

2.6

Properties of Equality and Congruence

Goal

Use properties of equality and congruence.

Key Words

- Reflexive Property
- Symmetric Property
- Transitive Property

Reflexive Property



Jean is the same height as **Jean**.

Symmetric Property



If **Jean** is the same height as **Pedro**, then **Pedro** is the same height as **Jean**.



Transitive Property



If **Jean** is the same height as **Pedro** and **Pedro** is the same height as **Chris**, then **Jean** is the same height as **Chris**.



The photos above illustrate the *Reflexive*, *Symmetric*, and *Transitive Properties* of Equality. You can use these properties in geometry with statements about equality and congruence.

Student Help

LOOK BACK

To review the difference between equality and congruence, see p. 30.

PROPERTIES OF EQUALITY AND CONGRUENCE

Reflexive Property

Equality $AB = AB$
 $m\angle A = m\angle A$

Congruence $\overline{AB} \cong \overline{AB}$
 $\angle A \cong \angle A$

Symmetric Property

Equality
If $AB = CD$, then $CD = AB$.
If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Congruence
If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive Property

Equality
If $AB = CD$ and $CD = EF$, then $AB = EF$.
If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Congruence
If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

EXAMPLE 1 Name Properties of Equality and Congruence

Name the property that the statement illustrates.

- If $\overline{GH} \cong \overline{JK}$, then $\overline{JK} \cong \overline{GH}$.
- $DE = DE$
- If $\angle P \cong \angle Q$ and $\angle Q \cong \angle R$, then $\angle P \cong \angle R$.

Solution

- Symmetric Property of Congruence
- Reflexive Property of Equality
- Transitive Property of Congruence

Checkpoint Name Properties of Equality and Congruence

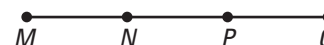
Name the property that the statement illustrates.

- If $DF = FG$ and $FG = GH$, then $DF = GH$.
- $\angle P \cong \angle P$
- If $m\angle S = m\angle T$, then $m\angle T = m\angle S$.

Logical Reasoning In geometry, you are often asked to explain why statements are true. Reasons can include definitions, theorems, postulates, or properties.

EXAMPLE 2 Use Properties of Equality

In the diagram, N is the midpoint of \overline{MP} , and P is the midpoint of \overline{NQ} . Show that $MN = PQ$.

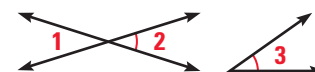
**Solution**

$MN = NP$	Definition of midpoint
$NP = PQ$	Definition of midpoint
$MN = PQ$	Transitive Property of Equality

Checkpoint Use Properties of Equality and Congruence

4. $\angle 1$ and $\angle 2$ are vertical angles, and $\angle 2 \cong \angle 3$. Show that $\angle 1 \cong \angle 3$.

$\angle 1 \cong \angle 2$? Theorem
$\angle 2 \cong \angle 3$	Given
$\angle 1 \cong \angle 3$? Property of Congruence



Student Help

STUDY TIP

In geometry, you can use properties of equality that you learned in algebra.

PROPERTIES OF EQUALITY

Addition Property

Adding the same number to each side of an equation produces an equivalent equation.

Example

$$\begin{aligned} x - 3 &= 7 \\ x - 3 + 3 &= 7 + 3 \end{aligned}$$

Subtraction Property

Subtracting the same number from each side of an equation produces an equivalent equation.

Example

$$\begin{aligned} y + 5 &= 11 \\ y + 5 - 5 &= 11 - 5 \end{aligned}$$

Multiplication Property

Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

Example

$$\begin{aligned} \frac{1}{4}z &= 6 \\ \frac{1}{4}z \cdot 4 &= 6 \cdot 4 \end{aligned}$$

Division Property

Dividing each side of an equation by the same nonzero number produces an equivalent equation.

Example

$$\begin{aligned} 8x &= 16 \\ \frac{8x}{8} &= \frac{16}{8} \end{aligned}$$

Substitution Property

Substituting a number for a variable in an equation produces an equivalent equation.

Example

$$\begin{aligned} x &= 7 \\ 2x + 4 &= 2(7) + 4 \end{aligned}$$

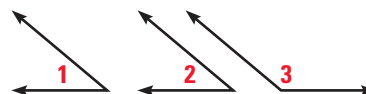


MORE EXAMPLES

More examples at classzone.com

EXAMPLE 3 Justify the Congruent Supplements Theorem

$\angle 1$ and $\angle 2$ are both supplementary to $\angle 3$. Show that $\angle 1 \cong \angle 2$.



Solution

$$m\angle 1 + m\angle 3 = 180^\circ$$

Definition of supplementary angles

$$m\angle 2 + m\angle 3 = 180^\circ$$

Definition of supplementary angles

$$m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$$

Substitution Property of Equality

$$m\angle 1 = m\angle 2$$

Subtraction Property of Equality

$$\angle 1 \cong \angle 2$$

Definition of congruent angles

Checkpoint Use Properties of Equality and Congruence

5. In the diagram, M is the midpoint of \overline{AB} . Show that $AB = 2 \cdot AM$.

$$MB = AM$$

Definition of ?

$$AB = AM + MB$$

 ? Postulate

$$AB = AM + AM$$

 ? Property of Equality

$$AB = 2 \cdot AM$$

Distributive property



2.6 Exercises

Guided Practice

Vocabulary Check

Match the statement with the property it illustrates.

- | | |
|--|--------------------------------------|
| 1. $m\angle DEF = m\angle DEF$ | A. Symmetric Property of Equality |
| 2. If $\overline{PQ} \cong \overline{ST}$, then $\overline{ST} \cong \overline{PQ}$. | B. Reflexive Property of Equality |
| 3. $\overline{XY} \cong \overline{XY}$ | C. Transitive Property of Equality |
| 4. If $\angle J \cong \angle K$ and $\angle K \cong \angle L$, then $\angle J \cong \angle L$. | D. Reflexive Property of Congruence |
| 5. If $PQ = QR$ and $QR = RS$, then $PQ = RS$. | E. Symmetric Property of Congruence |
| 6. If $m\angle X = m\angle Y$, then $m\angle Y = m\angle X$. | F. Transitive Property of Congruence |

Skill Check

Name the property that the statement illustrates.

- $\angle ABC \cong \angle ABC$
- If $m\angle B = m\angle D$ and $m\angle D = m\angle F$, then $m\angle B = m\angle F$.
- If $\overline{GH} \cong \overline{JK}$, then $\overline{JK} \cong \overline{GH}$.

Practice and Applications

Extra Practice

See p. 676.

Completing Statements Use the property to complete the statement.

- Reflexive Property of Equality: $JK = \underline{\hspace{1cm}}?$
- Symmetric Property of Equality: If $m\angle P = m\angle Q$, then $\underline{\hspace{1cm}}? = \underline{\hspace{1cm}}?$.
- Transitive Property of Equality: If $AB = BC$ and $BC = CD$, then $\underline{\hspace{1cm}}? = \underline{\hspace{1cm}}?$.
- Reflexive Property of Congruence: $\underline{\hspace{1cm}}? \cong \angle GHJ$
- Symmetric Property of Congruence: If $\underline{\hspace{1cm}}? \cong \underline{\hspace{1cm}}?$, then $\angle XYZ \cong \angle ABC$.
- Transitive Property of Congruence: If $\overline{GH} \cong \overline{IJ}$ and $\underline{\hspace{1cm}}? \cong \underline{\hspace{1cm}}?$, then $\overline{GH} \cong \overline{PQ}$.

Naming Properties Name the property that the statement illustrates.

- If $AB = CD$, then $AB + EF = CD + EF$.
- If $m\angle C = 90^\circ$, then $2(m\angle C) + 15^\circ = 2(90^\circ) + 15^\circ$.
- If $XY = YZ$, then $3 \cdot XY = 3 \cdot YZ$.

Homework Help

Example 1: Exs. 10–18

Example 2: Exs. 19–24

Example 3: Exs. 19–24

- 19. Using Properties** In the diagram, $m\angle 1 + m\angle 2 = 132^\circ$, and $m\angle 2 = 105^\circ$. Complete the argument to show that $m\angle 1 = 27^\circ$.

$$m\angle 1 + m\angle 2 = 132^\circ$$

Given

$$m\angle 2 = 105^\circ$$

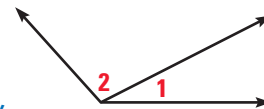
Given

$$m\angle 1 + 105^\circ = 132^\circ$$

? Property of Equality

$$m\angle 1 = 27^\circ$$

? Property of Equality



- 20. Using Properties of Congruence** In the diagram, $\overline{AB} \cong \overline{FG}$, and \overleftrightarrow{BF} bisects \overline{AC} and \overline{DG} . Complete the argument to show that $\overline{BC} \cong \overline{DF}$.

$$\overline{BC} \cong \overline{AB}$$

Definition of ?

$$\overline{AB} \cong \overline{FG}$$

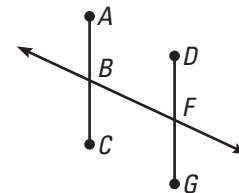
?

$$\overline{FG} \cong \overline{DF}$$

Definition of ?

$$\overline{BC} \cong \overline{DF}$$

? Property of Congruence



- 21. Unscramble the Steps** In the diagram below, $PQ = RS$. Copy the diagram and arrange the statements and reasons in order to make a logical argument to show that $PR = QS$.



$$PR = QS$$

Given

$$PQ + QR = RS + QR$$

$$PQ + QR = PR$$

Addition Property of Equality

Segment Addition Postulate

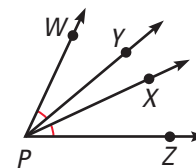
$$RS + QR = QS$$

$$PQ = RS$$

Substitution Property of Equality

Segment Addition Postulate

- 22. Using Properties of Equality** In the diagram at the right, $m\angle WPY = m\angle XPZ$. Complete the argument to show that $m\angle WPX = m\angle YPZ$.



$$m\angle WPY = m\angle XPZ$$

Given

$$m\angle WPX = m\angle WPY + m\angle YPX$$

?

$$m\angle YPZ = m\angle YPX + m\angle XPZ$$

?

$$m\angle WPY + m\angle YPX = m\angle YPX + m\angle XPZ$$

?

$$m\angle WPX = m\angle YPZ$$

?

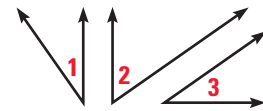
Student Help

LOOK BACK

To review the Congruent Complements Theorem, see p. 69.

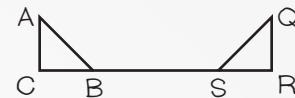
- 23. Congruent Complements Theorem** Show that the Congruent Complements Theorem is true. Use Example 3 on page 90 as a model. Provide a reason for each step.

In the diagram, $\angle 1$ is complementary to $\angle 2$, and $\angle 3$ is complementary to $\angle 2$.
Show that $\angle 1 \cong \angle 3$.



- 24. Error Analysis** In the diagram, $\overline{SR} \cong \overline{CB}$ and $\overline{AC} \cong \overline{QR}$. Explain what is wrong with the student's argument.

Because $\overline{SR} \cong \overline{CB}$ and $\overline{AC} \cong \overline{QR}$,
then $\overline{CB} \cong \overline{AC}$ by the Transitive
Property of Congruence. ✗



- Using Algebra** Find the value of the variable using the given information. Provide a reason for each step.

- 25.** $AB = BC$, $BC = CD$



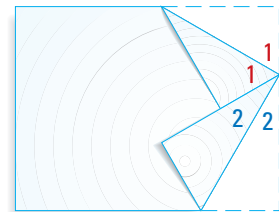
- 26.** $QR = RS$, $ST = RS$



- 27. Challenge** Fold two corners of a piece of paper so their edges match as shown at the right.

What do you notice about the angle formed by the fold lines?

Show that the angle measure is always the same. Provide a reason for each step.



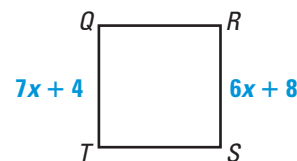
Standardized Test Practice

- 28. Multiple Choice** Which statement illustrates the Symmetric Property of Congruence?

- (A) If $\overline{AD} \cong \overline{BC}$, then $\overline{DA} \cong \overline{CB}$.
(B) If $\overline{WX} \cong \overline{XY}$ and $\overline{XY} \cong \overline{YZ}$, then $\overline{WX} \cong \overline{YZ}$.
(C) If $\overline{AB} \cong \overline{GH}$, then $\overline{GH} \cong \overline{AB}$.
(D) $\overline{AB} \cong \overline{BA}$

- 29. Multiple Choice** In the figure below, $\overline{QT} \cong \overline{TS}$ and $\overline{RS} \cong \overline{TS}$. What is the value of x ?

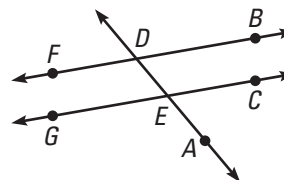
- (F) 4 (G) 12
(H) 16 (J) 32



Mixed Review

Naming Collinear Points Use the diagram to name a point that is collinear with the given points. (Lesson 1.3)

30. G and E 31. F and B
 32. A and D 33. B and D



Sketching Intersections Sketch the figure described. (Lesson 1.4)

34. Three lines that do not intersect but lie in the same plane.
 35. Two lines that intersect at one point, and another line that intersects both of those lines at different points.

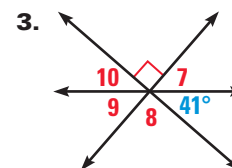
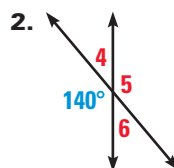
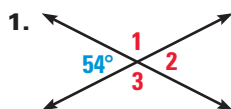
Algebra Skills

Plotting Points Plot the point in a coordinate plane. Then determine which quadrant, if any, the point lies in. (Skills Review, p. 664)

36. $(5, 2)$ 37. $(0, -7)$ 38. $(1, -4)$ 39. $(-8, -3)$
 40. $(-6, 7)$ 41. $(10, 2)$ 42. $(-1, 1)$ 43. $(9, -4)$

Quiz 2

Find the measures of the numbered angles. (Lesson 2.4)



In Exercises 4 and 5, rewrite the statement as an if-then statement. (Lesson 2.5)

4. A square is a four-sided figure. 5. The value of x^2 is 25 if $x = 5$.
 6. Use the Law of Syllogism to write the statement that follows from the pair of true statements. (Lesson 2.5)
 If we charter a boat, then we will go deep sea fishing.
 If we go deep sea fishing, then we will be gone all day.

7. In the diagram, \overrightarrow{KM} bisects $\angle JKN$, and \overrightarrow{KN} bisects $\angle MKL$. Complete the argument to show that $m\angle JKM = m\angle NKL$. (Lesson 2.6)

$$m\angle JKM = m\angle MKN$$

Definition of ?

$$m\angle MKN = m\angle NKL$$

Definition of ?

$$m\angle JKM = m\angle NKL$$

 ? Property of Equality

