key

Topic: FACTORING (math skill for a lifetime of math!) without the calculator AGAIN!

Types of factoring: GCF

- Monomial
- Non-binomial

Binomials

- Difference of squares
- Sum of cubes
- Difference of cubes

Trinomials (leading coefficient 1, leading coefficient not 1)

- Trial and error (guess and check)
- Split the middle term
- Using the calculator (for honors students with weak math skills which is a contradiction!)
- Recognizing perfect square trinomials

Four or more terms

- Four terms- group by twos
- Four terms- group three by one or one by three
- More than four- look for patterns

Complete factoring

Prime factors

The best strategy for factoring a polynomial:

1. If there is a common factor, factor the GCF out **first**.
2. Determine the number of terms in the polynomial and try factoring as follows.
   a. If there are two terms, can the binomial be factored by one of the following special forms?
      Difference of two squares: $A^2 - B^2 = (A + B)(A - B)$
      Sum of two cubes: $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$
      Difference of two cubes: $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$
   b. If there are three terms, is the trinomial a perfect square trinomial? If so factor by one of the following special forms:
      Binomial sum squared: $A^2 + 2AB + B^2 = (A + B)^2$
      Binomial difference squared: $A^2 - 2AB + B^2 = (A - B)^2$
      If the trinomial is not a perfect square trinomial, try factoring by trial and error or grouping (split the middle term).
   c. If there are four or more terms, try factoring by grouping.
3. Check to see if any factors with more than one term in the factored polynomial can be factored further. If so, factor completely.
4. Check by multiplying, if time permits.

**Trinomials, leading coeff. not 1!**

26. $3y^2 + 17y + 10$

$(3y + 2)(y + 5)$

27. $5m^2 - 17m + 6$

$(5m - 2)(m - 3)$

28. $4x^2 + 4x - 15$

$(2x - 3)(2x + 5)$

29. $5y^2 + 11y + 4$

not factorable

30. $8x^2 + 8x - 6$

$(4x - 2)(2x + 3)$

31. $2x^2 + 11x + 5$

$(2x + 1)(x + 5)$

32. $15y^2 + 13y - 2$

$(15y - 2)(y + 1)$

33. $3x^2 - 5x + 1$

not factorable

34. $16y^2 - 46y + 15$

$(2y - 5)(8y - 3)$

35. $8x^2 - 22x + 5$

$(4x + 1)(2x - 5)$

36. $9y^2 + 5y - 4$

$(9y - 4)(y + 1)$

9* 37. $15w^2 - 19w + 6$

$(5w - 3)(3w - 2)$

37. $3x^2 + 4xy + y^2$

$(3x + y)(x + y)$

38. $6x^2 - 7xy - 5y^2$

$(3x - 5y)(2x + y)$

39. $12x^2 - 25x + 12$

$(3x - 4)(4x - 3)$

40. $15m^2 - 31m + 10$

$(5m - 2)(3m - 5)$

41. $12w^2 - 33w + 21$

$(3w - 3)(4w - 7)$

42. $3x^3 + 14x^2 + 8x$

$x(3x^2 + 14x + 8)$

$x(3x + 2)(x + 4)$

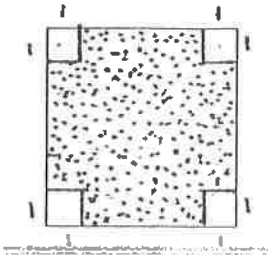
43. $24x^2 + 23x - 12$

not factorable

44. $50m^2 + 35m + 6$

$(5m + 2)(10m + 3)$

45. Find a formula for the area of the shaded region and express it in factored form.




shaded Area $= (x)(x) - 4 = x^2 - 4$

Name: _____

Four term polynomials.

1. $x^2 + 10x + 25 - y^2$

$(x+5)(x+5) - y^2$

$(x-y+5)(x+y+5)$

2. $4m^2 + 20m + 25 - k^2$

$(2m+5-k)(2m+5+k)$

3. $w^2 - 10w + 25 - 100z^2$

$(w-5)(w-5) - 100z^2$

$(w-10z-5)(w+10z-5)$

4. $49y^2 - 14y + 1 - 36x^2$

$(7y-1)(7y-1) - 36x^2$

$(7y-1-6x)(7y-1+6x)$

5. $k^2 + 18k + 81 - 25m^2$

$(k+9)(k+9) - 25m^2$

$(k+9+5m)(k+9-5m)$

6. $16x^2 - 56x + 49 - 64y^2$

$(4x-7)(4x-7) - 64y^2$

$(4x-7+8y)(4x-7-8y)$

7. $x^2 - 2xy + y^2 - 1$

$(x-y)(x-y) - 1$

$(x-y+1)(x-y+1)$

8. $x^4 + 16x^2 + 64 - 121p^2$

$(x^2+8)(x^2+8) - 121p^2$

$(x^2+8-11p)(x^2+8+11p)$

9. $y^2 - x^2 - 10x - 25$

$x^2+10x+25 - y^2$

$(x+5)(x+5) - y^2$

$(x+5-y)(x+5+y)$

10. $k^2 - 4m^2 - 20m - 25$

$4m^2+20m+25 - k^2$

$(2m+5)(2m+5) - k^2$

$(2m+5-k)(2m+5+k)$

11. $100z^2 - w^2 + 10w - 25$

$w^2-10w+25 - 100z^2$

$(w-5)(w-5) - 100z^2$

$(w-5-10z)(w-5+10z)$

12. $36x^2 - 49y^2 + 14y - 1$

$49y^2-14y+1 - 36x^2$

$(7y-1)(7y-1) - 36x^2$

$(7y-1-6x)(7y-1+6x)$

13. $25m^2 - k^2 - 18k - 81$

$k^2+18k+81 - 25m^2$

$(k+9)(k+9) - 25m^2$

$(k+9-5m)(k+9+5m)$

14. $64y^2 - 16x^2 + 56x - 49$

$16x^2-56x+49 - 64y^2$

$(4x-7)(4x-7) - 64y^2$

$(4x-7-8y)(4x-7+8y)$

15. $m^2 - x^2 + 2xy - y^2$

$x^2-2xy+y^2 - m^2$

$(x-y)(x-y) - m^2$

$(x-y-m)(x-y+m)$

16. $121p^2 - x^4 - 16x^2 - 64$

$x^4+16x^2+64 - 121p^2$

$(x^2+8)(x^2+8) - 121p^2$

$(x^2+8-11p)(x^2+8+11p)$

More practice . . .

Sum and Difference of Cubes

① $3x^2(x-7) + 2x(x-7) - 5(x-7)$

$(3x^2+2x-5)(x-7)$

② $4a^2 - (a-2b)^2$ $\quad A^2-B^2=(A+B)(A-B)$

$(2a)^2-(a-2b)^2$

$(2a+a-2b)(2a-(a-2b))$

$(3a-2b)(a-2b)$

③ $x^2 - 4x + 4 - 9y^2 + 6y - 1$

$(x-2)(x-2)+(-3y+1)(3y-1)$

$(x-2)^2+-(3y-1)(3y-1)$

$(x-2)^2-(3y-1)^2$

④ $m^4 - 13m^2 + 36$

$(m^2-4)(m^2-9)$

$(m-2)(m+2)(m-3)(m+3)$

same

⑤ $x^4 - (2x-1)^2$ $\quad A^2-B^2=(A+B)(A-B)$

$(x^2)^2-(2x-1)^2$

$(x^2+2x-1)(x^2-(2x-1))$

$(x^2+2x-1)(x^2-2x+1)$

$(x^2+2x-1)(x-1)(x-1)$

⑥ $w^4 - 13w^2 + 36$

$(w^2-4)(w^2-9)$

$(w-2)(w+2)(w-3)(w+3)$

⑦ $8(2p+q)^2 - 10(2p+q) + 3$

$8x^2-10x+3$

$(4x-1)(2x-3)$

$(4(2p+q)-1)(2(2p+q)-3)$

$(8p+4q-1)(4p+2q-3)$

⑧ $x^2 - xz - xy + yz$

$x(x-z)-y(x-z)$

$(x-y)(x-z)$

⑨ $125x^3 - 1$ diff of two cubes

$(A-B)(A^2+AB+B^2)$

$(5x-1)(25x^2-5x+1)$

⑩ $8y^3 + 27$ sum of two cubes

$(2y+3)(4y^2-6y+9)$

$(A+B)(A^2-AB+B^2)$

⑪ $64m^3 + 27n^3$

$(4m+3)(16m^2-12m+9)$

⑫ $1000 - 343w^3$

$(10-7w)(100-70w+49w^2)$

⑬ $x^3 + a^3$

$(x+a)(x^2-xa+a^2)$

⑭ $8 + (a+b)^3$ $\quad a^3+b^3=(a+b)(a^2-ab+b^2)$

$(a+b+2)(a+b+2)(a+b+2)$

⑮ $(x+1)^3 - (y+2)^3$

$a^3-b^3$

$(a-b)(a^2+ab+b^2)$

$(x+1-(y+2))((x+1)^2+(x+1)(y+2)+(y+2)^2)$

$(x+1-y-2)(x^2+2x+1+xy+2x+y+2+y^2+4y+4)$

$(x-y-1)(x^2+4x+7+xy+5y+y^2)$

⑯ $x^3 + y^3 + x^2 - y^2$

$(x+y)(x^2-xy+y^2)+(x+y)(x-y)$

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More mixed practice

7. $(2x + 3y + a)^2 - (x - y + a)^2$

difference of 2 squares

$(2x+3y+a+x-y+a)(2x+3y+a-x+y-a)$

$(3x+2y+2a)(x+4y)$

8. $343 - 7(x+3)^2$

$343-7(x^2+6x+9) = 343-7x^2-42x-63$

$= -7x^2-42x+280$

$= (-7x+28)(x+10)$

$= -7(x+4)(x+10)$

9. $x^2 - 13x - 30$

$(x-15)(x+2)$

10. $y^2 - 6y + 7$

not factorable

Factoring disguised difference of squares $A^2 - B^2 = (A+B)(A-B)$

1. $x^2 + 6x + 9 - y^2$

$x(x+6) - (y+3)(y-3)$

2. $a^2 - b^2 + 2b - 1$

$(a-b)(a+b) + 2b - 1$

3. $p^2 - 14p + 49 - 9k^2$

$p(p-14) + (7-3k)(7+3k)$

4. $r^2 - s^2 - 4s - 4$

$(r-s)(r+s) - 4(s+1)$

5. $81 - x^2 + 2xy - y^2$

$(9-x)(9+x) - y(-2x+y)$

Factoring by completing the square!

Example: $x^4 + 2x^2 + 9 = x^4 + 6x^2 + 9 + 2x^2 - 6x^2 = x^4 + 6x^2 + 9 - 4x^2 = (x^2 + 3)^2 - 4x^2$

$= ((x^2 + 3) - 2x)((x^2 + 3) + 2x) = (x^2 - 2x + 3)(x^2 + 2x + 3)$

1. $x^4 - x^2 + 16$

2. $x^4 + 11x^2 + 36$

3. $x^4 - 19x^2 + 9$

4. $x^4 + 4$

5. $x^4 - 26x^2 + 25$

(1) $x^4+8x^2-8x^2-x^2+16$

$x^4+8x^2+16-8x^2-x^2$

$(x^2+4)(x^2+4) - 9x^2$

$(x^2+4)^2 - 9x^2$

$((x^2+4)-3x)(x^2+4+3x)$

$(x^2-3x+4)(x^2+3x+4)$

(2) $x^4+11x^2+36$

$x^4+12x^2-12x^2+11x^2+36$

$x^4+12x^2+36 - x^2$

$(x^2+6)(x^2+6) - x^2$

$(x^2+6)^2 - x^2$

$((x^2+6)-x)((x^2+6)+x)$

$(x^2-x+6)(x^2+x+6)$

(3) $x^4-19x^2+9$

$x^4+6x^2-6x^2-19x^2+9$

$(x^2+3)(x^2+3) - 25x^2$

$(x^2+3)^2 - 25x^2$

$((x^2+3)-5x)((x^2+3)+5x)$

$(x^2-5x+3)(x^2+5x+2)$

(4) $x^4+4$

$x^4+4x-4x+4$

$(x^2+2)(x^2+2) - 4x$

$((x^2+2)-2x)((x^2+2)+2x)$

$(x^2-2x+2)(x^2+2x+2)$

(5) $x^4-26x^2+25$

$x^4+10x^2-10x^2-26x^2+25$

$(x^2+5)^2 - 36x^2$

$((x^2+5)-6x)((x^2+5)+6x)$

$(x^2-6x+5)(x^2+6x+5)$

✓

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# Algebra 2: Final Exam Review Answers

1. $(2x^2+3x+5)+(4x^2-10x-2)$

$2x^2+4x^2+3x-10x+5-2$

$\boxed{6x^2-7x+3}$

2. $(2x-5)^2$

$(2x-5)(2x-5)$ FOIL

$4x^2-10x-10x+25$

$\boxed{4x^2-20x+25}$

3. $(3x+5)(9x^2-15x+25)$

$27x^3-45x^2+75x+45x-75x+125$

$\boxed{27x^3+125}$

4. $(6x-5y)(6x+5y)$ conjugate factors!

$36x^2+30xy-30xy-25y^2$

$\boxed{36x^2-25y^2}$

5. $xy^3(xy^2+xy-1)$

$\boxed{x^2y^5+x^2y^4-xy^3}$

6. $x^3-27$ $(x)^3-(3)^3$

$\boxed{(x-3)(x^2+3x+9)}$

7. $3x^2+10x+3$

$\boxed{(3x+1)(x+3)}$

8. $x^3+2x^2-4x-8$

$x^2(x+2)-4(x+2)$

$(x+2)(x^2-4)$

$\boxed{(x+2)(x+2)(x-2)}$

or $\boxed{(x+2)^2(x-2)}$

9. $x^4-4x^2$ GCF ... difference of squares!

$x^2(x^2-4)$

$\boxed{x^2(x+2)(x-2)}$

10. $x^3+4x^2+4x+1$

$-1$ | 1 | 4 | 4 | 1

| | −1 | −3 | −1 |
|---|---|---|---|
| 1 | 3 | 1 | 0 |

$\boxed{(x+1)(x^2+3x+1)}$

11a. $(\swarrow, \nearrow)$

11b.

| | | |
|---|---|---|
| −2 | 3 | flattens out ↗ |
| −1 | 1 | passes through + |
| 0 | 1 | passes through + |
| 2 | 2 | touches, doesn't flatten ↘ |

11c. $f(x)=\frac{1}{4}(x+2)^3(x+1)(x)(x-2)^2$

12. $f(x)=-1(x-2)(x+3)(x+3)$

$=-1(x-2)(x^2+6x+9)$

$=-1(x^3+6x^2+9x-2x^2-12x-18)$

$=-x^3-4x^2+3x+9$

a) E.B. $(\nwarrow, \searrow)$ "−1" so down on right, odd degree opposite on left.

b) local maximum: $(\frac{1}{3}, 18.52)$

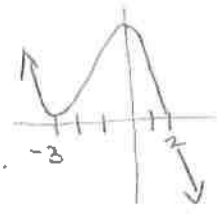
local minimum: $(-3,0)$

c) x-intercepts (zeros): $(-3,0), (2,0)$

d) $(-3,0) \rightarrow$ multiplicity 2

$(2,0) \rightarrow$ multiplicity 1

e) $(0,9)$ is y-intercept.



# Algebra 2: Final Exam Review Answers (page 2)

13. $2x^2+3x-5=0$

$(2x+5)(x-1)=0$

$2x+5=0$ or $x-1=0$

$2x=-5$ $x=1$

$x=-5/2$ or $x=1$

14. $y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$y^2-3y+4=0$ $\begin{cases} a=1 \\ b=-3 \\ c=4 \end{cases}$

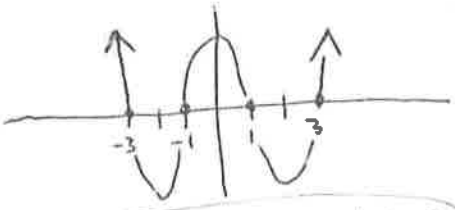
$y=\frac{3\pm\sqrt{9-4(1)(4)}}{2}$

$y=\frac{3\pm\sqrt{9-16}}{2}$

$y=\frac{3\pm\sqrt{-7}}{2}=\frac{3\pm i\sqrt{7}}{2}$

15. graph $y=x^4-10x^2+9$

solutions to $0=y=x^4-10x^2+9$ will be the x-intercepts




solutions: $x=-3,-1,1,3$

16. $1x^3+4x^2+4x+1=0$ ← change to "1"

possible zeros: $\frac{\text{factors of } 1}{\text{factors of } 1}$

possible: $\frac{\pm1}{\pm1}\rightarrow\pm1$

$-1$ | 1 4 4 1

↓ −1 −3 −1

1 3 1 | 0

16. (continued.)

$x^3+4x^2+4x+1=0$

$(x+1)(x^2+3x+1)=0$

use quadratic formula!

$x+1=0$ $x=-1$ $x=\frac{-3\pm\sqrt{9-4}}{2}=\frac{-3\pm\sqrt{5}}{2}$

solutions: $-1, \frac{-3\pm\sqrt{5}}{2}$

17. $-2$ | 1 0 −4 2

↓ −2 4 0

1 −2 0 | 2

so $(x^3-4x+2)\div(x+2)=x^2-2x$ R2

18. Since it is degree 4, there will be 4 solutions.
- 4 real solutions or
- 2 real and 2 complex (conjugates) or
- 4 complex solutions.

19. $y=2x^3-x^2+2x-5$

possible rational roots: $\frac{\text{factors of } 5}{\text{factors of } 2}$

$\frac{\pm1,\pm5}{\pm1,\pm2}\rightarrow\frac{\pm1}{\pm1},\frac{\pm5}{\pm1},\frac{\pm1}{\pm2},\frac{\pm5}{\pm2}$

$=\pm1,\pm5,\pm\frac{1}{2}$ or $\pm5/2$

20. hole at $x=-2$ b/c factor will cancel; vertical asymptote at $x=-3$ b/c factor is in denominator, horizontal asymptote at $y=1$ b/c $\frac{x}{x}=1$ (degree is same in numerator and denominator) and x-int at 1

... is R.

# Algebra 2 : Final Exam Review Answers (page 3)

21. Need to factor, cancel!

$$\frac{x^2+4x+4}{x^2+3x+2} = \frac{(x+2)(x+2)}{(x+2)(x+1)}$$

$$= \frac{x+2}{x+1}$$

22. Has a common denominator. Add, then reduce...

$$\frac{x^2}{x+5} + \frac{5x}{x+5} = \frac{x^2+5x}{x+5} = \frac{x(x+5)}{(x+5)}$$

$$= x$$

23. Get a common denominator...

$$\frac{y}{y+2} + \frac{3}{y+1} = \frac{y+1}{y+1}\left(\frac{y}{y+2}\right) + \frac{y+2}{y+2}\left(\frac{3}{y+1}\right)$$

$$= \frac{y^2+y}{(y+1)(y+2)} + \frac{3y+2}{(y+1)(y+2)} = \frac{y^2+4y+2}{(y+1)(y+2)}$$

(does not reduce :)

24. factor, then cancel...

$$\frac{m^2-m}{m^2-2m+1} \cdot \frac{m^3-1}{m^2+3m}$$

$$= \frac{m(m-1)}{(m-1)(m-1)} \cdot \frac{(m-1)(m^2+m+1)}{m(m+3)}$$

$$= \frac{m^2+m+1}{m+3}$$

25. Change to multiply by reciprocal...

$$\frac{2x+4}{x^2-4} \div \frac{x^2+2x}{3x^2-6x} = \frac{2x+4}{x^2-4} \cdot \frac{3x^2-6x}{x^2+2x}$$

$$= \frac{2(x+2)}{(x+2)(x-2)} \cdot \frac{3x(x-2)}{x(x+2)} = \frac{6x}{x(x+2)}$$

$$= \frac{6}{x+2}$$

26. B

27. A

28. C

29a) $\sqrt{12} = \sqrt{4\cdot3} = 2\sqrt{3}$

b) $\sqrt{8} + \sqrt{50} = 2\sqrt{2} + 5\sqrt{2}$ (4, 2 and 25, 2)

$$= 7\sqrt{2}$$

c) $\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

30a) $\sqrt[3]{24} = \sqrt[3]{8\cdot3} = 2\sqrt[3]{3}$

b) $\sqrt[3]{2x^3} + x\sqrt[3]{16}$ (8, 2)

$$x\sqrt[3]{2} + 2x\sqrt[3]{2} = 3x\sqrt[3]{2}$$

c) $\frac{1}{\sqrt[3]{4x}} = \frac{1}{\sqrt[3]{2\cdot2\cdot x}} \cdot \frac{\sqrt[3]{2\cdot x\cdot x}}{\sqrt[3]{2\cdot x\cdot x}}$

$$= \frac{\sqrt[3]{2x^2}}{2x}$$

31a) $\sqrt{4x}\cdot\sqrt{3x} = \sqrt{12x^2}$ (4, 3, x, x) $= 2x\sqrt{3}$

b) $(\sqrt{2}+\sqrt{3})^2$

$$= (\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{3})$$

$$= \sqrt{4} + \sqrt{6} + \sqrt{6} + \sqrt{9}$$

$$= 2 + \sqrt{6} + \sqrt{6} + 3$$

$$= 5 + 2\sqrt{6}$$

c) $\sqrt[4]{9x}\cdot\sqrt[4]{9x^3} = \sqrt[4]{81x^4} = 3x$

# Algebra 2: Final Exam Review Answers (page 4)

32a) $x^2 \cdot x^3 = x^{2+3} = x^5$

b) $y^{1/2} \cdot y^{1/3} = y^{1/2+1/3}$
$= y^{3/6+2/6} = y^{5/6}$

c) $(2m^{1/2} \cdot 3x) \div x^{1/2}$
$2 \cdot m^{1/2} \cdot 3 \cdot x^1 \div x^{1/2}$
$2 \cdot 3 \cdot m^{1/2} \cdot x^{1-1/2}$
$6m^{1/2}x^{1/2}$

33. $x^{\frac{3}{2}}$ ← index
$= \sqrt{x}^3$ or $\sqrt{x^3}$
or $x\sqrt{x}$

34. $(3x^2)^{1/5}$ or
$3^{1/5} \cdot x^{2/5}$

35a) $4^{1/2} = \sqrt[2]{4^1} = \sqrt{4} = 2$

b) $8^{1/3} = \sqrt[3]{8^1} = \sqrt[3]{8} = 2$

c) $(\frac{1}{2})^{-2} = (\frac{2}{1})^2 = 2^2 = 4$

d) $9^{1.5} = 9^{\frac{3}{2}} = (\sqrt{9})^3$
$= 3^3 = 27$

36. $P = 100(10)^{-0.175 \times 5}$
$P = 13.335 \approx 13.4\%$

37. • Compare table values
• Find the inverse of one + compare
• See if $f^{-1}(f(x)) = x$

38. $\log_3 \frac{1}{9} = -2$

39a) $10^3 = 1000$

b) $2^{-4} = \frac{1}{16}$

c) $e^x = 12$

40. $\log_2 2 + 2\log_2 x - \frac{1}{3}\log_2 y$
or $1 + 2\log_2 x - \frac{1}{2}\log_2 y$

41. $\log_2\left(\frac{6x}{y^3 z^{1/2}}\right)$

42. $b = 1+r$ is growth, $b = 1-r$ is decay
Halflife problems are decay b/c
$y = a(\frac{1}{2})^{t/\text{halflife}} \rightarrow \frac{1}{2} = 1 - .50$,
or 50% decrease!

43. $y = 1365(1+.12)^{10} \rightarrow y = 4239$ people

44. $1{,}000{,}000 = 100{,}000\, e^{.095t}$
$10 = e^{.095t} \rightarrow \log_e 10 = .095t$
$\rightarrow \frac{\ln 10}{.095} = t \approx 24.2$ years

45. Factor! $x = 2, -2$ or $1$

46. Square both sides! FOIL on right side! $x = 0$

47. $\sqrt[3]{(x+3)^2} = 9$ cube both sides!
$x = -30$ or $x = 24$

48. divide by 5000 first! put into log form... $x \approx 11.58$

49. Write as single log.
change to ~~exponent~~ form!
factor... $x = 9$ ~~$x = 1$~~

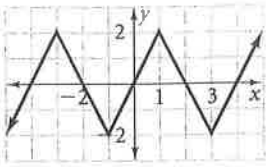
50. $x = \frac{1}{2}$

Chapter 13

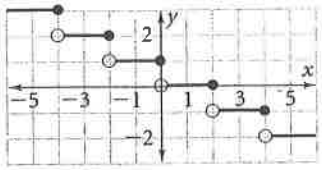
# Chapter Test

Go Online PHSchool.com
For: Chapter Test
Web Code: aga-1352

**Determine whether each function *is* or *is not* periodic. If it is periodic, find the period and amplitude.**

1. 

periodic; 4, 2

2. 

not periodic

**Find the measure of an angle between 0° and 360° coterminal with the given angle.**

3. $-32°$ 328°
4. $-229°$ 131°
5. 375° 15°

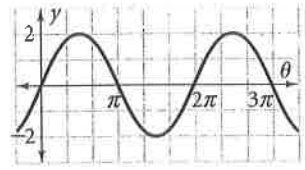
**Write each measure in radians. Express the answer in terms of $\pi$ and also as a decimal rounded to the nearest hundredth.** 6–8. See margin.

6. $-225°$
7. 120°
8. 600°

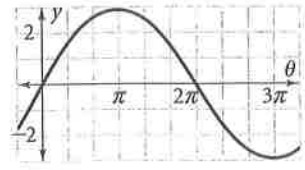
**Write each measure in degrees. If necessary, round your answer to the nearest degree.**

9. $\frac{5\pi}{6}$ 150°
10. $-2.5\pi$ −450°
11. 0.8 46°

**How many cycles does each sine function have in the interval from 0 to $2\pi$? Find the amplitude and period of each function.**

12. 

1 cycle; 2, $2\pi$

13. 

$\frac{1}{2}$ cycle; 3, $4\pi$

14. Open-Ended Sketch a function with period 7. Check students' work.

15. Writing Explain how to convert an angle measure in radians to an angle measure in degrees. Include an example. See margin.

16. Physics On each swing, a pendulum 18 inches long travels through an angle of $\frac{3\pi}{4}$ radians. How far does the tip of the pendulum travel in one swing? Round your answer to the nearest inch. 42 in.

17–18. See margin.
**Find the amplitude and period of each function. Then sketch one cycle of the graph of each function.**

17. $y = 4 \sin (2x)$
18. $y = 2 \sin (4x)$

**Solve each equation in the interval from 0 to $2\pi$. Give an exact answer and an answer rounded to the nearest hundredth.** 19–21. See margin p. 775.

19. $\cos t = \frac{1}{2}$
20. $2 \sin t = \sqrt{3}$
21. $3 \tan 2t = \sqrt{3}$
22. $\cos \frac{t\pi}{4} = 1$ 0

**Graph each function in the interval from 0 to $2\pi$.**

23. $y = 2 \cos x$
24. $y = -\cos \frac{\theta}{\pi}$
25. $y = 4 \sin x - 2$
26. $y = \cos (x + \pi)$
27. $y = \tan \frac{\theta}{3}$
28. $y = \tan \frac{\pi}{3}\theta$

23–28. See back of book.
**Write an equation for each translation.**

29. $y = \sin x$, 1 unit down $y = \sin x - 1$
30. $y = \cos x$, 7.5 units to the right
31. $y = \sin x$, 3 units to the left, 1.5 units down
32. $y = \cos x$, $\frac{\pi}{2}$ units to the right, 8 units up

30–32. See margin p. 775.
**Evaluate each expression. Write your answer in exact form. If the expression is undefined, write *undefined*.**

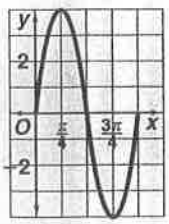
33. $\sin 30°$ $\frac{1}{2}$
34. $\cos 60°$ $\frac{1}{2}$
35. $\sin (-330°)$ $\frac{1}{2}$
36. $\csc (-330°)$ 2
37. $\sec 270°$ undefined
38. $\tan 60°$ $\sqrt{3}$
39. $\cos 45°$ $\frac{\sqrt{2}}{2}$
40. $\cot (-60°)$ $-\frac{\sqrt{3}}{3}$

**Graph each function in the interval from 0 to $2\pi$.**
41–44. See back of book.

41. $y = \cot \theta$
42. $y = \sec \theta + 1$
43. $y = \csc \frac{\theta}{2}$
44. $y = \csc (\theta + 1)$



15. Answers may vary. Sample: Multiply the radian measure by $\frac{180}{\pi}°$. Example: $\frac{2\pi}{3}$ radians $\cdot \frac{180°}{\pi} = 120°$

17. 4, $\pi$ 


18. 2, $\frac{\pi}{2}$