1.6 Angles and Their Measures

Goal

Measure and classify angles. Add angle measures.

Key Words

- angle
- sides and vertex of an angle
- measure of an angle
- degree
- congruent angles
- acute, right, obtuse, and straight angle

An **angle** consists of two rays that have the same endpoint.

The rays are the **sides** of the angle.

The endpoint is the **vertex** of the angle.

In the photograph at the right, the sides of $\angle BAC$ are $\overrightarrow{AB}$ and $\overrightarrow{AC}$. The vertex of $\angle BAC$ is point A.

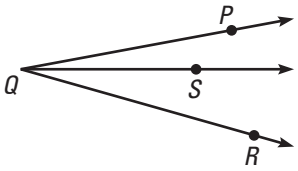
You can also write $\angle BAC$ as $\angle CAB$. Notice that the middle letter in the name of the angle is always the vertex of the angle.

You can write simply $\angle A$ if there are no other angles that have this vertex.

Mountaineers approaching Gasherbrum II, Karakorum Range, Himalayas.

EXAMPLE 1 Name Angles

Name the angles in the figure.



Solution

There are three different angles.

$\angle PQS$ or $\angle SQP$

$\angle SQR$ or $\angle RQS$

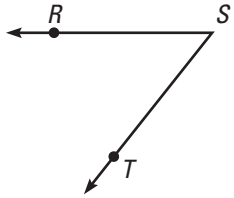
$\angle PQR$ or $\angle RQP$

You should not name any of these angles as $\angle Q$, because all three angles have Q as their vertex. The name $\angle Q$ would not distinguish one angle from the others.

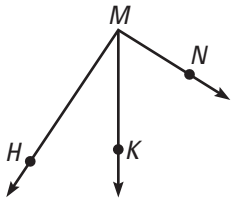
Checkpoint Name Angles

Name the angles in the figure.

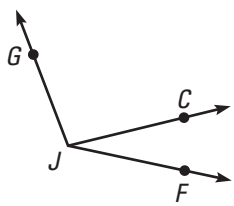
1.



2.



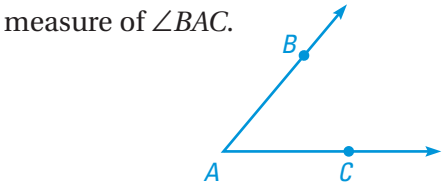
3.



The **measure** of an angle is written in units called **degrees** (°). The measure of $\angle A$ is denoted by $m\angle A$.

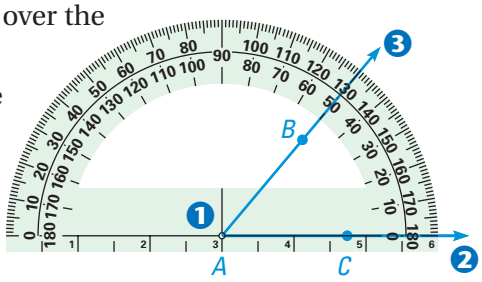
EXAMPLE 2 Measure Angles

Use a protractor to approximate the measure of $\angle BAC$.

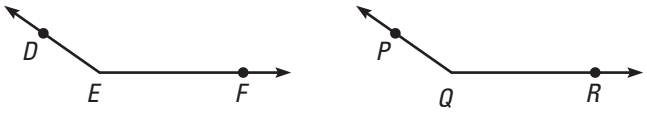


Solution

1. Place the center of the protractor over the vertex point A.
2. Align the protractor with one side of the angle.
3. The second side of the angle crosses the protractor at the 50° mark. So, $m\angle BAC = 50°$.



Two angles are **congruent angles** if they have the same measure. In the diagram below, the two angles have the same measure, so $\angle DEF$ is congruent to $\angle PQR$. You can write $\angle DEF \cong \angle PQR$.

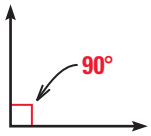


Angles are classified as **acute**, **right**, **obtuse**, or **straight**, according to their measures. Angles have measures greater than 0° and less than or equal to 180°.

Student Help

VISUAL STRATEGY
Add these words to the vocabulary pages in your notebook, as shown on p. 2.

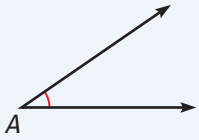
Visualize It!



A small corner mark in an angle means that the angle is a right angle.

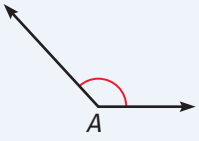
SUMMARY CLASSIFYING ANGLES BY THEIR MEASURES

Acute angle



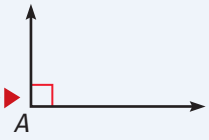
Measure is between 0° and 90°.

Obtuse angle



Measure is between 90° and 180°.

Right angle



Measure is 90°.

Straight angle

Measure is 180°.

EXAMPLE 3 Classify Angles

Classify each angle.

a. $m\angle A = 130°$ **b.** $m\angle B = 90°$ **c.** $m\angle C = 45°$

Solution

a. $\angle A$ is *obtuse* because its measure is greater than 90°.

b. $\angle B$ is *right* because its measure is 90°.

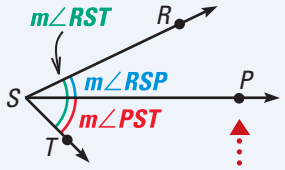
c. $\angle C$ is *acute* because its measure is less than 90°.

POSTULATE 6

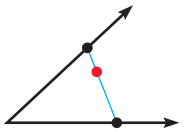
Angle Addition Postulate

Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is the sum of the measures of $\angle RSP$ and $\angle PST$.

Symbols If P is in the interior of $\angle RST$, then $m\angle RSP + m\angle PST = m\angle RST$.



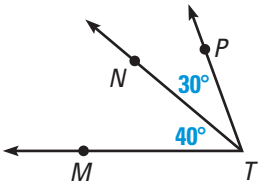
Visualize It!



A point is in the *interior* of an angle if it is between points that lie on each side of the angle.

EXAMPLE 4 Add Angle Measures

Find the measure of $\angle PTM$.



Solution

$m\angle PTM = m\angle PTN + m\angle NTM$ Angle Addition Postulate

$= 30° + 40°$ Substitute 30° for $m\angle PTN$ and 40° for $m\angle NTM$.

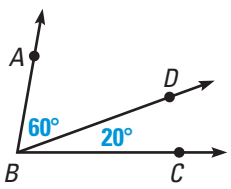
$= 70°$ Add angle measures.

ANSWER The measure of $\angle PTM$ is 70°.

Checkpoint Add and Subtract Angle Measures

Find the measure of $\angle ABC$.

4.



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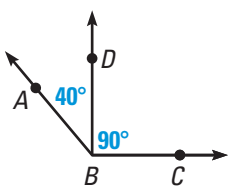

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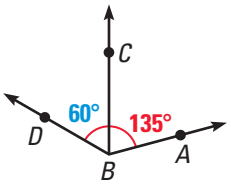


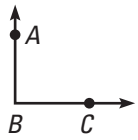
1.6 Exercises

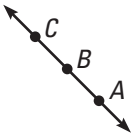
Guided Practice

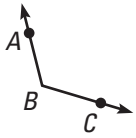
Vocabulary Check

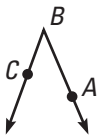
Match the angle with its classification.

A. acute **B.** obtuse **C.** right **D.** straight

1. 

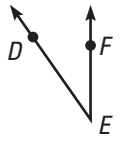
2. 

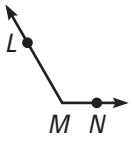
3. 

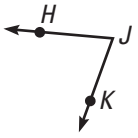
4. 

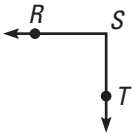
Skill Check

Name the vertex and the sides of the angle. Then estimate the measure of the angle.

5. 

6. 

7. 

8. 

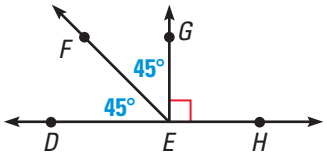
Classify the angle as *acute*, *right*, *obtuse*, or *straight*.

9. $m\angle A = 180°$ **10.** $m\angle B = 34°$ **11.** $m\angle C = 100°$ **12.** $m\angle D = 9°$

Use the diagram at the right to answer the questions. Explain your answers.

13. Is $\angle DEF \cong \angle FEG$?

14. Is $\angle DEG \cong \angle HEG$?

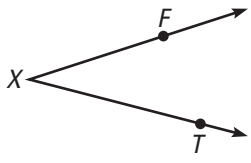


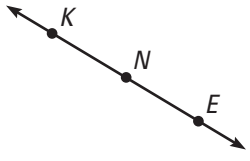
Practice and Applications

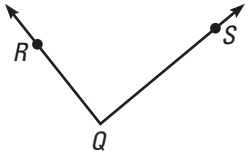
Extra Practice

See p. 676.

Naming Parts of an Angle **Name the vertex and the sides of the angle.**

15. 

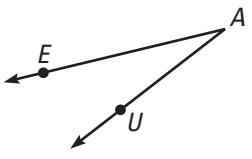
16. 

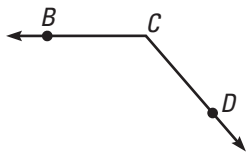
17. 

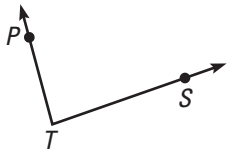
Homework Help

Example 1: Exs. 15–20
Example 2: Exs. 21–23
Example 3: Exs. 24–26, 30–33
Example 4: Exs. 27–29

Naming Angles **Write two names for the angle.**

18. 

19. 

20. 

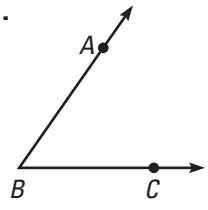
Link to Careers

SURVEYOR Surveyors use a tool called a theodolite, which can measure angles to the nearest 1/3600 of a degree.

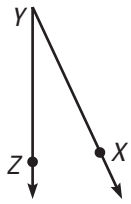
Career Links CLASSZONE.COM

Measuring Angles **Copy the angle and use a protractor to measure it to the nearest degree. Extend the sides of the angle if necessary.**

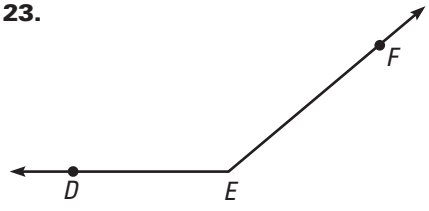
21.



22.

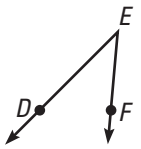


23.

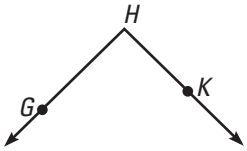


Classifying Angles **State whether the angle appears to be *acute, right, obtuse,* or *straight*. Then estimate its measure.**

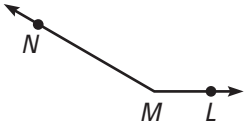
24.



25.

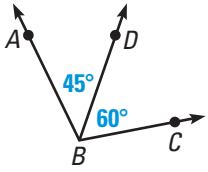


26.

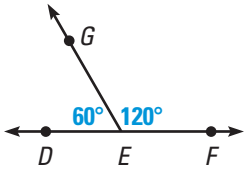


Angle Addition Postulate **Find the measure of the angle.**

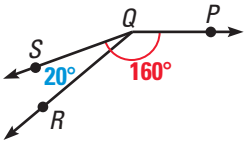
27. $m\angle ABC =$?



28. $m\angle DEF =$?



29. $m\angle PQR =$?



EXAMPLE **Angles on the Coordinate Plane**

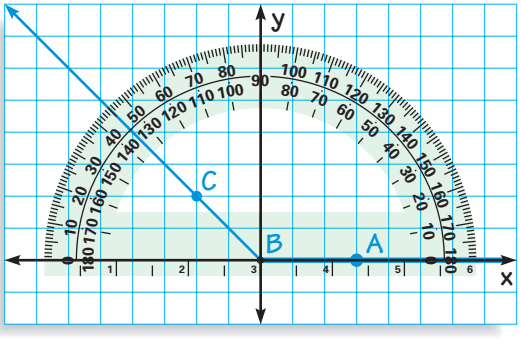
Plot the points $A(3, 0)$, $B(0, 0)$, $C(-2, 2)$ and sketch $\angle ABC$. Classify the angle.

Solution

Plot the points. Use a protractor to estimate the angle measure.

This angle has a measure of 135°.

So, $\angle ABC$ is obtuse.



Visualize It! **In Exercises 30–33, use the example above as a model. Plot the points and sketch $\angle ABC$. Classify the angle.**

30. $A(3, 0)$, $B(0, 0)$, $C(0, 3)$

31. $A(3, 0)$, $B(0, 0)$, $C(4, -4)$

32. $A(-3, 0)$, $B(0, 0)$, $C(2, -2)$

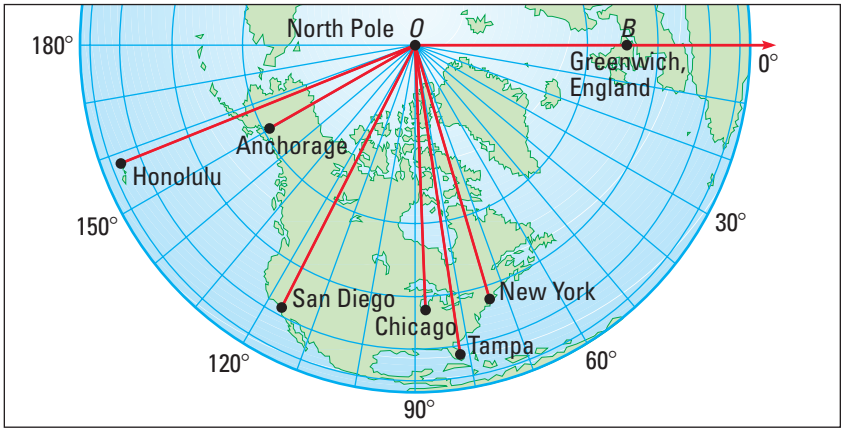
33. $A(0, 4)$, $B(0, 0)$, $C(2, 2)$

Link to Geography

LONGITUDE The 0° line of longitude runs through a telescope at the Royal Observatory in Greenwich, England.

Application Links CLASSZONE.COM

Geography **For each city, estimate the measure of $\angle BOA$, where B is on the 0° longitude line, O is the North Pole, and A is the city.**



34. New York, NY

35. Tampa, FL

36. Chicago, IL

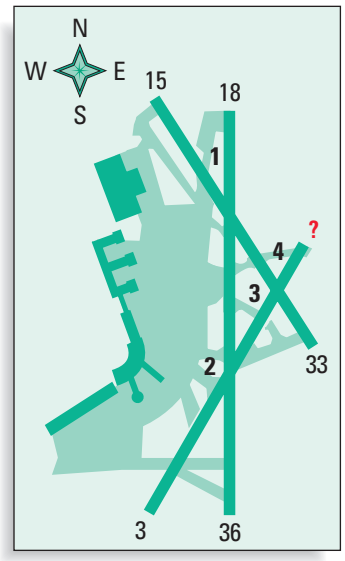
37. San Diego, CA

38. Honolulu, HI

39. Anchorage, AK

Airport Runways **In Exercises 40–44, use the diagram of Ronald Reagan Washington National Airport and the information about runway numbering on the page facing page 1.**

An airport runway number is its *bearing* (the angle measured clockwise from due north) divided by 10. Because a full circle contains 360°, runway numbers range from 1 to 36.



40. Find $m\angle 1$.

41. Find $m\angle 2$.

42. Find $m\angle 3$.

43. Find $m\angle 4$.

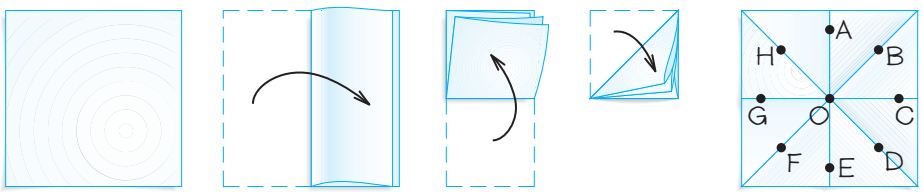
44. What is the number of the unlabeled runway?

Student Help

LOOK BACK
For an example of runway numbers, see the page facing p. 1.

Standardized Test Practice

45. **Multi-Step Problem** Fold a piece of paper in half three times and label it as shown.



a. Name eight congruent acute angles.

b. Name eight right angles.

c. Name eight congruent obtuse angles.

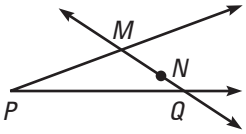
Mixed Review

Naming Rays **Name the ray described.** *(Lesson 1.3)*

46. Name a ray that contains M.

47. Name a ray that has N as an endpoint.

48. Name two rays that intersect at P.



Betweenness **Draw a sketch of the three collinear points. Then write the Segment Addition Postulate for the points.** *(Lesson 1.5)*

49. Y is between X and Z.

50. Q is between P and R.

51. B is between A and C.

52. K is between J and L.

Algebra Skills

One-Step Equations **Solve the equation.** *(Skills Review p. 672)*

53. $x + 3 = 15$ **54.** $y + 4 = 2$ **55.** $z - 7 = 9$ **56.** $w - 5 = -2$

57. $2p = 24$ **58.** $-9 = 3q$ **59.** $5r = 125$ **60.** $-12 = 6s$

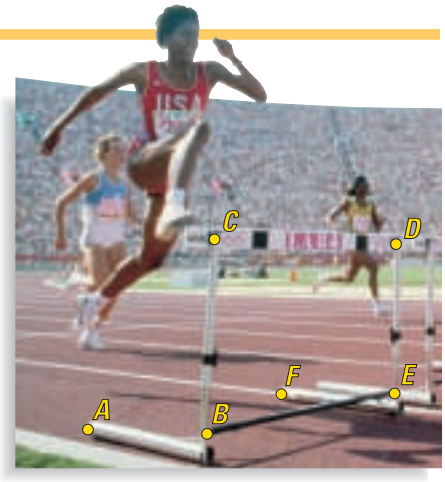
Quiz 2

In Exercises 1–3, use the photo shown at the right. *(Lesson 1.4)*

1. Name two lines that do not appear to intersect.

2. Name two lines that intersect at B.

3. Name two lines that intersect at E.



Judi Brown competing at the Summer Olympics.

Plot the points in a coordinate plane. Decide whether $\overline{AB}$ and $\overline{CD}$ are congruent. *(Lesson 1.5)*

4. $A(0, 0)$, $B(5, 0)$, $C(2, 4)$, $D(2, -1)$ **5.** $A(-3, 2)$, $B(3, 2)$, $C(6, 0)$, $D(-6, 0)$

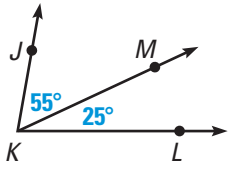
Classify the angle as *acute, right, obtuse,* or *straight*. *(Lesson 1.6)*

6. $m\angle Z = 90°$ **7.** $m\angle Y = 126°$ **8.** $m\angle X = 180°$

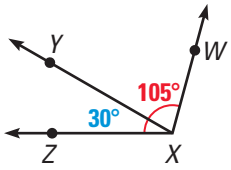
9. $m\angle W = 35°$ **10.** $m\angle V = 5°$ **11.** $m\angle U = 45°$

Use the Angle Addition Postulate to find the measure of the angle. *(Lesson 1.6)*

12. Find $m\angle JKL$.



13. Find $m\angle WXY$.



14. Find $m\angle VUW$.