

# TEF GEOMETRY

Summer 2019

Semester 2

NOTES & ASSIGNMENTS

<b>Monday</b>	<b>Tuesday</b>	<b>Wednesday</b>	<b>Thursday</b>	<b>Friday</b>
June 17 Hr 1:Translations Hr 2:Reflections Hr 3: Rotations Hr 4: Unit 1 Review (Kaleidoscope?)	June 18 Hr 1: Unit 1 Review Hr 2: <b>Unit 1 Test</b> Hr 3: Points, Lines, and Planes Hr 4: Segments and their Measures	June 19 Hr 1: Angles and their measures Hr 2: Segment and Angle Bisectors Hr 3: Directed Line Segments & Lines in Coordinate Plane (slope, midpoint, distance) Hr 4: Constructions	June 20 Hr 1: Review Constructions & Unit 2 Review Hr 2: <b>Unit 2 Test</b> Hr 3: Identifying Angles Hr 4: Identifying Angles HW 2	June 21 Hr 1: Constructions of Parallel & Perpendicular Lines & Activity Hr 2: Proving Angle Relationships Hr 3: Proving Angle Relationships 2 Hr 4 :Midunit Review
June 24 Hr 1: Proving Lines Parallel Hr 2: Lines in the Coordinate Plane & Equation Writing Hr 3: Unit 3 Review Hr 4: <b>Unit 3 Test</b>	June 25 Hr 1: Congruent Triangles & CPCTC; Hr 1: Classifying Triangles Hr 2: SSS, SAS, ASA, AAS Hr 3: Identifying Congruent Triangles HW Hr 4:Proving Congruent Triangles	June 26 Hr 1: Proving Congruent Triangles Hr 2: Midsegment Hr 3 : Medians and Altitudes Hr 4: Unit 4 Review	June 27 Hr 1: <b>Unit 4 Test</b> Hr 2: Properties of Parallelograms Hr 3: Props of Quads Hr 4: Final Review	June 28 Hr 1: Final Review Hr 2: <b>Final (8-9:30am)</b> Hr 3: Ratios and Proportions Hr 4: Similar Figures

July 1	July 2	July 3	July 4	July 5
Hr 1: AA~SAS~ and SSS~ Hr 2: Side Splitter Hr 3:Dilations Activity/Notes Hr 4: Unit 6 Review	Hr 1: <b>Unit 6 Test</b> Hr 2: Pyth Thm Hr 3: Sp Rights (45-45-90 and 30-60-90) Hr 4: Pythagorean Snail Project	Hr 1: Intro Trig Hr 2: Finding Missing Sides Hr 3: Finding Missing Angles Hr 4: Trig Applications	<b>No School Holiday</b>	Hr 1: Angles of Elevation and Depression Hr 2: Unit 7 Review Hr 3: <b>Unit 7 Test</b> Hr 4: Circles & Tangents & Arcs and Central Angles
July 8	July 9	July 10	July 11	July 12
Hr 1: Inscribed Angles Hr 2: Other Angles Hr 3: Freaky Friends Hr 4: Equations of Circles	Hr 1: Unit 8 Review Hr 2: <b>Unit 8 Test</b> Hr 3: Volume Hr 4: Compound Vol	Hr 1: Volume Practice, Scale Factor Hr 2: Rotations Hr 3: Cross Sections (play-doh optional) Hr4: Unit 9 Review	Hr 1: <b>Unit 9 Test</b> Hr 2: Basic Prob Hr 3: Geo Prob Hr 4: Compound Prob	Hr 1: Conditional Prob Hr 2: Final Review Hr 3: Final Review/Final Hr 4: <b>Final</b>

# Unit 6

## Similarity

# Ratios and Proportions

*Learning Targets: Students will be able to define and identify ratios and proportions.  
Students will be able to apply proportions to solve word problems.*

**Ratio:** a comparison of two "objects"  
usually written as a fraction  $\frac{3}{4}$  or 2:3  
or 2 to 3

**Proportion:** TWO OR MORE EQUAL RATIOS

1. How do we determine if two ratios are equal?

$$\frac{2}{7} = \frac{14}{49} \quad 2 \cdot 7 = 14 \quad 7 \cdot 2 = 14 \quad \frac{14}{49} \sim \frac{2}{7}$$

2.  $\frac{6}{5} = \frac{x}{15}$

$$6 \cdot 3 = \frac{18}{15} \quad x = 18$$

3.  $\frac{6}{x} = \frac{3}{x-6}$

$$3x = 6(x-6) \quad 3x = 6x - 36 \quad -3x = -36 \quad x = 12$$

4.  $\frac{21-x}{x} = \frac{2}{1}$

$$1(21-x) = 2x \quad 21-x = 2x \quad 21 = 3x \quad 7 = x$$

5. The ratio of tall students to short students in the class is 4:3. If there are 20 tall students in the class, how many short students are there?

$$\begin{array}{l} t \\ sh \end{array} \frac{4}{3} = \frac{20}{x} \quad 4x = 60 \quad x = 15 \quad 15 \text{ short stud}$$

6. The ratio of tall students to short students in the class is 1:3. If there are 28 students in the class, how many short students are there?

$$\begin{array}{l} t \\ s \end{array} \frac{1}{3} = \frac{x}{sh} \quad 3 \cdot 7 = x \rightarrow x = 21$$

7. A triangle has angles in a ratio of 1:2:3, what is the size of each angle? [Hint: sum of interior angles is 180 degrees]

$$1:2:3 \rightarrow 1x + 2x + 3x = 180 \quad 6x = 180 \quad x = 30$$

$$30:60:90$$

$$\frac{2}{5} \cdot \frac{3}{3} = \frac{6}{15}$$

$$\frac{2}{5}x$$

What are all the possible names for a quadrilateral with sides 3:1:3:1?

1. White rice needs to cook for 20 minutes, while brown rice cooks for 25 minutes. What is the ratio of cooking times for white rice to brown rice?
  
  
  
  
  
  
  
  
  
  
2. A soccer team won 22 games and lost 8. What is their win-loss ratio?
  
  
  
  
  
  
  
  
  
  
3. Charlotte's essay on pigs was 824 words in length. Wilbur's essay was only 360 words long. What is the ratio of Charlotte's essay to Wilbur's?
  
  
  
  
  
  
  
  
  
  
4. A recipe instructs the cook to use 4 cups of water for each 3 cups of powder. If you used 10 cups of water, how much powder should be added?
  
  
  
  
  
  
  
  
  
  
5. The soccer club has a 3:2 of juniors to freshman. If the club has 45 students attend that are freshman and juniors, how many juniors are there?
  
  
  
  
  
  
  
  
  
  
6. The sides of a triangle are in the ratio of 3:8:7. If the perimeter of the triangle is 63 cm, how long is the shortest side? [Hint: Perimeter is the sum of all side lengths. Draw & label the figure.]

7. The perimeter of rectangle JKLM is 56 centimeters. The ratio of JK:KL is 4:3. Find the length and width of the rectangle. **Draw & label the figure.**

8. The perimeter of an isosceles trapezoid is 100 inches. The ratio of one leg to the bigger base is 5:8, and the ratio of the smaller base to the bigger base is 1:4. Find the side lengths. **Draw & label the figure.**

**Solve for x.**

9.  $8(x+6) = 24$

10.  $-10(y+8) = -40$

11.  $7(2-x) = 5x$

12.  $\frac{1}{2}(10-9x) = \frac{3}{2}$

**Solve the proportion.**

13.  $\frac{2}{q} = \frac{4}{18}$

14.  $\frac{t}{27} = \frac{4}{9}$

15.  $\frac{w}{6} = \frac{7}{17}$

16.  $\frac{6}{45} = \frac{2x+10}{15}$

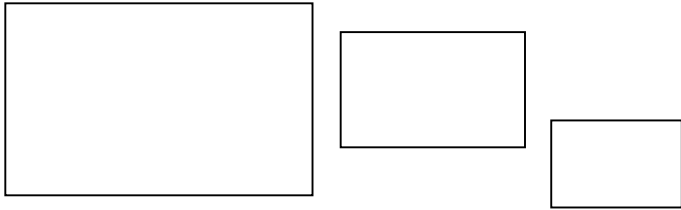
17.  $\frac{-3}{8} = \frac{21}{2(y+1)}$

18.  $\frac{3}{m-6} = \frac{1}{m}$

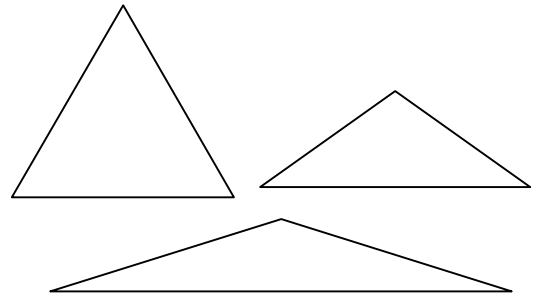
# Similar Figures

**Learning Targets:** *Students will be able to identify if two figures are similar.*  
*Students will be able to identify the scale factor "k" of two similar figures.*

These figures are similar



These are not similar



**Similar Figures** ~ Two polygons are similar if and only if the Corresp. angles are IR and the measures of the Corresp. sides are proportional  
 The symbol ~ means similar.

$\triangle ABC \sim \triangle DEF$  ("triangle ABC is **similar** to triangle DEF")

$\overline{BC} \sim \overline{EF}$

Similarity statement

Corresponding Angles

Corresponding Sides

are IR

have are proportional

$\angle A \Leftrightarrow \angle D$

$\overline{AB} \Leftrightarrow \overline{DE}$

$\angle B \Leftrightarrow \angle E$

$\overline{BC} \Leftrightarrow \overline{EF}$

$\angle C \Leftrightarrow \angle F$

$\overline{AC} \Leftrightarrow \overline{DF}$

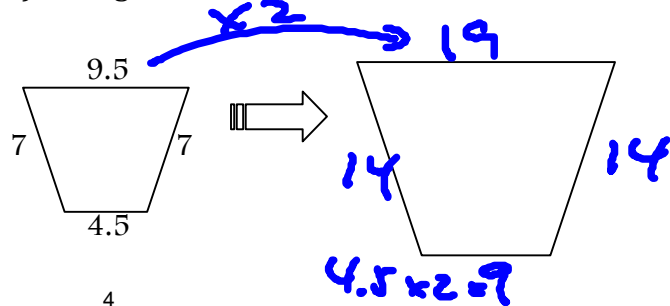
**Scale Factor -**

If the scale factor  $> 1$ , Enlargement

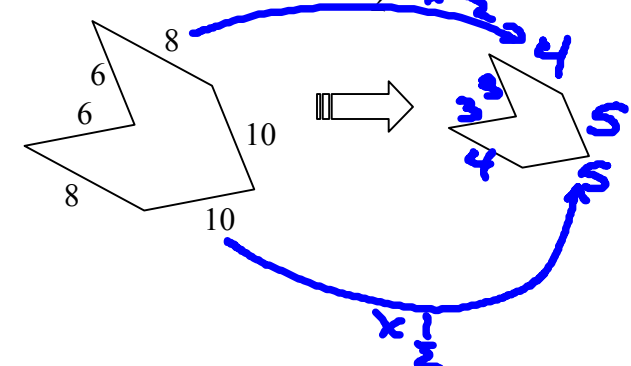
If the scale factor  $< 1$ , Reduction

Example: Find the dimensions of the figure ...

a) using a scale factor of 2.



b) using a scale factor of  $\frac{1}{2}$ .



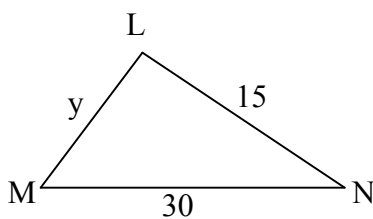
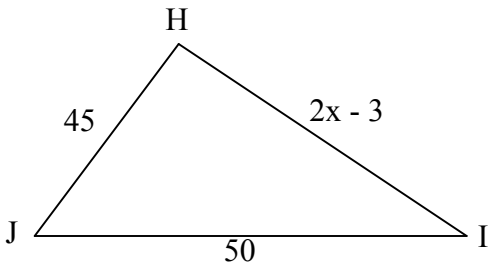


Similar figures are enlargements *or* reductions of each other. The amount of enlargement or reduction needed to change one figure to the other is called the scale factor. The ratio of the lengths of the corresponding sides of similar figures is the scale factor.

Determine if the polygons are similar. Show work to justify your answer.

<p>1)</p> <p>Handwritten work:  <math>\frac{12}{9} = \frac{16}{12} = \frac{4}{3}</math>  <math>\frac{12}{9} = \frac{4}{3}</math>  <math>\frac{16}{12} = \frac{4}{3}</math>  <b>yes</b></p>	<p>2)</p> <p>Handwritten work:  <math>8 \cdot 3 = 24</math>  <math>7 \cdot 3 \neq 14</math>  <b>NO</b></p>	<p>3)</p> <p>Handwritten work:  <b>NOT</b></p>
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Find the values of  $x$  and  $y$  if  $\triangle JHI \sim \triangle MLN$ .



Statement of proportionality

a) Write proportions for the corresponding sides.

$$\frac{JH}{ML} = \frac{HI}{LN} = \frac{JI}{MN}$$

b) Write the proportion to solve for  $x$ .

$$\frac{2x-3}{15} = \frac{50}{30}$$

Same  $\Delta$

$$3(2x-3) = 15(5)$$

$$6x-9 = 75$$

$$6x = 84$$

$$x = 14$$

c) Write the proportion to solve for  $y$ .

$$\frac{y}{45} = \frac{30}{50}$$

$\times 9$

$$y = 3 \cdot 9 = 27$$

ABCDE is similar to QRSTU <sup>← num</sup> <sup>← den</sup>  $\frac{3}{15} = \frac{1}{5}$   
 The similarity ratio of ABCDE to QRSTU is  $\frac{3}{15}$ .

The scale factor of ABCDE to QRSTU is 5.

Find the length of each side.

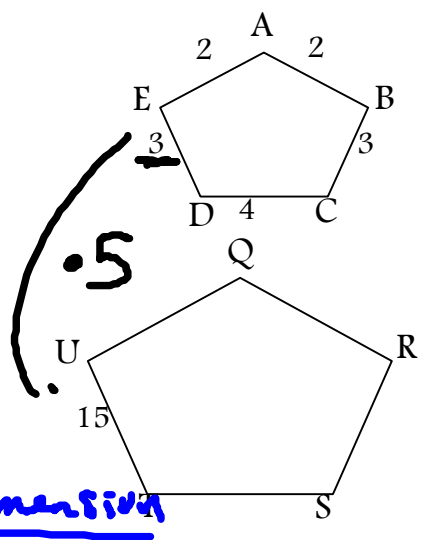
~~QU = \_\_\_\_\_  
 QR = \_\_\_\_\_  
 RS = \_\_\_\_\_  
 ST = \_\_\_\_\_~~

$3 + 2 + 2 + 3 + 4$

Perimeter of ABCDE = 14 → 1 dimension

Perimeter of QRSTU =  $14 \cdot 5 = 70$

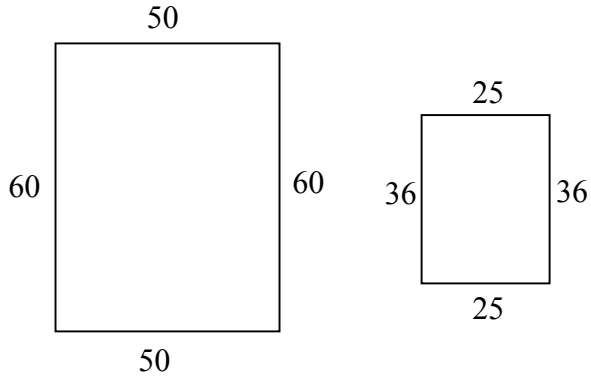
Ratio of perimeter of ABCDE to perimeter of QRSTU =  $\frac{14}{70} = \frac{1}{5}$



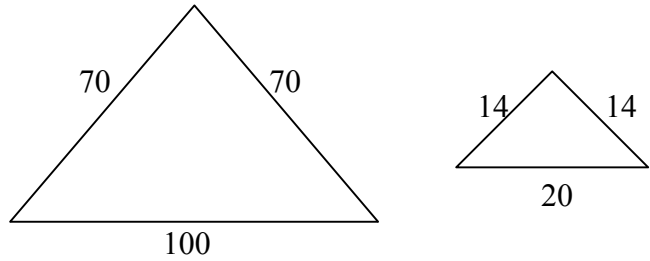
# Similar Figures-Classwork

Determine whether the figures are similar. If yes, what is the scale factor that transforms the figure on the left to the figure on the right? Assume the angles are congruent.

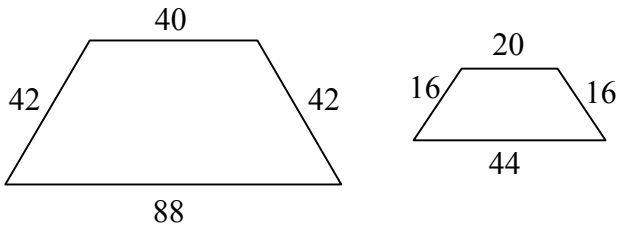
1. **Similar ?**    yes    no  
If yes, scale factor (left to right) \_\_\_\_\_



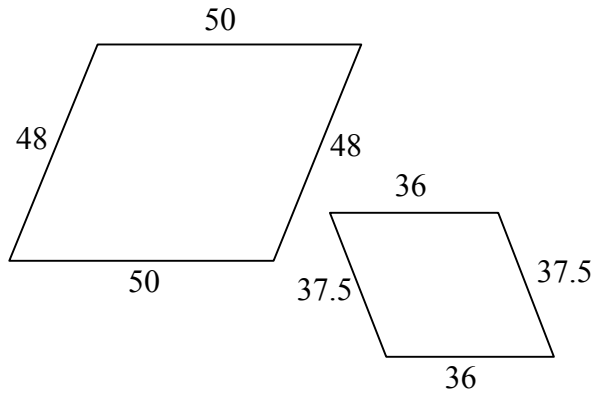
2. **Similar?**    yes    no  
If yes, scale factor (left to right) \_\_\_\_\_



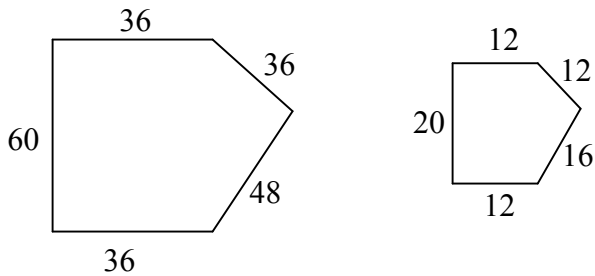
3. **Similar ?**    yes    no  
If yes, scale factor (left to right) \_\_\_\_\_



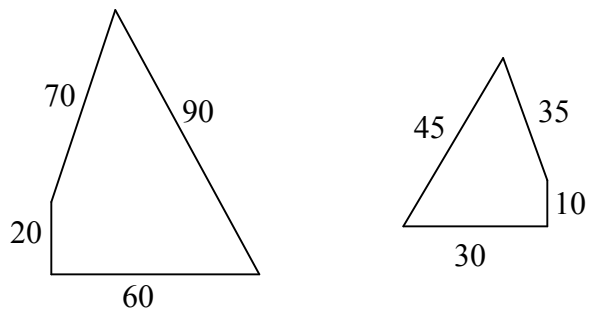
4. **Similar?**    yes    no  
If yes, scale factor (left to right) \_\_\_\_\_



5. **Similar ?**    yes    no  
If yes, scale factor (left to right) \_\_\_\_\_



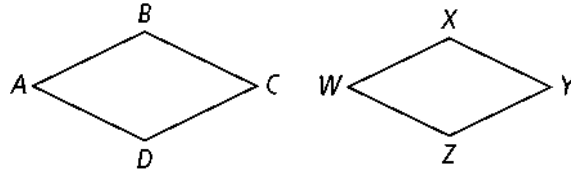
6. **Similar?**    yes    no  
If yes, scale factor (left to right) \_\_\_\_\_



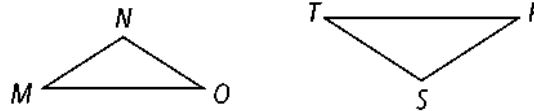
Similar Polygons HW

List the pairs of congruent angles and the extended proportion that relates the corresponding sides for the similar polygons.

1.  $ABCD \sim WXYZ$

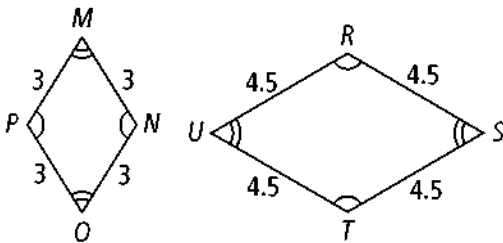


2.  $\triangle MNO \sim \triangle RST$

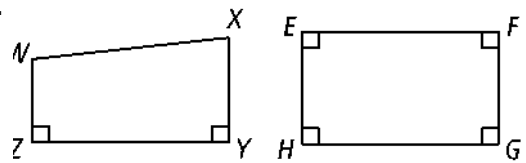


Determine whether the polygons are similar. If so, write a similarity statement and give the scale factor. If not, explain.

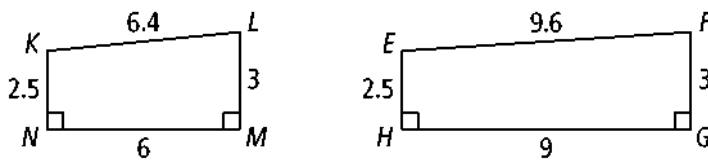
3.



4.



5.



Draw the following and determine whether the polygons are similar.

6. a square with side length 4 and a rectangle with width 8 and length 8.5

7. a rhombus with side lengths 8 and consecutive angles  $50^\circ$  and  $130^\circ$ , and a rhombus with side lengths 13 and consecutive angles  $50^\circ$  and  $130^\circ$

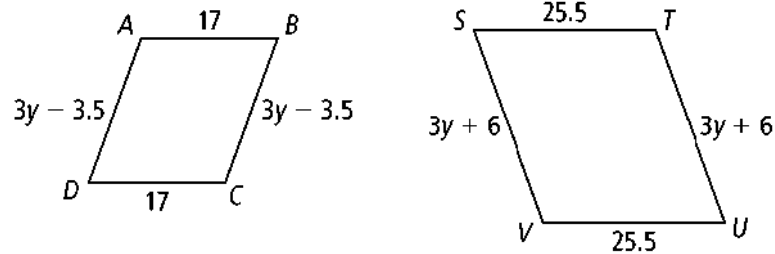
8. An architect is making a scale drawing of a building. She uses the scale 1 in. = 15 ft.
- If the building is 48 ft tall, how tall should the scale drawing be?
  - If the building is 90 ft wide, how wide should the scale drawing be?

Determine whether each statement is *always*, *sometimes*, or *never* true.

- Two squares are similar.
- Two hexagons are similar.
- Two similar triangles are congruent.
- A rhombus and a pentagon are similar.

**Algebra** Find the value of  $y$ . Give the scale factor of the polygons.

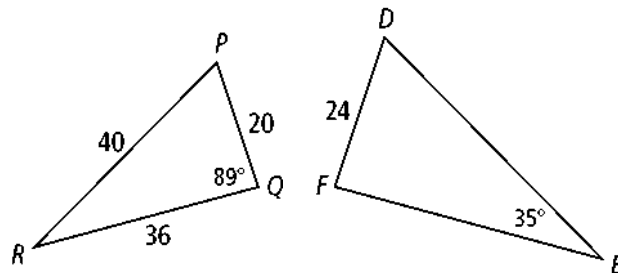
13.  $ABCD \sim TSVU$



14. The scale factor of  $RSTU$  to  $VWXY$  is 14 : 3. What is the scale factor of  $VWXY$  to  $RSTU$ ?

In the diagram below,  $\triangle PRQ \sim \triangle DEF$ . Find each of the following.

- the scale factor of  $\triangle PRQ$  to  $\triangle DEF$
- $m\angle D$
- $m\angle R$
- $m\angle P$
- $DE$
- $FE$



# Similar Triangles (AA~)

Learning Targets: Students will be able to identify similar triangles (using AA~).

Students will be able to use similar triangles to solve application problems.

## KEY TERMS

Similar Figures:

*(Angles)  
Same Shape w/d  
proportional side*

Similarity Statement:

$$\triangle ABC \sim \triangle DEF$$

Statement of Proportionality:

*proportional sides*

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

Scale Factor:

*multiplier which you mult the sides of one  $\Delta$  to obtain the sides of the other  $\Delta$*

1. In the diagram,  $\triangle GST \sim \triangle GNP$ .

a) Write a statement of proportionality.

$$\frac{GS}{GN} = \frac{ST}{NP} = \frac{GT}{GP}$$

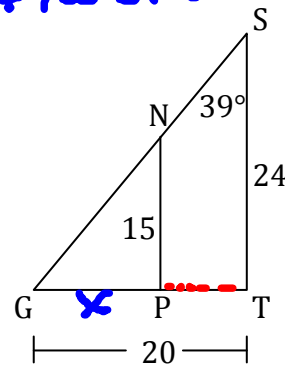
b) Find  $m\angle GNP$ .

$$39^\circ$$

c) Find GP.

$$\begin{aligned} \triangle GNP &\rightarrow \frac{x}{20} = \frac{15}{24} \\ \triangle GST &\rightarrow \end{aligned}$$

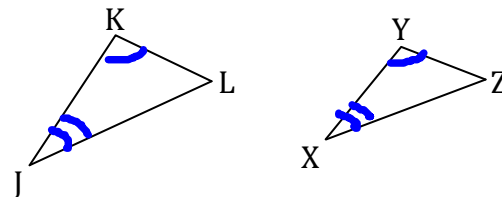
$$\begin{aligned} 24x &= 300 \\ x &= \frac{300}{24} = \frac{150}{12} \\ &= \frac{75}{6} = \frac{25}{2} = 12\frac{1}{2} \end{aligned}$$



## ANGLE-ANGLE SIMILARITY POSTULATE (AA~)

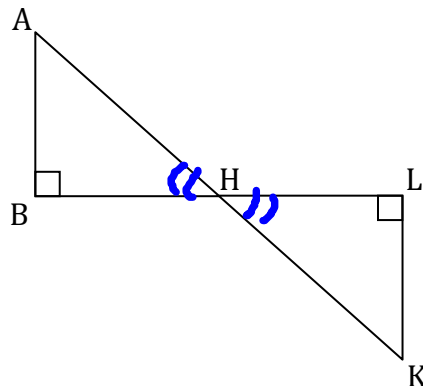
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If  $\angle JKL \cong \angle XYZ$  and  $\angle KJL \cong \angle YXZ$ , then  $\triangle JKL \sim \triangle XYZ$ .



2. Determine if the triangles are similar. If they are similar, write a similarity statement and explain why.

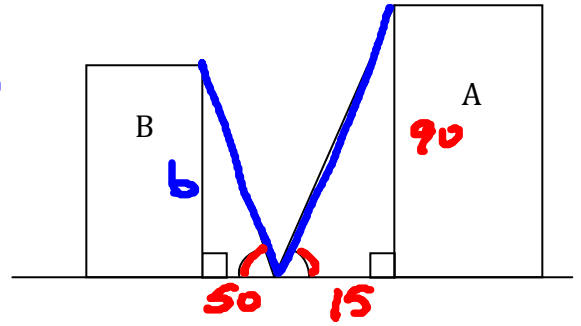
$\angle B \cong \angle L$  Both Rt  $\angle$ 's  
 $\angle AHB \cong \angle KHL$   
*Vertical  $\angle$ 's are  $\cong$*   
 SO  $\sim$  by AA~  
 $\triangle AHB \sim \triangle KHL$



3. Bill is standing 15 m from building A and 50 m from building B. Building A is 90 m tall. Find the height of building B.

$$\frac{b}{50} = \frac{90}{15} \quad \text{or} \quad \frac{b}{90} = \frac{50}{15}$$

$$b = 6(30) \\ = 300 \text{ m}$$

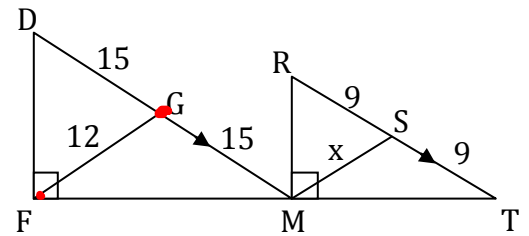


4.  $\triangle DFM \sim \triangle RMT$ . Find MS.

$$\frac{x}{12} = \frac{9}{15}$$

$$5x = 36$$

$$x = \frac{36}{5}$$

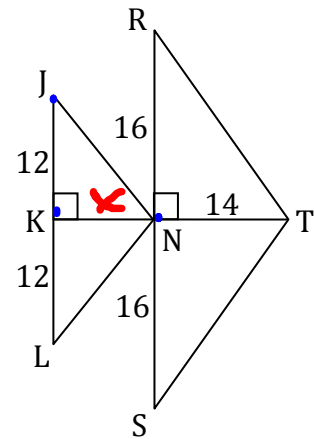


5.  $\triangle JNL \sim \triangle RTS$ . Find KN.

$$\frac{x}{14} = \frac{12}{16}$$

$$4x = 42$$

$$x = \frac{42}{4} = 10\frac{1}{2}$$



The girls' soccer team won 10 games and lost 2, and the boys' soccer team won 12 games and lost 3.

1. What is the ratio of the girls' wins to their losses?
2. What is the ratio of the boys' wins to their losses?
3. What is the ratio of the girls' wins to the total number of games played?
4. What is the ratio of the boys' wins to their total number of games played?
5. Which team had the greater winning ratio?

**Rewrite the ratio so that the numerator and denominator have the same units. Then simplify.**

6.  $\frac{5 \text{ weeks}}{30 \text{ days}}$

7.  $\frac{2 \text{ days}}{56 \text{ mins}}$

8.  $\frac{20 \text{ in}}{3 \text{ yd}}$

**Solve the proportion.**

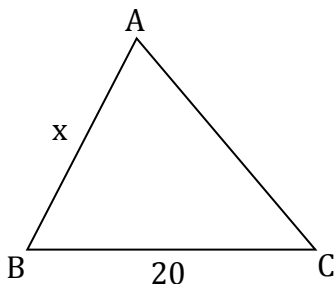
9.  $\frac{x}{3} = \frac{10}{15}$

10.  $\frac{2}{y-3} = \frac{3}{y}$

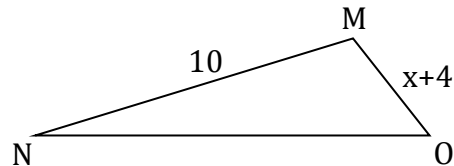
11.  $\frac{4}{x+2} = \frac{16}{x+5}$

**The ratio of the two side lengths of the triangle is given. Solve for the variable.**

12.  $AB : BC$  is 2:5.

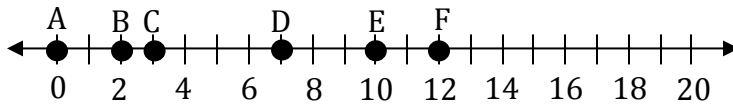


13.  $MN : MO$  is 3:5.





Use the number line to find the ratios of the distances.



14.  $\frac{AB}{CD} =$

15.  $\frac{BC}{DE} =$

16. A triangle's three angles are in the ratio of 5:7:8. What is the measure of the smallest angle?

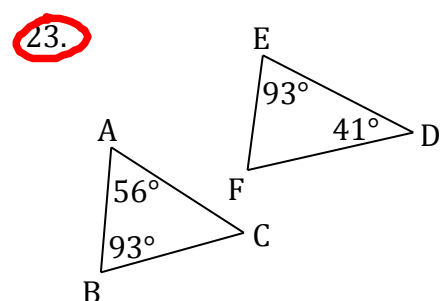
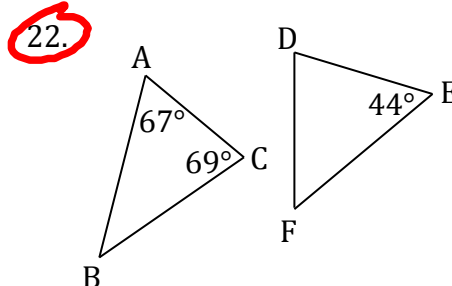
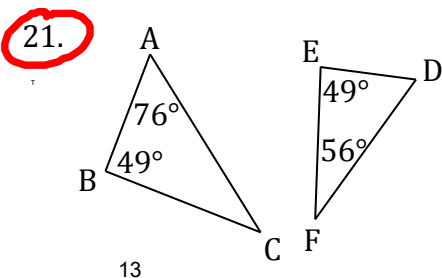
17. A 6 foot high school student casts a shadow of 24 inches. At the same time of day a student at the elementary school park casts a shadow of 14 inches. How tall is the elementary student (in feet)?

18. The ratio of two supplementary angles is 4:5. Find the measures of each angle.

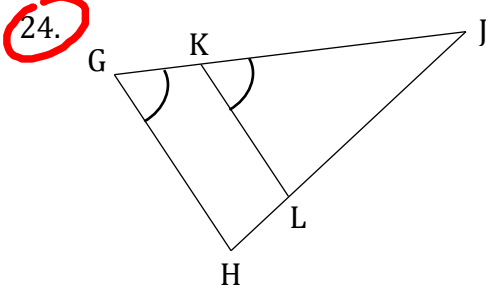
19. A 3-foot stick is broken into two pieces. The ratio of the two pieces is 5:7. How big are the two pieces?

20. What are the possible names for the quadrilateral with sides  $\sqrt{5}:\sqrt{5}:\sqrt{5}:\sqrt{5}$  ?

Decide if the triangles are similar, not similar, or cannot be determined for the given information.



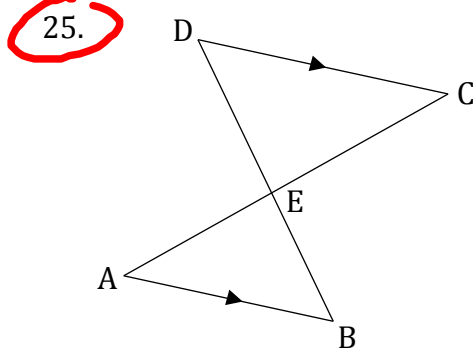
Are the triangles similar? If yes, then write the similarity statement and statement proportionality.



Similar: Yes or No

Similarity Statement: \_\_\_\_\_

Statement of Proportionality: \_\_\_\_\_



Similar: Yes or No

Similarity Statement: \_\_\_\_\_

Statement of Proportionality: \_\_\_\_\_

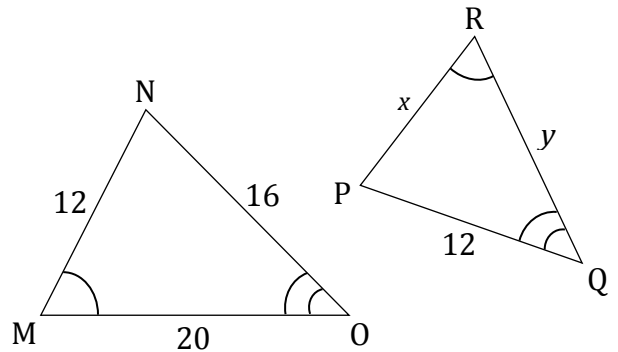
Use the diagram to complete the following.

26. Write a similarity statement.

27. Write a statement of proportionality.

28. Find the scale factor.

29. Solve for  $x$ .



30. Solve for  $y$ .

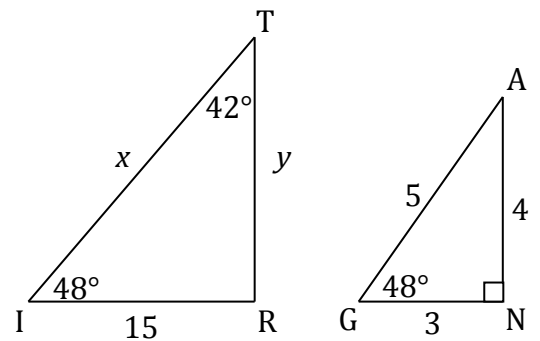
Use the diagram to complete the following.

31. Write a similarity statement.

32. Write a statement of proportionality.

33. Find the scale factor.

34. Solve for  $x$ .



35. Solve for  $y$ .

## Similar Triangles (SSS~ and SAS~)

*Learning Targets: Students will be able to identify similar triangles (using SSS~ and SAS~).*

*Students will be able to use similar triangles to solve application problems.*

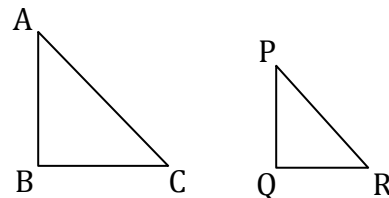
Remember... we already learned one theorem to show triangles are similar:

### SIDE-SIDE-SIDE SIMILARITY POSTULATE (SSS~)

If the lengths of corresponding sides of two triangles are proportional, then the two triangles are similar.

If  $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ , then  $\triangle ABC \sim \triangle PQR$ .

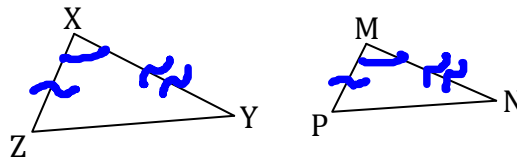
IS TRUE



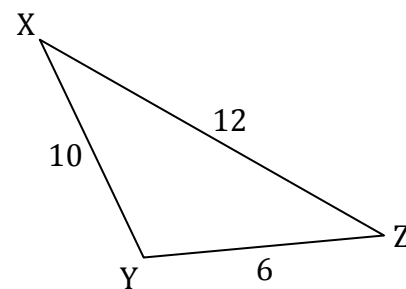
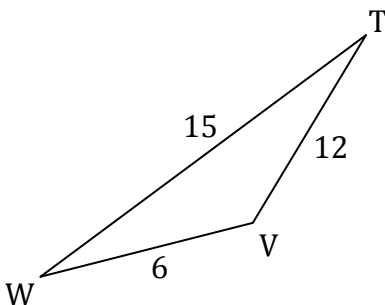
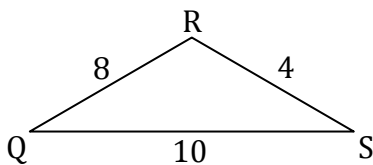
### SIDE-ANGLE-SIDE SIMILARITY POSTULATE (SAS~)

If an angle of one triangle is congruent to a corresponding angle of a second triangle and the lengths of the corresponding sides including those angles are proportional, then the triangles are similar.

If  $\angle X \cong \angle M$  and  $\frac{ZX}{PM} = \frac{XY}{MN}$ , then  $\triangle XYZ \sim \triangle MNP$ .



1. Which of the following triangles are similar?



$\triangle RQS \sim \triangle WVT?$

$$\frac{4}{6} = \frac{8}{12} = \frac{10}{15}$$

$48 = 48$  ✓  $120 = 120$  ✓  
yes! by SSS~

$\triangle WVT \sim \triangle XYZ?$

$$\frac{6}{6} = \frac{12}{10} = \frac{15}{12}$$

$1 \neq \frac{12}{10}$  NO

Explain why

2. Use the lengths given to ~~prove~~ that  $\triangle DFR \sim \triangle MNR$ .

Since  $\angle RNW$  and  $\angle RDF$  are both RT  
 $\angle$ 's  $MN \parallel DF$  (Corresp.  $\angle$  converse)  
 and they are Congruent (All Right  $\angle$ 's are  $\cong$ )

Also,  $\angle MNR \cong \angle DFR$   
 (Corresp  $\angle$  Post)

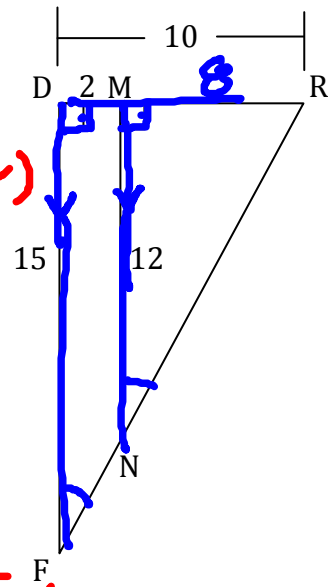
So,  $\triangle DFR \sim \triangle MNR$  by AA  $\sim$

$$\frac{DR}{MR} = \frac{DF}{MN}$$

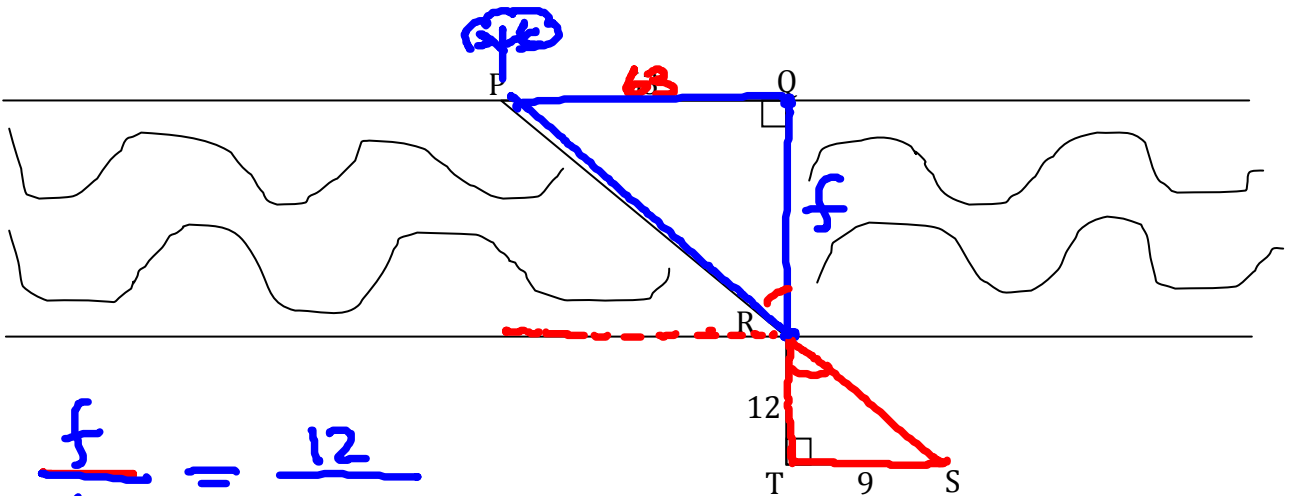
$$\frac{10}{8} = \frac{15}{12}$$

is TRUE  
 $120 = 120$

and  $\angle MNW \cong \angle RDF$   
 so similar by SAS  $\sim$



3. To measure the width of a river, you use a surveying technique which includes similar triangles. Find the length of the river (RQ) given the lengths shown (measured in feet).



$$\frac{f}{63} = \frac{12}{9}$$

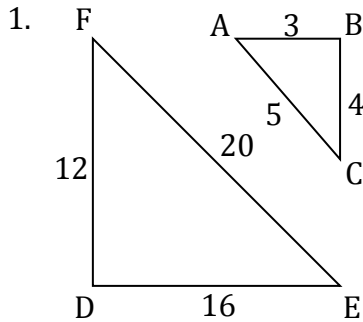
$\leftarrow \times 7$

$$12 \cdot 7 = f$$

$$f = 84 \text{ ft}$$

# Geometry

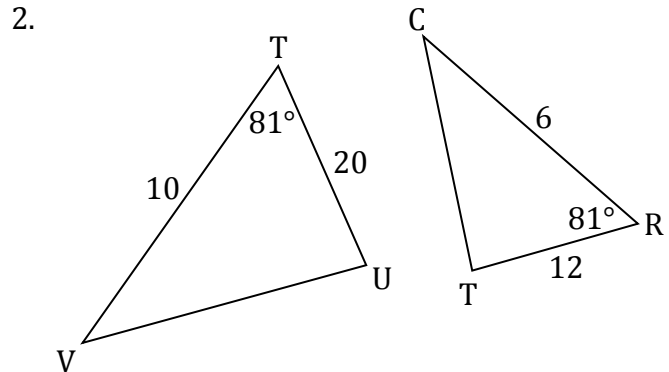
Name the postulate or theorem that can prove the two triangles similar. Then write a similarity statement and statement of proportionality.



Theorem/Postulate: \_\_\_\_\_

Similarity Statement: \_\_\_\_\_

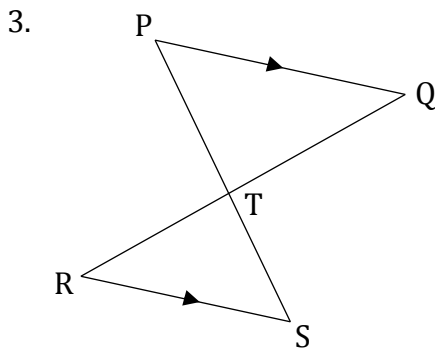
Statement of Proportionality: \_\_\_\_\_



Theorem/Postulate: \_\_\_\_\_

Similarity Statement: \_\_\_\_\_

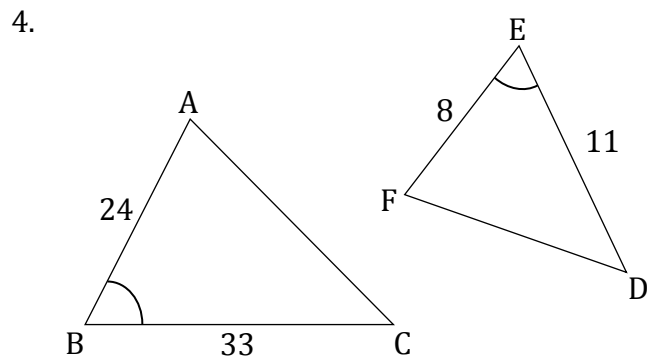
Statement of Proportionality: \_\_\_\_\_



Theorem/Postulate: \_\_\_\_\_

Similarity Statement: \_\_\_\_\_

Statement of Proportionality: \_\_\_\_\_

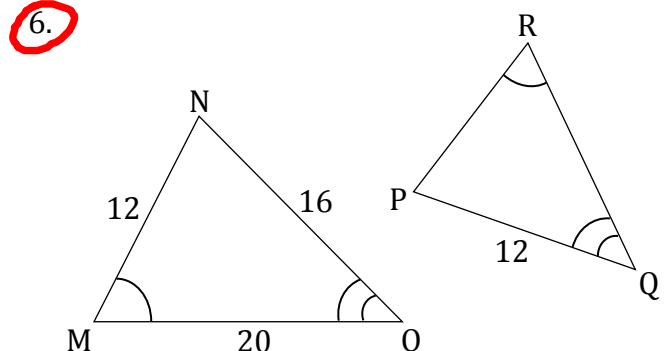
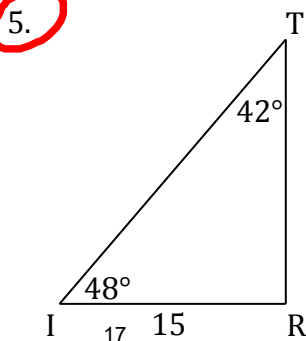


Theorem/Postulate: \_\_\_\_\_

Similarity Statement: \_\_\_\_\_

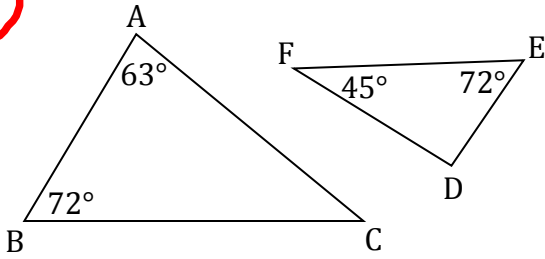
Statement of Proportionality: \_\_\_\_\_

Find the scale factor of the two similar triangles.



Are the triangles similar? If so, state the postulate or theorem that proves your answer.

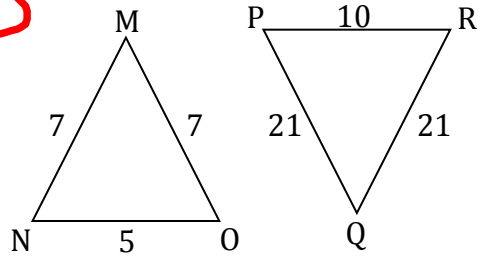
7.



Similar: Yes or No

Postulate/Theorem: \_\_\_\_\_

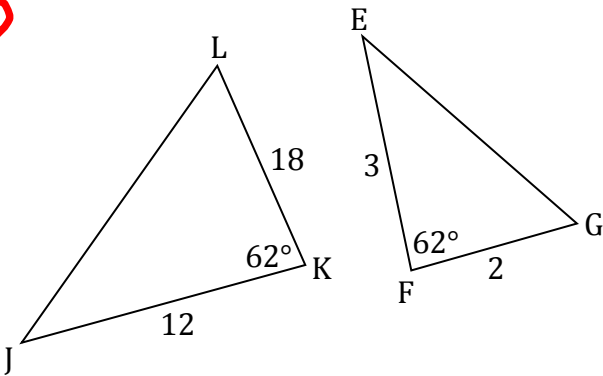
8.



Similar: Yes or No

Postulate/Theorem: \_\_\_\_\_

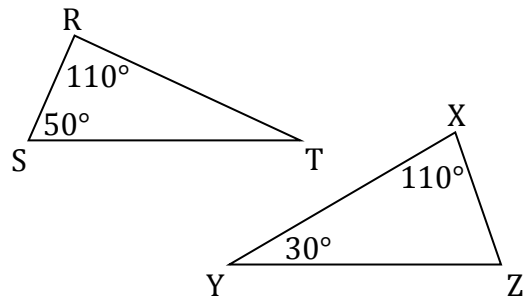
9.



Similar: Yes or No

Postulate/Theorem: \_\_\_\_\_

10.

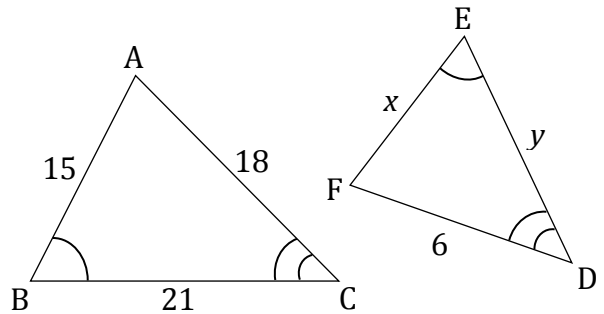


Similar: Yes or No

Postulate/Theorem: \_\_\_\_\_

Use the diagram to complete the following.

11. Write a similarity statement.
12. Write a statement of proportionality.
13. Solve for  $x$ .
14. Solve for  $y$ .



## Side-Splitter Theorem

**Learning Targets:** Students will be able to use proportionality theorems to calculate segment lengths.

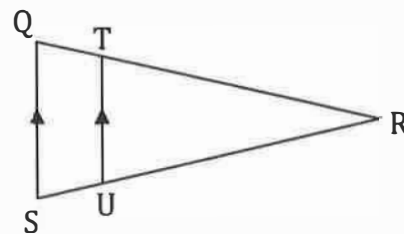
### SIDE SPLITTER THEOREM

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

$$\frac{QT}{TR} = \frac{SU}{UR}$$

If  $\overline{TU} \parallel \overline{QS}$ , then \_\_\_\_\_.

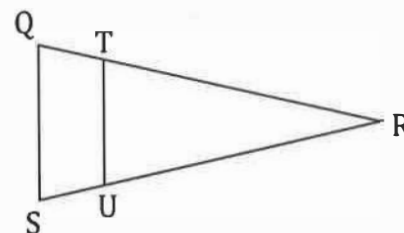
$$\text{or } \frac{RT}{TQ} = \frac{RU}{US}$$



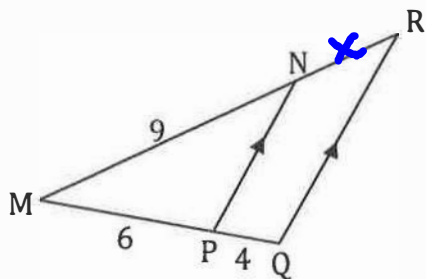
### CONVERSE OF THE SIDE SPLITTER THEOREM

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

If  $\frac{RT}{TQ} = \frac{RU}{US}$ , then  $\overline{QS} \parallel \overline{TU}$  \_\_\_\_\_.



1. What is the length of  $\overline{NR}$ ?

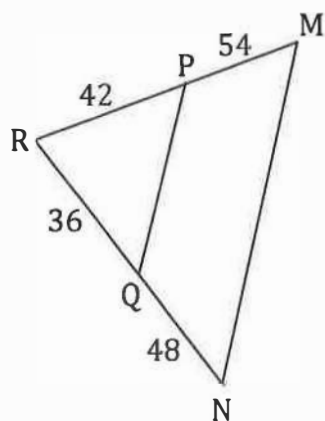


$$\frac{9}{x} = \frac{6}{4}$$

$$6x = 36$$

$$x = 6$$

2. Given the diagram, determine whether  $\overline{MN}$  is parallel to  $\overline{PQ}$ .



$$\frac{42}{54} \stackrel{?}{=} \frac{36}{48}$$

$$1944 \stackrel{?}{=} 2016$$

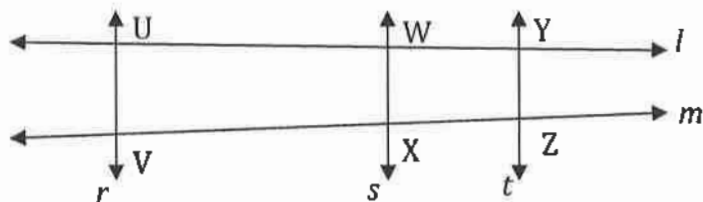
NO

So  $\overline{MN} \not\parallel \overline{PQ}$

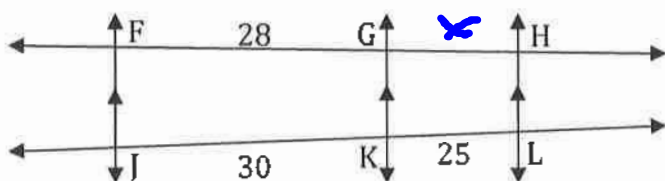
### THEOREM

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

If  $r \parallel s$  and  $s \parallel t$ , and  $l$  and  $m$  intersect  $r, s$ , and  $t$ , then  $\frac{UW}{WY} = \frac{VX}{YZ}$



3. What is the length of  $\overline{GH}$ ?



$$\frac{28}{30} = \frac{x}{25}$$

$$\frac{28}{x} = \frac{30}{25}$$

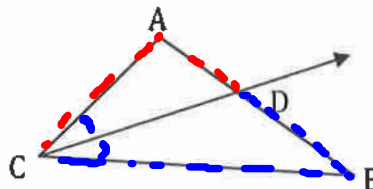
$$6x = 140$$

$$x = \frac{140}{6} = \frac{70}{3}$$

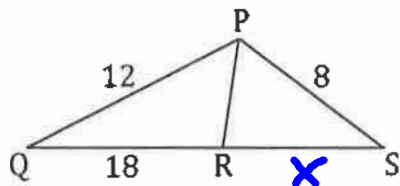
### THEOREM

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

If  $\overline{CD}$  bisects  $\angle ACB$ , then  $\frac{DB}{CB} = \frac{DA}{AC}$



4. In the diagram,  $\angle QPR \cong \angle RPS$ . Use the given side lengths to find the length of  $\overline{RS}$ .



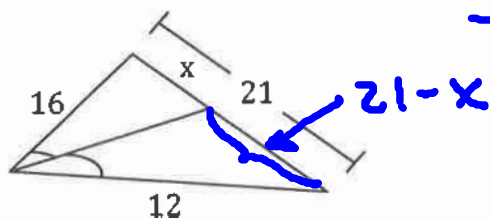
$$\frac{318}{212} = \frac{x}{8}$$

$$\frac{18}{x} = \frac{12}{8}$$

$$2x = 24$$

$$x = 12$$

5. Find the value of  $x$ .



$$\frac{21-x}{12} = \frac{x}{16}$$

$$336 - 16x = 12x$$

$$336 = 28x$$

$$x = \frac{336}{28} = 12$$



1. Complete the proportions.

a)  $\frac{AB}{BC} = \frac{DE}{\square}$

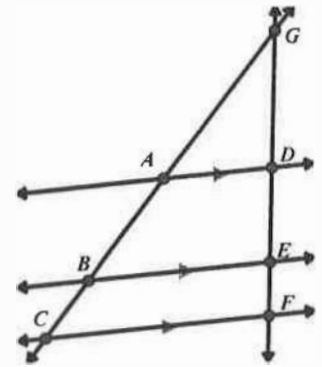
b)  $\frac{AC}{DF} = \frac{AB}{\square}$

c)  $\frac{GE}{DF} = \frac{\square}{AC}$

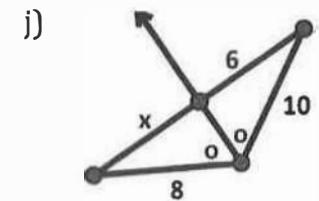
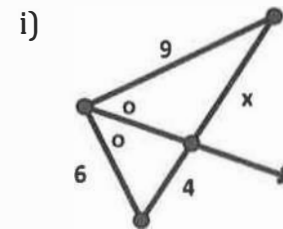
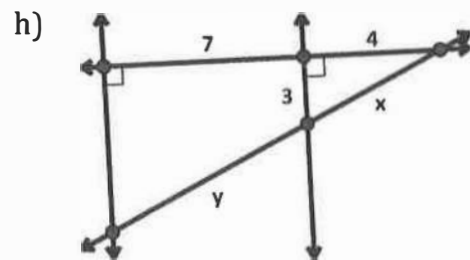
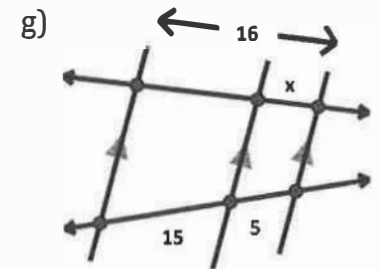
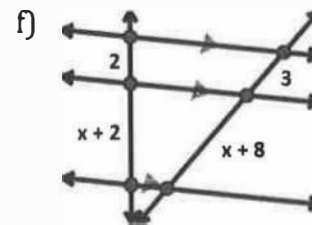
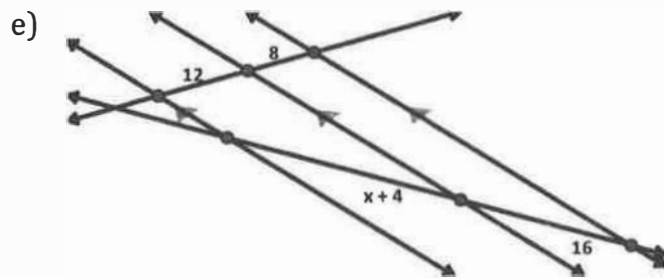
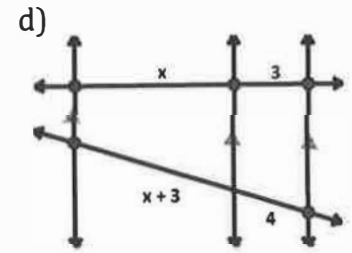
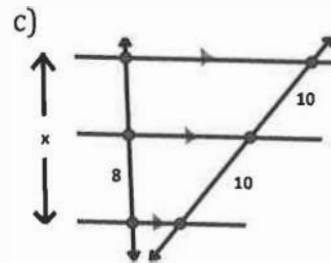
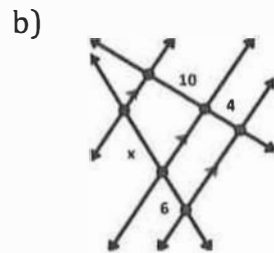
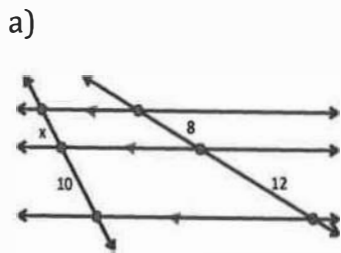
d)  $\frac{GF}{DE} = \frac{GC}{\square}$

e)  $\frac{\square}{DF} = \frac{BC}{EF}$

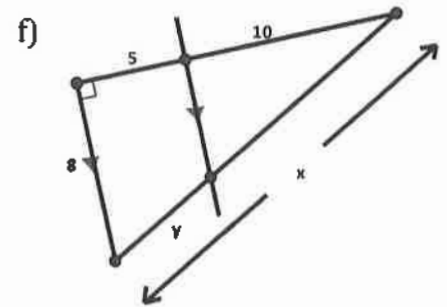
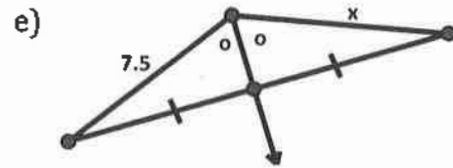
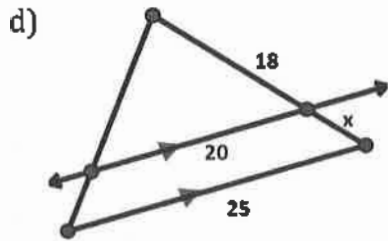
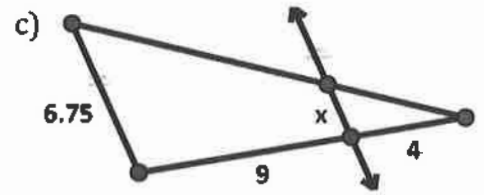
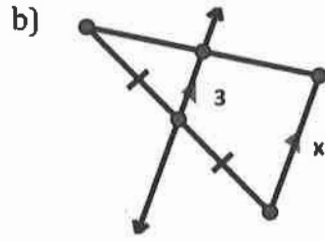
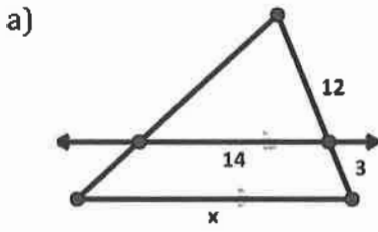
f)  $\frac{CB}{CG} = \frac{FE}{\square}$



2. Find the values for the missing variables.



3. Find the values for the missing variables.



## Dilations

---

*Learning Targets: Students will be able to dilate figures with a compass and straightedge.  
Students will be able to dilate figures on the coordinate plane.*

Dilation: A transformation that enlarges or reduces a figure starting at a center point with a single scale factor.

Scale Factor:

$k$ , amount of the enlargement or reduction

Center of Dilation:

where the dilation starts from

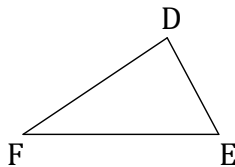
Enlargement:

$$k > 1$$

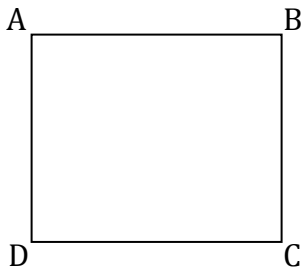
Reduction:

$$0 < k < 1$$

1. Using a compass and straightedge dilate the given figure with a scale factor of 3.

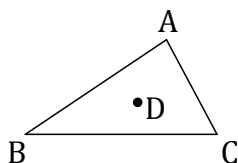


2. Using a compass and straightedge dilate the given figure with a scale factor of  $\frac{1}{2}$ .

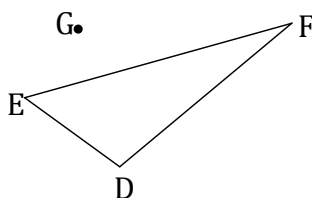


Using a compass and a straightedge, dilate each figure with the given scale factor and center.

3.  $k = 3$ ; Center: D

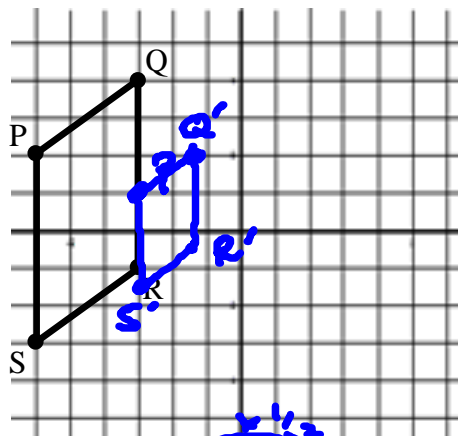


4.  $k = 2$ ; Center: G



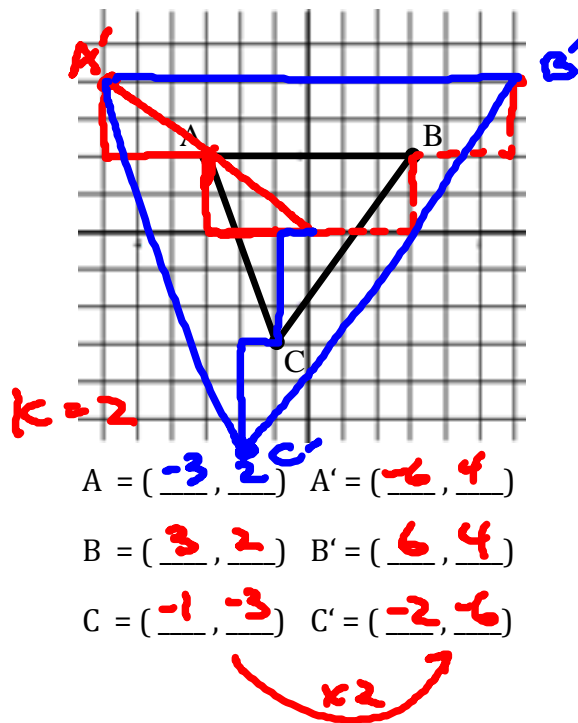
5. Using the origin as the center of the dilation and the given scale factor, find and plot the coordinates of the vertices of the image.

a)  $k = \frac{1}{2}$



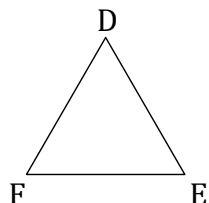
$$\begin{aligned}
 P &= (-6, 2) & P' &= (-3, 1) \\
 Q &= (3, 4) & Q' &= (1.5, 2) \\
 R &= (-3, -1) & R' &= (-1.5, -0.5) \\
 S &= (-6, -3) & S' &= (-3, -1.5)
 \end{aligned}$$

b)  $k = 2$

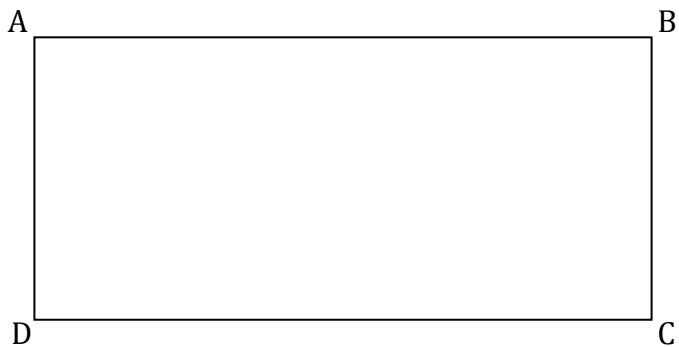


$$\begin{aligned}
 A &= (-3, 2) & A' &= (-6, 4) \\
 B &= (3, 2) & B' &= (6, 4) \\
 C &= (-1, -3) & C' &= (-2, -6)
 \end{aligned}$$

1. Using a compass and straightedge dilate the given figure with a scale factor of 2.5.

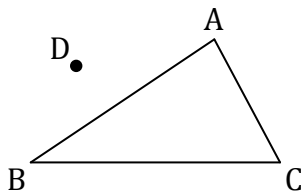


2. Using a compass and straightedge dilate the given figure with a scale factor of  $\frac{3}{4}$ .

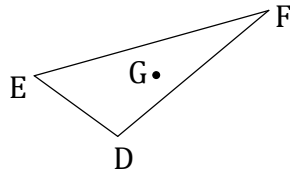


**Using a compass and a straightedge, dilate each figure with the given scale factor and center.**

3.  $k = 4$ ; Center: D

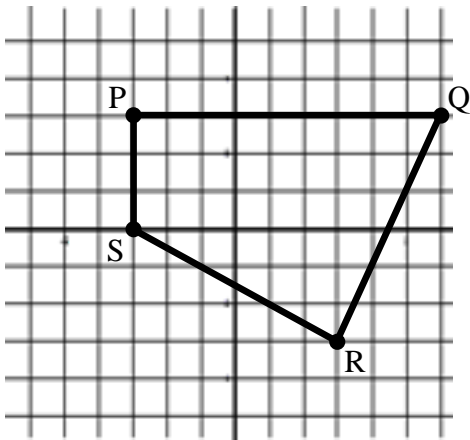


4.  $k = 3$ ; Center: G



5. Using the origin as the center of the dilation and the given scale factor, find and plot the coordinates of the vertices of the image.

a)  $k = 1/3$



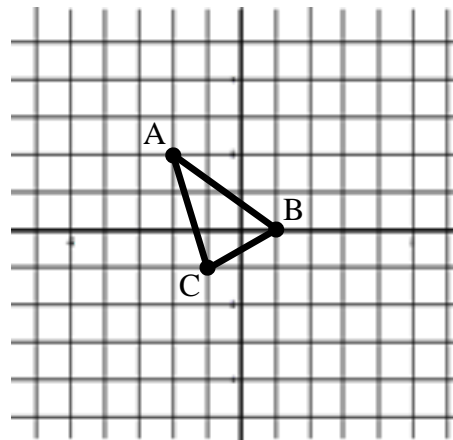
$P = (\underline{\quad}, \underline{\quad})$   $P' = (\underline{\quad}, \underline{\quad})$

$Q = (\underline{\quad}, \underline{\quad})$   $Q' = (\underline{\quad}, \underline{\quad})$

$R = (\underline{\quad}, \underline{\quad})$   $R' = (\underline{\quad}, \underline{\quad})$

$S = (\underline{\quad}, \underline{\quad})$   $S' = (\underline{\quad}, \underline{\quad})$

b)  $k = 3$



$A = (\underline{\quad}, \underline{\quad})$   $A' = (\underline{\quad}, \underline{\quad})$

$B = (\underline{\quad}, \underline{\quad})$   $B' = (\underline{\quad}, \underline{\quad})$

$C = (\underline{\quad}, \underline{\quad})$   $C' = (\underline{\quad}, \underline{\quad})$

**A deck of cards has 52 cards.**

1. What is the ratio of red cards to the total number of cards?
  
2. What is the ratio of Kings, Queens, and Jacks to the total number of cards?
  
3. What is the ratio of Aces to the total number of cards?
  
4. What is the ratio of Kings to Queens?

**Solve the proportion.**

5.  $\frac{x}{3} = \frac{18}{27}$

6.  $\frac{2}{y-2} = \frac{4}{14}$

7.  $\frac{4}{x-2} = \frac{8}{x+3}$

**Rewrite the ratio so that the numerator and denominator have the same units. Then simplify.**

8.  $\frac{1 \text{ day}}{200 \text{ seconds}}$

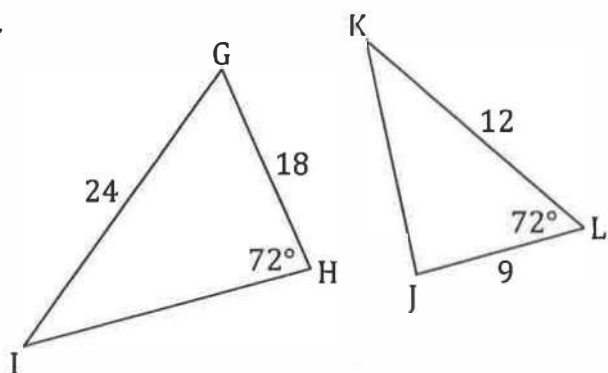
9.  $\frac{64 \text{ hours}}{2 \text{ weeks}}$

10.  $\frac{2 \text{ yds}}{30 \text{ inches}}$

11. Three numbers are in the ratio of 2:5:3. If the largest number is 65, then what is the smallest number?
  
12. Cameron has been eating 2 dollar menu burgers every week (7 days). At that rate, how many hamburgers will he eat in 4 weeks?
  
13. Is the largest angle acute, right or obtuse in a triangle that has angles measures in ratio, 2:3:4?

Are the triangles similar? If so, state the postulate or theorem that proves your answer.

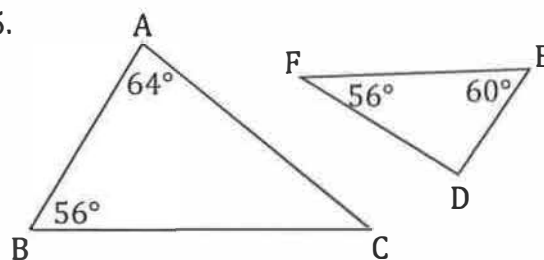
14.



Similar: Yes or No

Postulate/Theorem: \_\_\_\_\_

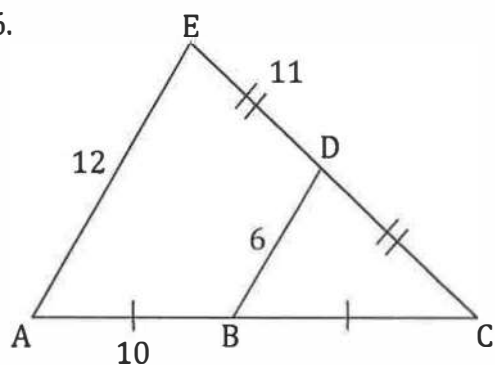
15.



Similar: Yes or No

Postulate/Theorem: \_\_\_\_\_

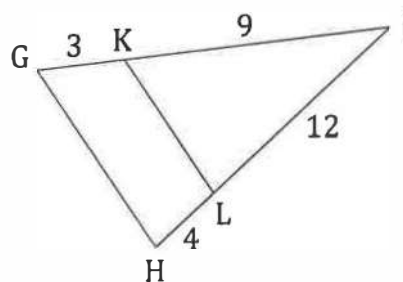
16.



Similar: Yes or No

Postulate/Theorem: \_\_\_\_\_

17.



Similar: Yes or No

Postulate/Theorem: \_\_\_\_\_

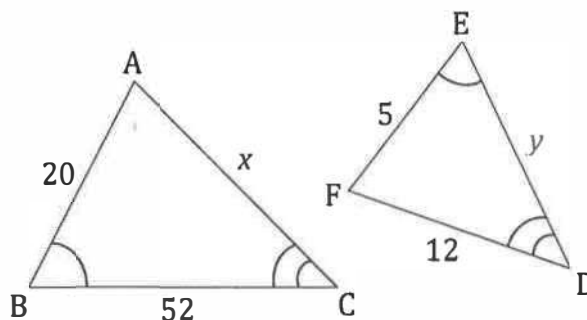
Use the diagram to complete the following.

18. Write a similarity statement.

19. Write a statement of proportionality.

20. Solve for  $x$ .

21. Solve for  $y$ .





# Unit 7

## Right Triangles & Trigonometry

Prime Factorization to Simplify Square Roots

Simplify.

1)  $\sqrt{8}$

$\sqrt{3 \cdot 3} = \sqrt{3^2}$

2)  $\sqrt{18}$

$\sqrt{18} = \sqrt{2 \cdot \underbrace{3 \cdot 3}_{3^2}}$ 
 $2 \sqrt{9}$ 
 $2 \sqrt{18}$



3)  $\sqrt{20}$

4)  $\sqrt{32}$

5)  $\sqrt{72}$

6)  $\sqrt{100}$

7)  $\sqrt{150}$

$\sqrt{192} = 13.86$

8)  $\sqrt{192} = \sqrt{4 \cdot 48}$

$\sqrt{3 \cdot 64}$   
 $= 8\sqrt{3}$

$= \sqrt{4 \cdot 16 \cdot 3}$

$= 2 \cdot 4 \sqrt{3}$

$= 8\sqrt{3}$

EXACT

9)  $\sqrt{384}$

10)  $\sqrt{448}$

**Extra OPTIONAL Practice. Simplify**

11)  $\sqrt{125}$

12)  $\sqrt{27}$

13)  $\sqrt{36}$

14)  $\sqrt{75}$

15)  $\sqrt{216}$

16)  $\sqrt{96}$

17)  $\sqrt{105}$

18)  $\sqrt{50}$

19)  $\sqrt{70}$

20)  $\sqrt{343}$

21)  $\sqrt{20}$

22)  $\sqrt{18}$

23)  $\sqrt{175}$

24)  $\sqrt{48}$

25)  $\sqrt{128}$

26)  $\sqrt{42}$

27)  $\sqrt{16}$

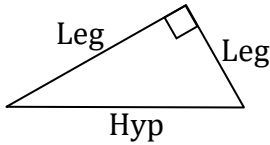
28)  $\sqrt{252}$

# Pythagorean Theorem

**Learning Targets:** Students will be able to identify families of right triangles and find missing lengths. Students will be able to classify triangles based on three sides given.

## PYTHAGOREAN THEOREM

In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse. **The hypotenuse is always the longest side, it is across the right angle.**



$$\underline{\text{leg}^2} + \underline{\text{leg}^2} = \underline{\text{hyp}^2}$$

A Pythagorean Triple is a set of three positive integers, such that  $L^2 + L^2 = H^2$ .

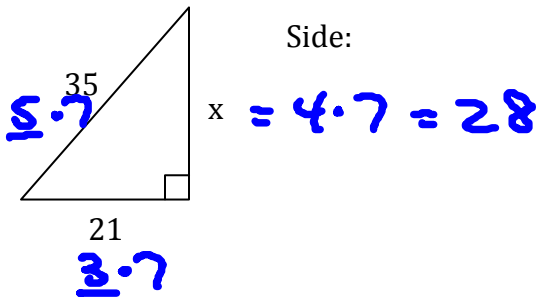
**Triple Families:** (3, 4, 5) (6, 8, 10) (1,  $\frac{3}{4}$ ,  $\frac{5}{4}$ ) (5, 12, 13) ( , , ) ( , , ) ( , , ) ( , , )

$$3^2 + 4^2 = 5^2$$

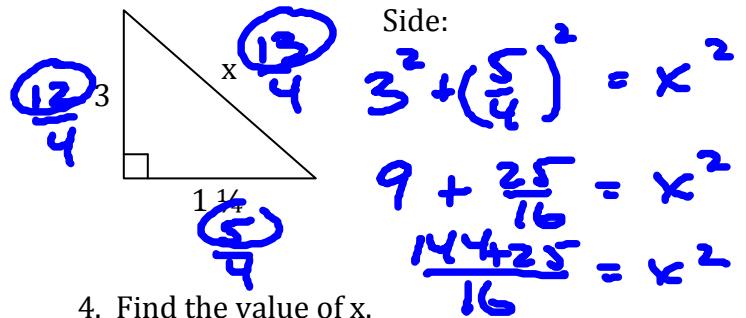
$$7^2 + 24^2 = 25^2$$

$$7, 24, 25$$

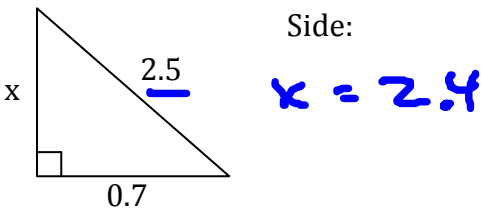
1. Find the value of x.



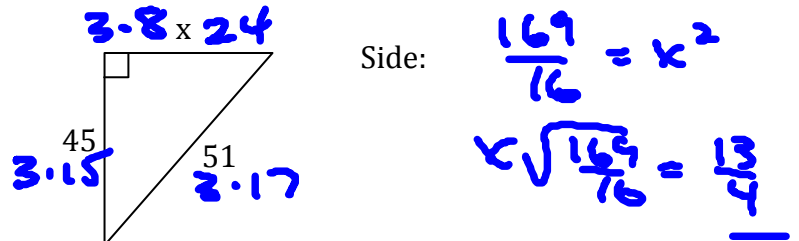
2. Find the value of x.



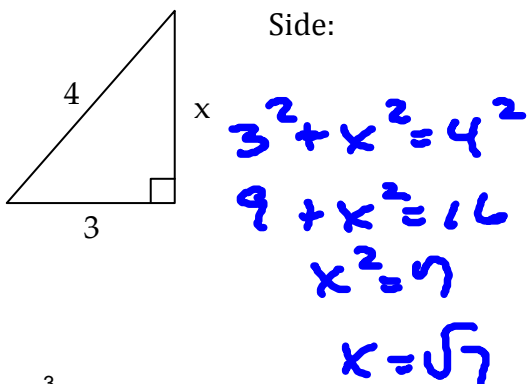
3. Find the value of x.



4. Find the value of x.



5. Find the value of x.



$$45^2 + x^2 = 51^2$$

$$2025 + x^2 = 2601$$

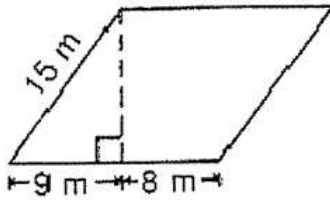
$$x^2 = 576$$

$$x = \sqrt{576} = 24$$

Handwritten notes:  $3 \cdot 8 \times 24$ ,  $8-15-17$

## Pythagorean Theorem HW

- 1) What is the height of the trapezoid?



- 2) A 50-ft cable is stretched from the top of an antenna to an anchor on the ground 15ft from the base of the antenna. How tall is the antenna?

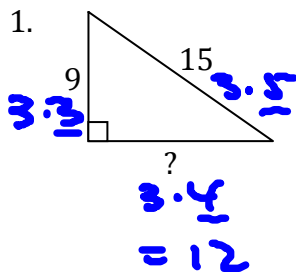
- 3) The bases on a softball diamond are 60 feet apart. How far is home plate from second base?

- 4) A state park is in the shape of a rectangle 8 miles long and 6 miles wide. Walking from corner to corner diagonally is shorter than going along the length and width of the park. By how much is it shorter?

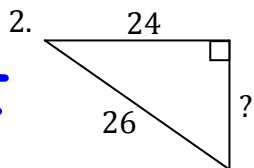
- 5) A newly-planted tree needs to be staked with three wires. Each wire is attached to the trunk 3 ft above the ground, and then anchored to the ground 4 ft from the base of the tree. How much wire is needed for 6 trees [Total amount of wire]?



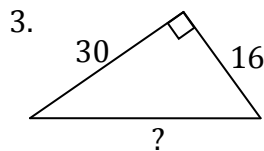
~~State the triple family, find the missing side, and state the scale factor.~~



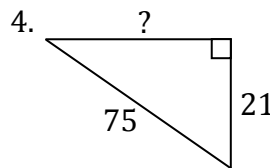
Side =



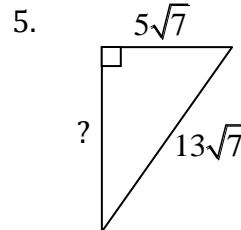
Side =



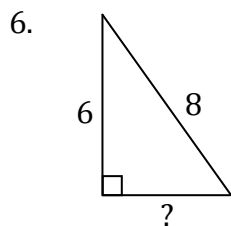
Side =



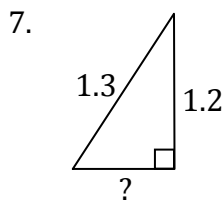
Side =



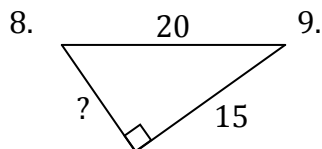
Side =



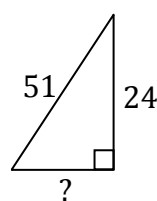
Side =



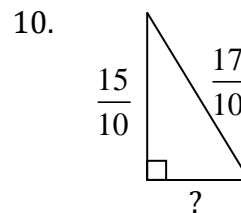
Side =



Side =



Side =

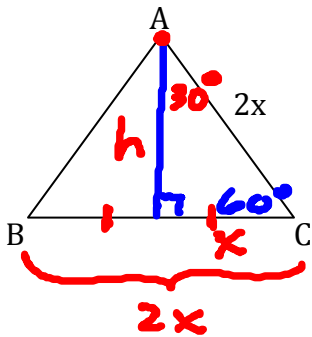


Side =

# Special Right Triangles

Learning Targets: Students will be able to use ratios of special right triangles to find missing sides.

$\triangle ABC$  is equilateral.



$$h^2 + x^2 = (2x)^2$$

$$h^2 + x^2 = 4x^2$$

$$h^2 = 3x^2$$

$$h = \sqrt{3x^2}$$

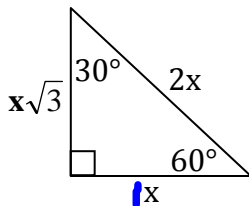
$$= x\sqrt{3}$$

$$\sqrt{x^2} = |x|$$

$$\sqrt{(-6)^2} = |-6|$$

$$= 6$$

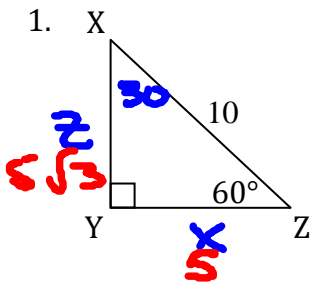
30°-60°-90°:



Find the missing sides

30 60 90

$$\frac{1}{x} = \frac{\sqrt{3}}{z} = \frac{2}{10}$$



$$\frac{1}{x} = \frac{2}{10}$$

$$x = 5$$

$$\frac{\sqrt{3}}{z} = \frac{2}{10}$$

$$z = 5\sqrt{3}$$

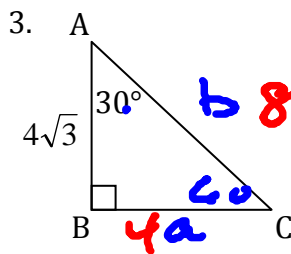
2.

$$\frac{1}{k} = \frac{\sqrt{3}}{j} = \frac{2}{6}$$

$$k\sqrt{3} = 6$$

$$k = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

$$j\sqrt{3} = 12$$

$$j = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$


30 60 90

$$\frac{1}{a} = \frac{\sqrt{3}}{b} = \frac{2}{8}$$

$$\frac{1}{a} = \frac{1}{4}$$

$$a = 4$$

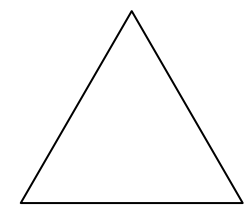
$$\frac{\sqrt{3}}{b} = \frac{2}{8}$$

$$\frac{\sqrt{3}}{b} = \frac{1}{4}$$

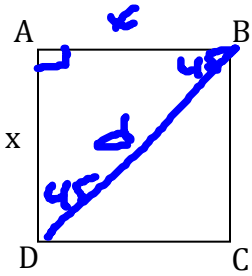
$$b = 4\sqrt{3}$$

4. Find the altitude of an equilateral triangle whose perimeter is 30.

*Rationalizing*



ABCD is a square. Find the length of the diagonal.



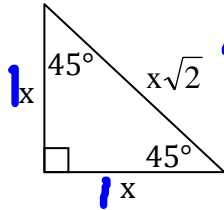
$$x^2 + x^2 = d^2$$

$$2x^2 = d^2$$

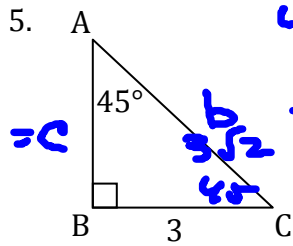
$$d = \sqrt{2x^2}$$

$$= x\sqrt{2}$$

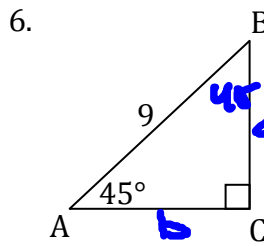
45°-45°-90°:



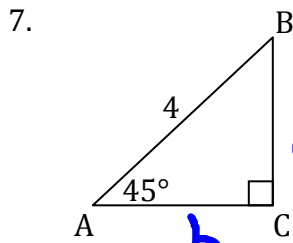
Isosceles Right  $\Delta$   
 45-45-90  
 1 1  $\sqrt{2}$



45-45-90  
 $\frac{1}{3} = \frac{1}{c} = \frac{\sqrt{2}}{b}$   
 $\frac{1}{3} = \frac{1}{c}$   
 $c = 3$   
 $\frac{1}{3} = \frac{\sqrt{2}}{b}$   
 $b = 3\sqrt{2}$

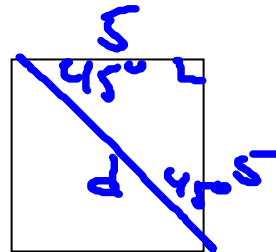


45 45 90  
 $\frac{1}{a} = \frac{1}{b} = \frac{\sqrt{2}}{9}$   
 $\frac{1}{a} = \frac{\sqrt{2}}{9}$   
 $a\sqrt{2} = 9$   
 $a = \frac{9}{\sqrt{2}} = \frac{9\sqrt{2}}{2}$   
 $b = \frac{9\sqrt{2}}{2}$



45 45 90  
 $\frac{1}{a} = \frac{1}{b} = \frac{\sqrt{2}}{4}$   
 $\frac{1}{a} = \frac{\sqrt{2}}{4}$   
 $a\sqrt{2} = 4$   
 $a = \frac{4}{\sqrt{2}}$   
 $b = \frac{4}{\sqrt{2}}$

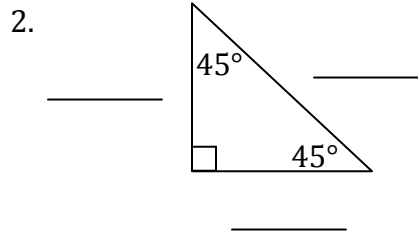
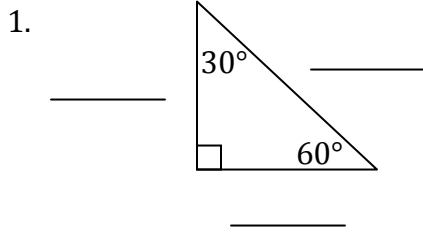
8. Find the diagonal of a square whose perimeter is 20.



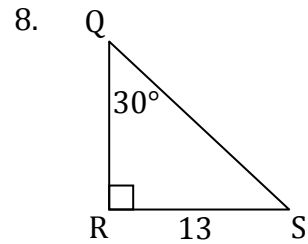
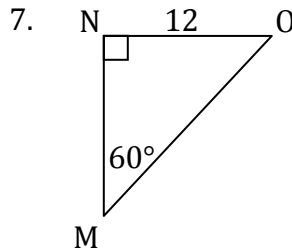
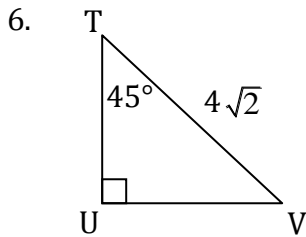
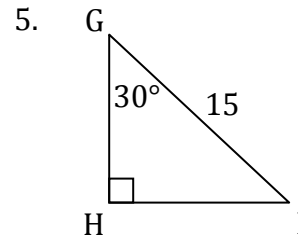
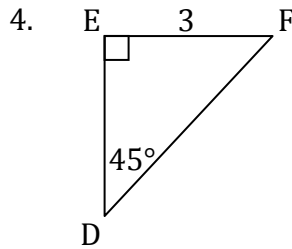
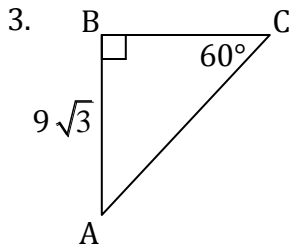
45 45 90  
 $\frac{1}{s} = \frac{1}{s} = \frac{\sqrt{2}}{d}$   
 $d = s\sqrt{2}$



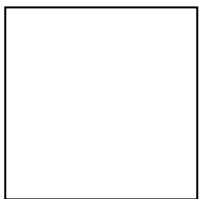
Write the ratios of the sides for the given triangles



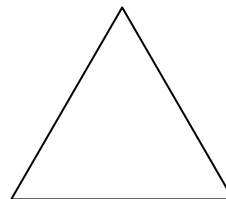
Find the missing sides for the given triangles.



9. Find the diagonal of a square whose perimeter is 44 km.



10. Find the altitude of an equilateral triangle whose perimeter is 18 mm.



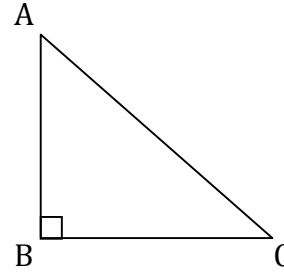
# Trigonometric Ratios

Learning Targets: Students will be able to find the trig ratios of sine, cosine, and tangent.

## KEY TERMS

Trig Ratios: **Ratio of The Sides of a Triangle**

Reference Angles: **Angle you are working with.**



### Three Basic Trig Ratios

Sine (sin) of an angle =

**opposite side / hypotenuse**

**SOH**  
1 P Y  
2 P P

**CAH**  
3 J Y  
5 J P

**TOA**  
4 P J  
5 P J

Cosine (cos) of an angle =

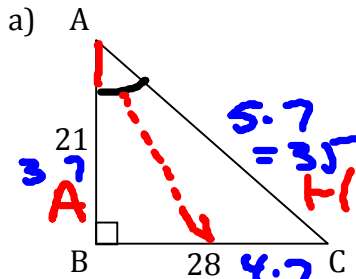
**adjacent side / hypotenuse**

**S O C A T O**  
**H H T A A**

Tangent (tan) of an angle =

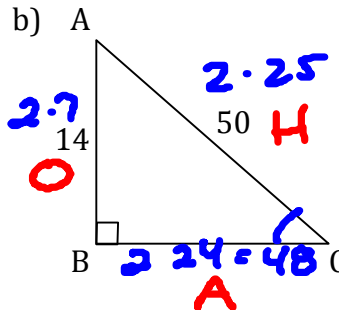
**opposite side / adjacent side**

1. For each of the following, find the given trig ratios.



$\sin A = \frac{28}{37}$        $\cos A = \frac{21}{37}$

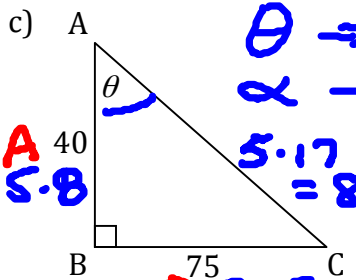
$\tan A = \frac{28}{21}$



$\sin C = \frac{14}{50}$        $\cos C = \frac{48}{50}$

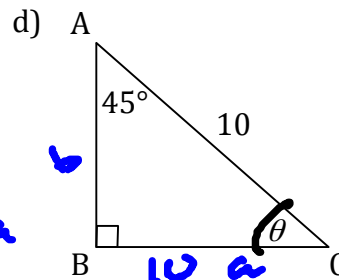
$\tan C = \frac{14}{48}$

$14^2 + A^2 = 50^2$   
 $196 + A^2 = 2500$   
 $A^2 = 2304$   
 $A = \sqrt{2304} = 48$



$\sin \theta = \frac{75}{85}$        $\cos \theta = \frac{40}{85}$

$\tan \theta = \frac{75}{40}$



$\sin \theta = \frac{a}{10} = \frac{1}{\sqrt{2}}$        $\cos \theta = \frac{b}{10} = \frac{1}{\sqrt{2}}$

$\tan \theta = \frac{a}{b} = 1$

$45 \quad 45 \quad 90$   
 $a \quad b \quad \sqrt{2}$   
 $b\sqrt{2} = 10$   
 $b = \frac{10}{\sqrt{2}}$   
 $\frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

The Sine Ratio (sin)	The Cosine Ratio (cos)	The Tangent Ratio (tan)
$\sin \theta =$	$\cos \theta =$	$\tan \theta =$

1. Match the following.

a) \_\_\_\_\_ Opposite Leg To  $\angle A$

b) \_\_\_\_\_ Sine Ratio Of  $\angle C$

c) \_\_\_\_\_ Opposite Angle To  $\overline{AB}$

d) \_\_\_\_\_ The Hypotenuse

e) \_\_\_\_\_ Adjacent Leg To  $\angle A$

f) \_\_\_\_\_ Tangent Ratio Of  $\angle C$

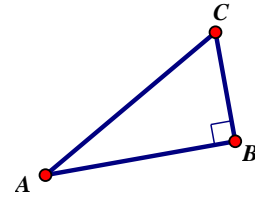
g) \_\_\_\_\_ Reference Angle Of  $\frac{BC}{AC}$  Is The Cosine Ratio

h) \_\_\_\_\_ Adjacent Leg To  $\angle C$

i) \_\_\_\_\_ Cosine Ratio Of  $\angle A$

j) \_\_\_\_\_ The Longest Side

k) \_\_\_\_\_ Reference Angle Of  $\frac{BC}{AC}$  Is The Sine Ratio



1.  $\angle A$

2.  $\angle B$

3.  $\angle C$

4.  $\overline{AB}$

5.  $\overline{BC}$

6.  $\overline{AC}$

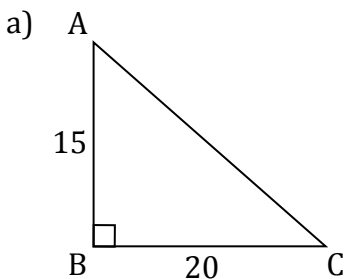
7.  $\frac{BC}{AC}$

8.  $\frac{AB}{AC}$

9.  $\frac{BC}{AB}$

10.  $\frac{AB}{BC}$

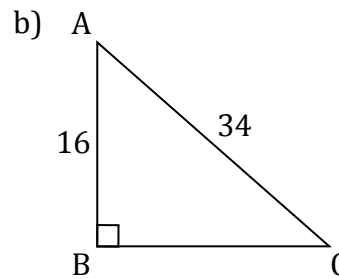
2. For each of the following, find the given trig ratios.



$\sin A =$

$\cos A =$

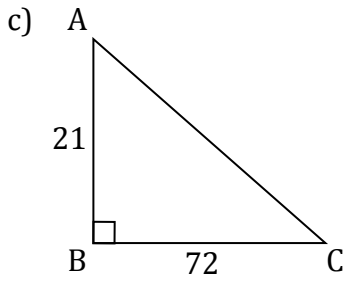
$\tan A =$



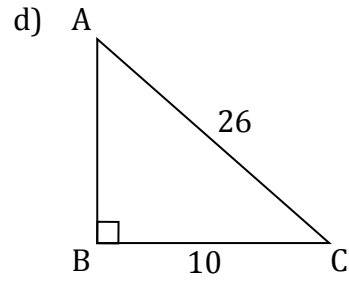
$\sin A =$

$\cos A =$

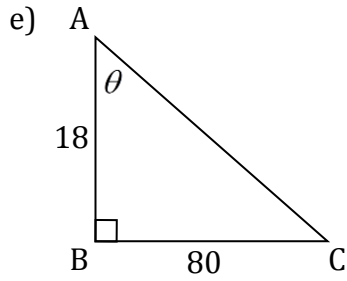
$\tan A =$



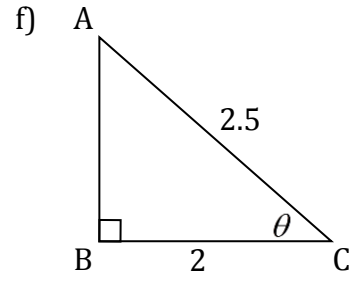
$\sin C =$        $\cos C =$        $\tan C =$



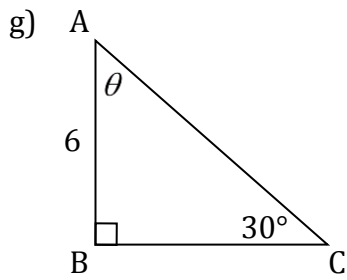
$\sin C =$        $\cos C =$        $\tan C =$



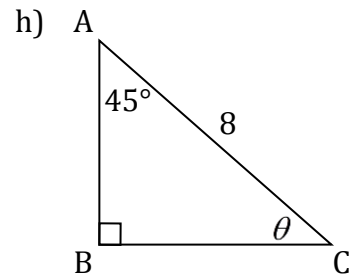
$\sin \theta =$        $\cos \theta =$        $\tan \theta =$



$\sin \theta =$        $\cos \theta =$        $\tan \theta =$



$\sin \theta =$        $\cos \theta =$        $\tan \theta =$



$\sin \theta =$        $\cos \theta =$        $\tan \theta =$

## Trig (Find Missing Sides)

Learning Targets: Students will be able to solve for missing sides of right triangles by using trig.

Use a calculator to find the value of the trigonometric expression to four decimal places.

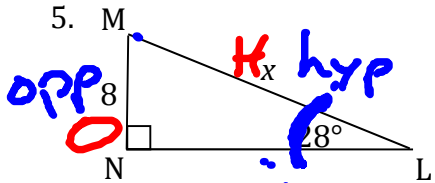
1.  $\cos 57^\circ =$  \_\_\_\_\_

2.  $\tan 39^\circ =$  \_\_\_\_\_

3.  $\sin 27^\circ =$  \_\_\_\_\_

4.  $\tan 2^\circ =$  \_\_\_\_\_

Use trigonometry to solve for each variable. Round decimals to the nearest hundredth.

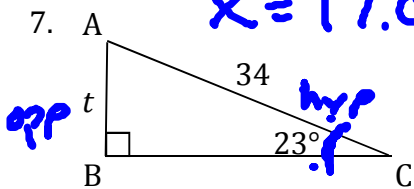


$$\frac{\sin 28^\circ}{1} = \frac{8}{x}$$

$$x \sin 28^\circ = 8$$

$$x = \frac{8}{\sin 28^\circ}$$

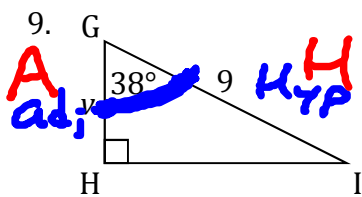
$$x = 17.04$$



$$\frac{\sin 23^\circ}{1} = \frac{t}{34}$$

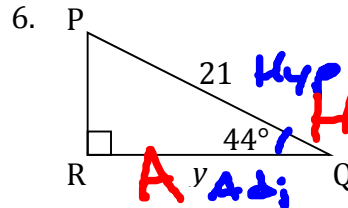
$$t = 34 \sin 23^\circ$$

$$t = 13.28$$



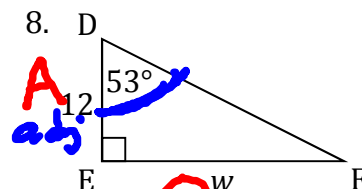
$$\frac{\cos 38^\circ}{1} = \frac{v}{9}$$

$$v = 9 \cos 38^\circ = 7.09$$



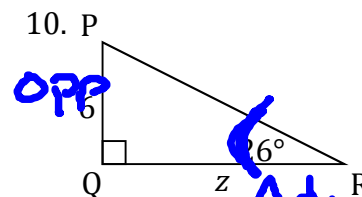
$$\frac{\cos 44^\circ}{1} = \frac{y}{21}$$

$$y = 21 \cos 44^\circ = 15.11$$



$$\frac{\tan 53^\circ}{1} = \frac{w}{12}$$

$$w = 12 \tan 53^\circ = 15.92$$



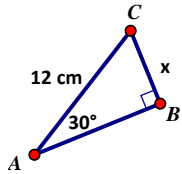
$$\frac{\tan 26^\circ}{1} = \frac{6}{z}$$

$$z \tan 26^\circ = 6$$

$$z = \frac{6}{\tan 26^\circ} = 12.30$$

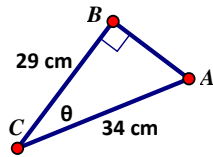
1. Choose which trigonometric ratio that you would use to solve for the missing info.

a)



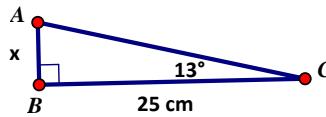
SIN COS TAN

b)



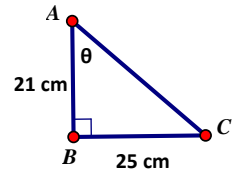
SIN COS TAN

c)



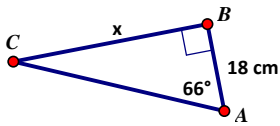
SIN COS TAN

d)



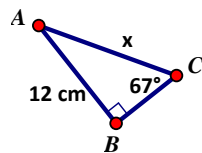
SIN COS TAN

e)



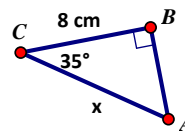
SIN COS TAN

f)



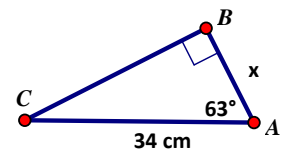
SIN COS TAN

g)



SIN COS TAN

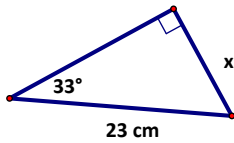
h)



SIN COS TAN

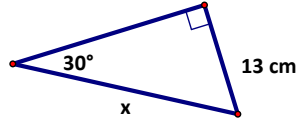
2. Solve for x (round all final answers to nearest hundredths).

a)



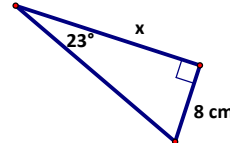
$x \approx$  \_\_\_\_\_

b)



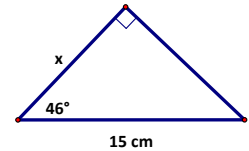
$x \approx$  \_\_\_\_\_

c)



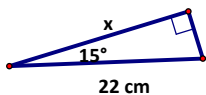
$x \approx$  \_\_\_\_\_

d)



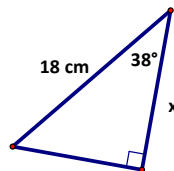
$x \approx$  \_\_\_\_\_

e)



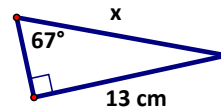
$x \approx$  \_\_\_\_\_

f)



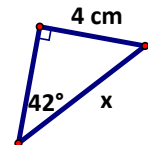
$x \approx$  \_\_\_\_\_

g)



$x \approx$  \_\_\_\_\_

h)



$x \approx$  \_\_\_\_\_

## Trig (Find Missing Angles)

Learning Targets: Students will be able to find the missing angle of a right triangle using trig.

### INVERSE TRIG FUNCTIONS:

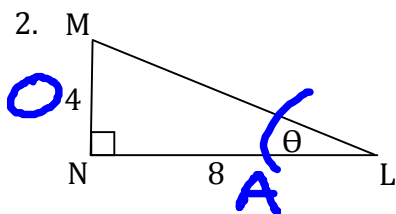
If  $\sin A = x$ , then  $\sin^{-1} x = A$  }  $\sin A = x$   
 If  $\cos A = x$ , then  $\cos^{-1} x = A$  }  $\sin^{-1}(\sin A) = \sin^{-1} x$   
 If  $\tan A = x$ , then  $\tan^{-1} x = A$  }  $\sin^{-1} x$   
 Inverse Sine  
 Angle

1. Use a calculator to find the measure of these angles to the nearest hundredth of a degree.

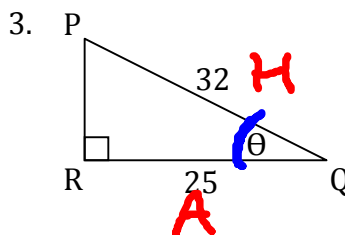
a.  $\sin A = .6947$       b.  $\cos B = .8988$       c.  $\tan C = 28.636$       d.  $\sin D = \frac{5}{13}$

$A = \sin^{-1}(.6947) = 44.00^\circ$        $B = \cos^{-1}(.8988) = 26.00^\circ$        $D = \sin^{-1}(\frac{5}{13}) = 22.62^\circ$

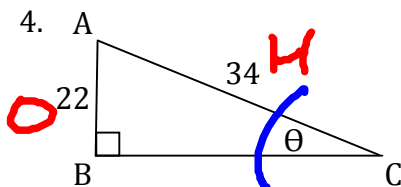
Use trigonometry to solve for  $\theta$ . Round decimals to the nearest hundredth.



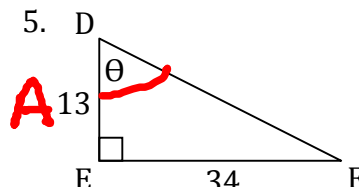
$\tan \theta = \frac{4}{8}$   
 $\theta = \tan^{-1}(\frac{4}{8})$   
 $\theta = 26.57^\circ$



$\cos \theta = \frac{25}{32}$   
 $\theta = \cos^{-1}(\frac{25}{32})$   
 $\theta = 38.62^\circ$



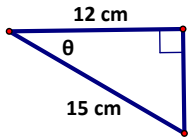
$\sin \theta = \frac{22}{34}$   
 $\theta = \sin^{-1}(\frac{22}{34})$   
 $\theta = 40.32^\circ$



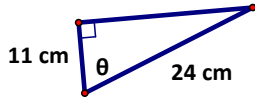
$\tan \theta = \frac{34}{13}$   
 $\theta = \tan^{-1}(\frac{34}{13})$   
 $\theta = 69.08^\circ$

1. Solve for  $\theta$  (round all final answers to nearest hundredths).

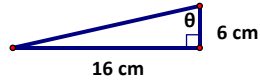
a)



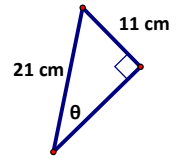
b)



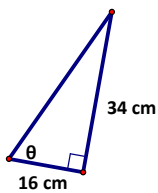
c)



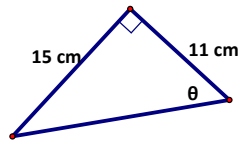
d)



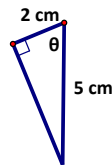
e)



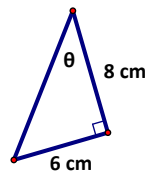
f)



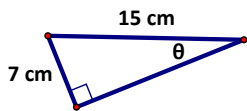
g)



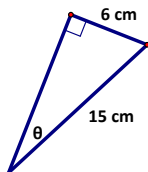
h)



i)



j)



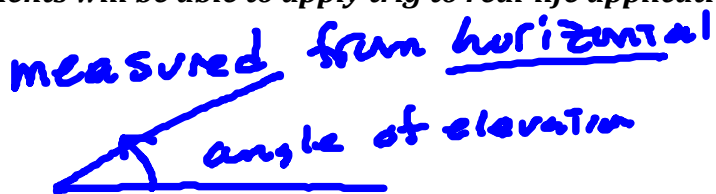


# Applications of Trig

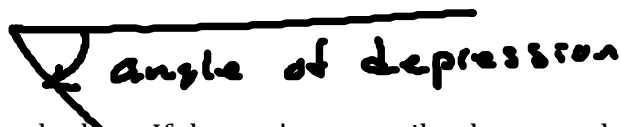
Learning Targets: Students will be able to apply trig to real-life applications.

## KEY TERMS

Angle of Elevation:



Angle of Depression:



1. A man casts a 3 feet long shadow. If the sun's rays strike the ground  $62^\circ$ , what is the height of the man?

2. A man in a lighthouse tower that is 30 feet. He spots a ship at sea at an angle of depression of  $10^\circ$ . How far is the ship from the base of the lighthouse?



$$\tan 10^\circ = \frac{30}{f}$$

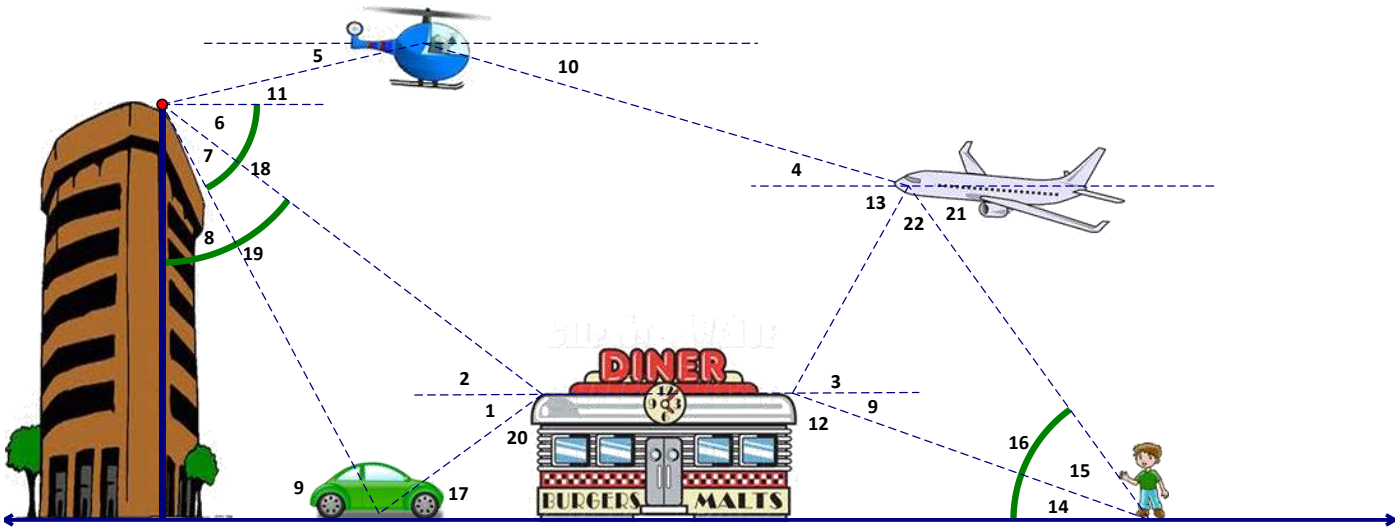
$$f \tan 10^\circ = 30$$

3. Two trees are 100 meters apart. From the exact middle between them, the angles of elevation of their tops are  $12^\circ$  and  $16^\circ$ . How much taller is one tree than the other?

$$f = \frac{30}{\tan 10} \approx 170.14 \text{ ft}$$



1. Choose the correct angle number for the provided description.



- a) The angle of elevation from the **CAR** to the top of the **DINER** is 17.
- b) The angle of depression from the top of the **TALL BUILDING** to the **DINER** is \_\_\_\_\_.
- c) The angle of elevation from the **PLANE** to the **HELICOPTER** is \_\_\_\_\_.
- d) The angle of depression from the top of the **DINER** to the **BOY** is \_\_\_\_\_.
- e) The angle of depression from the **HELICOPTER** to the **PLANE** is \_\_\_\_\_.
- f) The angle of depression from the **PLANE** to the top of the **DINER** is \_\_\_\_\_.
- g) The angle of elevation from the **BOY** to the top of the **DINER** is \_\_\_\_\_.
- h) The angle of depression from the top of the **TALL BUILDING** to the top of the **CAR** is \_\_\_\_\_.
- i) The angle of depression from the **HELICOPTER** to the top of the **TALL BUILDING** is \_\_\_\_\_.
- j) The angle of elevation from the top of the **DINER** to the top of the **TALL BUILDING** is \_\_\_\_\_.
- k) The angle of elevation from the top of the **DINER** to the **PLANE** is \_\_\_\_\_.
- l) The angle of depression from the top of the **DINER** to the **CAR** is \_\_\_\_\_.
- m) The angle of elevation from the **BOY** to the front of the **PLANE** is \_\_\_\_\_.
- n) The angle of depression from the front of the **PLANE** to the **BOY** is \_\_\_\_\_.
- o) the angle of elevation from the **TALL BUILDING** to the **HELICOPTER** is \_\_\_\_\_.

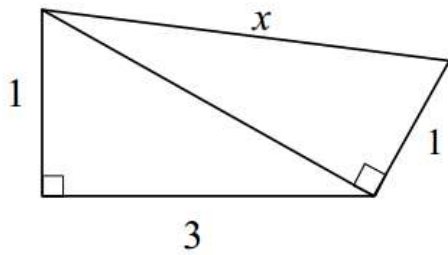
**For each problem, draw a diagram and then solve for the requested information. Round to the nearest hundredth, unless otherwise instructed.**

- Sharon is flying a kite on a string 130 meters long. Determine the height of the kite if the string is at an angle of  $37^\circ$  to the ground.
- An airplane is flying at an altitude of 6000 meters over the ocean directly toward an island. When the angle of depression of the coastline from the airplane is  $14^\circ$ , how much farther does the airplane have to fly before it crosses the coast?
- A loading ramp is 25 meters long with a height of 10 meters. What is the horizontal distance of the ramp and what is the angle of incline that the ramp forms with the ground?
- A telephone pole casts a shadow 18 meters long when the sun's rays strike the ground at an angle of  $70^\circ$ . How tall is the pole?
- John looks out the attic window of his home, which is 22 feet above the ground. At an angle of elevation of  $35^\circ$  he sees a bird sitting at the very top of the large high rise apartment building down the street. How tall is the high rise apartment building, if the two buildings are 75 feet apart?

**Unit 7 Review Part 1**

**Pythagorean Theorem**

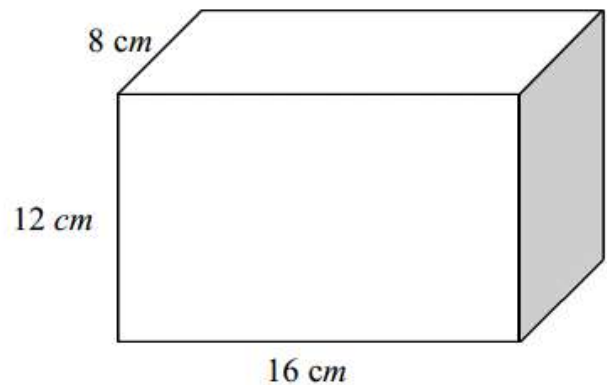
1. Solve for  $x$ .



2. A 13-foot ladder is placed 5 feet away from the base of a wall. The distance from the ground straight up to the top of the wall is 13 feet. How far up the wall will the ladder reach?

3. A pencil box, pictured to the right, measures 16 cm by 12 cm by 8 cm. Find the length of the longest pencil that could fit inside the box.

[Hint: Diagonally inside the box is possible]

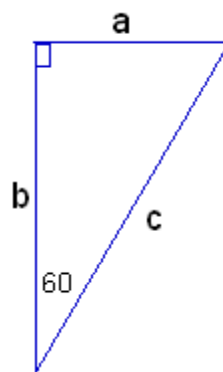


4. An isosceles triangle has congruent sides of 20 cm. The base is 10 cm. Find the height (altitude) of the triangle.

### Special Right Triangles

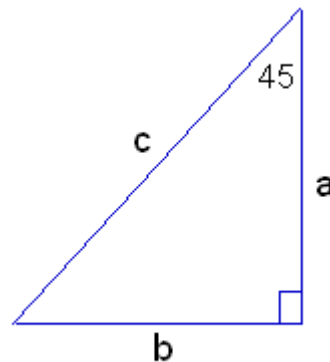
Use the figure to the right to answer the questions #5-7. Each question is separate.

5. If  $a=5$ , then the exact value of  $c$  is...
6. If  $a=2\sqrt{3}$ , then the exact value of  $c$  is...
7. If  $b=7$ , then the value  $a$  to the nearest tenth is...

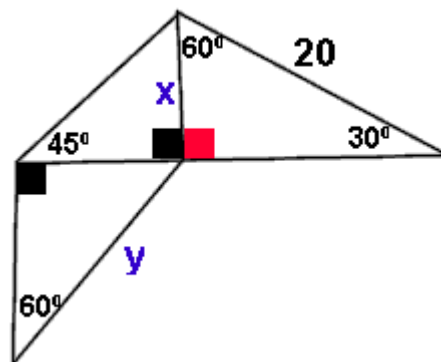


Use the figure to the right to answer the questions # 8-10.

8. If  $a=5$ , then the exact value of  $c$  is...
9. If  $a=2\sqrt{3}$ , then the exact value of  $c$  is...
10. If  $c=7$ , then find the value of  $a$ . Keep your answer in simplest radical form.

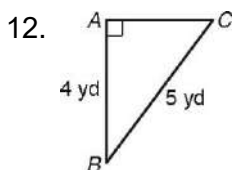


11. Use the figure to the right to solve for  $x$  and  $y$ .



### Trigonometry

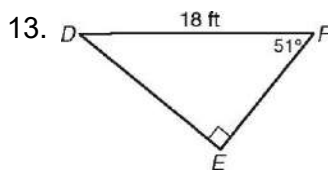
Use a calculator and inverse trigonometric ratios to find the unknown side lengths and angle measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.



$AC =$  \_\_\_\_\_

$m\angle B =$  \_\_\_\_\_

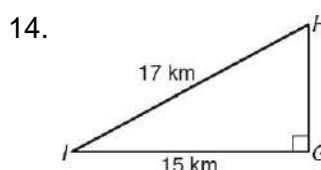
$m\angle C =$  \_\_\_\_\_



$DE =$  \_\_\_\_\_

$EF =$  \_\_\_\_\_

$m\angle D =$  \_\_\_\_\_

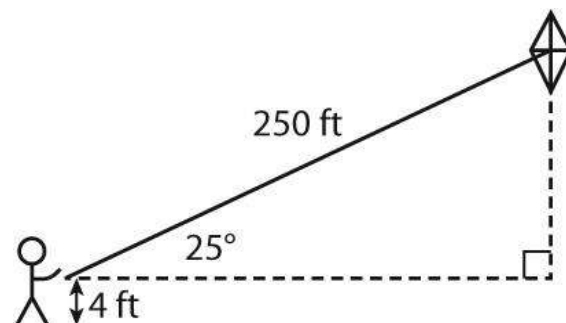


$GH =$  \_\_\_\_\_

$m\angle H =$  \_\_\_\_\_

$m\angle I =$  \_\_\_\_\_

15. Raul is flying a kite as shown. How high is the kite off the ground? Round your answer to the nearest tenth. Explain your reasoning.



Use the figure to the right for 16-17.

16. Explain how cosine can be used to determine  $m\angle F$ . Then find  $m\angle F$  to the nearest tenth, using cosine.

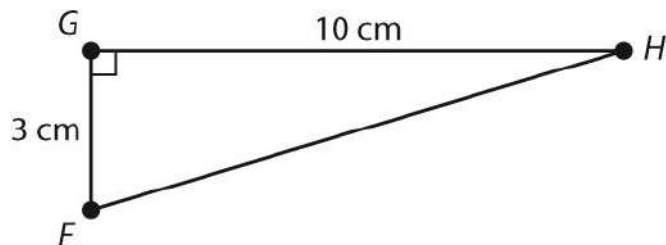
\_\_\_\_\_

\_\_\_\_\_

17. What is the perimeter of  $\triangle FGH$  to the nearest tenth of a centimeter?

\_\_\_\_\_

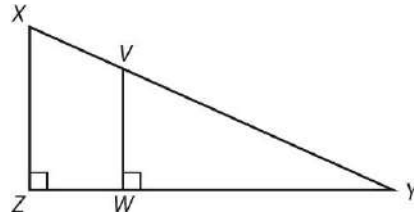
\_\_\_\_\_



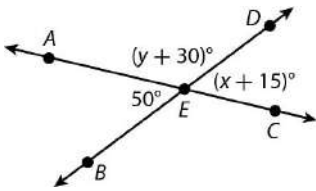
Use the figure to answer Problems # 18-19.

18.  $\sin \angle YXZ =$

19.  $\sin \angle YVW =$



Use the figures for Problems 20-21.



20. supplement of  $\angle AEB$

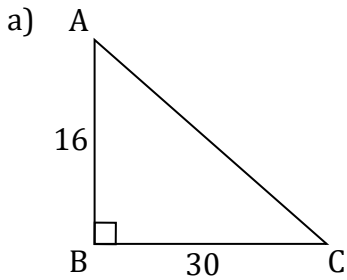
21. complement of  $\angle AEB$

22.. The slope, or grade, of a road or a ramp can be given as a percent. The grade of a treadmill ramp is 7%, which means that it would rise 7 inches over a horizontal distance of 100 inches. If the length of the ramp itself is 53 inches, to the nearest 0.1 inch, how many inches does it rise vertically? Show your work. \_\_\_\_\_

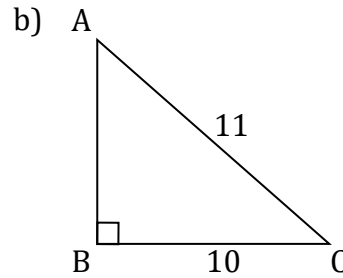
23. A wheelchair ramp has a slope of 1:12 (1 foot of rise over a horizontal distance of 12 feet). To the nearest 0.1 foot, how many feet of ramp will be needed to rise 3 feet? (Round the angle of incline to the nearest 0.01°.) Show your work. \_\_\_\_\_

24. The hypotenuse of a right triangle measures 9 inches, and one of the acute angles measures 36°. To the nearest square inch, what is the area of the triangle? Show your work. \_\_\_\_\_

1. For each of the following, find the given trig ratios.



$$\sin A = \quad \cos A = \quad \tan A =$$



$$\sin C = \quad \cos C = \quad \tan C =$$

**Solve the given right triangle. Round decimals to the nearest hundredth.**

2.  $\triangle ABC$  is a right triangle. Draw a diagram to represent the given information.

**GIVEN:**

$$\angle A = \underline{\hspace{2cm}}$$

$$\angle B = 90^\circ$$

$$\angle C = 21^\circ$$

$$a = \underline{\hspace{2cm}}$$

$$b = 18 \text{ in}$$

$$c = \underline{\hspace{2cm}}$$

**For each problem, draw a diagram and then solve for the requested information. Round to the nearest hundredth, unless otherwise instructed.**

3. A helicopter is hovering over a landing pad 100 meters from where you are standing. The helicopter's angle of elevation with the ground is  $15^\circ$ . What is the altitude of the helicopter?



4. A lighthouse operator sights a sailboat at an angle of depression of  $25^\circ$ . If the lighthouse is 40 feet tall, how far is the boat from the base of the lighthouse?
5. A 15 meter pole is leaning against a wall. The foot of the pole is 10 meters from the wall. Find the angle that the pole makes with the ground.
6. A blue bird sitting in its nest at the top of a tree spots a large red apple in Janice's hands at an angle of depression of  $23^\circ$ . If the nest is 20 feet off the ground and the apple is 4 feet off the ground, how far is Janice from the tree?

### Answers

$$1a. \sin A = \frac{15}{17} \quad \cos A = \frac{8}{17} \quad \tan A = \frac{15}{8}$$

$$1b. \sin C = \frac{\sqrt{21}}{11} \quad \cos C = \frac{10}{11} \quad \tan C = \frac{\sqrt{21}}{10}$$

$$\angle A = 69^\circ$$

$$2. a = 16.80 \text{ in} \\ c = 6.45 \text{ in}$$

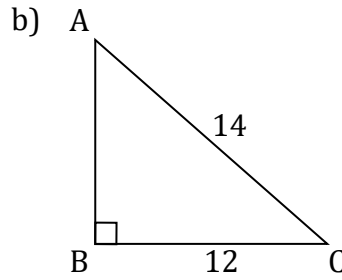
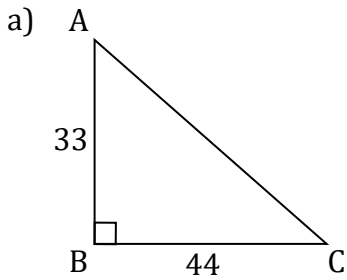
$$3. 26.79 \text{ m}$$

$$4. 85.78 \text{ ft}$$

$$5. 48.19^\circ$$

$$6. 37.69 \text{ ft}$$

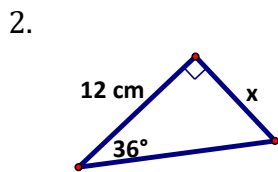
1. For each of the following, find the given trig ratios.



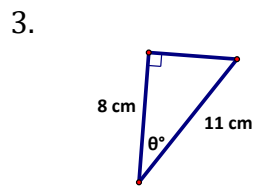
$\sin A =$        $\cos A =$        $\tan A =$

$\sin C =$        $\cos C =$        $\tan C =$

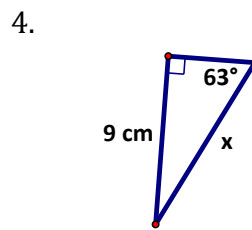
Solve for the missing information (round all final answers to nearest hundredths).



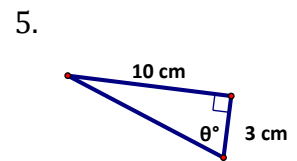
$x \approx$  \_\_\_\_\_



$\theta =$  \_\_\_\_\_

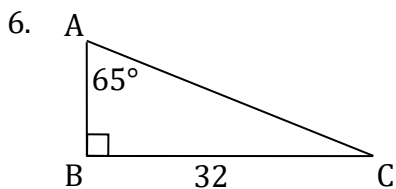


$x \approx$  \_\_\_\_\_



$\theta =$  \_\_\_\_\_

Solve the given right triangle. Round decimals to the nearest hundredth.



Solve the given right triangle. Round decimals to the nearest hundredth.

7.  $\triangle ABC$  is a right triangle. Draw a diagram to represent the given information.

**GIVEN:**

$\angle A = \underline{\hspace{2cm}}$

$\angle B = 90^\circ$

$\angle C = \underline{\hspace{2cm}}$

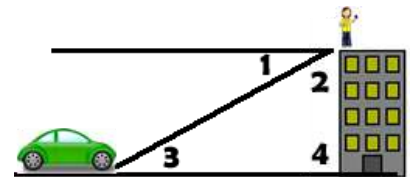
$a = 7 \text{ in}$

$b = \underline{\hspace{2cm}}$

$c = 10 \text{ in}$

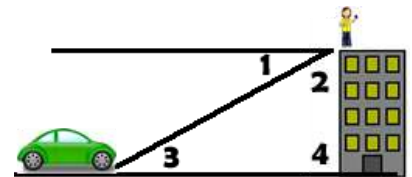
8. The angle of depression from the girl to the car is:

- A)  $\angle 1$       B)  $\angle 2$       C)  $\angle 3$       D)  $\angle 4$



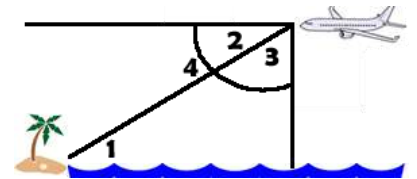
9. The angle of elevation from the car to the girl is:

- A)  $\angle 1$       B)  $\angle 2$       C)  $\angle 3$       D)  $\angle 4$



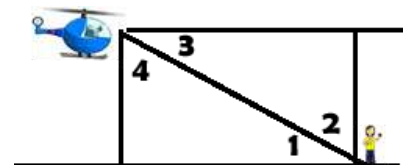
10. The angle of depression from the airplane to the island is:

- A)  $\angle 1$       B)  $\angle 2$       C)  $\angle 3$       D)  $\angle 4$



11. The angle of elevation from the girl to the helicopter is:

- A)  $\angle 1$       B)  $\angle 2$       C)  $\angle 3$       D)  $\angle 4$



For each problem, draw a diagram and then solve for the requested information. Round to the nearest hundredth, unless otherwise instructed.

12. Tommy has caught his kite at the top of a 16 foot tree. From where Tommy is standing the elevation to the top of the tree is  $29^\circ$ , what is the length of string (round to the nearest foot)?

13. A plane at an altitude of 7000 feet is flying in the direction of an island. If angle of depression is  $21^\circ$  from the plane to the island, what is the horizontal distance until the plane flies over the island?
14. An 18 foot tree casts a 15 foot long shadow. What is the angle formed by the sun's rays and the ground?
15. A ladder reaches a window 12 feet above the ground and the foot of the ladder is 4.8 feet from the wall. How long is the ladder?
16. The house is 30 feet tall and the building is 120 feet tall. If the distance between them is 100 feet, what is the angle of elevation from the top of the house to the top of the building (round to the nearest degree)?

# Unit 8

## Circles

# Circles and Tangents

Learning Targets: Students will be able to identify segments and lines related to circles.

## KEY TERMS

Circle: The set of all points equidistant from a center point

Radius: The distance from the center to the circle

Diameter: The distance between two points through the diameter  $d = 2r$

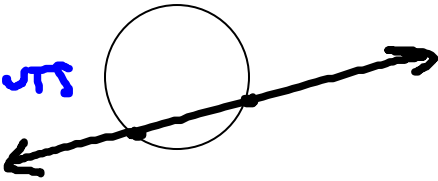
Chord: A segment between two points on the circle (diameter is the longest chord)

Interior: all the points contained in the circle

Exterior: all the points not contained by the circle

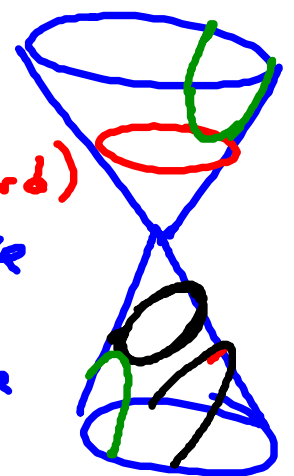
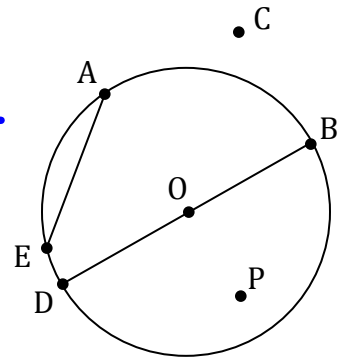
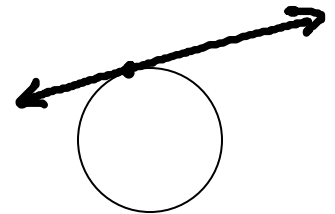
Secant: a line intersecting the circle at 2 points

(creates a chord)



Tangent:

a line that intersects the circle at 1 point



1. Give the most specific name for each of the following.

a)  $\overline{CG}$

radius

b)  $\overline{BE}$

chord

c)  $\overline{AD}$

tangent

d) C

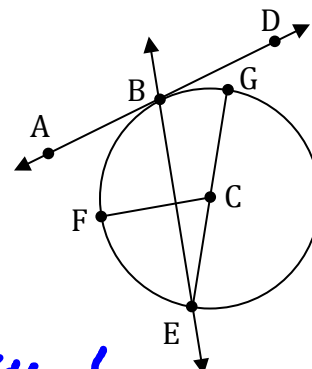
center  
circle C  
1

e)  $\overline{BE}$

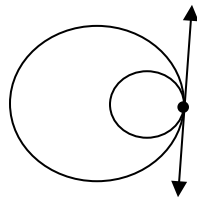
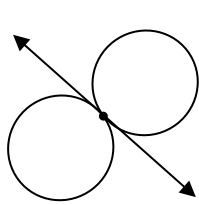
secant

f) B

point of tangency (tangent point)

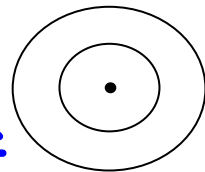


Coplanar circles that intersect at **one point** are called \_\_\_\_\_.

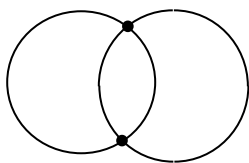


**Tangent circles**

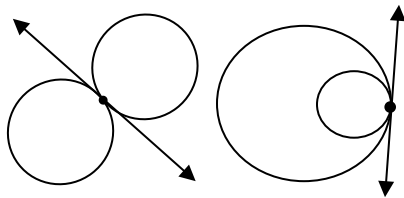
Coplanar circles that have a **common center** are called **Concentric**



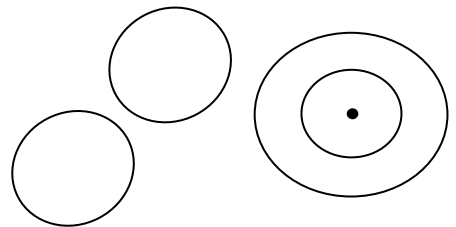
**2 points of intersection**



**1 point of intersection**



**No points of intersection**



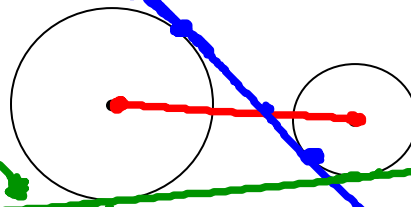
A line or segment that is tangent to two coplanar circles is called a **Common Tangent**

**Common internal tangent:**

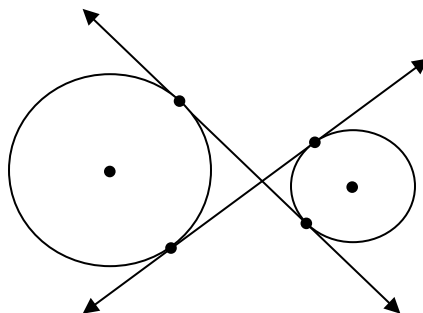
**Will intersect the segment connecting the two centers**

**Common external tangent:**

**Will NOT intersect the segment connecting the two centers**



2. Tell whether the common tangents are internal or external.



**GEOMETRY**

**Circles and Tangents HW**

Name: \_\_\_\_\_

Date: \_\_\_\_\_ Period: \_\_\_\_\_

**The diameter of a circle is given. Find the radius.**

1.  $d = 6$  in.

2.  $d = 24$  cm

3.  $d = 15$  ft

4.  $d = 9$  in.

**The radius of a circle is given. Find the diameter.**

5.  $r = 11$  cm

6.  $r = 8$  ft

7.  $r = 10$  in.

8.  $r = 4.6$  cm

**Give the most specific name for each of the following.**

9.  $U$

10.  $Z$

11.  $\overline{UW}$

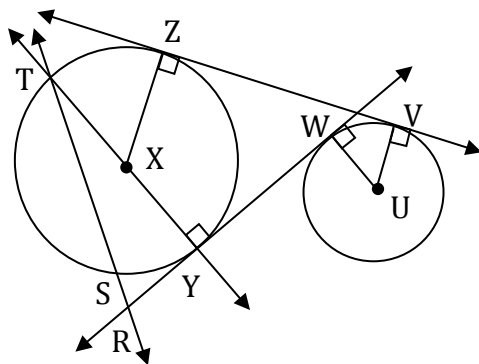
12.  $\overline{TS}$

13.  $\overline{YW}$

14.  $\overline{TS}$

15.  $\overline{ZV}$

16.  $\overline{TY}$



9. \_\_\_\_\_

10. \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

14. \_\_\_\_\_

15. \_\_\_\_\_

16. \_\_\_\_\_

**Give the most specific name for each of the following.**

17.  $D$

18.  $\overline{FH}$

19.  $\overline{CD}$

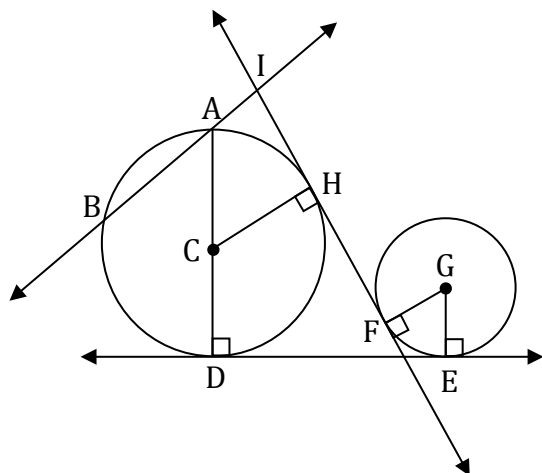
20.  $\overline{AB}$

21.  $C$

22.  $\overline{AD}$

23.  $\overline{AB}$

24.  $\overline{DE}$



17. \_\_\_\_\_

18. \_\_\_\_\_

19. \_\_\_\_\_

20. \_\_\_\_\_

21. \_\_\_\_\_

22. \_\_\_\_\_

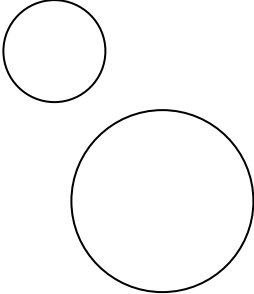
23. \_\_\_\_\_

24. \_\_\_\_\_

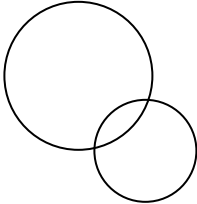


Tell how many common tangents the circles have. Then sketch the tangents.

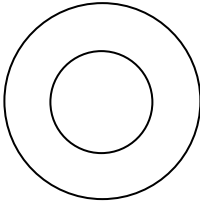
25.



26.



27.



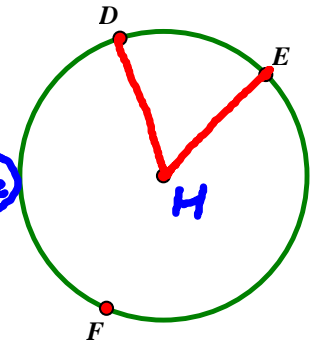
# Arcs and Central Angles

Learning Targets: Students will be able to classify arcs of circles and find their measure.

## KEY TERMS

Central Angle:

$0^\circ < \theta < 360^\circ$   
 An angle whose vertex is the center of the circle and intersects the circle at 2 points (contains an arc)



Arc:

The part of the circle between two points on the circle.  $\widehat{DE}$

Minor Arc:

→ an arc  $0^\circ < \theta < 180^\circ$  named with 2 points

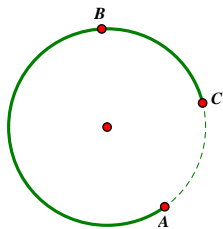
Major Arc:

→ an arc  $> 180^\circ$  ( $180^\circ < \theta < 360^\circ$ ) name w/ 3 letters

Semicircle:

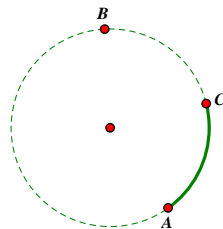
→ an arc =  $180^\circ$  generally named w/ 2 letters  
 USE 3 letters for specificity  $\widehat{DFE}$

### Major Arc



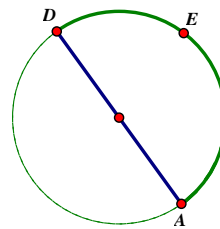
$\widehat{ABC}$   
 OR  $\widehat{CBA}$

### Minor Arc



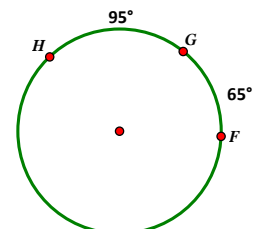
$\widehat{AC}$   
 OR  $\widehat{CA}$

### Semi-Circle



$\widehat{DEA}$   
 OR  $\widehat{AED}$

### Arc Addition



$\widehat{HG} + \widehat{GF} = \widehat{HF}$   
 $95^\circ + 65^\circ = 160^\circ$

1. Determine the arc measure.

$m\widehat{CE} = 53 + 34 = 87^\circ$

$m\widehat{EF} = 180 - 87 = 93^\circ$

$m\angle CAK = 60^\circ + 60^\circ = 120^\circ$

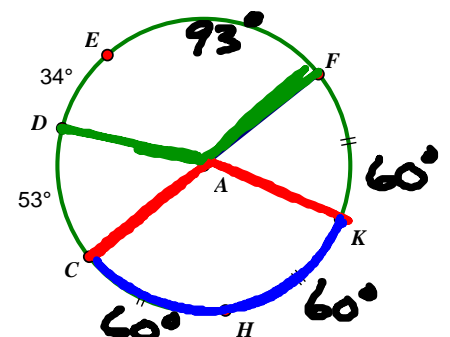
$m\widehat{ECK} = 34 + 53 + 120 = 207^\circ$

$m\angle DAF = 34 + 93 = 127^\circ$

$m\widehat{DFC} = 360 - 53 = 307^\circ$

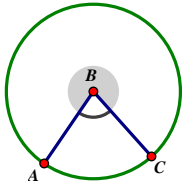
$\widehat{CE}$  ← the arc itself  
 $m\widehat{CE}$  ← measurement of the arc (in degrees)

Central Angle equals the intercepted Arc (the arc inside it)

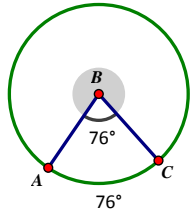


$m\widehat{ECF} = 360 - 93 = 267^\circ$

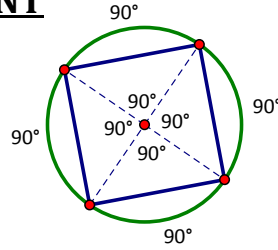
# CENTRAL ANGLE VS ARC MEASUREMENT



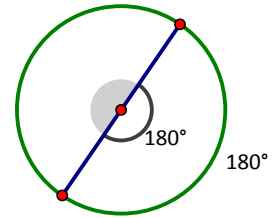
$\angle ABC$  is a central angle.



Central angle is equal to the intercepted arc measure.



The square has four congruent arcs and four congruent central angles.



A diameter has a central angle of  $180^\circ$ , thus the arc is also  $180^\circ$ .

2. Determine the missing information. Given circle B with  $\overline{EC}$  as a diameter.

$m\widehat{AC} =$  \_\_\_\_\_

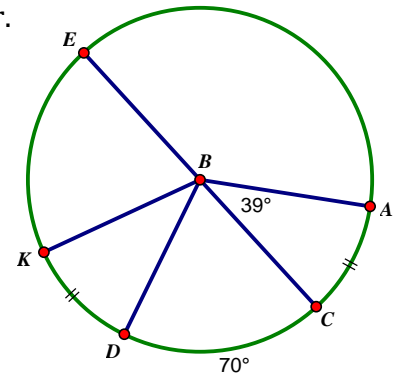
$m\angle ABK =$  \_\_\_\_\_

$m\widehat{AE} =$  \_\_\_\_\_

$m\angle KBD =$  \_\_\_\_\_

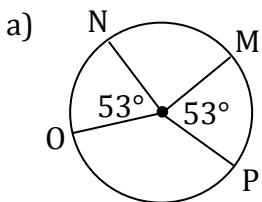
$m\widehat{EK} =$  \_\_\_\_\_

$m\angle BDC =$  \_\_\_\_\_

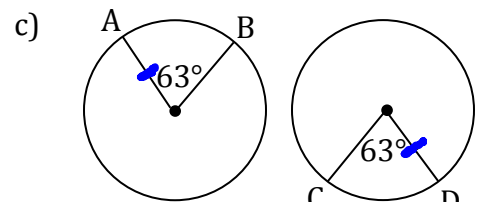
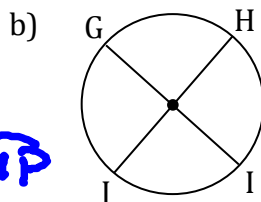


Two arcs of the same circle or of congruent circles are congruent arcs if they have the same measures. So, two minor arcs of the same circle or of congruent circles are congruent if their central angles are congruent.

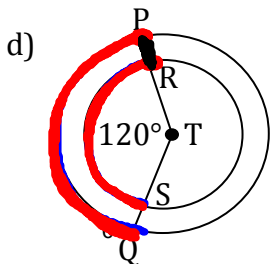
3. Name the congruent arcs if there are any.



$\widehat{NO} \cong \widehat{MP}$   
 since they both measure  $53^\circ$



$\widehat{AB} \cong \widehat{CD}$



$m\widehat{PQ} = m\widehat{RS}$   
 $120^\circ = 120^\circ$   
 But  $\widehat{PQ} \not\cong \widehat{RS}$

The circles are not congruent

**GEOMETRY**

**Arcs and Central Angles HW**

Name: \_\_\_\_\_

Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Determine the arc measure.

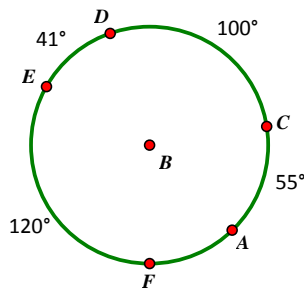
a)

$m\widehat{DF} =$  \_\_\_\_\_

$m\widehat{ECA} =$  \_\_\_\_\_

$m\widehat{AF} =$  \_\_\_\_\_

$m\widehat{CFD} =$  \_\_\_\_\_



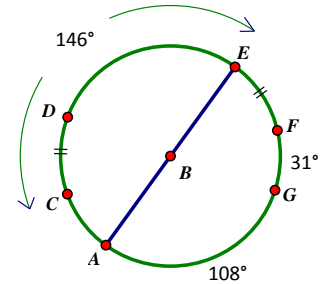
b)

$m\widehat{AC} =$  \_\_\_\_\_

$m\widehat{DAG} =$  \_\_\_\_\_

$m\widehat{AD} =$  \_\_\_\_\_

$m\widehat{DAF} =$  \_\_\_\_\_



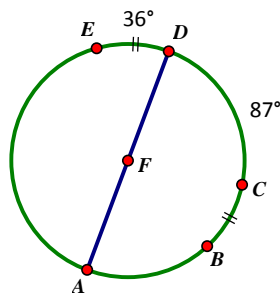
c)

$m\widehat{AE} =$  \_\_\_\_\_

$m\widehat{AB} =$  \_\_\_\_\_

$m\widehat{CDB} =$  \_\_\_\_\_

$m\widehat{BD} =$  \_\_\_\_\_



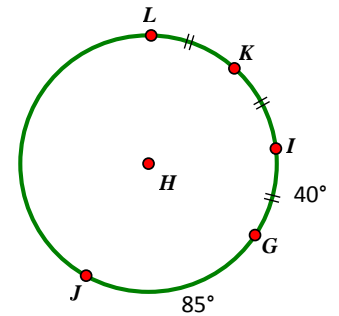
d)

$m\widehat{LJ} =$  \_\_\_\_\_

$m\widehat{KJ} =$  \_\_\_\_\_

$m\widehat{GJK} =$  \_\_\_\_\_

$m\widehat{KLI} =$  \_\_\_\_\_



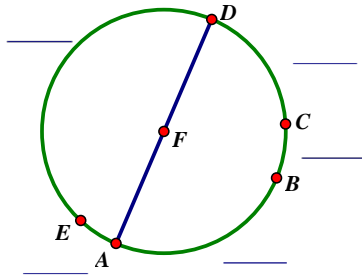
2. Determine the measure of the missing arcs on the circle.

a) Circle F

$m\widehat{AC} = 117^\circ$

$m\widehat{BE} = 111^\circ$

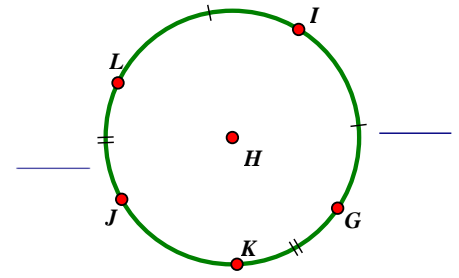
$m\widehat{BD} = 91^\circ$



b) Circle H

$m\widehat{LK} = 302^\circ$

$m\widehat{LG} = 168^\circ$



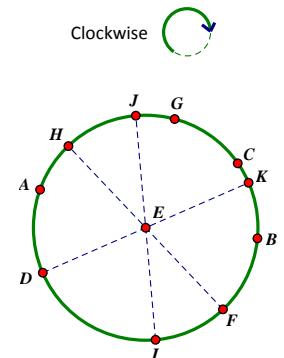
3. From the given diagram, determine whether the arcs are Major, Minor or Semi-Circle. To describe the arc without giving it way through notation we will refer to clockwise and counterclockwise (counter cw).

a) F to G, clockwise Major Minor Semi    b) A to F, clockwise Major Minor Semi

c) J to C, clockwise Major Minor Semi    d) K to D, clockwise Major Minor Semi

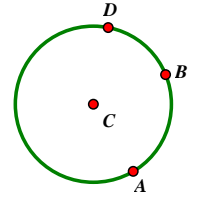
e) D to I, counter cw Major Minor Semi    f) C to A, counter cw Major Minor Semi

g) F to J, clockwise Major Minor Semi    h) G to I, counter cw Major Minor Semi



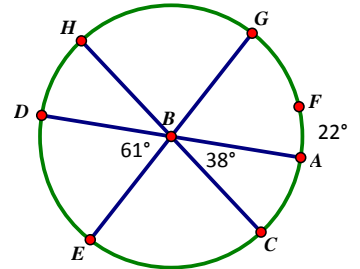
4. The teacher asks a student to write the name for the arc from A to B on the board.

Jackie comes up writes  $m\widehat{AB}$  or  $m\widehat{BA}$  Jeff raises his hand and says that he has a different answer. What might his answer be if it is different than Jackie's?



5. Given Circle B with diameters  $\overline{HC}$ ,  $\overline{EG}$ , and  $\overline{DA}$ .

- a)  $m\angle DBH =$  \_\_\_\_\_      b)  $m\widehat{DCE} =$  \_\_\_\_\_  
 c)  $m\widehat{HG} =$  \_\_\_\_\_      d)  $m\widehat{HCF} =$  \_\_\_\_\_  
 e)  $m\angle HBA =$  \_\_\_\_\_      f)  $m\angle DBA =$  \_\_\_\_\_

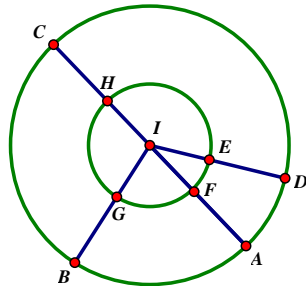


6. Given circle M,  $m\widehat{HG} = m\widehat{HTG}$ . How could this be?

7. Determine the missing information.

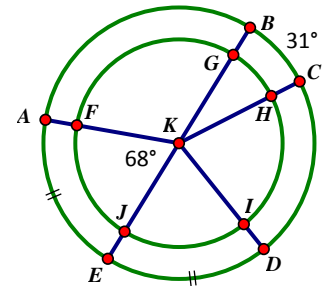
a) Given concentric circles with  $m\widehat{GF} = 76^\circ$ ,  $m\angle HIE = 147^\circ$  and  $\overline{CA}$  &  $\overline{FH}$  are diameters.

- $m\widehat{CB} =$  \_\_\_\_\_  
 $m\widehat{HE} =$  \_\_\_\_\_  
 $m\widehat{BDC} =$  \_\_\_\_\_  
 $m\angle CIB =$  \_\_\_\_\_



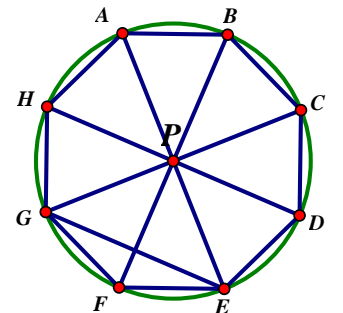
b) Given concentric circles with  $m\widehat{BC} = 31^\circ$ ,  $m\angle FKJ = 68^\circ$  and  $\overline{EB}$  is a diameter.

- $m\widehat{ED} =$  \_\_\_\_\_  
 $m\angle GKH =$  \_\_\_\_\_  
 $m\widehat{ABD} =$  \_\_\_\_\_  
 $m\angle AKB =$  \_\_\_\_\_



8. Given a regular octagon. Determine the missing information.

- a)  $m\angle APB =$  \_\_\_\_\_      b)  $m\angle HPF =$  \_\_\_\_\_  
 c)  $m\widehat{AE} =$  \_\_\_\_\_      d)  $m\widehat{GEA} =$  \_\_\_\_\_  
 e)  $m\angle GPF =$  \_\_\_\_\_      f)  $m\angle PAH =$  \_\_\_\_\_  
 g)  $m\angle PGE =$  \_\_\_\_\_      h) If  $HD = 12$  cm, what is  $GE =$  \_\_\_\_\_



9. Points A, B, C, D, and E are placed on circle R in this order such that there are five congruent arcs. What is the  $m\widehat{BCE}$ ?

# Inscribed Angles

Learning Targets: Students will be able to apply properties of inscribed angles to solve problems.

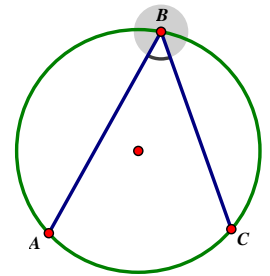
## KEY TERMS

Inscribed Angle: The vertices are on the circle  $\angle ABC$   
 $\rightarrow$  The vertex is on the circle

Intercepted Arc: The arc inside the angle on the circle  $\widehat{AC}$

## THEOREM

If an angle is inscribed in a circle, then its measure is  
 $\frac{1}{2}$  the measure of the intercepted arc  
 $m\angle ABC = \frac{1}{2} m\widehat{AC}$

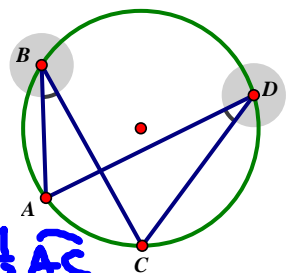


## THEOREM

If two inscribed angles of a circle intercept the same arc, then the two  $\angle$ 's are  $\cong$ .

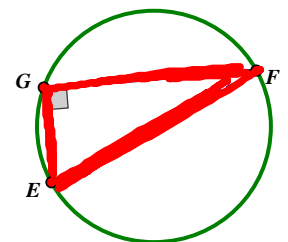
$\angle ABC \cong \angle ADC$        $\angle BAD \cong \angle BCD$

$\left\{ \begin{array}{l} \angle ABC = \frac{1}{2} \widehat{AC} \\ \angle ADC = \frac{1}{2} \widehat{AC} \end{array} \right.$



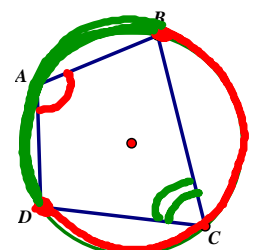
## THEOREM

If one side of an inscribed triangle is a diameter, then the angle opposite it is a right  $\angle$ .



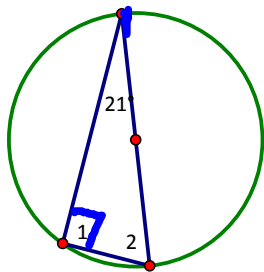
## THEOREM

If a quadrilateral is inscribed in a circle, then the opposite angles are Supplementary.



1. Determine the requested value(s).

a)



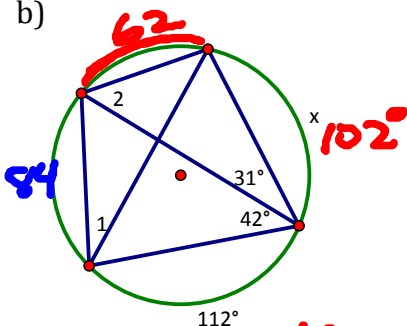
$$m\angle 1 = \underline{90^\circ}$$

$$m\angle 2 = \underline{69^\circ}$$

$$\angle 2 = 90 - 21$$

$$\angle 2 + 21 = 90$$

b)



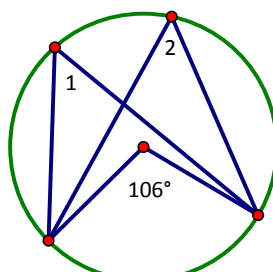
$$m\angle 1 = \underline{31^\circ}$$

$$m\angle 2 = \underline{51^\circ}$$

$$m\hat{x} = \underline{102^\circ}$$

$$\begin{array}{r} 62 \\ 184 \\ \hline 258 \\ 360 \\ -258 \\ \hline 102 \end{array}$$

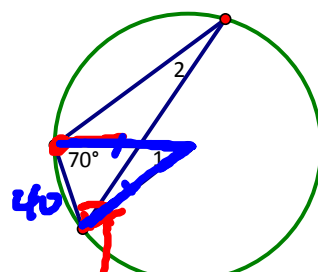
c)



$$m\angle 1 = \underline{53}$$

$$m\angle 2 = \underline{53}$$

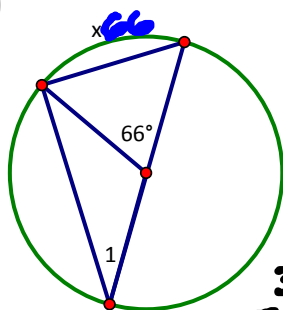
d)



$$m\angle 1 = \underline{40^\circ}$$

$$m\angle 2 = \underline{20^\circ}$$

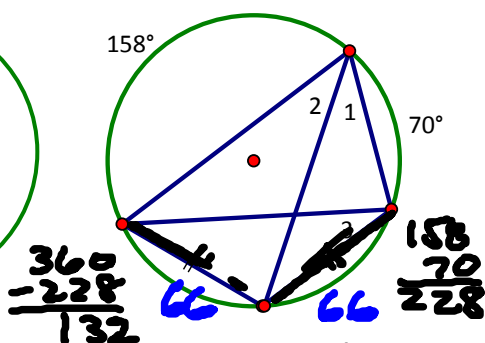
e)



$$m\angle 1 = \underline{33^\circ}$$

$$m\hat{x} = \underline{66^\circ}$$

f)



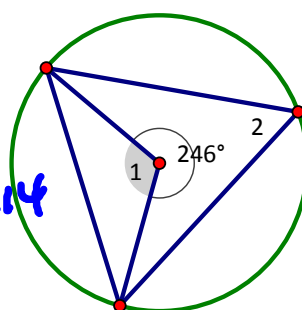
$$m\angle 1 = \underline{33^\circ}$$

$$m\angle 2 = \underline{33^\circ}$$

$$m\angle 3 = \underline{33^\circ}$$

$$\begin{array}{r} 158 \\ 70 \\ \hline 228 \\ 360 \\ -228 \\ \hline 132 \\ 66 \end{array}$$

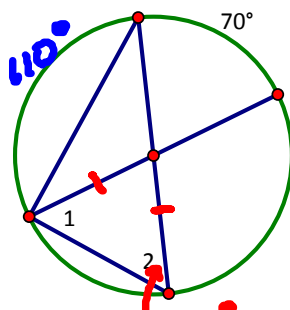
g)



$$m\angle 1 = \underline{114^\circ}$$

$$m\angle 2 = \underline{57^\circ}$$

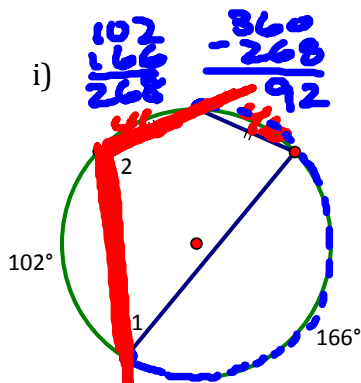
h)



$$m\angle 1 = \underline{55^\circ}$$

$$m\angle 2 = \underline{55^\circ} \quad \angle 2 = \frac{110}{2}$$

i)



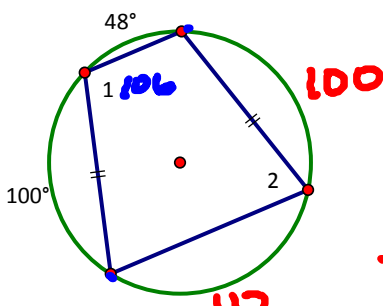
$$m\angle 1 = \frac{1}{2} 92 = 46$$

$$m\angle 2 = \underline{106}$$

$$\frac{1}{2}(166 + 46)$$

$$\frac{1}{2}(212)$$

j)



$$m\angle 1 = \frac{1}{2}(100 + 112) = 106$$

$$m\angle 2 = \underline{74}$$

$$\begin{array}{r} 360 \\ -248 \\ \hline 112 \end{array}$$

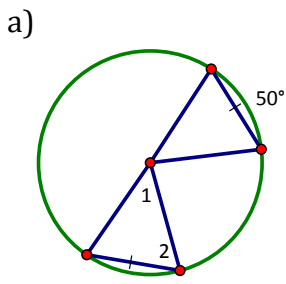
EQUAL Chords  
Create equal Arc

**GEOMETRY**  
**Inscribed Angles HW**

Name: \_\_\_\_\_

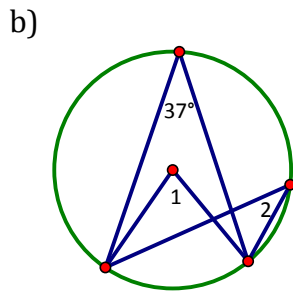
Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Determine the requested value(s).



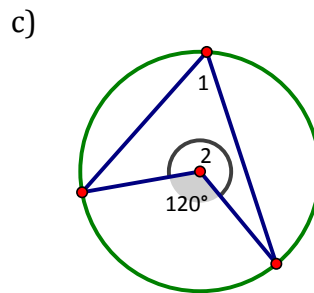
$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$



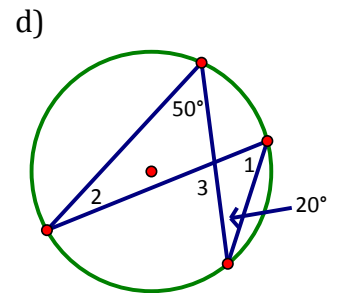
$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$



$m\angle 1 = \underline{\hspace{2cm}}$

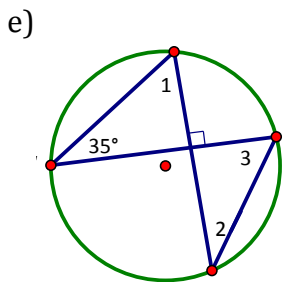
$m\angle 2 = \underline{\hspace{2cm}}$



$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$

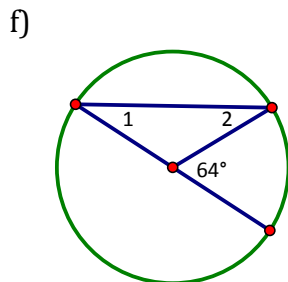
$m\angle 3 = \underline{\hspace{2cm}}$



$m\angle 1 = \underline{\hspace{2cm}}$

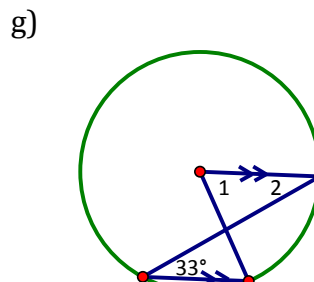
$m\angle 2 = \underline{\hspace{2cm}}$

$m\angle 3 = \underline{\hspace{2cm}}$



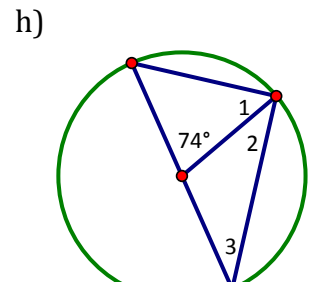
$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$



$m\angle 1 = \underline{\hspace{2cm}}$

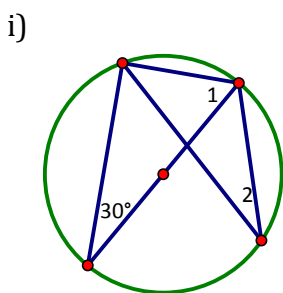
$m\angle 2 = \underline{\hspace{2cm}}$



$m\angle 1 = \underline{\hspace{2cm}}$

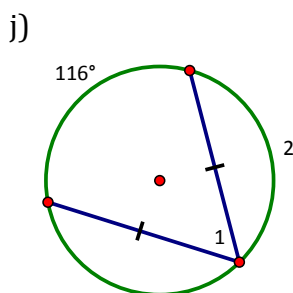
$m\angle 2 = \underline{\hspace{2cm}}$

$m\angle 3 = \underline{\hspace{2cm}}$



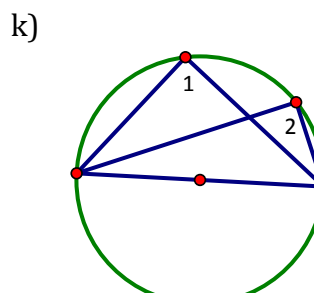
$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$



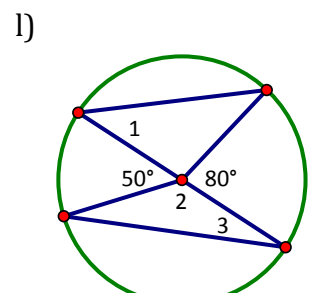
$m\angle 1 = \underline{\hspace{2cm}}$

$m\hat{2} = \underline{\hspace{2cm}}$



$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$



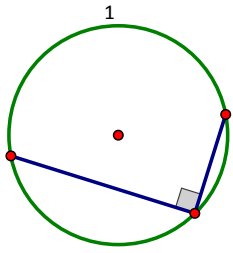
$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$

$m\angle 3 = \underline{\hspace{2cm}}$

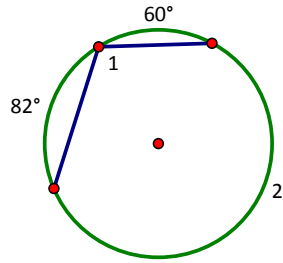


m)



$m\hat{1} = \underline{\hspace{2cm}}$

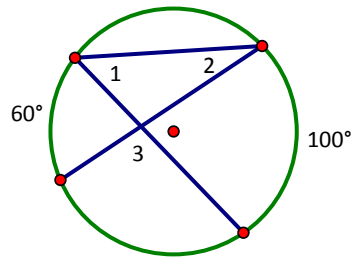
n)



$m\angle 1 = \underline{\hspace{2cm}}$

$m\hat{2} = \underline{\hspace{2cm}}$

o)

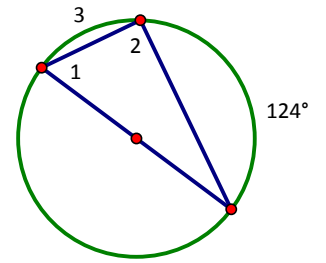


$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$

$m\angle 3 = \underline{\hspace{2cm}}$

p)

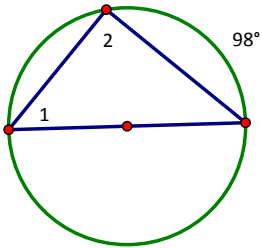


$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$

$m\hat{3} = \underline{\hspace{2cm}}$

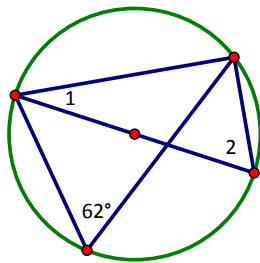
q)



$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$

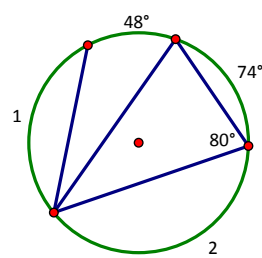
r)



$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$

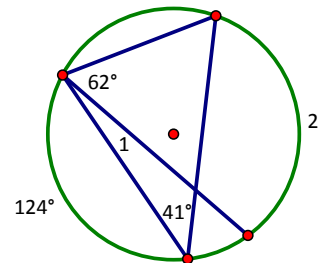
s)



$m\hat{1} = \underline{\hspace{2cm}}$

$m\hat{2} = \underline{\hspace{2cm}}$

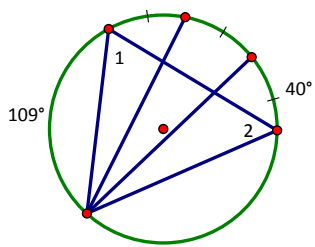
t)



$m\angle 1 = \underline{\hspace{2cm}}$

$m\hat{2} = \underline{\hspace{2cm}}$

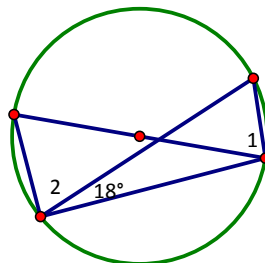
u)



$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$

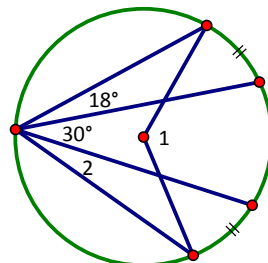
v)



$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$

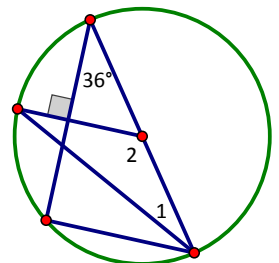
w)



$m\angle 1 = \underline{\hspace{2cm}}$

$m\angle 2 = \underline{\hspace{2cm}}$

x)



$m\angle 1 = \underline{\hspace{2cm}}$

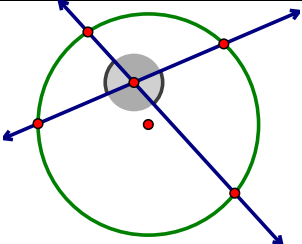
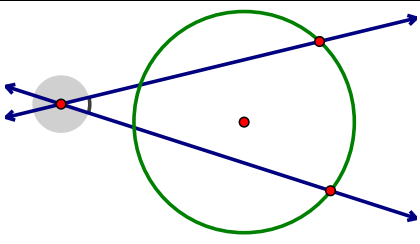
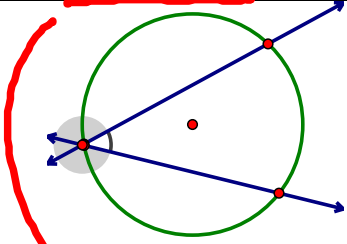
$m\angle 2 = \underline{\hspace{2cm}}$

# Other Angle Relationships in Circles

**Learning Targets:** Students will be able to apply properties of angles formed by chords, secants, and tangents to solve problems in circles.

## Internal, External and Tangential Angle Properties

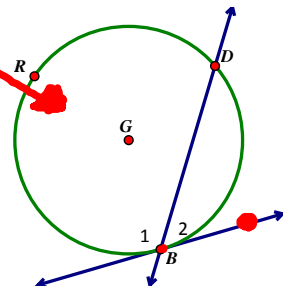
Inscribed and central angles are **NOT** the only types of angles that are formed when dealing with circles. Lines can intersect to form angles in the exterior, interior and on the circle.

Interior Angle (In)	Exterior Angle (Out)	Inscribed Angle (On)
		

### THEOREM (On)

If a tangent and a secant (chord) intersect at a point **on** a circle, then the measure of each angle formed is  $\frac{1}{2}$  of intercepted arc.

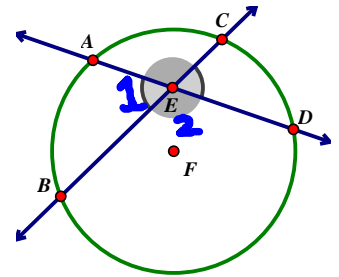
$$m\angle 1 = \frac{1}{2} m\widehat{DRB} \quad m\angle 2 = \frac{1}{2} m\widehat{DB}$$



### THEOREM (Inside)

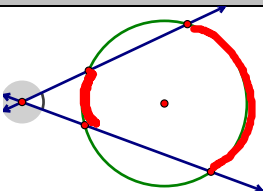
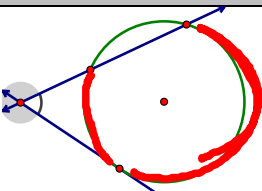
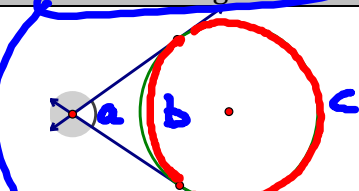
If two chords intersect in the **interior** of the circle, then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

$$m\angle 1 = \frac{1}{2} (m\widehat{AB} + m\widehat{CD}) \quad m\angle 2 = \frac{1}{2} (m\widehat{AC} + m\widehat{BD})$$



### THEOREM (Outside)

If a tangent and a secant, two tangents, or two secants intersect in the **exterior** of the circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.

Two Secants	One Secant and One Tangent	Two Tangents
		

$$c = 360 - b$$

$$a = \frac{1}{2} (c - b)$$

$$a = \frac{1}{2} (360 - b - b)$$

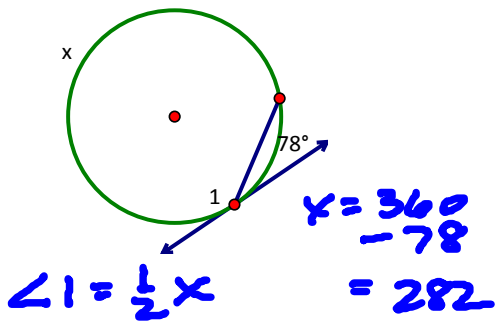
$$a = \frac{1}{2} (360 - 2b)$$

$$a = 180 - b$$

OR

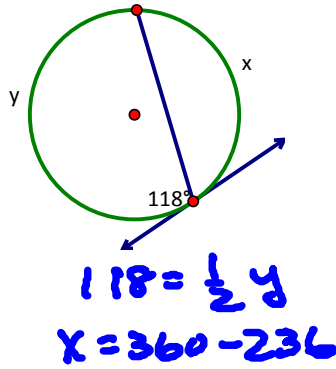
$$a + b = 180$$

1. Find  $x$  and  $m\angle 1$ .



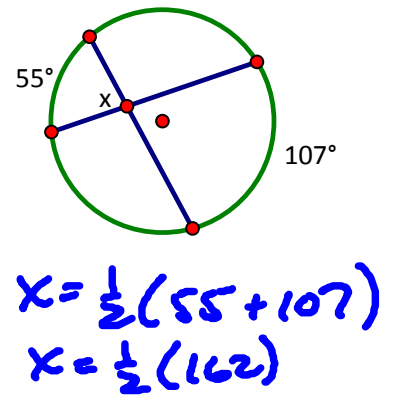
$x = \underline{282^\circ}$      $m\angle 1 = \underline{141^\circ}$

2. Find  $x$  and  $y$ .



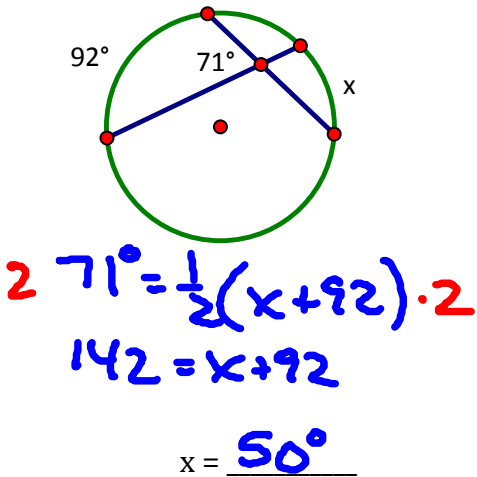
$x = \underline{124^\circ}$      $y = \underline{236^\circ}$

3. Find  $x$ .

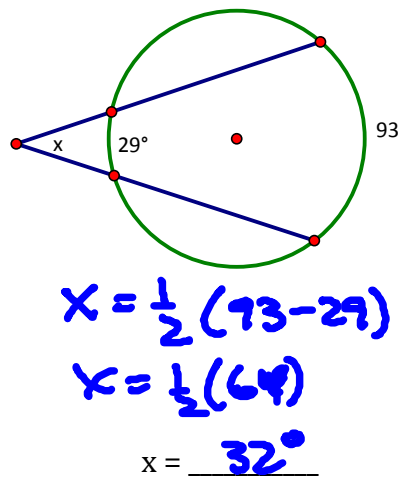


$x = \underline{81^\circ}$

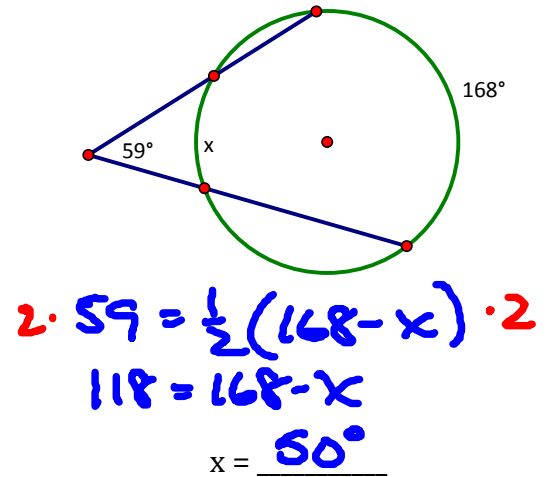
4. Find  $x$ .



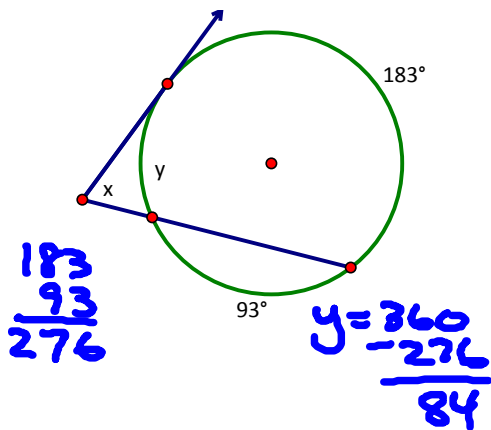
5. Find  $x$ .



6. Find  $x$ .



7. Find  $x$  and  $y$ .

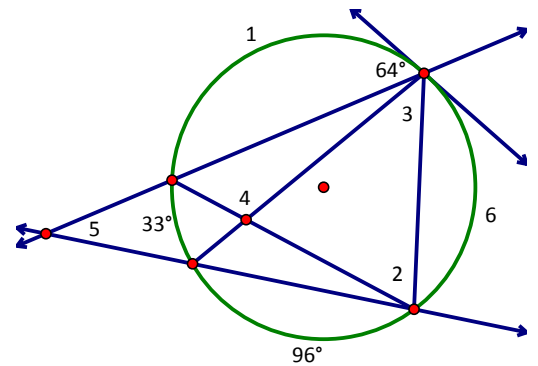


$x = \underline{49\frac{1}{2}}$      $y = \underline{84}$

$x = \frac{1}{2}(183 - 84)$   
 $x = \frac{1}{2}(99)$

8. Solve for the missing values.

- a)  $m\hat{1} = \underline{\hspace{2cm}}$
- b)  $m\angle 2 = \underline{\hspace{2cm}}$
- c)  $m\angle 3 = \underline{\hspace{2cm}}$
- d)  $m\angle 4 = \underline{\hspace{2cm}}$
- e)  $m\angle 5 = \underline{\hspace{2cm}}$
- f)  $m\hat{6} = \underline{\hspace{2cm}}$



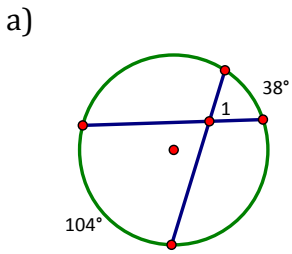
**GEOMETRY**

**Other Angle Relationships in Circles HW**

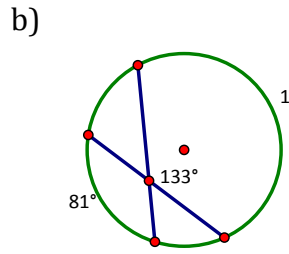
Name: \_\_\_\_\_

Date: \_\_\_\_\_ Period: \_\_\_\_\_

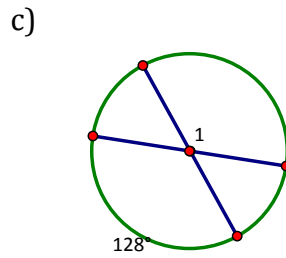
1. Determine the requested value(s).



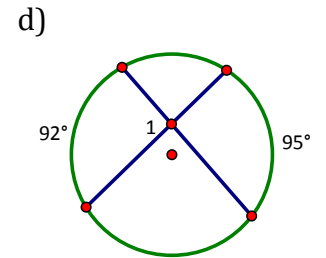
$m\angle 1 = \underline{\hspace{2cm}}$



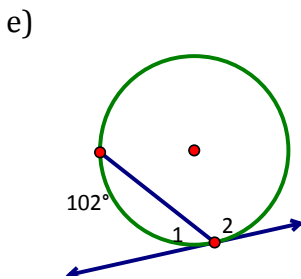
$m\hat{1} = \underline{\hspace{2cm}}$



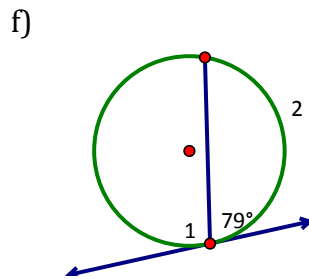
$m\angle 1 = \underline{\hspace{2cm}}$



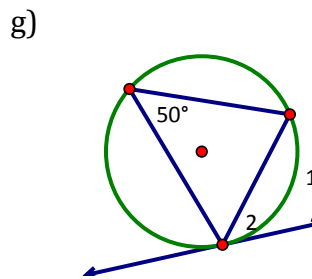
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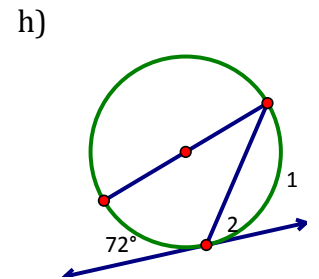
$m\angle 1 = \underline{\hspace{1cm}}$   $m\angle 2 = \underline{\hspace{1cm}}$



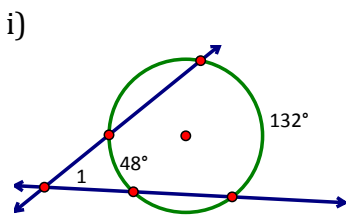
$m\angle 1 = \underline{\hspace{1cm}}$   $m\hat{2} = \underline{\hspace{1cm}}$



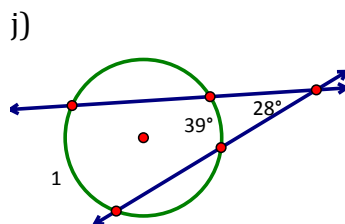
$m\hat{1} = \underline{\hspace{1cm}}$   $m\angle 2 = \underline{\hspace{1cm}}$



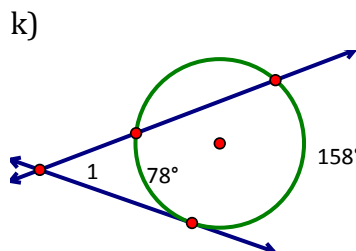
$m\hat{1} = \underline{\hspace{1cm}}$   $m\angle 2 = \underline{\hspace{1cm}}$



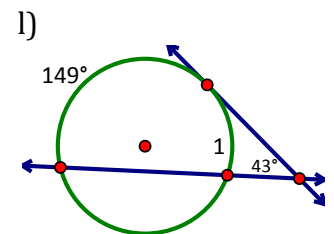
$m\angle 1 = \underline{\hspace{2cm}}$



$m\hat{1} = \underline{\hspace{2cm}}$

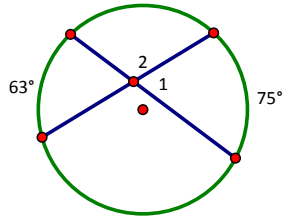


$m\angle 1 = \underline{\hspace{2cm}}$



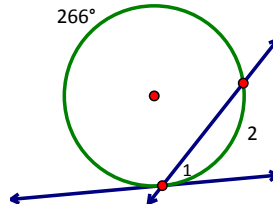
$m\hat{1} = \underline{\hspace{2cm}}$

m)



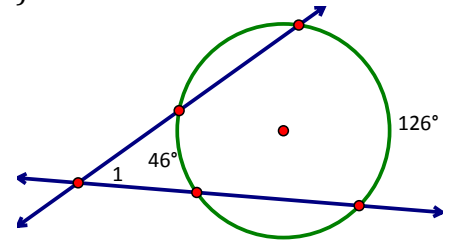
$m\angle 1 = \underline{\hspace{2cm}}$     $m\angle 2 = \underline{\hspace{2cm}}$

n)



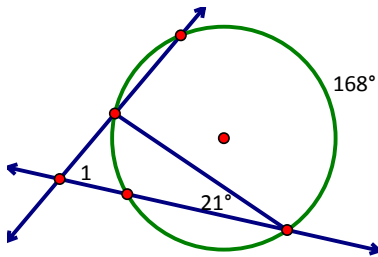
$m\angle 1 = \underline{\hspace{2cm}}$     $m\widehat{2} = \underline{\hspace{2cm}}$

o)



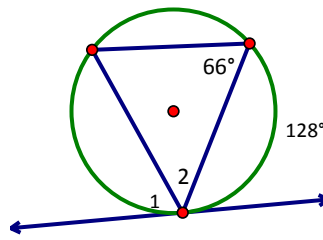
$m\angle 1 = \underline{\hspace{2cm}}$

p)



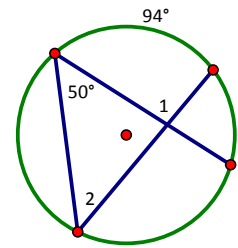
$m\angle 1 = \underline{\hspace{2cm}}$

q)



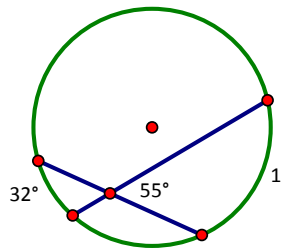
$m\angle 1 = \underline{\hspace{2cm}}$     $m\angle 2 = \underline{\hspace{2cm}}$

r)



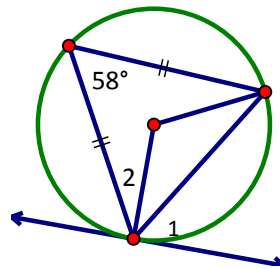
$m\angle 1 = \underline{\hspace{2cm}}$     $m\angle 2 = \underline{\hspace{2cm}}$

s)



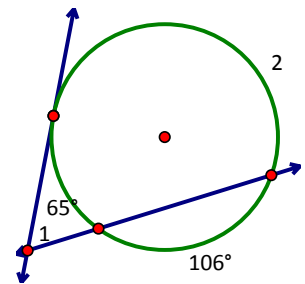
$m\widehat{1} = \underline{\hspace{2cm}}$

t)



$m\angle 1 = \underline{\hspace{2cm}}$     $m\angle 2 = \underline{\hspace{2cm}}$

u)



$m\angle 1 = \underline{\hspace{2cm}}$     $m\widehat{2} = \underline{\hspace{2cm}}$

2. Solve for the missing values.

a)  $m\angle 1 = \underline{\hspace{2cm}}$

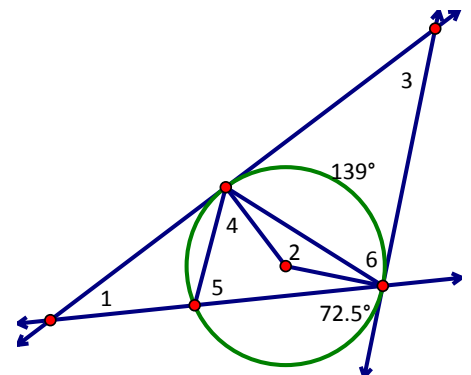
b)  $m\angle 2 = \underline{\hspace{2cm}}$

c)  $m\angle 3 = \underline{\hspace{2cm}}$

d)  $m\angle 4 = \underline{\hspace{2cm}}$

e)  $m\angle 5 = \underline{\hspace{2cm}}$

f)  $m\angle 6 = \underline{\hspace{2cm}}$

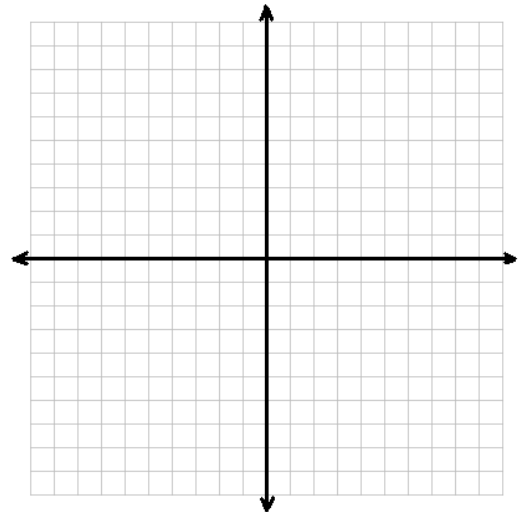


## Freaky Friends

### Finding Friends

*Miley only has friends that are a fixed distance from her house. She is so superstitious that she won't talk to any one that lives closer than the given distance. She gets car sick if she has to drive any further. Miley is friends with Lindsay who lives exactly 3 miles east and four miles north of Miley's house. Assume Miley lives on the origin (0, 0). At what other locations, exactly the same number of miles from her house (crow's flight), can Miley potentially find a new friend?*

1 Plot at least six other locations (order pairs) that are exactly the same distance as Lindsay is from Miley's house. How did you find these points?



2. How far is Lindsay from Miley? \_\_\_\_\_ miles

3. If the location of all of Miley's possible dates were plotted, what shape would be formed?

4. a) Write the equation of this shape: \_\_\_\_\_

b) Your equation is based on what other formulas/principles that you might know?

5. Find the equation of this shape, for a radius of ..

a) 2

b) 3

c) 7

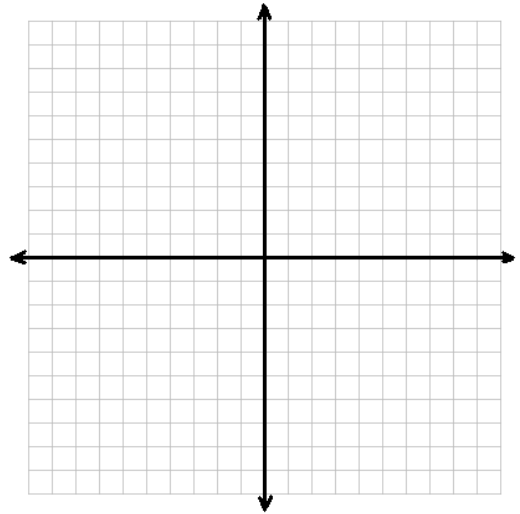
d) r

## Freaky Friends (cont'd)

### Finding New Friends

Britney is a friend of Miley. She lives at the location  $(-4, 3)$  and has the same distance restriction because she is just as interesting.

6. Find an equation that will give the location of all of the people who are exactly that distance from Britney



7. Write the equation of the circles for the given center and radius:

a)  $C(2, 5), r = 7$

b)  $C(-1, 7), r = 2$

c)  $C(0, 3), r = 3$

### Finding Any Friends For Anybody

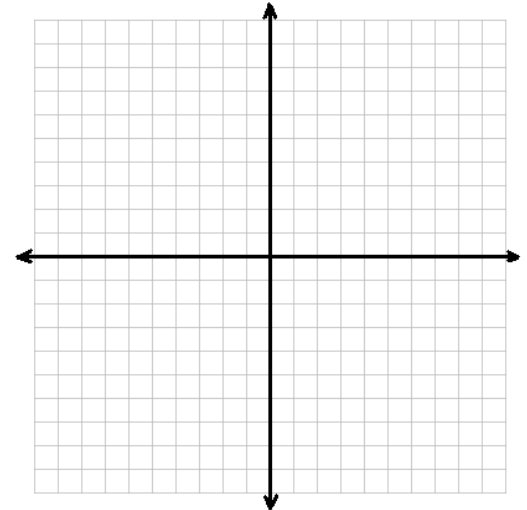
8. Write the equation of the circle that represents all the locations  $(x, y)$  that are  $r$  miles away from someone living at  $(h, k)$ .

### Freaky Friends (cont'd)

#### Taking it to the Streets

*In Miley's neighborhood there is a street named Main Street that runs north-south and is two miles east of Miley's house.*

9. a) Plot Miley's house, her circle of friends and all the potential homes on Main Street. Rewrite the equation for Miley's circle of friends.



- b) What is the equation for the graph of Main Street?

- c) Use the two equations to find the locations of Miley's friends who can live on Main Street. Why are there two answers? Plot both location on the graph.

10. a) For which streets (equations) will Miley have only one friend? Graph each. Verify one.

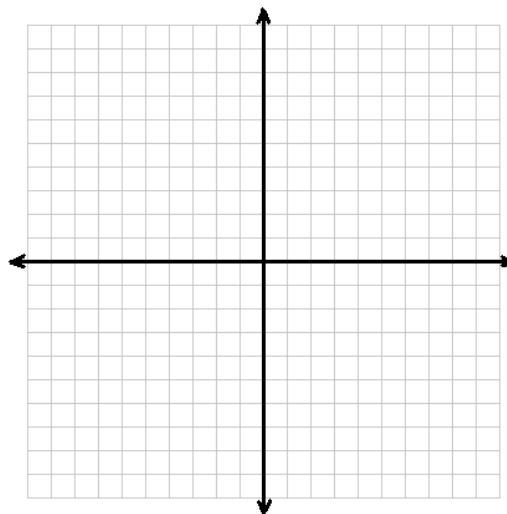
- b) For which streets (values) will Miley not have any friends? Verify one of these.



## Freaky Friends (cont'd)

In Britney's neighborhood, there is an east-west street, called Central Ave, that is 4 miles south of her house.

- 11 a) Plot Britney's house, her circle of friends and all the potential homes on Central Ave. Rewrite the equation for Britney's circle of friends.



- b) What is the equation for the graph of Central Ave?

- c) Use the two equations to find the locations of Britney's friends who can live on Central Ave. Plot both location on the graph.

- d) Algebraically and graphically determine the location of any of Britney's friends who live on Main St.

- e) Algebraically and graphically determine the location of any of Britney's friends who live on Broadway, 6 miles south of her home.

# Equations of Circles

Learning Targets: Students will be able to write an equation of a circle and graph it.

## STANDARD EQUATION OF A CIRCLE:

radius = r and center (h, k)

$$(x-h)^2 + (y-k)^2 = r^2$$

If the center is the origin, the standard equation is  $x^2 + y^2 = r^2$ .

Parabola

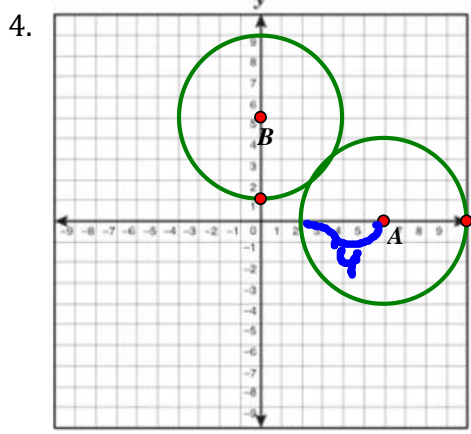
$$y = a(x-h)^2 + k$$

Vertex (h, k)

Determine the center and radius of the given circles.

- |                              |                                 |  |
|------------------------------|---------------------------------|--|
| 1. $(x-2)^2 + (y+5)^2 = 100$ | Center ( <u>2</u> , <u>-5</u> ) | Radius = <u>10</u> $r^2 = 100$         |
| 2. $(x-8)^2 + y^2 = 25$      | Center ( <u>8</u> , <u>0</u> )  | Radius = <u>5</u>                      |
| 3. $14 = (x+3)^2 + (y-1)^2$  | Center ( <u>-3</u> , <u>1</u> ) | Radius = <u><math>\sqrt{14}</math></u> |

Determine the equation of the circle.



- |   |   |
|---|---|
| <b>CIRCLE A:</b>                                  | <b>CIRCLE B:</b>                                  |
| Center ( <u>6</u> , <u>0</u> )                    | Center ( <u>0</u> , <u>5</u> )                    |
| Radius = <u>4</u>                                 | Radius = <u>4</u>                                 |
| Equation: <u><math>(x-6)^2 + y^2 = 4^2</math></u> | Equation: <u><math>x^2 + (y-5)^2 = 4^2</math></u> |

5. The point (-1, 1) is on a circle whose center is (-3, 4). Write the standard equation of the circle.

$$r = \sqrt{(2)^2 + (3)^2}$$

$$r = \sqrt{4 + 9}$$

$$r = \sqrt{13}$$

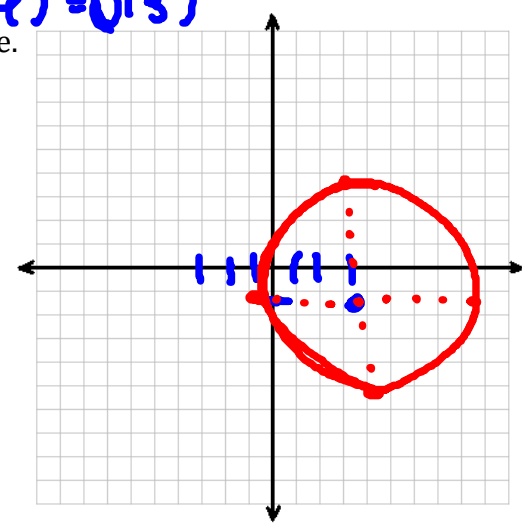
$$(x+3)^2 + (y-4)^2 = 13$$

$$(x - (-3))^2 + (y - 4)^2 = (\sqrt{13})^2$$

6. The equation of a circle is  $(x-3)^2 + (y+1)^2 = 16$ . Graph the circle.

CTR (3, -1)

$$r = \sqrt{16} = 4$$

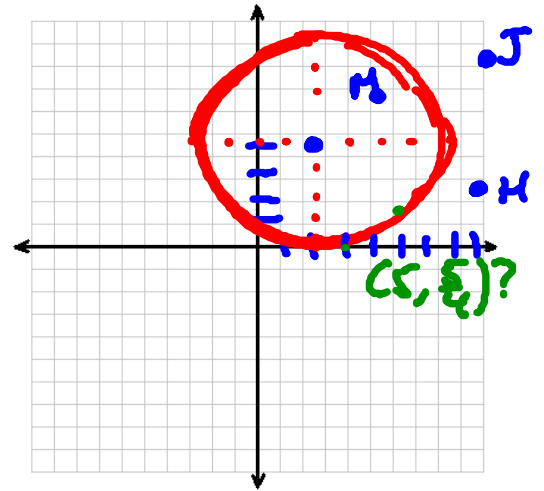


7. A bank of lights is arranged over a stage. Each light illuminates a circular area on the stage. A coordinate plane is used to arrange the lights, using the corner of the stage as the origin. The equation  $(x - 2)^2 + (y - 4)^2 = 16$  represents one of the disks of light.

a) Graph the disk of light.

$$\text{CTR } (2, 4)$$

$$r = \sqrt{16} = 4$$



- b) Three actors are located as follows: Henry is at (8, 2), Jolene is at (8, 5), and Martin is at (4, 6). Which actors are in the disk of light?

Henry - No  
 Jolene - no  
 Martin - yes

$$(x-2)^2 + (y-4)^2 = 16$$

$$(8-2)^2 + (2-4)^2 \square 16$$

$$6^2 + (-2)^2 \square 16$$

$$36 + 4 = 40 > 16$$

outside

$$(4-2)^2 + (6-4)^2 \square 16$$

$$2^2 + 2^2 \square 16$$

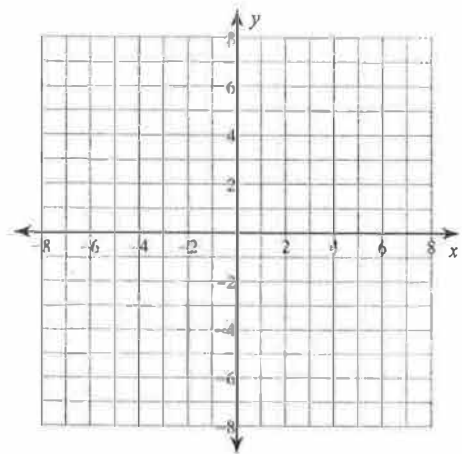
$$8 < 16$$

inside

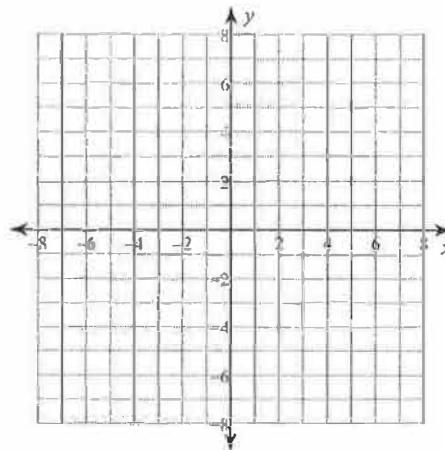
Equations of Circles HW

Identify the center and radius of each. Then sketch the graph.

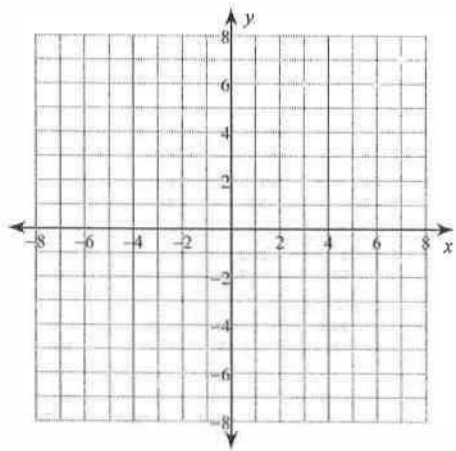
1)  $(x - 1)^2 + (y - 4)^2 = 9$



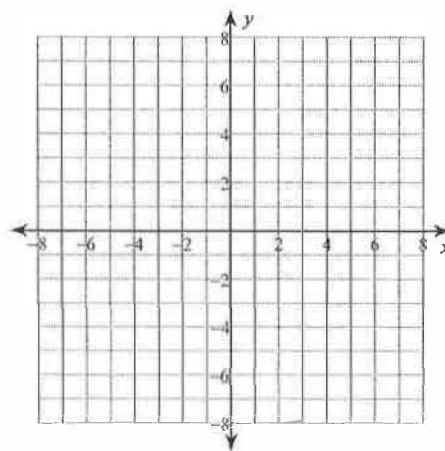
2)  $(x - 1)^2 + (y - 1)^2 = 4$



3)  $(x + 4)^2 + y^2 = 4$



4)  $(x - 2)^2 + (y - 4)^2 = 1$



Identify the center and radius of each.

5)  $(x - 9)^2 + y^2 = 16$

6)  $(x - 8)^2 + (y + 8)^2 = 49$

7)  $(x - 13)^2 + (y - 5)^2 = 10$

8)  $(x - 12)^2 + (y + 16)^2 = 9$

Use the information provided to write the equation of each circle.

9) Center:  $(0, 0)$   
Radius: 10

10) Center:  $(0, 0)$   
Radius:  $5\sqrt{2}$

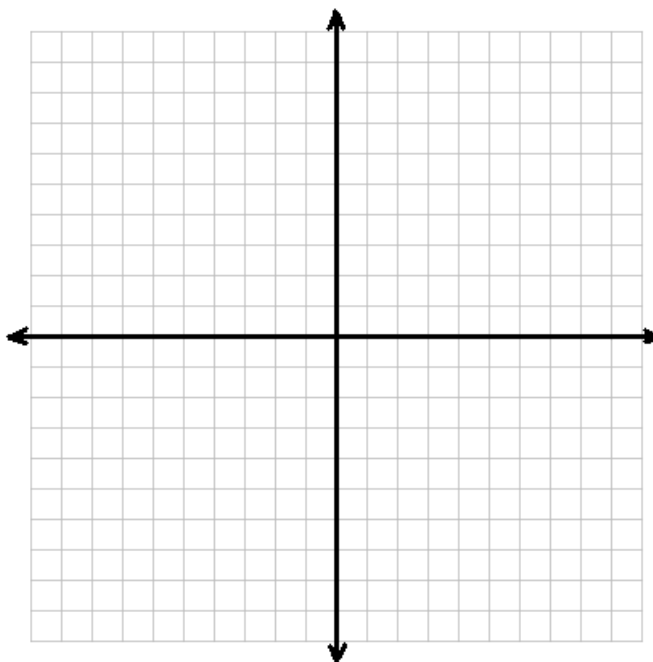
11) Center:  $(0, 0)$   
Radius: 5

12) Center:  $(1, -13)$   
Point on Circle:  $(1, -9)$

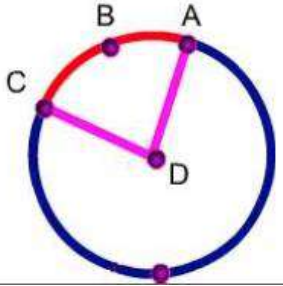
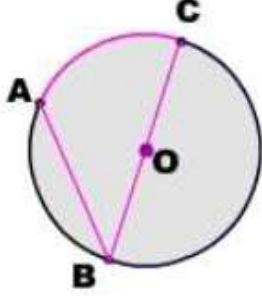
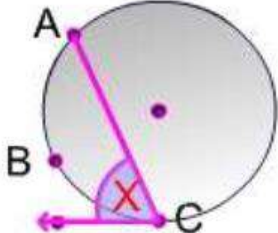
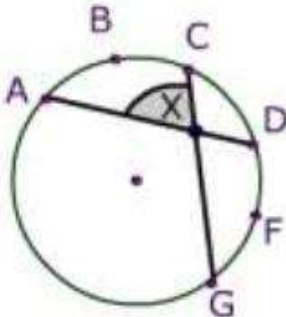
13) Center:  $(11, 12)$   
Point on Circle:  $(14, 12)$

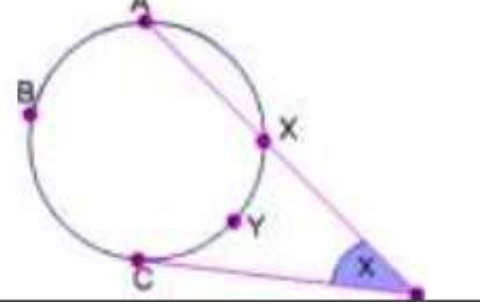
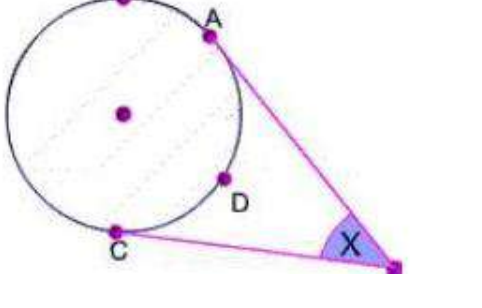
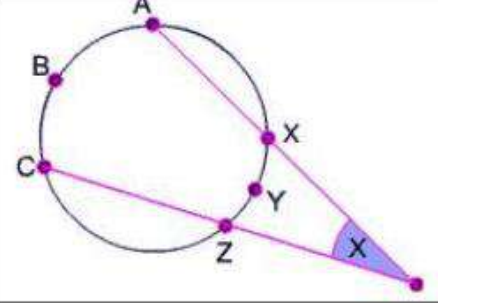
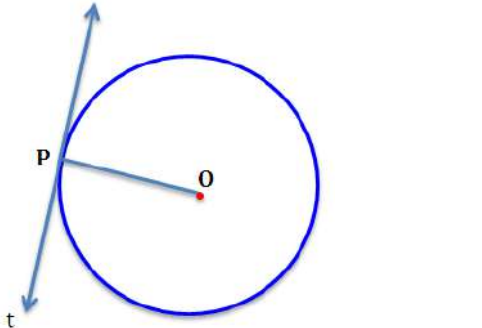
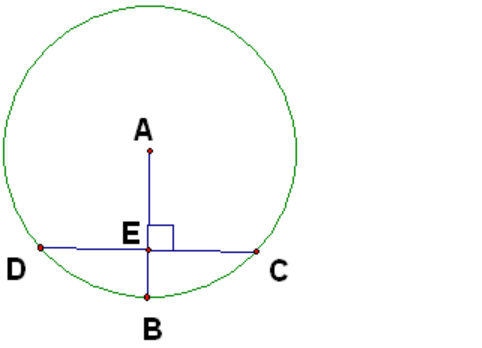
14) Center:  $(-4, -3)$   
Point on Circle:  $(-5, -6)$

15) Center:  $(-12, -8)$   
Point on Circle:  $(-13, -13)$



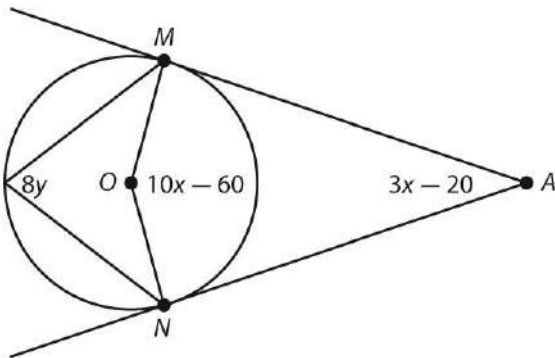
# Theorem, Postulates, Definitions Practice/Review

Labeled Diagram	Formula
	$m\angle D =$ $m\widehat{CBA} =$
	$m\angle ABC =$ $m\widehat{AC} =$
	$m\angle ACB =$ $m\widehat{ABC} =$
	$m\angle AXC =$ $m\angle AXG =$

	$m\angle AXC =$
	$m\angle AXC =$
	$m\angle AXC =$
	<p>What is the relationship between the tangent line <math>t</math> and the radius <math>\overline{OP}</math>?</p>
	<p>What is the relationship between <math>\overline{DE}</math> and <math>\overline{EC}</math>?</p>

### Unit 8 Review

Use the figure below to answer questions 1-4.



1. What is the measure of  $\angle MON$ ?

2. What is the value of  $y$ ?

3. If a line segment is drawn from point  $O$  to point  $A$ , is  $\angle OMA \cong \angle ONA$ ? Explain why or why not.

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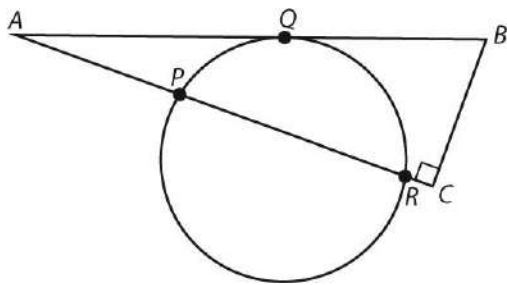
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4. What is the measure of  $\widehat{MN}$ ? \_\_\_\_\_

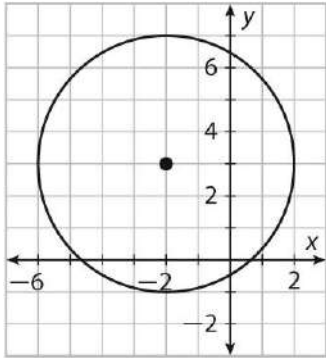
5. In the figure below  $m\angle ABC = 70^\circ$  and  $\overline{AB}$  is tangent to the circle. If  $m\widehat{PQ} = 60$ , what is  $m\widehat{QR}$ ?



6. What is the equation of a circle that is centered at  $(0, 5)$  and has a diameter of 18 units?



7. Write the equation of the circle pictured below.



8. Write the equation of a circle with a center at (3, 4) that goes through the point (3, 7).

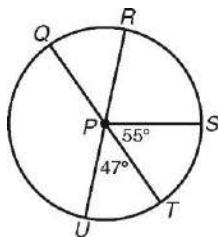
a) Find the radius of the circle. Show your work. Radius: \_\_\_\_\_

b) Write the equation of the circle.

c) Does the point (5, 6) lie within the circle?

**For each figure, determine the indicated measures.**

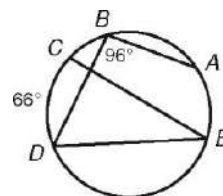
9.



$m\angle QPS$  \_\_\_\_\_

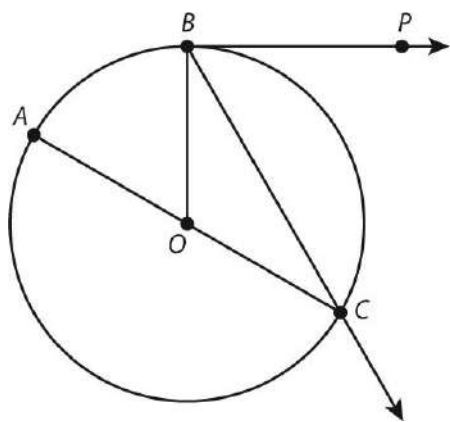
$m\angle RPT$  \_\_\_\_\_

10.



$m\angle CED$  \_\_\_\_\_

$m\angle DEA$  \_\_\_\_\_



For 11-14, use the circle below centered at point O.

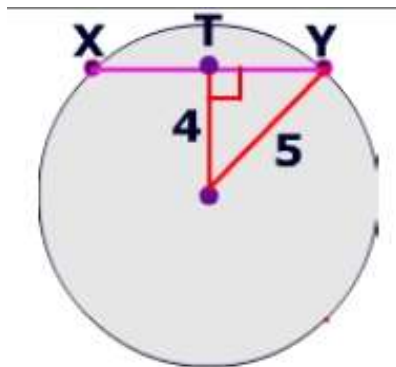
11. If  $\angle ACB = 2x + 4$  and  $\angle AOB = 6x - 10$ , then find the value of  $x$ .

12. Find  $m\angle ACB$ .

13. Given the  $m\widehat{BC} = 136^\circ$ , is  $\overline{AC}$  a diameter of the circle? Explain.

14. Find  $m\angle CBP$ .

15. Use the diagram to the right to find the distance of the chord  $\overline{XY}$ .



# Unit 9

## 3-Dimensional Modeling

# Volume of Prisms

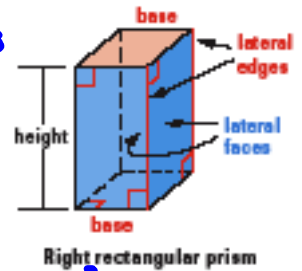
Learning Targets: Students will be able to find the volume of a prism.

## KEY TERMS

Prism: A space figure w/ 2  $\cong$  bases where the bases are connected by rectangles (named after the bases)

Base:  $\hookrightarrow$  2  $\cong$  faces

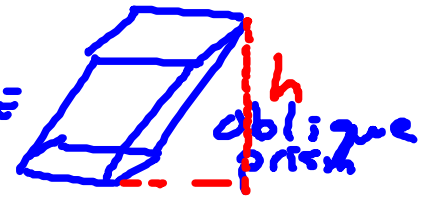
Height: distance between the faces at a Right  $\angle$



## VOLUME OF A RIGHT PRISM

$$V = Bh$$

B: AREA of the BASE  
P: Perimeter of base  
h: height



$$SA = Ph$$

Find the volume of the given right prisms.

$h^2 + 8^2 = 10^2$   
 $h^2 + 64 = 100$   
 $h^2 = 36$   
 $h = 6$

$B_{\Delta} = \frac{bh}{2}$   
 $= \frac{8 \cdot 6}{2}$   
 $= 24$

$V = Bh \cdot p$   
 $= 24(15)$   
 $= 360 \text{ m}^3$

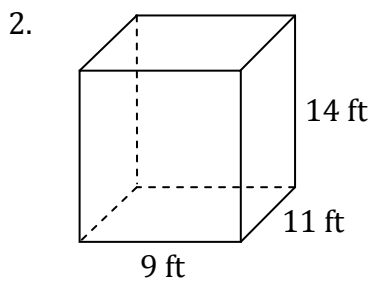
$3-4-5$   
 $6-8-10$

Name: Triangular Prism  
 Surface Area: 360 m<sup>2</sup>  
 Volume: 360 m<sup>3</sup>

$$SA = Ph$$

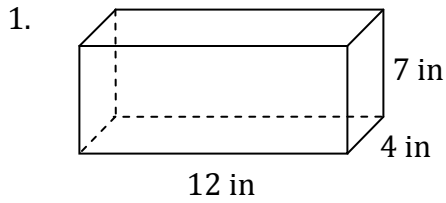
$$= (6 + 8 + 10)(15)$$

$$= 24(15)$$

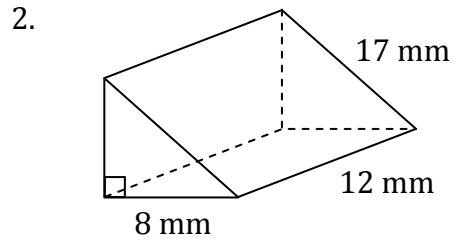


Name: \_\_\_\_\_  
 Surface Area: \_\_\_\_\_  
 Volume: \_\_\_\_\_

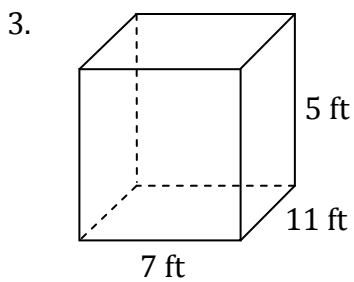
Find the volume of the given right prisms.



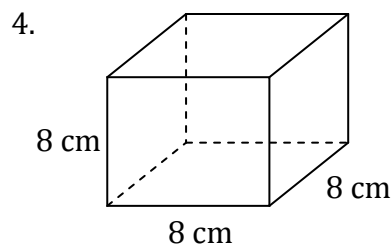
Volume: \_\_\_\_\_



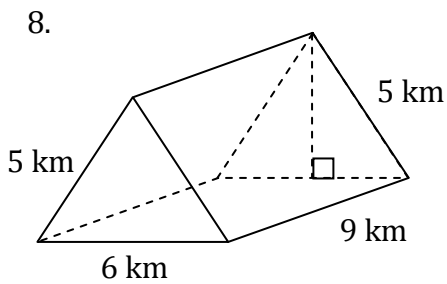
Volume: \_\_\_\_\_



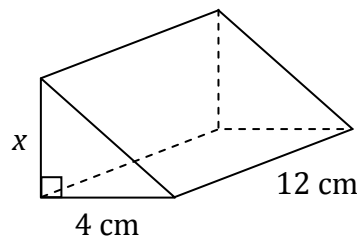
Volume: \_\_\_\_\_



Volume: \_\_\_\_\_



9. **Solve for x.** Volume =  $144 \text{ cm}^3$



# Volume of Cylinders

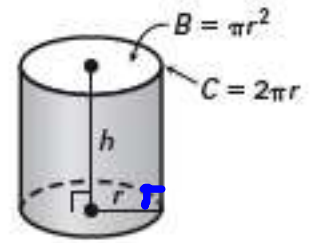
Learning Targets: Students will be able to find the volume of a cylinder.

## KEY TERMS

Area Of A Circle:  $A = \pi r^2$

Cylinder: 2 circles connected by a rectangle that wraps around the circle

Right Cylinder: "edge" is  $\perp$  base



## VOLUME OF A RIGHT CYLINDER

$$V = Bh = \pi r^2 h$$

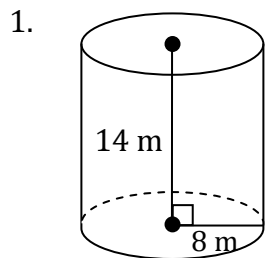
B: Area of base

C:

h: height

$$3.8\sqrt{5} = 24\sqrt{5}$$

Find the volume of the given cylinders.



$$V = \pi r^2 h$$

$$= \pi (8)^2 (14)$$

Surface Area: \_\_\_\_\_

Volume: 2814.87 m<sup>3</sup>

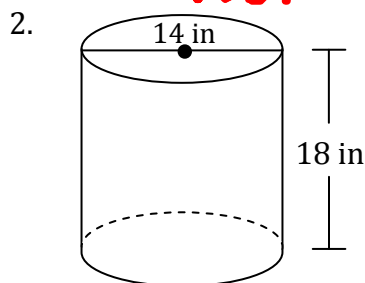
$$= \underline{3.14(8)^2(14)} = \underline{2812.44}$$

NO!

USE  $\pi$  key

$$896\pi \text{ m}^3$$

EXACT OR IN TERMS OF  $\pi$

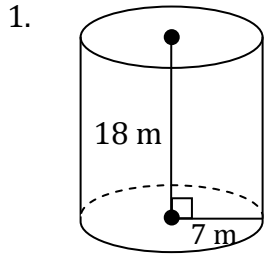


Surface Area: \_\_\_\_\_

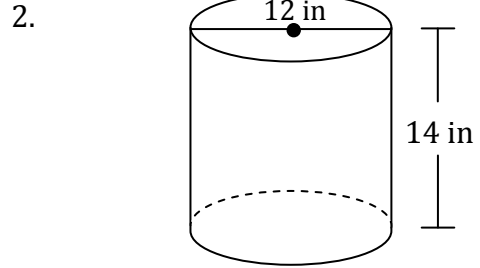
Volume: \_\_\_\_\_

3. The surface area of a cylinder is 879.65 cm<sup>2</sup> and has a height of 23 cm what is the radius?

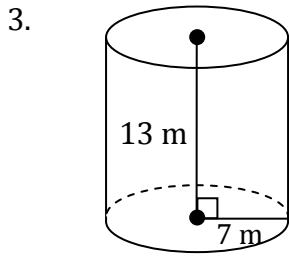
Find the volume of the given solids.



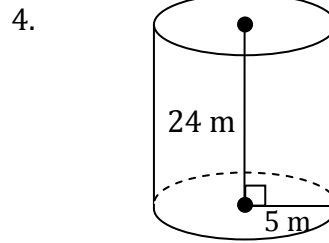
Volume: \_\_\_\_\_



Volume: \_\_\_\_\_



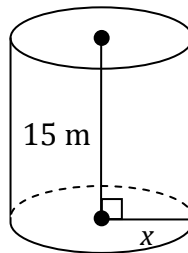
Volume: \_\_\_\_\_



Volume: \_\_\_\_\_

5. Solve for  $x$ .

Volume =  $540 \text{ m}^3$



# Volume of Pyramids and Cones

Learning Targets: Students will be able to find the surface area and volume of a pyramid and cone.

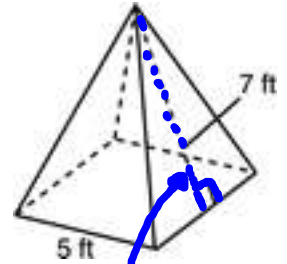
## KEY TERMS

Pyramid:

1 BASE The other faces are  $\Delta$ 's which are lateral faces which are  $\Delta$ 's which are lateral faces which are  $\Delta$ 's

Height:

Vertex  $\perp$  to the base (the distance)



## VOLUME OF A PYRAMID

$$V = \frac{Bh}{3} \text{ or } \frac{1}{3} Bh$$

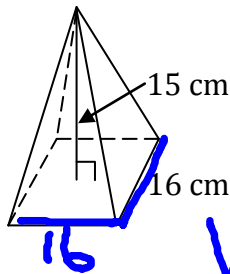
B: Area of base

h: height

Slant height

Find the volume of the regular pyramids.

1.

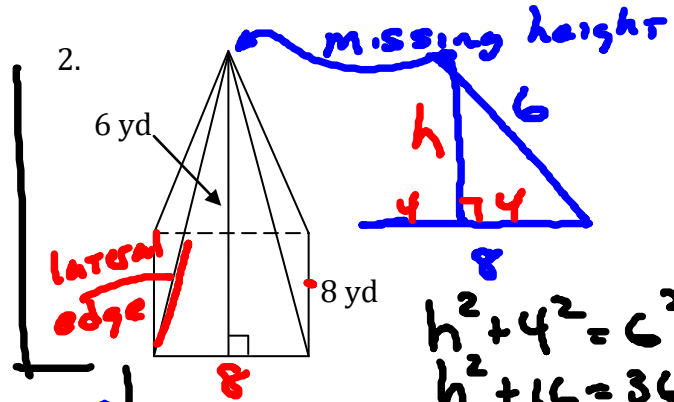


base is regular

$$V = \frac{Bh}{3}$$

$$= \frac{16^2 \cdot 15}{3} = 1280 \text{ cm}^3$$

2.



$$h^2 + 4^2 = 6^2$$

$$h^2 + 16 = 36$$

$$h^2 = 20$$

$$h = \sqrt{20}$$

$$V = \frac{Bh}{3}$$

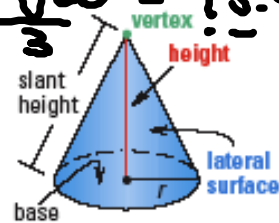
$$= \frac{8^2 \sqrt{20}}{3} = 95.41 \text{ yd}^3$$

## VOLUME OF A CONE

$$V = \frac{Bh}{3} = \frac{\pi r^2 h}{3}$$

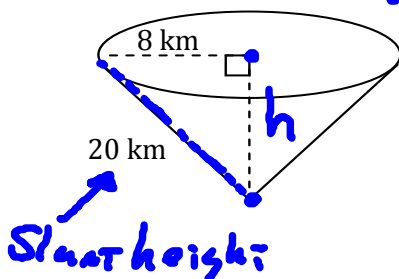
r: radius

h: height



Find the volume of the given cones.

3.



$$V = \frac{\pi r^2 h}{3}$$

$$h^2 + 8^2 = 20^2$$

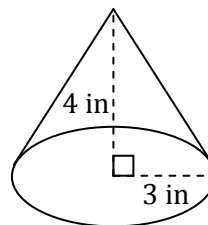
$$h^2 + 64 = 400$$

$$h^2 = 336$$

$$h = \sqrt{336}$$

$$V = \frac{\pi (8)^2 \sqrt{336}}{3} = 1228.51 \text{ km}^3$$

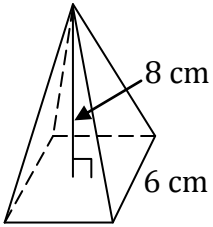
4.



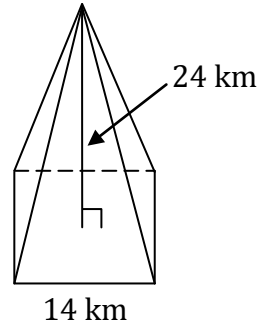


Find the volume of the given solids.

1.



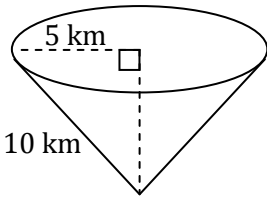
2.



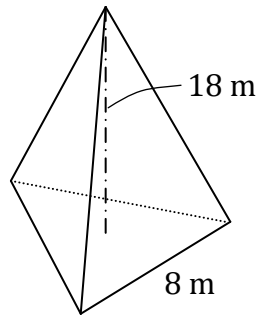
Volume: \_\_\_\_\_

Volume: \_\_\_\_\_

3.



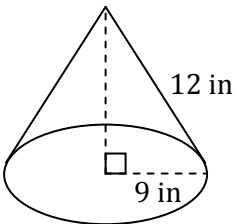
4.



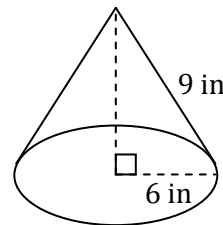
Volume: \_\_\_\_\_

Volume: \_\_\_\_\_

5.



6.



Volume: \_\_\_\_\_

Volume: \_\_\_\_\_

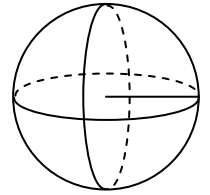
# Volume of Spheres

Learning Targets: Students will be able to find the volume of a sphere.

## KEY TERMS

Sphere: all the points the same distance from a center point in 3 dimensions

Hemisphere:  $\frac{1}{2}$  of a sphere



## VOLUME OF A SPHERE

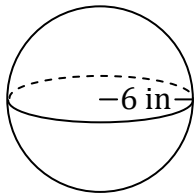
$$V = \frac{4}{3}\pi r^3$$

$r$ : radius of sphere

$$SA_{sph} : 4\pi r^2$$

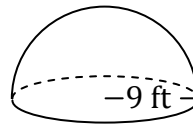
Find the volume of the given solids.

1.



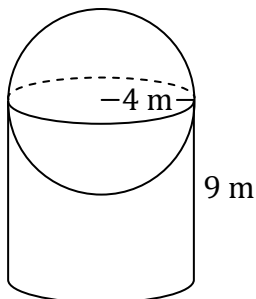
$$V = \frac{4}{3}\pi (6)^3$$
$$= 904.78$$

2.



Composite Volume: \_\_\_\_\_

3.



1

What is the radius of a cylinder with a diameter of 36?

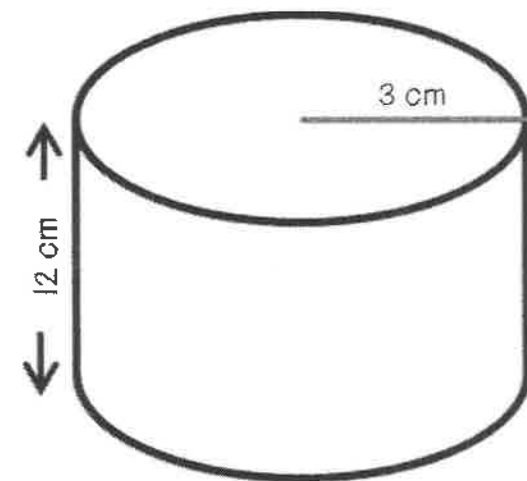


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2

What is the approximate volume of this cylinder?

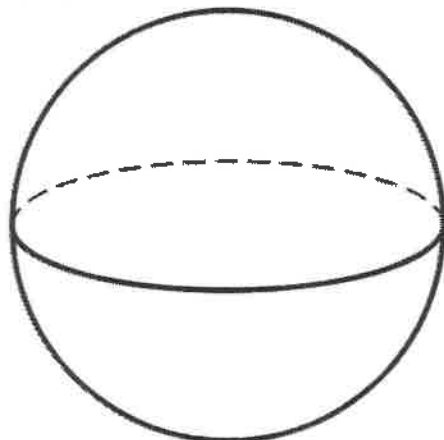
- a.  $36 \text{ cm}^3$
- b.  $339 \text{ cm}^3$
- c.  $226 \text{ cm}^3$
- d.  $324 \text{ cm}^3$



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3

What is the volume of this sphere, with a radius of 4 inches, to the nearest tenth?



Radius = 4 inches

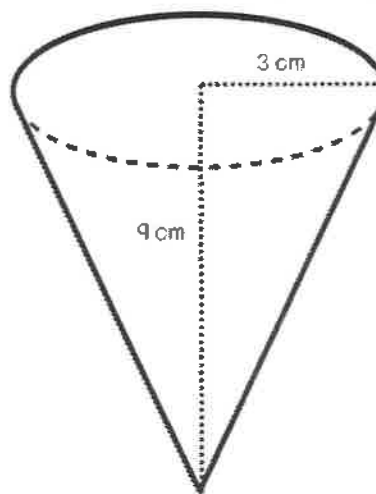


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4

Sarah wants to melt chocolate and pour it into a cone shaped mold with a radius of 3 cm and a height of 9 cm. Which is the closest to the volume?

- a.  $84 \text{ cm}^3$
- b.  $254 \text{ cm}^3$
- c.  $169 \text{ cm}^3$
- d.  $56 \text{ cm}^3$



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5

What is the exact volume of a cylinder with a diameter of 10 cm and a height of 4 cm?

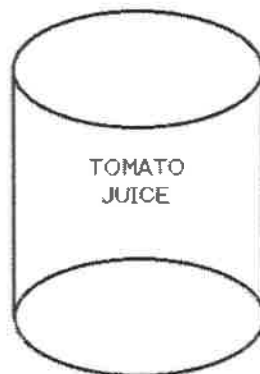
- a.  $25\pi \text{ cm}^3$
- b.  $40\pi \text{ cm}^3$
- c.  $100\pi \text{ cm}^3$
- d.  $314\pi \text{ cm}^3$



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6

A can of tomato juice has a diameter of 14 cm and a height of 20 cm. What is the volume?



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7

It is snowing and salt trucks are dumping a mixture to help. When the mixture is dumped from a truck, it forms a mound with a conical shape. If the mound has a diameter of 15 feet and a height of 10 feet, how much salt-sand mixture is there?



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8

The diameter of a beach ball is 3 feet. What is the volume of the beach ball?



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9

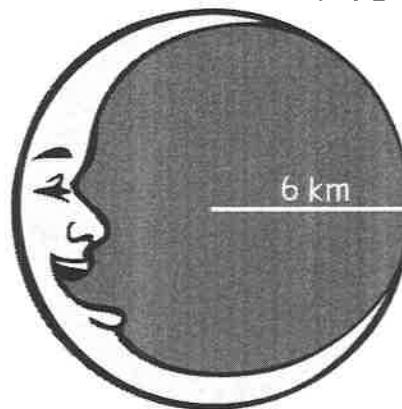
An ice cream cone has a diameter of 3 inches and a height of 6 inches. How much ice cream can just the cone hold?



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10

A distant moon's radius is 6 km. What is the moon's volume?



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11

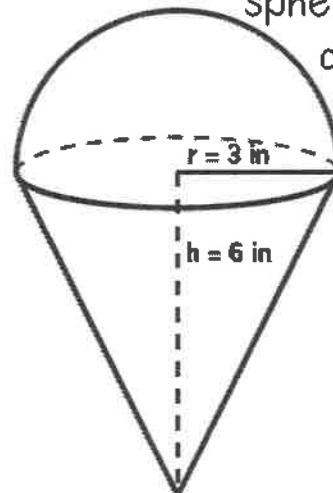
Sam is stacking cans of vegetables at the Store. His shelf is 10 inches tall, 10 inches deep, and 50 inches wide. The cans are 4 inches tall and each has a volume of  $50.24 \text{ in}^3$ . How many cans will fit on the shelf?



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12

Kim has an ice cream cone with a scoop of ice cream. The cone is completely filled with ice cream and has half a sphere on top. How much ice cream does Kim have in cubic inches?



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13

Explain how the formula for the volume of a cone is derived.



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14

What is the total volume of Diet Coke in this six pack? Each can is 5 inches tall and has a diameter of 3 inches.

- a.  $47.1 \text{ in}^3$
- b.  $141.3 \text{ in}^3$
- c.  $35.3 \text{ in}^3$
- d.  $211.95 \text{ in}^3$



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15

A basketball has volume of  $288\pi \text{ in}^3$ . What is the length of the radius?

- a. 6 in
- b. 12 in
- c. 3 in
- d. 4 in

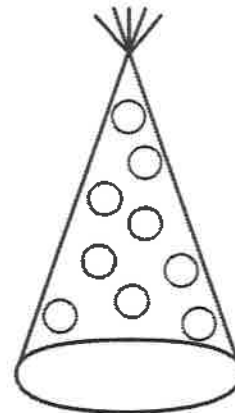


13

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16

A party hat has a volume of  $75\pi$  cubic inches. If the radius is 5 inches, what is the height of the party hat?



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17

What is the formula for the volume of the following figures?

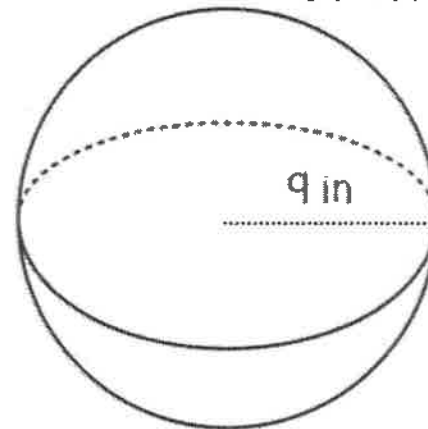
CONE  
CYLINDER  
SPHERE



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18

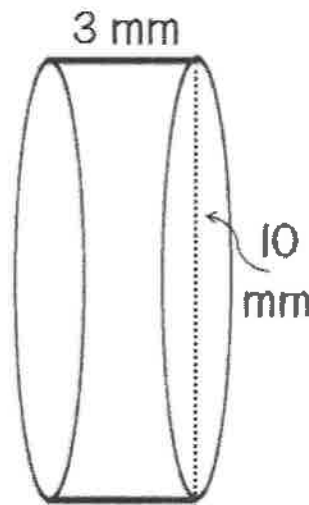
Find the volume of the sphere to the nearest tenth with a radius of 9 in.



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19

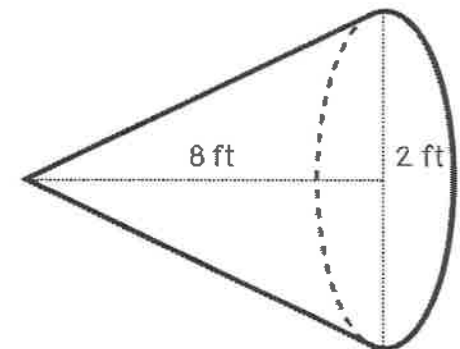
Find the volume of the cylinder to the nearest tenth with a diameter of 10 mm and a height of 3 mm.



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20

Find the volume of the cone to the nearest tenth with a diameter of 2 ft and a height of 8 ft.



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## Question 1

### Applying Volume to Problems

A rectangular flower bed measured 3 yards by 2 yards. The ground must be dug out and filled with topsoil to a height of 0.5 yards.

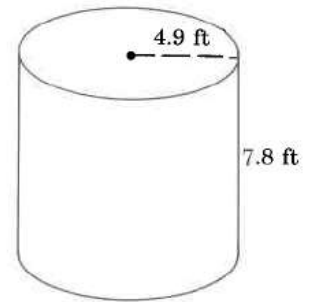
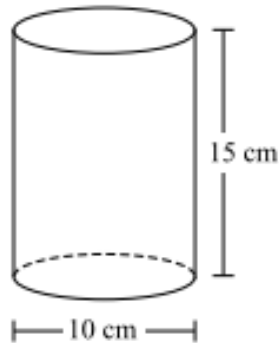
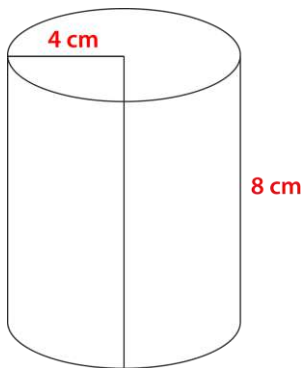
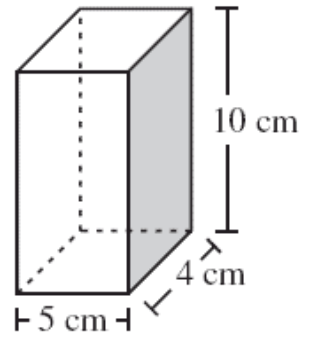
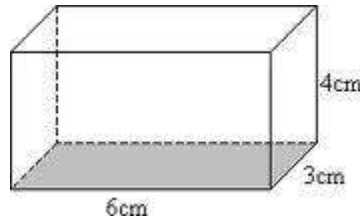
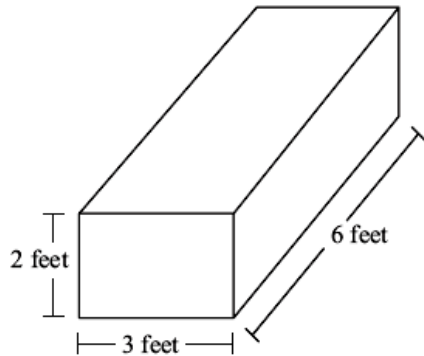
a) Calculate the volume of topsoil required to fill the flower bed.

b) Topsoil is delivered for \$72 per cubic yard. Calculate the cost of the topsoil to be delivered for this flower bed.



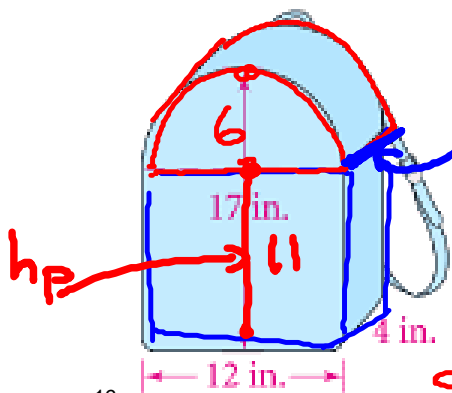
**Question 2**

Calculate the volume of the following geometric shapes.



**Question 3**

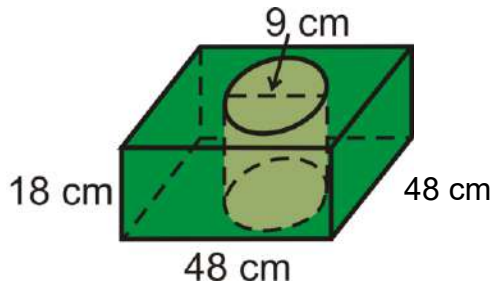
Find the volume of the composite shape.



$$\begin{aligned}
 V &= \text{Rect Prism} + \frac{1}{2} \text{Cylinder} \\
 V &= Bh + \frac{1}{2} \pi r^2 h \\
 &= 12 \cdot 4 \cdot 11 + \frac{1}{2} \pi (6)^2 \cdot 4 \\
 &= 754.19 \text{ in}^3
 \end{aligned}$$

**Question 4**

Find the volume of the space in the square prism not occupied by the cylinder.

**Question 5**

Janelle is building a water garden. The pond is circular and 1.5 m in diameter. It will be filled with water to a height of 0.75 m. Calculate the volume of water in the pond.

**Question 6**

An aquarium weighs 22.5 pounds when empty. The aquarium is 30 inches long, 14 inches wide and is filled with water to a height of 18 inches. Water weighs 0.036 pounds per cubic inch. How much does the aquarium weigh when it is full of water?

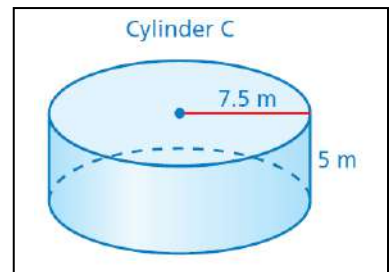
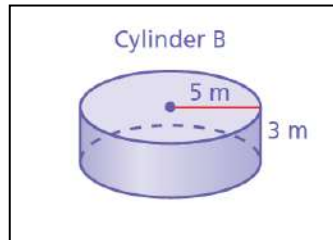
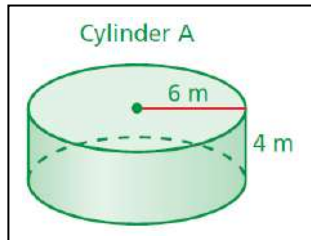
## Similar Solids

**Learning Targets:** Students will be able to use the ratios (areas and volumes) of similar solids to solve real-life problems.

### Similar Solids

- \_\_\_\_\_
- \_\_\_\_\_

Example 1:



Example 1: Which cylinder is similar to Cylinder A?

Cylinder A: Cylinder B

Cylinder A: Cylinder C

\*\*\*Remember that Area is always units \_\_\_\_\_ and Volume is units \_\_\_\_\_

**Areas and Volumes of Similar Solids:**

If the scale factor of two similar solids is  $a:b$ , then

- The ratio of their corresponding areas is \_\_\_\_\_
- The ratio of their corresponding volumes is \_\_\_\_\_

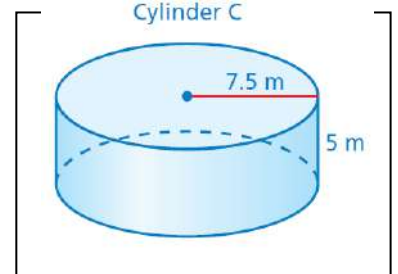
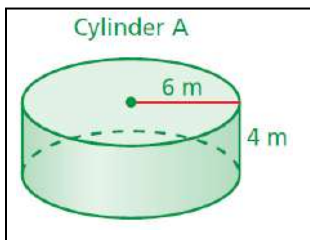
$r = \frac{a}{b}$

$r^2 = \frac{a^2}{b^2}$

$r^3 = \frac{a^3}{b^3}$

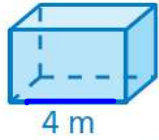
$r, r^2, r^3$  Theorem

Extension of Example 1: Find the Volume of Cylinder A and Cylinder C? Compare them.

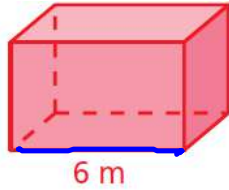


**Example 2:** The solids are similar. Find the surface area or the volume of the second solid.

a.



Surface Area = 336 m<sup>2</sup>



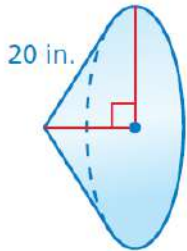
6 m

Area → use  $r^2$   
 $r = \frac{4}{6} = \frac{2}{3}$      $r^2 = \frac{4}{9}$

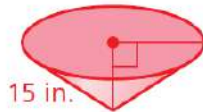
$$\frac{4}{9} = \frac{336}{b} \rightarrow 4b = 9(336)$$

$$b = \frac{9(336)}{4} = 756 \text{ m}^2$$

b.

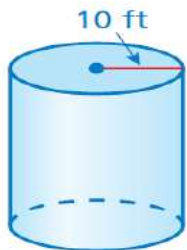


Surface Area = 1800 in.<sup>2</sup>

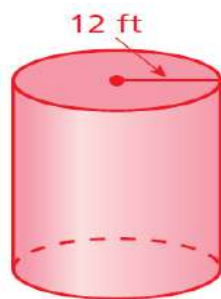


15 in.

c.



Volume = 7850 ft<sup>3</sup>



12 ft

$$r = \frac{10}{12} = \frac{5}{6}$$

$$r^3 = \frac{5^3}{6^3} = \frac{125}{216}$$

$$\frac{125}{216} = \frac{7850}{V}$$

$$125V = 7850(216)$$

$$V = \frac{7850(216)}{125}$$

$$= 13564.8 \text{ ft}^3$$

Ex 3. The ratio of two similar cans of fruit is 4 to 7. The smaller can has a surface area of 220 square centimeters. Find the surface area of the larger can.

Ex 4. Two pyramids have the lateral area of 20 square feet and 45 square feet. The volume of the smaller pyramid is 8 cubic feet. Find the volume of the larger pyramid.

$$r^2 = \frac{20}{45} \quad r^2 = \frac{4}{9} \quad r = \frac{2}{3} \quad r^3 = ? \quad \frac{8}{27}$$

$$\frac{8}{27} = \frac{8}{V_b}$$

Ex 5. Two spheres have a volume of 729 cubic inches and 27 cubic inches.

- What is the ratio of the radii?
- Find the ratio of their surface areas.

$$V_b = 27 \text{ in}^3$$

# Similar Polygons & Solids

Name \_\_\_\_\_

\*\*If the scale factor between 2 similar polygons is  $\frac{a}{b}$ , then

- the ratio of their perimeters is  $\frac{a}{b}$  and the ratio of their areas is  $\frac{a^2}{b^2}$ .

\*\*So...in 3-dimensions: If the scale factor between 2 similar solids is  $\frac{a}{b}$ , then

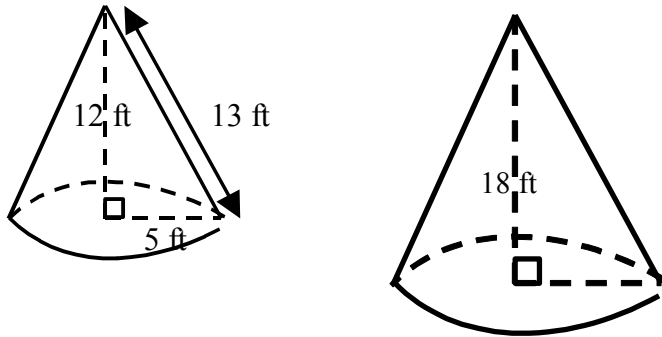
- the ratio of their surface areas is  $\frac{a^2}{b^2}$  and the ratio of their volumes is  $\frac{a^3}{b^3}$ .

Shape	Scale Factor/ Ratio of Perimeters	Ratio of Surface Areas	Ratio of Volumes
Cone	$\frac{2}{3}$		
Sphere	$\frac{4}{6} = \frac{2}{3}$	$(\frac{2}{3})^2 = \frac{4}{9}$	$(\frac{2}{3})^3 = \frac{8}{27}$
Pyramid	$\sqrt{\frac{9}{16}} = \frac{3}{4}$	$r^2 = \frac{9}{16}$	$(\frac{3}{4})^3 = \frac{27}{64}$
Prism	$\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$	$r^2 = (\frac{1}{2})^2 = \frac{1}{4}$	$r^3 = \frac{8}{64} = \frac{1}{8}$
Cylinder		$\frac{49}{64}$	
Cube	$\sqrt[3]{\frac{125}{216}} = \frac{5}{6}$	$r^2 = (\frac{5}{6})^2 = \frac{25}{36}$	$\frac{125}{216}$

- Triangle A is similar to Triangle B. If the scale factor of  $\Delta A$  to  $\Delta B$  is 4 to 5, what is the ratio of the perimeters of  $\Delta A$  to  $\Delta B$ ? \_\_\_\_\_ What is the ratio of the areas of  $\Delta A$  to  $\Delta B$ ? \_\_\_\_\_
- Pyramid X is similar to Pyramid Y. If the scale factor of X:Y is 3:7, what is the ratio of the surface areas of X:Y? \_\_\_\_\_ What is the ratio of the volumes of X:Y? \_\_\_\_\_
- The ratio of the surface areas of two similar cones is 16:49. What is the scale factor between the similar cones? \_\_\_\_\_ What is the ratio of the volumes of the similar cones? \_\_\_\_\_

4. Two spheres have a scale factor of 1:3. The smaller sphere has a surface area of  $16 \text{ ft}^2$ . Find the surface area of the larger sphere.

5. The cones below are similar. What is the volume of the larger cone?



6. Two rectangular prisms are similar and the ratio of their sides is 2:3. The surface area of the larger rectangular prism is  $1944 \text{ cm}^2$ . What is the surface area of the smaller rectangular prism?

7. The ratio of the sides of two similar cubes is 3:4. The smaller cube has a volume of  $729 \text{ m}^3$ . What is the volume of the larger cube?

8. Pyramid X is similar to pyramid Y. The Surface area of pyramid X is  $135 \text{ cm}^2$ , and the surface area of pyramid Y is  $240 \text{ cm}^2$ . If the volume of pyramid X is  $189 \text{ cm}^3$ , then what is the volume of pyramid Y?

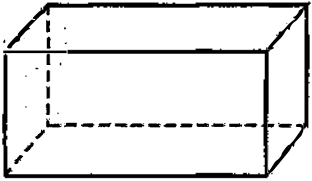
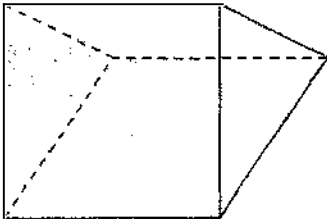
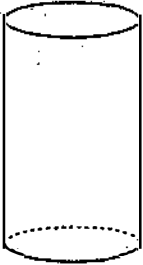
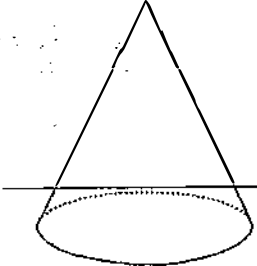
# GEOMETRY

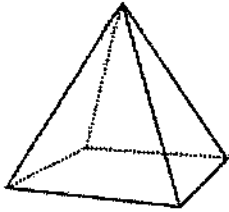
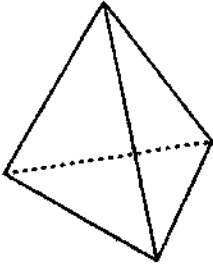
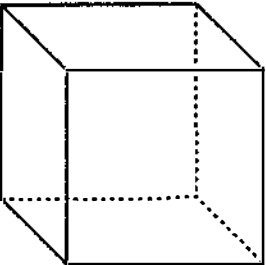
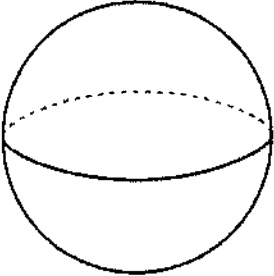
## Cross Sections Activity

Name \_\_\_\_\_

Hour \_\_\_\_\_ Date \_\_\_\_\_

When a solid is cut by a plane, the resulting two-dimensional figure is called a **cross section**.

<b>SOLID</b>	Draw a cross section that is <b>parallel</b> to the base; identify the polygon.	Draw a cross section that is <b>perpendicular</b> to the base; identify the polygon.	Draw a cross section that is <b>slanted</b> to the base; identify the polygon.
			
			
			
			

<b>SOLID</b>	Draw a cross section that is <b>parallel</b> to the base; identify the polygon.	Draw a cross section that is <b>perpendicular</b> to the base; identify the polygon.	Draw a cross section that is <b>slanted</b> to the base; identify the polygon.
			
			
			
			



Name: \_\_\_\_\_

## G.GMD.4 Identifying Three-Dimensional Figures by Rotating Two-Dimensional Figures

For questions 1-3 go the website <http://www.shodor.org/interactivate/activities/3DTransmographer>

### Question 1: The Right Triangle

Create a Polygon with 3 vertices. Use the following points as the vertices.

1: (9, 0)

2: (0, 0)

3: (0, 10)

Click the “Graph” button to graph the polygon.

A. Predict and sketch what three-dimensional shape will be formed when you rotate the right triangle around the y-axis.

B. Under the “Revolve” box, click the last button that says, “across  $x = 0$ .” Then, click the “Revolve” button. What three-dimensional figure is formed by rotating the right triangle around the y-axis? Was your prediction accurate? Explain your reasoning and sketch a picture.

C. Under the “Revolve” box, click the first button that says, “across  $y = 0$ .” Then, click the “revolve” button. What three-dimensional figure is formed by rotating the right triangle around the x-axis? Sketch a picture.

### Question 2: The Rectangle

Create a Polygon with 4 vertices. Use the following points as the vertices.

1: (10, 0)

2: (10, 6)

3: (0, 6)

4: (0, 0)

Click the “Graph” button to graph the polygon.

A. Predict and sketch what three-dimensional shape will be formed when you rotate the rectangle around the y-axis.

B. Under the “Revolve” box, click the last button that says, “across  $x = 0$ .” Then, click “Revolve.” What three-dimensional figure is formed by rotating the rectangle around the y-axis? Was your prediction accurate? Explain your reasoning and sketch a picture.

C. Under the “Revolve” box, click the first button that says, “across  $y = 0$ .” Then, click “Revolve.” What three-dimensional figure is formed by rotating the rectangle around the  $x$ -axis? Sketch a picture.

Question 3: The Trapezoid

Create a Polygon with 4 vertices. Use the following points as the vertices.

- 1: (10, 0)
- 2: (4, 8)
- 3: (0, 8)
- 4: (0, 0)

Click the “Graph” button to graph the polygon.

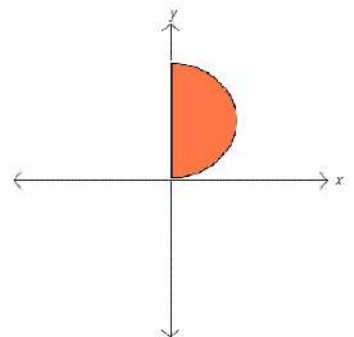
A. Predict and sketch what three-dimensional shape will be formed when you rotate the trapezoid around the  $y$ -axis.

B. Under the “Revolve” box, click the last button that says, “across  $x = 0$ .” Then, click “revolve.” What three-dimensional figure is formed by rotating the trapezoid around the  $y$ -axis? Was your prediction accurate? Explain your reasoning and sketch a picture.

Question 4: The Semicircle

Given the semicircle to the right.

A. What three-dimensional figure is formed when the semicircle is rotated around the  $y$ -axis?



B. What three-dimensional figure is formed when the semicircle is rotated around the  $x$ -axis?

Question 5: Working Backwards

A. What two-dimensional figure is rotated around the  $x$ -axis to form a cone?

B. What two-dimensional figure is rotated around the  $y$ -axis to form a hemisphere?

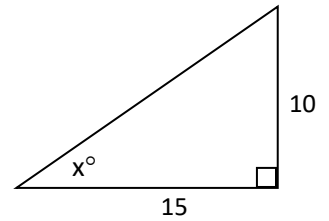
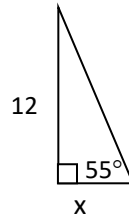
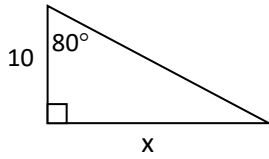
Question 6: Summary

A. What two-dimensional figure would you rotate and around which axis to make an upside down cone? Identify the figure and sketch the picture.

B. Create a three-dimensional figure and describe what two-dimensional shape you rotated to form your figure. With a partner, switch three-dimensional figures. Determine what two-dimensional shape your partner used to create their figure.

Remember this...

1. Using the tangent ratio to solve for missing sides and angles.

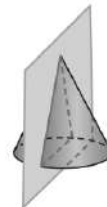
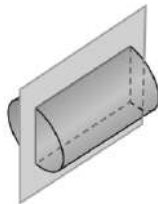
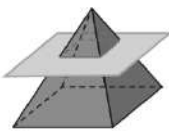


2. A cylinder's volume must be 850 cubic feet while the radius of the cylinder cannot exceed 6 feet. What is the smallest possible height for the cylinder? Round your answer to the nearest tenth of a foot.

3. The length of the radius of a circle is tripled. If the resulting circle has an area of  $72\pi$ , then what was the exact circumference of the original circle? Simplify your answer completely and write it in terms of  $\pi$ .

4. Mrs. Lira is redesigning her backyard garden. If she would like to quadruple the area of her current garden, then by what scale factor should she increase its dimensions?

5. Describe the following cross sections.



6. Sketch and describe the figure that is generated by rotating an isosceles right triangle around a line that contains the triangle's hypotenuse in three-dimensional space.

7. Name a polygon that can be rotated through 3 dimensional space to form a sphere. Sketch the rotation.

8. Which of the following shapes could be formed by the intersection of a plane and a cube?  
Select all that apply.

- a. Equilateral Triangle
- b. Scalene Triangle
- c. Square
- d. Rectangle
- e. Circle

9. Look at the cylinder below.



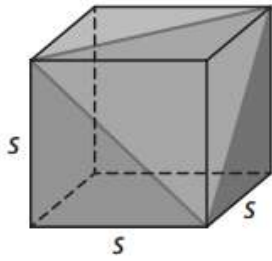
Which of these could NOT be a cross section of the cylinder?

- A circle
- B square
- C triangle
- D rectangle

10. Each side of a square is multiplied by 5. State how each transformation affects the perimeter and area.

11. Describe the effect on the volume of multiplying the length, the width, and the height of a rectangular prism by 2.

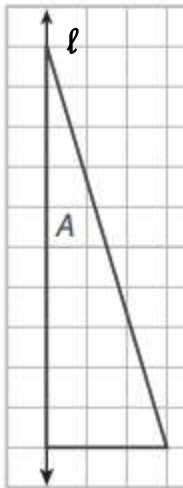
12. Use the figure below to answer the following question.



A cube with sides of length  $s$  is intersected by a plane that passes through three of the cube's vertices, forming the cross section shown. Let  $s = 4$  inches. What type of triangle is in the cross section? Explain.

b. Find the perimeter of the cross section.

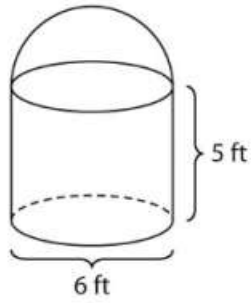
13. The following triangle is rotated 360 degrees around line  $\ell$  to form a 3 dimensional



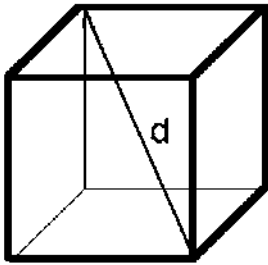
a. Calculate the area of the base of the object.

b. Calculate the volume of the solid formed.

14. A hemisphere sits on top of a cylinder, pictured below.  
Find the volume of the composite figure.



15. Find the diagonal, line  $d$ , if each edge of the cube is 8in long.



16. A sphere sits inside a cylinder and touches all sides. The radius of the sphere is 10 cm.  
What is the volume of the cylinder outside the sphere? Round your answer to the nearest tenth.



# Unit 10

## Probability



# Experimental and Theoretical Probability

*Learning Targets: Students will be able to find the probability of events occurring.*

## KEY TERMS

**Set:**

**Sample Space/Universal Set:**

**Event:**

**Outcome:**

**Probability:**

A probability is expressed as:

$$\frac{\begin{array}{l} \# \text{ of successful} \\ \text{outcomes} \end{array} \boxed{\phantom{000}}}{\begin{array}{l} \# \text{ of total outcomes} \\ \text{in the sample space} \end{array} \boxed{\phantom{000}}} \quad \begin{array}{l} n(A) \\ n(S) \end{array}$$

The numerator is the number of elements in the sample space that meet the desired criteria.

$n(A)$  reads the "number of" outcomes of event A.

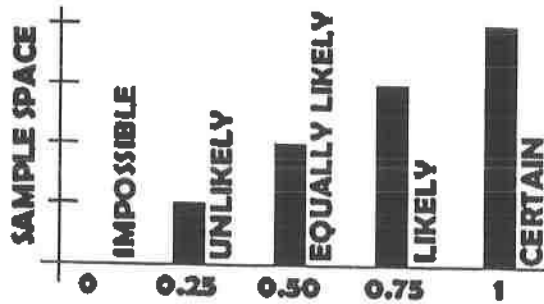
The denominator is the total number of possible outcomes in the sample space.

$n(S)$  reads the "number of" outcomes in the sample space S.

$$*** P(A) = \frac{n(A)}{n(S)} ***$$

Typically, we express probability as a \_\_\_\_\_. However, it is acceptable and common to also use \_\_\_\_\_ or \_\_\_\_\_.

**Complement:**



**Probabilities range from:**

Probability = 0 means impossible

Probability = 1 means certain

**1. An eight sided die is rolled, and the outcome noted. Determine each probability.**

P(3):

P(>3):

P(<10):

P(12):

**2. A six-sided die is rolled, and the outcome noted. Determine each probability.**

a) What is the probability of rolling a 2?

b) Probability of rolling an even #?

c) Probability of rolling a 7?

d) Probability of rolling an integer?

**3. There are 6 green, 3 red, 4 yellow and 5 brown M&M's in a bag. Suppose you select one M&M at random. Find each probability.**

a) P(red)

b) P(yellow or brown)

c) P(orange)

d) P(green or orange)

# Geometric Probability

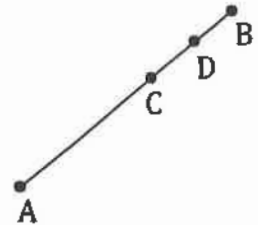
*Learning Targets: Students will be able to find a geometric probability.*

## KEY TERMS

**Geometric Probability:**

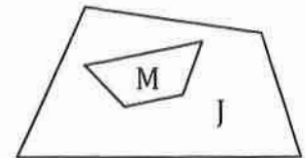
### PROBABILITY AND LENGTH:

Let  $\overline{AB}$  be a segment that contains the segment  $\overline{CD}$ . If a point  $K$  on  $\overline{AB}$  is chosen at random, then the probability that it is on  $\overline{CD}$  is as follows:

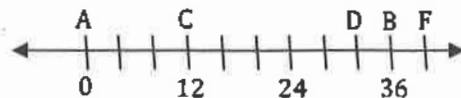


### PROBABILITY AND AREA:

Let  $J$  be a region that contains region  $M$ . If a point on  $K$  in  $J$  is chosen at random, then the probability that it is in region  $M$  is as follows:

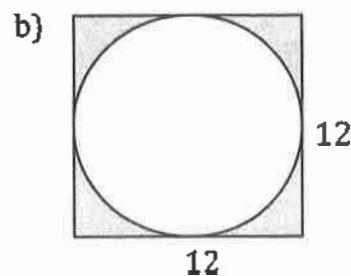
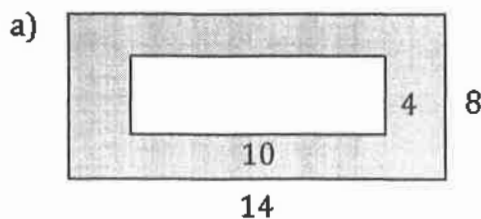


1. Find the probability that a point  $K$ , selected randomly on  $\overline{AF}$ , is on the given segment.

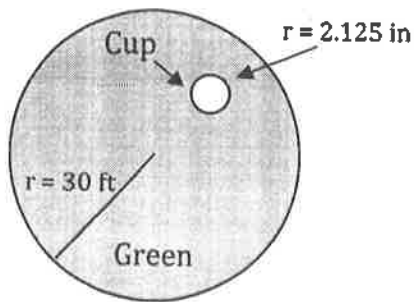


- a)  $\overline{AB}$                       b)  $\overline{CD}$                       c)  $\overline{BD}$                       d)  $\overline{CF}$

2. Find the probability that a randomly chosen point in the figure lies in the shaded region.



3. A golf ball is hit and lands on the circular green shown. The ball is equally likely to land on any point on the green. Find the probability that the ball lands in the cup.



4. You are expecting a visit from a friend anytime between 3:00 pm and 5:00 pm. During this time, you know that you will need to spend 20 minutes cleaning your room. What is the probability that your friend will arrive while you are cleaning your room?

5. A game at the state fair has a circular target with a radius of 10.7 cm on a square board measuring 30 cm on a side. Players win prizes if they throw a dart and hit the circular area only. What is the probability of a player winning with one dart?

Extra Geometric Probability HW

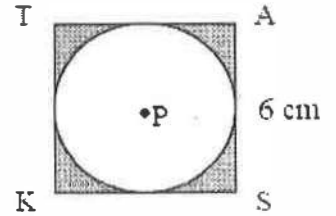
**Remember:** The probability of the event is:  $\frac{\text{the area of the region of the event}}{\text{the area of the entire region}}$

Round to the nearest hundredth and use 3.14 for Pi.

1. a. Find the area of the circle.

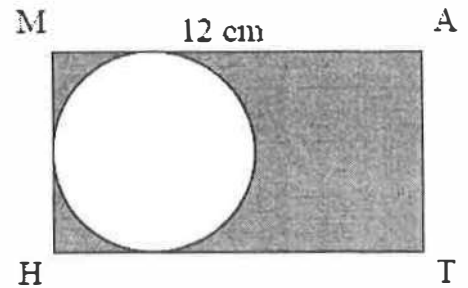
b. Find the area of the square.

c. Find the probability that a dart thrown randomly will hit the circle.  
Give your answer as a fraction, decimal and percent.



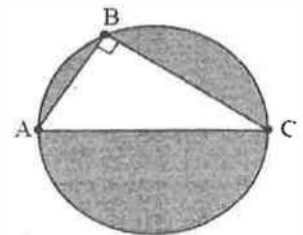
2. In the following diagram MATH is a rectangle with an inscribed circle. The circle has a diameter of 8 centimeters and the rectangle has a height of 12 centimeters (as shown).

Find the probability that a dart thrown randomly will hit the circle. Give your answer as a fraction, decimal and percent.



3. In the following diagram, right triangle ABC is inscribed in a circle. It is given that AC = 26, BC = 24, AB=10 and AC is the diameter of the circle.

Find the probability that a dart thrown randomly will hit the triangle. Give your answer as a fraction, decimal and percent.



Use the picture at the right for Questions 4 - 8.



4. A rectangular field measures 27 feet by 15 feet. Find the area of the field.
  
5. A small shed is on the field. Its dimensions are 8 feet by 10 feet. What is its area?
  
6. What is the probability that a single drop of rain that lands in the field would hit the shed? Give your answer as a fraction, decimal and percent.
  
7. What is the probability that a single drop of rain that lands in the field would *not* hit the shed? Give your answer as a fraction, decimal and percent.
  
8. **CHALLENGE:** There is a large oak tree in one corner whose branches have a diameter of 20 feet. What is the probability that a single drop of rain that lands in the field would miss both the shed and the tree? (Assume the shed is not under the tree.)

Use the dartboard at the right for Questions 9 - 15.

A dartboard is made up of concentric circles with the following radii:

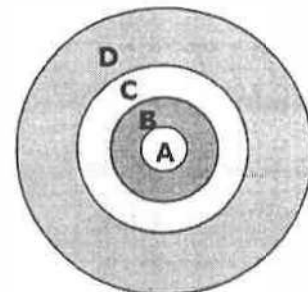
Circle A:  $r = 2$  inches Circle B:  $r = 4$  inches Circle C:  $r = 6$  inches Circle D:  $r = 10$  inches

9. Find the area of circle A.

10. Find the area of circle B that is *not* covered by circle A

11. Find the area circle C that is *not* covered by circle A or B.

12. Find the area of the dartboard that is *not* covered by circles A, B, or C.



The circles on the dartboard are painted on a rectangular piece of corkboard that is 2 feet by 30 inches. Find the probability of each event, assuming the dart always lands on the corkboard.

13. A random dart lands on circle B or C.

14. A random dart lands just on circle B.

15. A random dart will make a bull's eye.

## **Compound Probability- Independent and Dependent**

*Learning Targets: Students will be able to find the probability of independent and dependent events.*

### **KEY TERMS**

**Compound Probability:**

**Independent:**

**Dependent:**

**Mutually Exclusive:**

1. What is the probability of rolling a 6 and then getting a head on a coin flip?
2. What is the probability of rolling a 6 and then rolling a 5?
3. Given a bag of marbles with 3 red, 2 green and 5 yellow. What is the probability of choosing a red, replacing it, and then choosing a green?
4. What is the probability of getting a head on a coin flip and then choosing a purple marble from a bag that has 2 purple, 1 green, and 2 orange marbles?



**REPLACEMENT AND NO REPLACEMENT:**

The terms **replacement** and **no replacement** get used a lot in compound probabilities problems because they describe what you did with the first thing that you selected.

Did you put it back or did you keep it?

P (Getting a green marble, **replacing it, and** getting a green marble)                      **Independent**

P (Picking a black queen, **not replacing it, and** getting an ace)                      **Not Independent**

These two words are huge clues as to whether the events are going to be independent or not.

**REPLACEMENT:**                      Because the item is replaced, it resets the event back to the original arrangement and no probabilities are altered. **Thus REPLACEMENT tells us that the events are INDEPENDENT.**

**NO REPLACEMENT:**                      Because the item is NOT replaced, the probabilities are altered. **Thus NO REPLACEMENT tells us that the events are NOT INDEPENDENT.**

7. What is the probability of drawing two Aces in a row?

**THE ADDITION RULE:**

8. Given a bag of marbles with 2 red, 4 green and 1 blue.

$P(\text{Red or Green}) =$

9. Given a standard deck of cards.

$P(\text{Red or Face Card}) =$

10. Given two dice are rolled.

$P(\text{Even Sum or Sum} < 5) =$

**Probability with Compound Events (Independent and Dependent)**

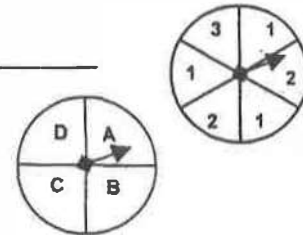
**HW**

Describe the events by writing **I** for *independent event* or **D** for *dependent event*.

1. Ann draws a colored toothpick from a jar. Without replacing it, she draws a second toothpick. \_\_\_\_\_
2. John rolls a six on a number cube and then flips a coin that comes up heads. \_\_\_\_\_
3. Susie draws a card from a deck of cards and replaces it. She then draws a second card. \_\_\_\_\_
4. Seth draws a colored tile from a bag, replaces it; draws a second tile from the bag, replaces it; and then draws a tile a third time from the bag. \_\_\_\_\_
5. You draw a red marble from a bag, and then another red marble (without replacing the first marble)? \_\_\_\_\_

Using the two spinners, find each **compound** probability.

6. P(A and 2) \_\_\_\_\_
7. P(D and 1) \_\_\_\_\_
8. P(B and 3) \_\_\_\_\_
9. P(A and not 2) \_\_\_\_\_



A box contains 3 red marbles, 6 blue marbles, and 1 white marble. The marbles are selected at random, one at a time, and are **not replaced**. Find each **compound** probability.

10. P(blue and red) \_\_\_\_\_
11. P(blue and blue) \_\_\_\_\_
12. P(red and white and blue) \_\_\_\_\_
13. P(red and red and red) \_\_\_\_\_
14. P(white and red and white) \_\_\_\_\_

Suppose that two tiles are drawn from the collection shown at the right. The first tile is replaced before the second is drawn. Find each **compound** probability.



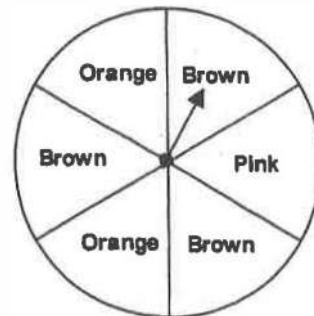
15. P(A and A) \_\_\_\_\_
16. P(R and C) \_\_\_\_\_
17. P(A and not R) \_\_\_\_\_

Suppose that two tiles are drawn from the same collection shown above. The first tile is **not** replaced before the second is drawn. Find each **compound** probability.

18. P(A and A) \_\_\_\_\_
19. P(R and C) \_\_\_\_\_
20. P(A and not R) \_\_\_\_\_

Use the spinner to the right for the next two problems.

21. If you spin the spinner twice, what is the probability of spinning orange then brown? \_\_\_\_\_
22. If you spin the spinner twice, what is the probability of spinning brown both times? \_\_\_\_\_



23. Kevin had 6 nickels and 4 dimes in his pocket. If he took out one coin and then a second coin without replacing the first coin ---
  - (a) what is the probability that both coins were nickels? \_\_\_\_\_
  - (b) what is the probability that both coins were dimes? \_\_\_\_\_
  - (b) what is the probability that the first coin was a nickel and the second a dime? \_\_\_\_\_

# Spring Fun

Two-way frequency tables Pop-QUIZ

name: \_\_\_\_\_

date: \_\_\_\_\_

A total of 176 people were surveyed about what outdoor fitness activity they preferred to do in the summer. The results can be shown in a two-way frequency table. Fill in the table below:

	<i>Swimming</i>	<i>Playing Tennis</i>	<i>Jogging</i>	<i>Total</i>
<i>Teenagers</i>		<b>8</b>	<b>16</b>	<b>62</b>
<i>College students</i>			<b>17</b>	<b>57</b>
<i>Adults</i>		<b>18</b>		
<i>Total</i>	<b>78</b>	<b>43</b>		

1. How many adults preferred swimming? \_\_\_\_\_
2. What was preferred most by adults? \_\_\_\_\_
3. What was preferred least by teenagers? \_\_\_\_\_
4. Overall, what was the most popular choice of all the people? \_\_\_\_\_
5. Were the different age groups of people about equally represented? Why or why not?  
\_\_\_\_\_
6. What is the meaning of the number 16 in the table? \_\_\_\_\_
7. What is the meaning of the number 43 in the table? \_\_\_\_\_
8. Explain the meaning of the marginal frequencies in this table.  
\_\_\_\_\_  
\_\_\_\_\_
9. Without naming each one, explain the meaning of the joint frequencies in this table.  
\_\_\_\_\_  
\_\_\_\_\_
10. How can you check your work to be sure you have filled in the table correctly?  
\_\_\_\_\_

**Activity**  
**Lollipops, anyone?**

name \_\_\_\_\_

date \_\_\_\_\_

Making a two-way frequency table

The Math Club is going to sell candy as a fundraiser. They surveyed 80 students about their favorite candy. The results are shown in the two-way frequency table. Fill in the missing information:

	Lollipop	Peanut butter cups	total
boys	19		
girls			43
total		35	

1. How many of the students surveyed preferred lollipops? \_\_\_\_\_
2. How many of the girls preferred peanut butter cups? \_\_\_\_\_
3. How many boys answered the survey? \_\_\_\_\_
4. Were boys and girls both evenly represented in this survey? Why or why not?  
\_\_\_\_\_
5. Explain what the number 19 means in this table.  
\_\_\_\_\_
6. Explain what the number 35 means in this table.  
\_\_\_\_\_
7. What is the meaning of each marginal frequency in this table?  
\_\_\_\_\_
8. What is the meaning of each joint frequency in this table?  
\_\_\_\_\_

## Conditional Probability

*Learning Targets: Students will be able to use two-way frequency tables to find conditional probability.*

### KEY TERMS

**Conditional Probability:**

**Two-Way Frequency Table:**

**Marginal Frequencies:**

1. A group of 50 students were asked about iPod and Smart phone ownership, 28 owned an iPod, 38 owned a smart phone, and 20 owned both. Construct the two-way frequency table.

2. Use the two-way frequency table created above to answer the following questions.

$$P(\text{iPod}) =$$

$$P(\text{Smart Phone}) =$$

$$P(\text{Not iPod}) =$$

$$P(\text{Not Smart Phone}) =$$

$$P(\text{Smart Phone and iPod}) =$$

$$P(\text{No Smart Phone and No iPod}) =$$

$$P(\text{No Smart Phone and iPod}) =$$

$$P(\text{Smart Phone or iPod}) =$$

$$P(\text{No Smart Phone or No iPod}) =$$

3. Use the two-way frequency table from above to answer the following questions.

$$P(\text{Smart Phone}|\text{iPod}) =$$

$$P(\text{No Smart Phone}|\text{iPod}) =$$

$$P(\text{iPod}|\text{No Smart Phone}) =$$

$$P(\text{No iPod}|\text{No Smart Phone}) =$$

4. Is owning an iPod independent of owning a smart phone?

5. If the sample space is the sides of 12 sided dice, create a two way table that compares the event of rolling an even number and rolling a multiple of 3.

	Not		
	Multiply of 3	Multiple of 3	Total
Even			
Not Even (Odd)			
Total			<b>12</b>

6. From the chart, determine the probabilities.

$$P(\text{Mult. 3}) =$$

$$P(\text{Mult. 3})^c =$$

$$P(\text{Even}) =$$

$$P(\text{Even})^c =$$

$$P(\text{Mult. 3 and Even}) =$$

$$P(\text{Not Mult. 3 and Not Even}) =$$

$$P(\text{Mult.3 or Even}) =$$

$$P(\text{Mult. 3}|\text{Even}) =$$

$$P(\text{Even}|\text{Mult. 3}) =$$

Is rolling an even independent of rolling a multiple of 3?

Conditional Probability and Tree Diagrams

- 1) A survey of CHS students found that 36% said that they would be interested in going to Saturn. Of those who wanted to go to Saturn, 60% were not seniors. Of those who did not want to go to Saturn, 30 % were seniors. Create a tree diagram for this situation.



What is the probability that a randomly selected

- a) Student wanted to go to Saturn?
  
- b) Student was a senior and wanted to go to Saturn?
  
- c) Student was a senior?
  
- d) Senior wanted to go to Saturn?
  
  
- e) Saturn wannabe was a senior?

Fill in the two-way (contingency) percent table with the information.

	Saturn	Not Saturn	Totals
Senior			
Not Senior			
Totals			100%

If 500 students were surveyed, fill in the two-way (contingency) counts table with the information

	Saturn	Not Saturn	Totals
Senior			
Not senior			
Totals			500

- 2) When the male students at CHS were asked, 50% said they do not date someone from CHS. When the female students were asked, 40% said they do not date someone from CHS. The male students make up 52 % of the student population. Draw a tree diagram to represent this situation.



Fill in the two-way (contingency) percent table with the information.

	Date CHS Student	Don't date CHS student	Totals
Male			
Female			
Totals			100%

What is the probability that a randomly selected

- Student does not date someone from CHS?
- Student is female?
- Student is female and does not date someone from CHS?
- Student who dates someone from CHS is male?

If 500 students were surveyed, fill in the two-way counts table with the information.

	Date CHS Student	Don't date CHS student	Totals
Male			
Female			
Totals			500

- How many were male?
- How many were females who did not date someone from CHS?



## Unit 10 Review

Name \_\_\_\_\_

1. Define and give an example of the following
  - a. Independent Events
  - b. Dependent Events
  - c. Mutually Exclusive Events
2. Tyler has 54 rock songs, 92 dance songs and 12 classical songs on her playlist. If Tyler's iPod randomly selects a song from the playlist, what is the probability that the song will **not** be a classical song?
3. If the probability of an event is 0.99, which of the following could describe the event? Select Yes or No.
  - A) The event might occur.  
 Yes    No
  - B) There is a small chance that the event will occur.  
 Yes    No
  - C) The event is likely to occur.  
 Yes    No
  - D) The event will definitely occur.  
 Yes    No
4. The probability that Amir will make a basket is 0.4. The probability that Alex will make a basket is 0.6. What is the probability that Amir and Alex each make the first basket they shoot?
5. In a round robin, tennis tournament, each player must play each other player once. If there are a total of 7 tennis players, how many tennis matches will there be in the tournament?

6. An airport employee collects data on 180 random flights that arrive at the airport. The data was collected and organized in the two-way table below.

	Late Arrival	On Time	Total
Domestic Flight	12	108	120
International Flight	6	54	60
Total	18	162	180

Is a late arrival independent of the flight being an international flight? Why or why not?

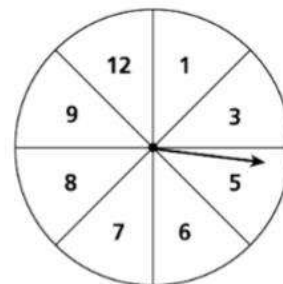
7. Fill out the table below and use the table below to answer the questions.

	Freshman	Sophomore	Junior	Senior	Total
Girl		104	100	94	396
Boy	102			108	412
Total	200	210	196		808

a. Find the probability of each of the following

- i. The student is a senior \_\_\_\_\_
- ii. The student is a boy \_\_\_\_\_
- iii. The student is a senior girl \_\_\_\_\_
- iv. The student is a senior or a girl \_\_\_\_\_
- v. The student is a girl given the student is a senior \_\_\_\_\_
- vi. If the student is a girl she is a sophomore \_\_\_\_\_

8. You spin the spinner to the right two times. What is the probability that you spin an even number on your first spin followed by an odd number on the second spin?

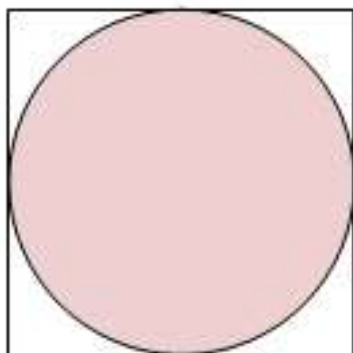


9. In a class of 36 students, one student is chosen at random to represent the class in ASB. Another student is chosen at random to be the alternate representative. Are these events independent or dependent? Explain your answer.

10. A bag contains 8 yellow marbles, 4 blue marbles, and 5 red marbles. You choose a marble, put it aside, and then choose another marble.

- a. What is the probability that you choose two yellow marbles?
- b. What is the probability that you choose two marbles that are **not** yellow?
- c. What is the probability that you choose a blue marble first and then a red marble?

11. A circle with a diameter of 20 cm is placed inside a square so that it is touching every side of the square, as pictured below.

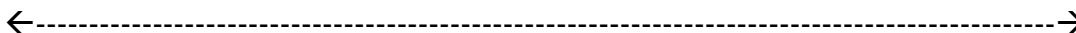
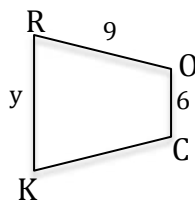
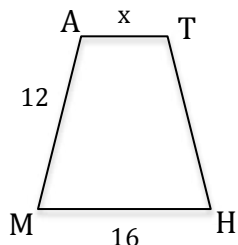


- a. Label the diagram below given the information above.
- b. Assuming a thrown dart must land within the square, what is the probability that a dart thrown will land inside the circle? Give your answer as a percent.
- c. Using your answer from part (b) as justification, is it more likely for the dart to land in the square or circle? Explain your answer.

# Final Review

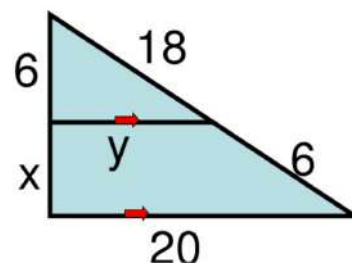
## Review Day 1 Similarity

1) Given that  $MATH \sim ROCK$ , place the following values on the number line:  $x, y, \frac{6}{x}, \frac{y}{16}$ .



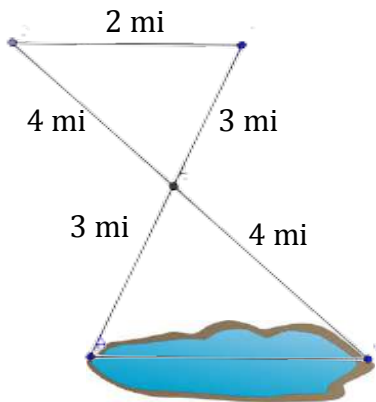
2) In the diagram to the right, the lines are parallel as shown.

- a) What theorem claims that the two triangles are similar.
- b) The proportion below is correct for solving for  $x$ , but the one for  $y$  is wrong. Correct it, and then solve for both  $x$  &  $y$ .

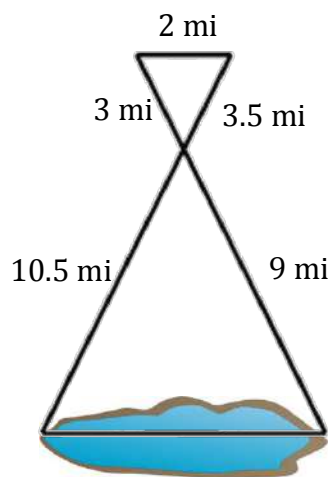


$$\frac{6}{x} = \frac{18}{6} \qquad \frac{y}{20} = \frac{18}{6}$$

3) What is the length of the lake in each diagram?



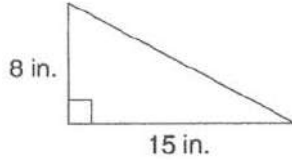
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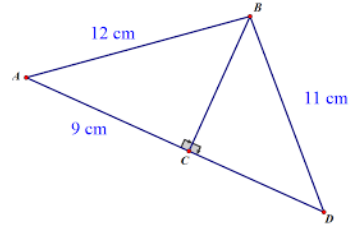
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4-5) Solve for the missing lengths of the triangles.

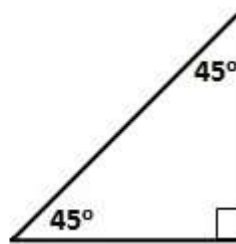
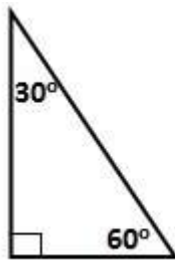
4)



5)

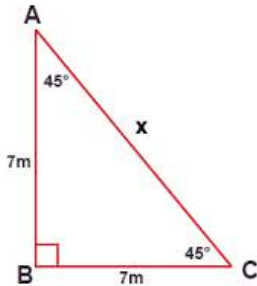


6) Show the ratios in the diagrams for the special right triangles below

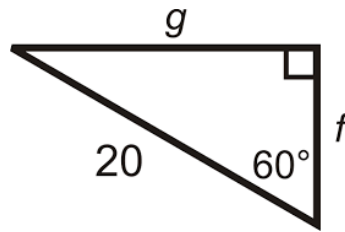


7-10) Solve for the indicated variables.

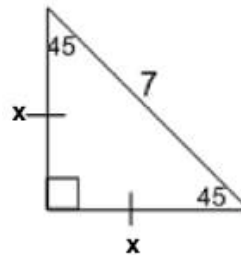
7)



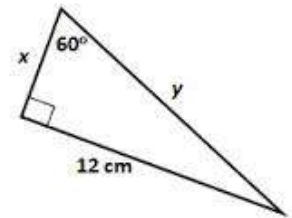
8)



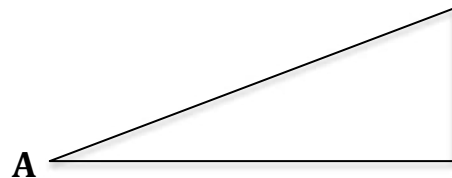
9)



10)



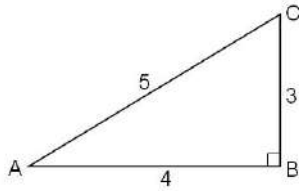
11) Measure angle A and the three sides of the triangle. Determine the tangent of angle A by the ratio of two sides. Then show that the tangent function on your calculator will yield the same answer.



Tan( \_\_\_ ) = \_\_\_\_\_ by your measurements.

Tan( \_\_\_ ) = \_\_\_\_\_ by the tangent function.

12) For the given triangle determine the three trigonometric ratios for each of the acute angles

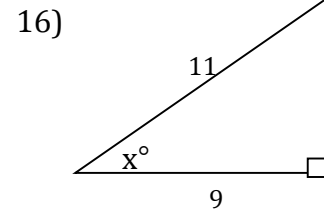
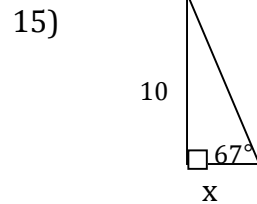
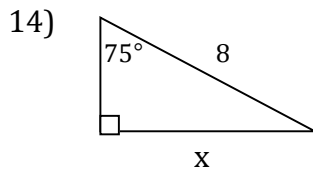


Sin A =                      Sin C =  
 Cos A =                      Cos C =  
 Tan A =                      Tan C =

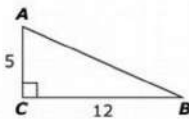
13) Consider a triangle ABC for which C is the right angle. Place the possible measures offered in the appropriate column of the chart.

$\cos A < \sin A$	$\cos A = \sin A$	$\cos A > \sin A$
Possible Measures of Angle A		
5°	25°	35°
55°	65°	75°
	45°	85°

14-16) Solve for the indicated variables.



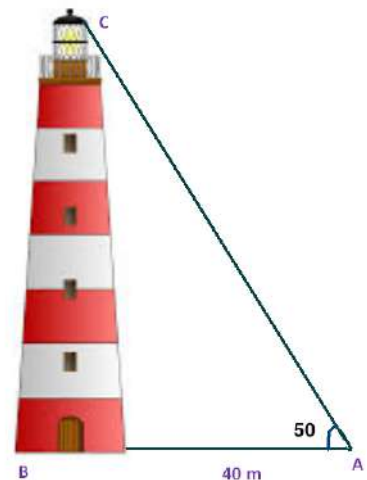
17) Consider this right triangle.



Determine if each expression is equivalent to the length of  $\overline{AC}$ . Select Yes or No for each expression.

	Yes	No
$13\sin(B)$	<input type="checkbox"/>	<input type="checkbox"/>
$13\cos(A)$	<input type="checkbox"/>	<input type="checkbox"/>
$12\tan(A)$	<input type="checkbox"/>	<input type="checkbox"/>
$12\tan(B)$	<input type="checkbox"/>	<input type="checkbox"/>

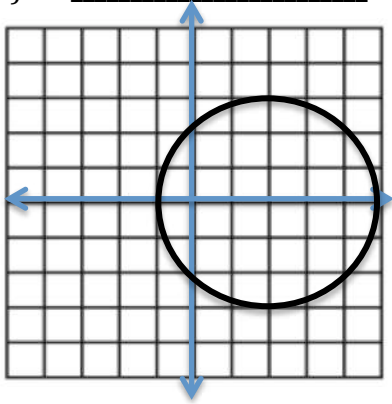
18) Find the height of the lighthouse.



## Review Day 2 Circles

19-21) Write the equation of the circle that is represented by the information given.

19) \_\_\_\_\_

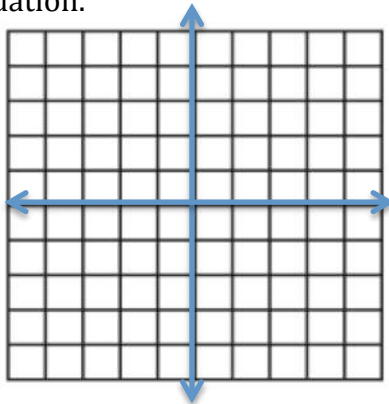


20) Center  $(-4, 3)$ , radius of 7 \_\_\_\_\_

21) Center  $(6, 2)$ , passing through the point  $(10, 5)$ .  
\_\_\_\_\_

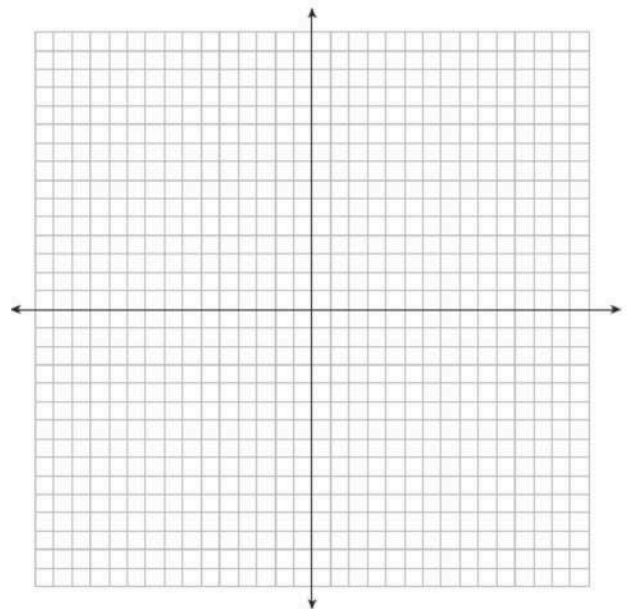
22-23) Given the equation of the circle:  $(x - 2)^2 + (y + 3)^2 = 4$

22) Graph equation.



23) Determine if  $(1, -1)$  is on the circle, both algebraically and graphically.

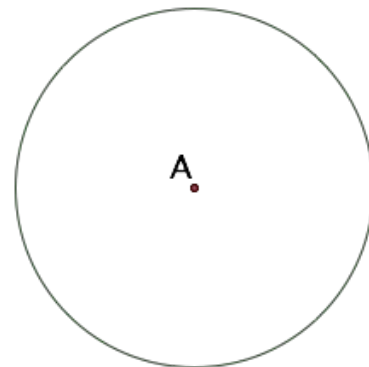
24) A circle has its center at  $(6, 7)$  and goes through the point  $(1, 4)$ . A second circle is tangent to the first circle at the point  $(1, 4)$  and has the same area. What are the possible coordinates for the center of the second circle? Show your work or explain how you found your answer.



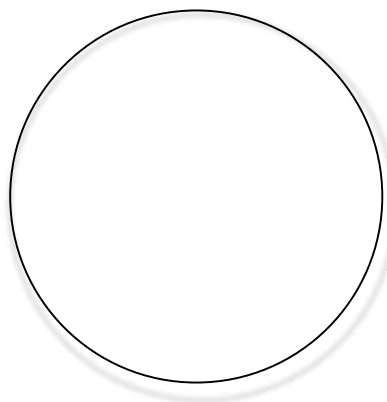
**Note:** After completing #19-23 by hand, check your answer with graphing software.



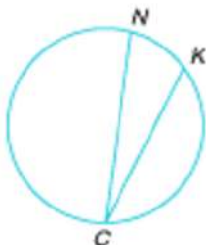
- 25) a) In the circle below, draw a central angle of any measure, and label the intercepted arc AB.  
 b) Draw an inscribed angle anywhere on the circle that intercepts arc AB.  
 c) Measure your central and inscribed angles. What do these measures say about the relationship between an inscribed angle and its intercepted arc?



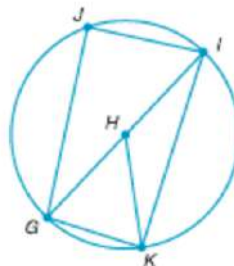
- 26) Use two right inscribed angles to find the center this circle.



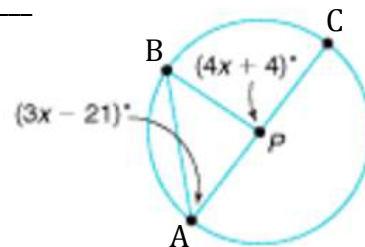
27)  $m\angle C = 23^\circ$ ,  $m\widehat{NK} = \underline{\hspace{1cm}}^\circ$



28) Center H,  $m\widehat{GK} = 50^\circ$ ,  
 $m\angle J = \underline{\hspace{1cm}}^\circ$ ,  $m\angle GKH = \underline{\hspace{1cm}}^\circ$



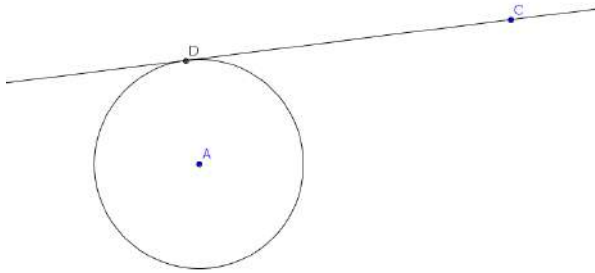
29)  $m\widehat{AB} = \underline{\hspace{1cm}}^\circ$



30-32) Draw and measure the designated segments and angles, then state the theorem your diagram supports.

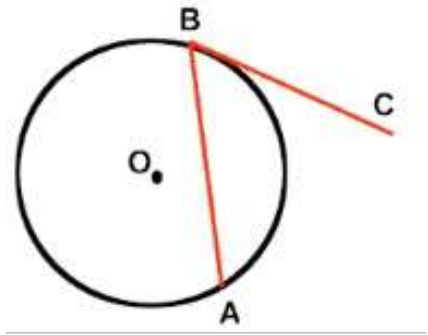
30) Draw  $\overline{OP}$ .  $m\angle ADC = \underline{\hspace{2cm}}^\circ$

Supports that \_\_\_\_\_



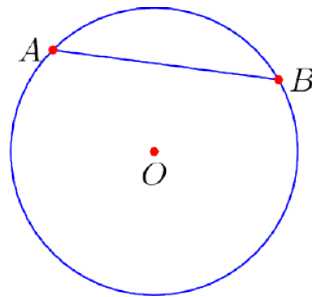
31) Draw  $\overline{OB}$  &  $\overline{OA}$ .  $m\angle AOB = \underline{\hspace{2cm}}^\circ$   
 $m\angle AOB = \underline{\hspace{2cm}}^\circ$ ,  $m\angle ABC = \underline{\hspace{2cm}}^\circ$

Supports that \_\_\_\_\_

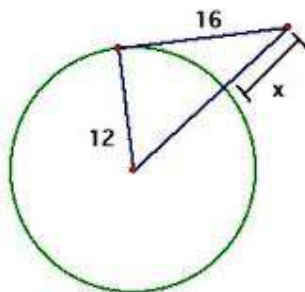


32) Draw M as the midpoint of  $\overline{AB}$ . Draw  $\overline{OM}$ .  $AM = \underline{\hspace{2cm}}$ ,  $BM = \underline{\hspace{2cm}}$ .

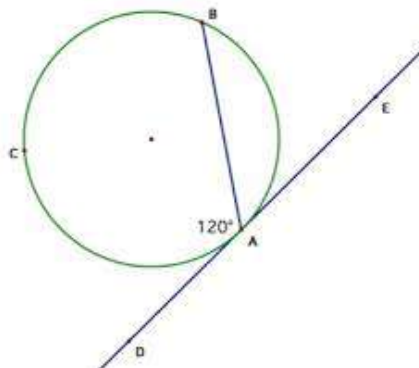
Supports that \_\_\_\_\_



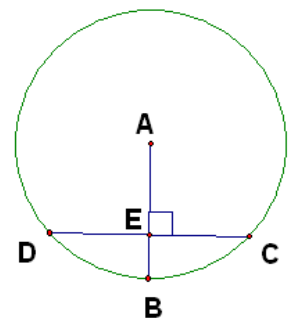
33)  $x = \underline{\hspace{2cm}}$



34)  $m\widehat{ACB} = \underline{\hspace{2cm}}^\circ$ ,  $m\widehat{AB} = \underline{\hspace{2cm}}^\circ$

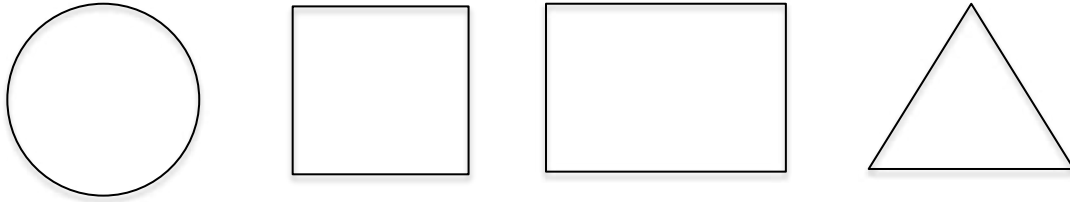


35)  $DE = 8$ ,  $AE = 6$ ,  $AC = \underline{\hspace{2cm}}$



## Review Day 3 3-D Modeling

36) Which of these cross sections can be made from a cylinder.



37) Which of these solids can yield a cross section of a rectangle (or square)?

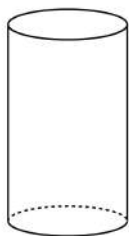


38) Match the 2D dimensional figures to the solid that it can form by a rotation. Also, circle any figures that do not form one of the solids by rotation, and circle any solids that cannot be formed by a rotation.

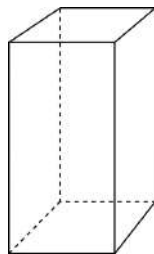
- |                         |                       |
|-------------------------|-----------------------|
| a. Right Triangle       | i. Triangular Prism   |
| b. Semi-Circle          | ii. Cylinder          |
| c. Square               | iii. Cone             |
| d. Equilateral Triangle | iv. Square Pyramid    |
| e. Trapezoid            | v. Sphere             |
| f. Circle               | vi. Rectangular Prism |

39-42) Given the class models of the following solids, collect their measurements and calculate their volumes.

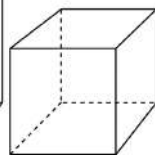
39)



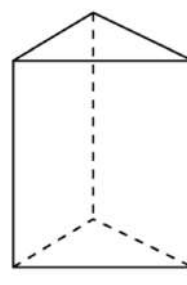
40)



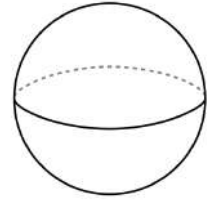
41)



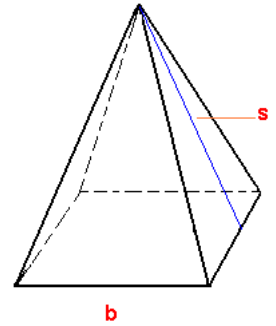
42)



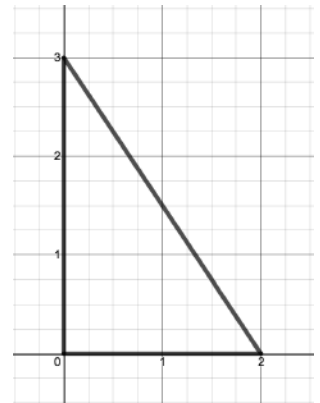
- 43) From the class model of the sphere, measure the circumference and use it to calculate the volume of the sphere.



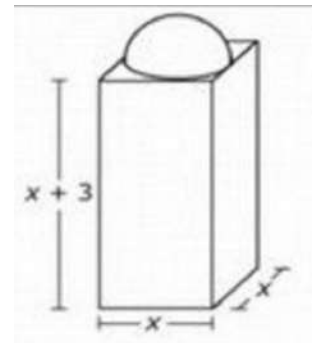
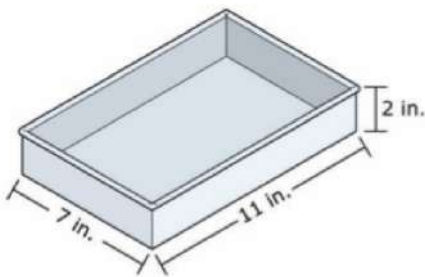
- 44) From the class model of the square pyramid, measure the base edge and the slant height. Use these measurements to calculate the volume.



- 45) Which will create a solid with a larger volume: Rotating the triangle about the y-axis, or about the x-axis?



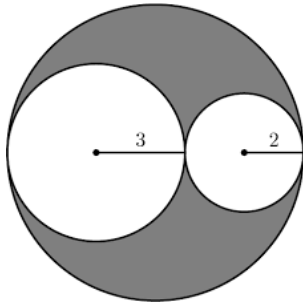
- 46) Hannah makes 6 cups of cake batter. She pours and levels all the batter into a rectangular cake pan with a length of 11 inches, a width of 7 inches, and a depth of 2 inches. One cubic inch is approximately equal to 0.069 cup. What is the depth of the batter in the pan when it is completely poured in? (Round to the nearest  $\frac{1}{8}$  of an inch.)



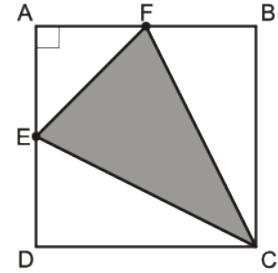
- 47) The figure shown is composed of a rectangular prism and half of a sphere. The diameter of the sphere is  $x$ . Write an expression for the volume of the solid in terms of  $x$  and  $\pi$ .

48-49) Find the ratio of the area of the shaded region to the area of the entire figure.

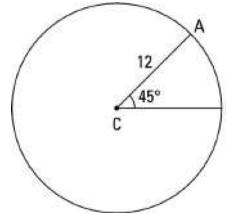
48)



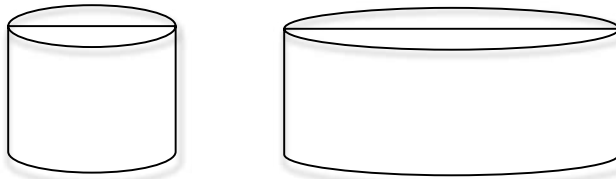
49) E & F are midpoints.



50) Find the arc length and area of the sector.



51) A cylinder has the diameter of its base doubled to form a new cylinder with the same height.

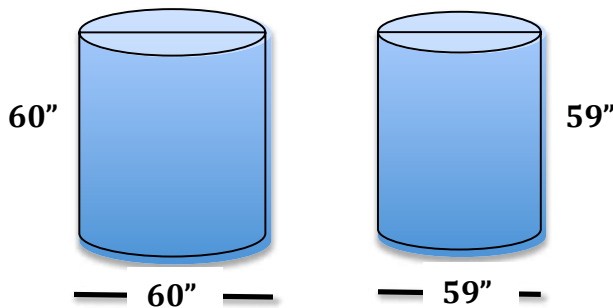


a) What is the ratio of the larger volume to the smaller? \_\_\_\_\_ : \_\_\_\_\_

b) What is the ratio of the smaller volume to the larger? \_\_\_\_\_ : \_\_\_\_\_

52) The volume of a 7" tube of toothpaste is 6.4 oz. Assuming that the tubes are similar in shape, how long is a travel tube that contains 0.85 oz?

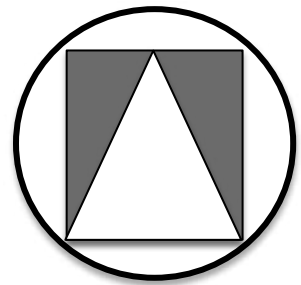
53) In the *Shorter Shovel* problem, two different size holes were dug. One hole was 60" in diameter and 60" deep. The second hole was 59" in diameter and 59" deep. Assume that the difference in the dirt dug between the two holes was placed in a pile. Calculate the height of the cone created by the extra dirt. (Note: A cone made by loose dirt will form a 45-degree angle between the base and the slant height.)



## ***Review Day 4*** **Probability**

54) What is the probability that a roll of two dice will result in both dice being even?

55) Inscribed in a circle is a square that has side lengths of 10 cm, and an inscribed triangle with the same base and height as the square. The probability of randomly choosing a point within the circle that is within the shaded area = \_\_\_\_%



Bonus Challenge: Prove that this probability is true for side length.

56) If you draw a card from a deck of 52 cards that has 4 Aces, what are the chances of drawing an Ace three times in a row? Assume that each card drawn is not replaced.

57)

Jaime randomly surveyed some students at his school to see what they thought of a possible increase to the length of the school day. The results of his survey are shown in the table below.

**Lengthening School Day Survey**

<b>Grade</b>	<b>In Favor</b>	<b>Opposed</b>	<b>Undecided</b>
9	12	6	9
10	15	3	11
11	8	12	10
12	5	16	9

**Part A**

A newspaper reporter will randomly select a Grade 11 student from this survey to interview. What is the probability that the student selected is opposed to lengthening the school day?

**Part B**

The newspaper reporter would also like to interview a student in favor of lengthening the school day. If a student in favor is randomly selected, what is the probability that this student is also from Grade 11?

**Part C)** Are the positions of being opposed and being a Senior independent or dependent?