3.7 Translations

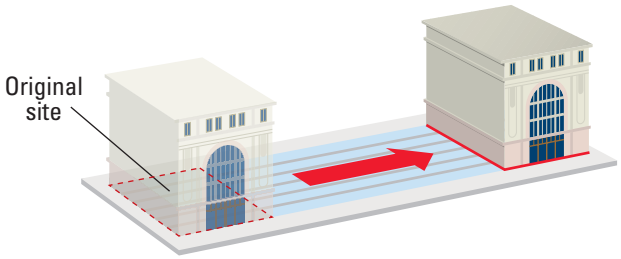
Goal

Identify and use translations.

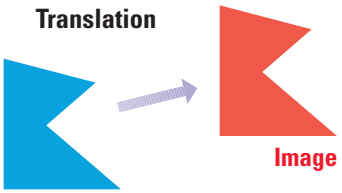
Key Words

- translation
- image
- transformation

In 1996, New York City's Empire Theater was slid 170 feet up 42nd Street to a new location.



A slide is also called a **translation**. The new figure after the translation is the **image**. In this book, the original figure is given in blue and its image in red, as shown at the right.



A translation is one kind of **transformation**. A transformation is an operation that *maps,* or moves, a figure onto an image. You will study other transformations in Lessons 5.7, 7.6, and 11.8.

Student Help

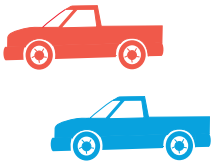
VOCABULARY TIP
Use the following relationship to help you remember that a translation is a slide:

translation
slide

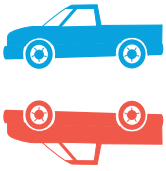
EXAMPLE 1 Compare a Figure and Its Image

Decide whether the red figure is a translation of the blue figure.

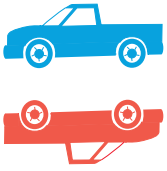
a.



b.



c.



Solution

a. Yes, this is a translation.

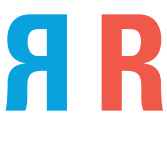
b. No, this is *not* a translation. The image is a mirror image of the original figure.

c. No, this is *not* a translation. The original figure is rotated.

Checkpoint ✓ Compare a Figure and Its Image

Decide whether the red figure is a translation of the blue figure.

1.



2.

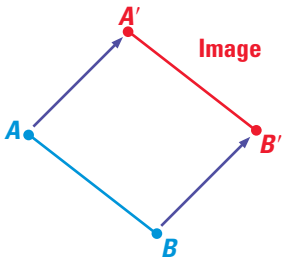
3.

Student Help

READING TIP
In the diagram at the right, A' is read as "A prime."

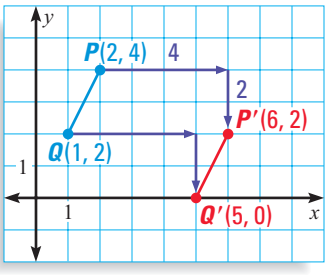
Labeling Translations When labeling points on the image, write the prime symbol ($'$) next to the letter used in the original figure, as shown at the right.

In a translation, segments connecting points in the original figure to their corresponding points in the image are congruent and parallel. For example, $\overline{AA'}$ and $\overline{BB'}$ at the right are congruent and parallel.



EXAMPLE 2 Describe Translations

Describe the translation of the segment.



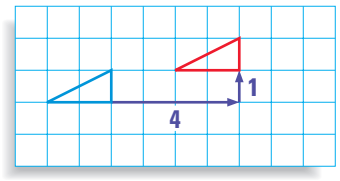
Solution

Point P is moved 4 units to the right and 2 units down to get to point P'. So, every point on $\overline{PQ}$ moves 4 units to the right and 2 units down.

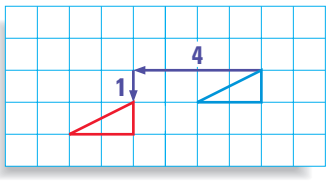
Translations in a coordinate plane can also be described using the following coordinate notation:

$$(x, y) \rightarrow (x + a, y + b)$$

Each point shifts a units horizontally (right or left) and b units vertically (up or down). When moving right or up, *add* the number of units. When moving left or down, *subtract* the number of units. Here are some examples:



$(x, y) \rightarrow (x + 4, y + 1)$



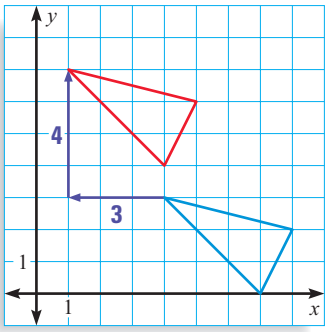
$(x, y) \rightarrow (x - 4, y - 1)$

Student Help
CLASSZONE.COM

MORE EXAMPLES
More examples at classzone.com

EXAMPLE 3 Use Coordinate Notation

Describe the translation using coordinate notation.



Solution

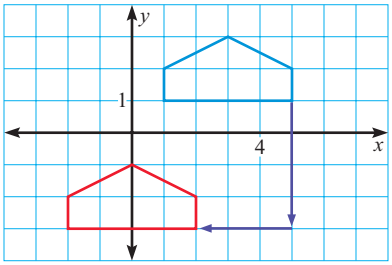
Each point is moved 3 units to the left and 4 units up.

ANSWER The translation can be described using the notation $(x, y) \rightarrow (x - 3, y + 4)$.

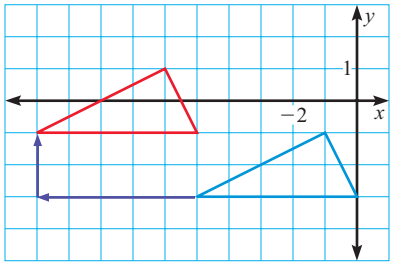
Checkpoint Describe Translations

Describe the translation using words and coordinate notation.

4.



5.

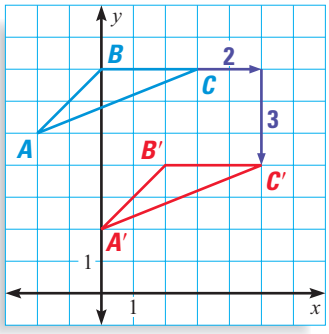


EXAMPLE 4 Draw Translated Figures

Draw the triangle given by points $A(-2, 5)$, $B(0, 7)$, and $C(3, 7)$. Then draw the image of the triangle after the translation given by $(x, y) \rightarrow (x + 2, y - 3)$.

Solution

First, sketch $\triangle ABC$ as shown. To find points A', B', and C', start at points A, B, and C, and slide each point 2 units to the right and 3 units down.



$\triangle ABC$	$\triangle A'B'C'$
$A(-2, 5)$	$A'(0, 2)$
$B(0, 7)$	$B'(2, 4)$
$C(3, 7)$	$C'(5, 4)$

Notice that each x-value of $\triangle A'B'C'$ is 2 units more than the corresponding x-value of $\triangle ABC$ and each y-value of $\triangle A'B'C'$ is 3 units less than the corresponding y-value of $\triangle ABC$.

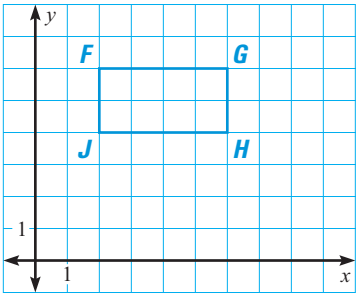
Student Help

READING TIP
In this book, shapes are named by listing in order the labels at their corners. For example, the blue triangle in Example 4 is named $\triangle ABC$.

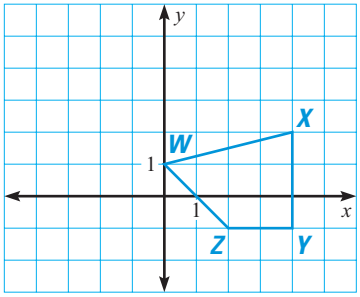
Checkpoint Draw Translated Figures

Draw the image of the figure after the given translation.

6. $(x, y) \rightarrow (x + 3, y - 2)$



7. $(x, y) \rightarrow (x - 3, y + 4)$



3.7 Exercises

Guided Practice

Vocabulary Check

1. What is a *translation*?

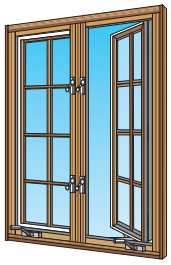
2. Complete the statement: A translation shows a blue triangle and a red triangle. The blue triangle is the original figure and the red triangle is the __?__.

Skill Check

Window Frames **Decide whether "opening the window" is a translation of the moving part.**

3. Double hung

4. Casement



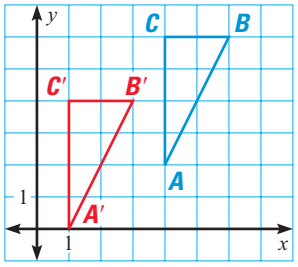
5. Sliding

Decide whether the statement is *true* or *false*. Explain.

6. The red figure is a translation of the blue figure.

7. To move from $\triangle ABC$ to $\triangle A'B'C'$, shift 3 units to the right and 2 units up.

8. The translation from $\triangle ABC$ to $\triangle A'B'C'$ is given by $(x, y) \rightarrow (x - 3, y - 2)$.



Practice and Applications

Extra Practice

See p. 680.

Compare a Figure and Its Image **Decide whether the red figure is a translation of the blue figure.**

9.

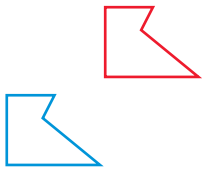
10.

11.

Homework Help

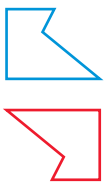
Example 1: Exs. 9–14
Example 2: Exs. 15–21
Example 3: Exs. 22, 23
Example 4: Exs. 38–41

12.



13.

14.



Matching Translations **Match the description of the translation with its diagram.**

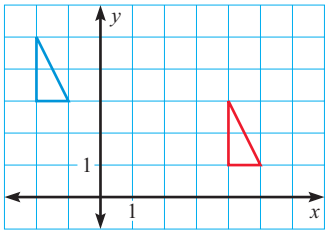
15. 4 units right and 3 units up

16. 6 units right and 2 units down

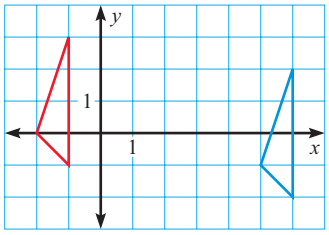
17. 7 units left and 1 unit up

18. 5 units right and 2 units down

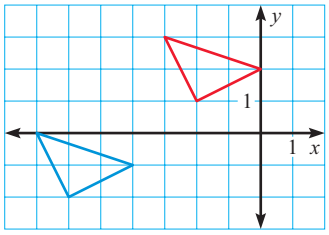
A.



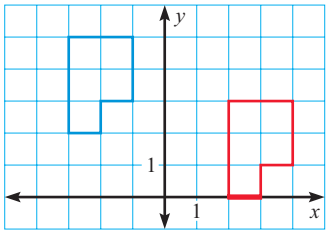
B.



C.

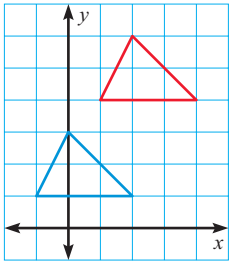


D.

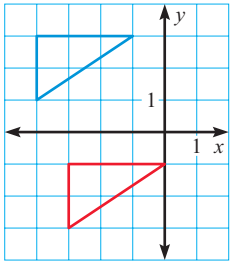


Describing Translations **Describe the translation using words.**

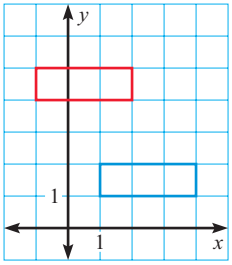
19.



20.

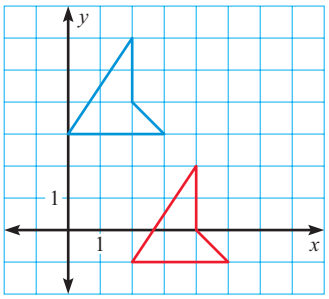


21.

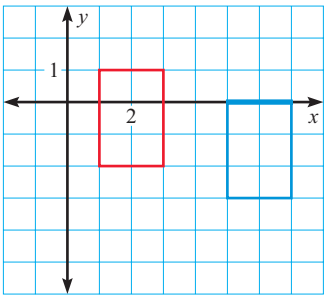


Coordinate Notation **Describe the translation using coordinate notation.**

22.



23.



A Point and Its Image **Find the image of the point using the translation $(x, y) \rightarrow (x + 4, y - 3)$.**

24. (2, 5)

25. (−3, 7)

26. (−1, −4)

27. (4, −6)

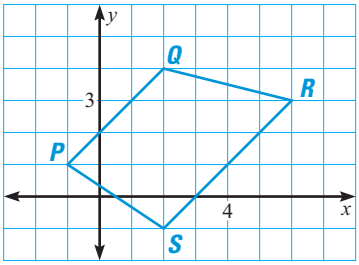
28. (0, 0)

29. (−4, 3)

30. (3, −4)

31. (−1, −1)

Finding an Image **Find the coordinates of P', Q', R', and S' using the given translation.**



32. $(x, y) \rightarrow (x + 1, y - 4)$

33. $(x, y) \rightarrow (x - 3, y + 2)$

34. $(x, y) \rightarrow (x + 5, y - 5)$

35. $(x, y) \rightarrow (x, y - 3)$

Link to Chess

CHESS TEAMS The chess team at the University of Maryland, Baltimore County (UMBC), has become a strong, nationally recognized team. This is due in part to the efforts of UMBC President Dr. Freeman A. Hrabowski, III (second from the left).

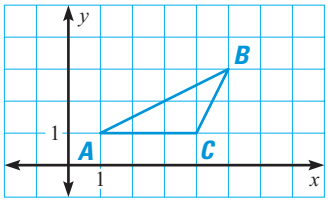
Chess **In chess, six different kinds of pieces are moved according to individual rules. The board below shows some moves for the Knight (the piece shaped like a horse).**

36. Describe the translation used by the White Knight to capture the Black Pawn.

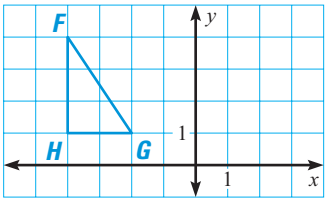
37. Assume that the White Knight has taken the place of the Black Pawn. Describe the translation used by the Black Knight to move to capture the White Knight at its new location.

Drawing Translated Figures **Draw the image of the figure after the given translation.**

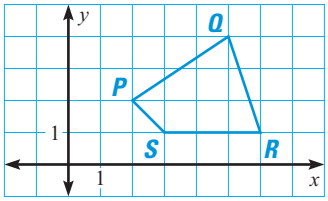
38. $(x, y) \rightarrow (x + 2, y + 1)$



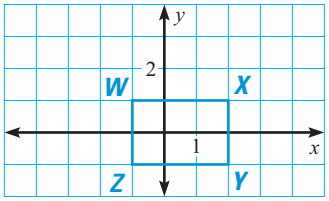
39. $(x, y) \rightarrow (x + 4, y - 5)$



40. $(x, y) \rightarrow (x - 5, y + 3)$



41. $(x, y) \rightarrow (x - 3, y + 8)$



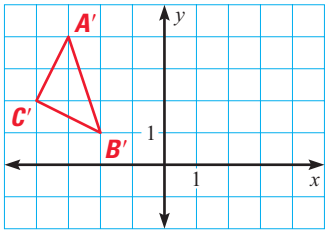
Use Points on an Image **A point on an image and the translation are given. Find the corresponding point on the original figure.**

42. Point on image: $(0, 3)$; translation: $(x, y) \rightarrow (x - 3, y + 2)$

43. Point on image: $(-2, 4)$; translation: $(x, y) \rightarrow (x + 5, y - 1)$

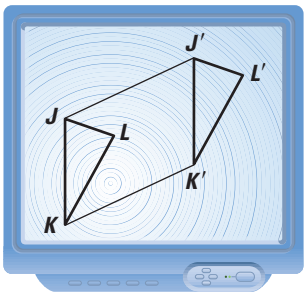
44. Point on image: $(6, -1)$; translation: $(x, y) \rightarrow (x + 3, y + 7)$

45. **You be the Judge** The figure on the grid shown at the right is the image after the translation $(x, y) \rightarrow (x - 6, y + 4)$. One of your classmates tells you that C on the original figure is $(2, -2)$. Do you agree? Explain your reasoning.



Technology **In Exercises 46 and 47, use geometry software to complete the steps below.**

1. Draw a triangle and translate it.
2. Construct $\overline{JJ'}$ and $\overline{KK'}$.



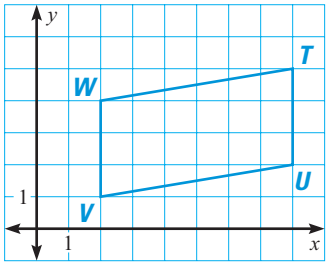
46. If two lines have the same slope, then they are parallel. Measure the slopes of $\overline{JJ'}$ and $\overline{KK'}$. Are $\overline{JJ'}$ and $\overline{KK'}$ parallel?

47. What should $m\angle KJJ' + m\angle K'KJ$ be? Measure the angles and check your answer.

48. **Challenge** Point C is located at $(1, 3)$. The translation that shifts C to C' is given by $(x, y) \rightarrow (x + 5, y - 4)$. The translation that shifts C' to C'' is given by $(x, y) \rightarrow (x - 1, y + 8)$. Give the coordinate notation that describes the translation directly from C to C''. (*Hint*: Start by plotting C, C', and C''.)

Standardized Test Practice

Multiple Choice **In Exercises 49 and 50, use the diagram below.**



49. Find the coordinates of T' using the translation $(x, y) \rightarrow (x - 5, y + 2)$.

Ⓐ $(3, 7)$ Ⓑ $(10, 0)$

Ⓒ $(3, 5)$ Ⓓ $(-5, 7)$

50. Find the coordinates of W' using the translation $(x, y) \rightarrow (x + 3, y - 3)$.

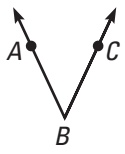
Ⓕ $(5, 1)$ Ⓖ $(-1, 7)$

Ⓗ $(5, 7)$ Ⓙ $(-1, 1)$

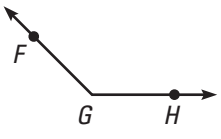
Mixed Review

Classifying Angles **State whether the angle appears to be *acute*, *right*, *obtuse*, or *straight*. Then estimate its measure.** *(Lesson 1.6)*

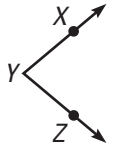
51.



52.



53.



Algebra Skills

Problem Solving **Use problem solving strategies to answer the question.** *(Skills Review, p. 653)*

54. Your telephone company charges $.15 per minute for all long distance calls. This month you paid $12.60 for long distance calls. How many minutes did you spend on long distance calls?

55. You just bought a CD single that has four tracks. In how many different orders can the songs be played?

Ordering Numbers **Write the numbers in order from least to greatest.** *(Skills Review, p. 662)*

56. $-0.4, 0.5, 0, 1.0, -0.1, 0.9$

57. $3.4, -1.2, 0.7, -1.5, 0, 1.1, -4$

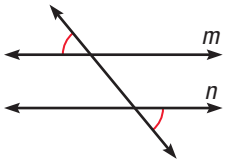
58. $6.7, 7.6, -0.77, 6.6, -0.7, -6.7$

59. $-6.12, 6.3, -6.8, -6.1, 6, 6.09$

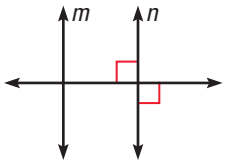
Quiz 3

Determine whether enough information is given to conclude that $m \parallel n$. Explain. *(Lesson 3.5)*

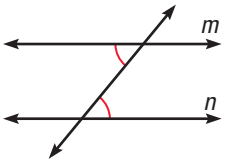
1.



2.

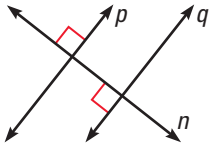


3.

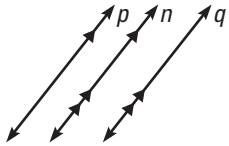


In Exercises 4–6, explain how you would show that $p \parallel q$. State any theorems or postulates that you would use. *(Lesson 3.6)*

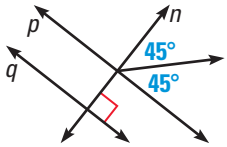
4.



5.



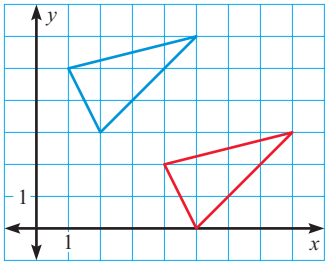
6.



7. Draw a vertical line ℓ and construct a line m perpendicular to it through a point P to the left of line ℓ. *(Lesson 3.6)*

In Exercises 8 and 9, describe the translation of the figure using coordinate notation. *(Lesson 3.7)*

8.



9.