to the beginning of the lesson you are working on. If you accessed a feature, this button will return you to the slide from where you accessed the feature.

Click the **Main Menu** button to return to the presentation main menu.

Click the **Help** button to access this screen.

EXIT

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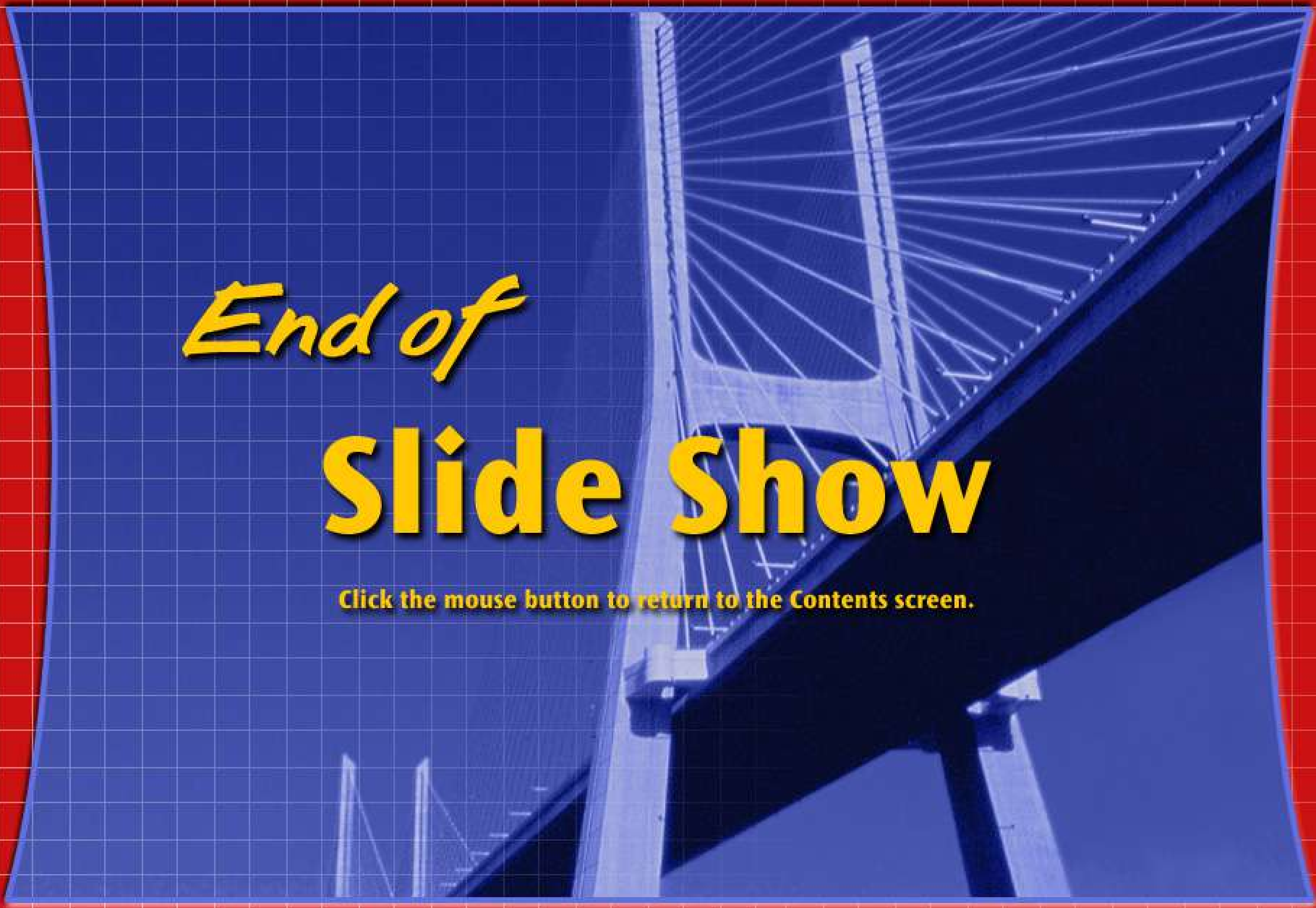
Extra Examples e **Extra Examples** button to access additional examples on the Internet.

5-Minute Check e **5-Minute Check** button to access the specific 5-Minute Check transparency that corresponds to each lesson.


End of
Slide Show
Click the mouse button to return to the Contents screen.