Session 2: Problem Set 3: Geometry, Trigonometry, Advanced Topics, Imaginary Numbers Solutions

1.

If h is a function defined over the set of all real numbers and $h(x - 4) = 6x^2 + 2x + 10$, then which of the following defines $h(x)$?

Ⓐ $h(x) = 6x^2 - 2x + 114$
Ⓑ $h(x) = 6x^2 - 46x + 98$
Ⓒ $h(x) = 6x^2 + 2x + 98$
Ⓓ $h(x) = 6x^2 + 50x + 114$

Category: Passport to Advanced Math / Functions

Strategic Advice: The key to answering this question is in having a conceptual understanding of function notation. Here, the input $(x - 4)$ has already been substituted and simplified in the given function. Your job is to determine what the function would have looked like had x been the input instead.

Getting to the Answer: To keep things organized, let $u = x - 4$, the old input. This means $x = u + 4$. Substitute this into $h(x - 4)$ and simplify:

$$h(x - 4) = 6x^2 + 2x + 10$$
$$h(u) = 6(u + 4)^2 + 2(u + 4) + 10$$
$$= 6(u^2 + 8u + 16) + 2u + 8 + 10$$
$$= 6u^2 + 48u + 96 + 2u + 8 + 10$$
$$= 6u^2 + 50u + 114$$

This means $h(u) = 6u^2 + 50u + 114$.

When working with function notation, you evaluate the function by substituting a given input value for the variable in the parentheses. Here, if the input value is x, then $h(x) = 6x^2 + 50x + 114$.

2.

$$\frac{4 + \sqrt{-16}}{2 + \sqrt{-4}}$$

Use the definition $\sqrt{-1} = i$ to simplify the expression above.

Difficulty: Medium

Category: Additional Topics / Imaginary Numbers

Strategic Advice: Because $\sqrt{-1} = i$, rewrite each number under the radical as a product of -1 and itself. Then take the square root of each. If possible, cancel any factors that are common to the numerator and the denominator.

Getting to the Answer:

$$\frac{4 + \sqrt{-16}}{2 + \sqrt{-4}} = \frac{4 + \sqrt{16 \times -1}}{2 + \sqrt{4 \times -1}}$$
$$= \frac{4 + 4i}{2 + 2i}$$
$$= \frac{2(2 + 2i)}{2 + 2i}$$
$$= 2$$

3.

$$\frac{1}{4}i^{42} + i^{60}$$

What is the value of the complex number given above?

Difficulty: Hard

Category: Additional Topics in Math / Imaginary Numbers

Strategic Advice: To evaluate a high power of i, look for patterns and use the definition $\sqrt{-1} = i$, which can be written in a more useful form as $i^2 = -1$.

Getting to the Answer: Write out enough powers of i that allow you to see the pattern:

$i^1 = i$
$i^2 = -1$ (by definition)
$i^3 = i \times i^2 = i \times -1 = -i$
$i^4 = i^2 \times i^2 = -1 \times -1 = 1$
$i^5 = i^4 \times i = 1 \times i = i$
$i^6 = i^4 \times i^2 = 1 \times -1 = -1$
$i^7 = i^6 \times i = -1 \times i = -i$
$i^8 = i^4 \times i^4 = 1 \times 1 = 1$

Notice that the pattern (i, -1, $-i$, 1, i, -1, $-i$, 1) repeats on a cycle of 4. To evaluate i^{42}, divide 42 by 4. The result is 10, remainder 2, which means 10 full cycles, and then back to i^2. This means i^{42} is equivalent to i^2, which is -1. Do the same for i^{60}: $60 \div 4 = 15$, remainder 0, which means stop on the 4th cycle to find that $i^{60} = 1$. Make these substitutions in the original equation:

$$\frac{1}{4}i^{42} + i^{60} = \frac{1}{4}(-1) + 1 = -\frac{1}{4} + 1 = \frac{3}{4}$$

Grid in the answer as 3/4 or .75.

4. Calculator

$$\begin{cases} y = 3x \\ -3x^2 + 2y^2 = 180 \end{cases}$$

If (x, y) is a solution to the system of equations above, what is the value of x^2?

Ⓐ 12
Ⓑ 20
Ⓒ 60
Ⓓ 144

Difficulty: Medium

**Category:** Passport to Advanced Math / Quadratics

Strategic Advice: Even though one of the equations in this system isn't linear, you can still solve the system using substitution.

Getting to the Answer: You already know that y is equal to $3x$, so substitute $3x$ for y in the second equation. Don't forget that when you square $3x$, you must square both the coefficient and the variable.

$-3x^2 + 2y^2 = 180$
$-3x^2 + 2(3x)^2 = 180$
$-3x^2 + 2(9x^2) = 180$
$-3x^2 + 18x^2 = 180$
$15x^2 = 180$
$x^2 = 12$

The question asks for the value of x^2, not x, so there is no need to take the square root of 12 to find the value of x. The answer is (A).

5.
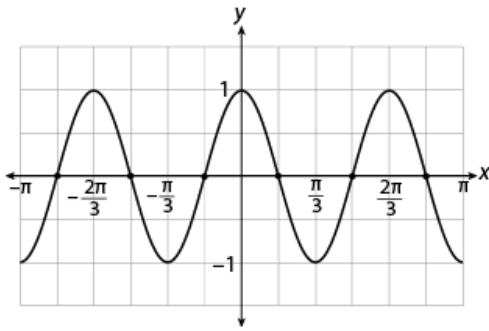
The graph of $g(x) = \cos(3x)$ is shown above. Which of the following lists represents the values of x for which $g(x) = 0$?

- (A) -180°, -120°, -60°, 60°, 120°, 180°
- (B) -165°, -105°, -45°, 45°, 105°, 165°
- (C) -150°, -90°, -30°, 30°, 90°, 120°
- (D) -120°, -80°, -40°, 40°, 80°, 120°
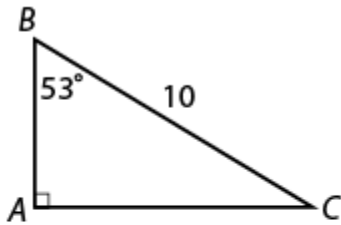
Difficulty: Hard

Category: Additional Topics in Math / Trigonometry

Strategic Advice: You don't need to know a lot of trig to answer this question. You will, however, need to know how to convert radians to degrees (multiply the radians by $\frac{180°}{\pi}$).

Getting to the Answer: Recall that $g(x) = 0$ means "crosses the x-axis," regardless of the type of function involved, which means you are looking for the x-intercepts. Study the graph carefully: The function crosses the x-axis six times, halfway between each of the labeled grid-lines. Rather than finding the points using the radians given in the graph, convert the radians to degrees and then determine the halfway points: $-\pi\left(\frac{180°}{\pi}\right) = -180°$, $\left(-\frac{2\pi}{3}\right)\left(\frac{180°}{\pi}\right) = -120°$, $\left(-\frac{\pi}{3}\right)\left(\frac{180°}{\pi}\right) = -60°$, and so on. Take a minute now to find the halfway points because chances are that you don't have to do all the conversions. Halfway between -180° and -120° is -150°. Stop—that's all you need to know. The leftmost x-intercept is at -150°, which means (C) must be correct. If you want to check another value just to be sure, halfway between -120° and -60° is -90°, which is the second value in (C), confirming that it is correct.

6.

Based on the figure above, what is the approximate length of side AB?

- (A) 6
- (B) 7.2
- (C) 8
- (D) 8.5

Difficulty: Medium

Category: Additional Topics in Math / Trigonometry

Strategic Advice: *ABC* is a right triangle. You know the length of one side and the measure of one of the acute angles, which means you can use SOH CAH TOA.

Getting to the Answer: You know the length of the hypotenuse (10) and you're looking for the length of the side adjacent to (touching) the 53° angle, so use cosine. Set up a trigonometric ratio and solve for the length of *AB*. Make sure your calculator is set to degree mode.

$$\cos(53°) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos(53°) = \frac{AB}{10}$$

$$10(0.601815) = AC$$

$$6.01815 = AC$$

The length of *AB* is approximately 6.

7.

In geology, the water table is the level below which the ground is saturated with water. Wells must be dug below this point to bring water up into the well. Except in cases of severe flooding, the water level in a well does not rise above the water table. Suppose a cylindrical well is 6 feet wide and 60 feet deep in an area where the water table is 40 feet below ground level. Assuming no unusual circumstances, what is the volume in cubic feet of the water in the well at any given time?

Ⓐ 180π
Ⓑ 360π
Ⓒ 540π
Ⓓ 720π

Difficulty: Hard

Category: Additional Topics in Math / Geometry

Strategic Advice: Use the formula for finding the volume of a cylinder, $V = \pi r^2 h$. Check the formula page rather than trying to recall it from memory. Don't forget to use the radius, not the diameter, which is given.
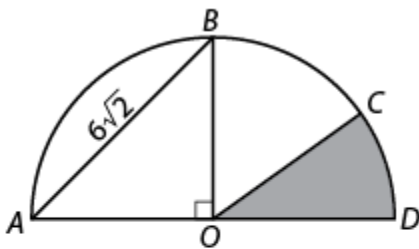
Getting to the Answer: The well is 6 feet wide; this is its diameter, so $r = 3$. The height of the well is 60 feet, but the water table is 40 feet below ground level, which means only $60 - 40 = 20$ feet of the well is below the water table and thus has water in it, so $h = 20$.

$$V = \pi(3)^2(20)$$

$$V = \pi(9)(20)$$

$$V = 180\pi$$

8.

If segment *AD* is a diameter of the circle shown above, and the length of arc *CD* is π, what is the area of the shaded region? Use 3.14 to approximate π and round your answer to the nearest tenth.

Category: Additional Topics in Math / Geometry

Strategic Advice: This question requires logical thinking, knowledge of special right triangles, and knowing how to find arc length. It's a challenging question, so if you're pushed for time, skip it and come back later.

Getting to the Answer: Finding the area of a sector of a circle (the shaded region) requires knowing the degree measure of the corresponding interior angle. Given that information, your first step is to find the area of the entire circle. Then you'll find the proportional amount represented by the sector. To find the area of a circle, the only thing you need is the radius. The radius is not shown in the figure, so you will have to think about special right triangles. In the figure, triangle *ABO* is formed by 2 radii and a 90° angle. This means the triangle must be a 45-45-90 triangle, and therefore its side lengths are in the ratio $1 : 1 : \sqrt{2}$. The hypotenuse is given as $6\sqrt{2}$, so the side lengths of the triangle, and therefore the radius of the circle, must be 6, and the area of the entire circle is $A = \pi r^2 = \pi(6) = 36\pi$. Now you need to find the portion of the circle represented by the shaded region by finding the measure of the angle inside the sector and dividing by 360. You'll need to use the given arc length, π, and the formula for finding arc length (arc length = θr, where θ is the interior angle and r is the length of the radius):

$$\pi = \theta(6)$$
$$\frac{\pi}{6} = \theta$$

If you know your unit circle, you know this corresponds to 30°. If you don't recall this fact, then you can convert radians to degrees by multiplying the radian measure by $\frac{180}{\pi}$ to get $\frac{\pi}{6} \times \frac{180}{\pi} = \frac{180}{6} = 30$.

This means the shaded region makes up $\frac{30}{360} = \frac{1}{12}$ of the total area of the circle, so divide the total area by 12 to get $36\pi \div 12$ = 3π. The question tells you to approximate π using 3.14 and to round to the nearest tenth, so the final answer is 9.4.

9. Calculator
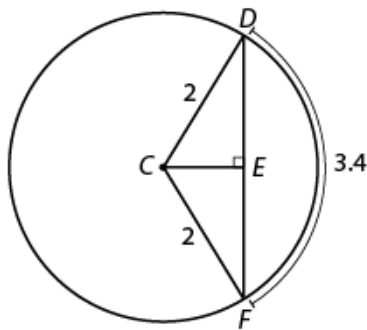
Which of the following gives the length of chord *DF* in the figure above?

Ⓐ 2cos(1.7)
Ⓑ 2sin(1.7)
Ⓒ 4cos(0.85)
Ⓓ 4sin(0.85)

Difficulty: Hard

Category: Additional Topics in Math / Trigonometry

Strategic Advice: This is a very difficult question involving arc length given in radians and answer choices that involve trig functions. If you're not familiar with these topics, you should guess and move on to the next question. (Don't forget—there is no penalty for wrong answers on the new SAT!)

Getting to the Answer: Take a peek at the answer choices—the angles of the trig functions are given in radians, rather than degrees (you know this because there is no degree symbol). This means you'll need to use the radian formula for finding arc length: $arcL = \theta \times r$, where θ is the central angle of the arc in radians and r is the radius of the circle. This will allow you to determine the measure of the central angle, half of which becomes one of the angles of a right triangle (*CDE*, for example). You know both the arc length (3.4) and the radius (2), so solve for the central angle.

$arcL = \theta \times r$

$3.4 = \theta \times 2$

$1.7 = \theta$

This means that angle *DCF* has a measure of 1.7 radians, and consequently, angle *DCE* has a measure of half that, or 0.85 radians. Add this measure to the triangle, or draw a quick right triangle off to the side like the one below:
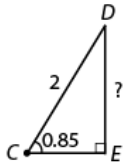

Now, if you can find the length of side *DE*, you can double it to find the length of chord *DF*. Side *DE* is opposite the angle measure that you found and you know the hypotenuse of the triangle; so, use the ratio $\sin x = \dfrac{\text{opposite}}{\text{hypotenuse}}$ to find the length of side *DE*:

$\sin(0.85) = \dfrac{DE}{2}$

$2\sin(0.85) = DE$

Multiply by 2 to find that $DF = 2 \times 2\sin(0.85) = 4\sin(0.85)$. Keep in mind that multiplying the angle (inside the parentheses) is not the same as multiplying the whole quantity by 2.

10. Calculator
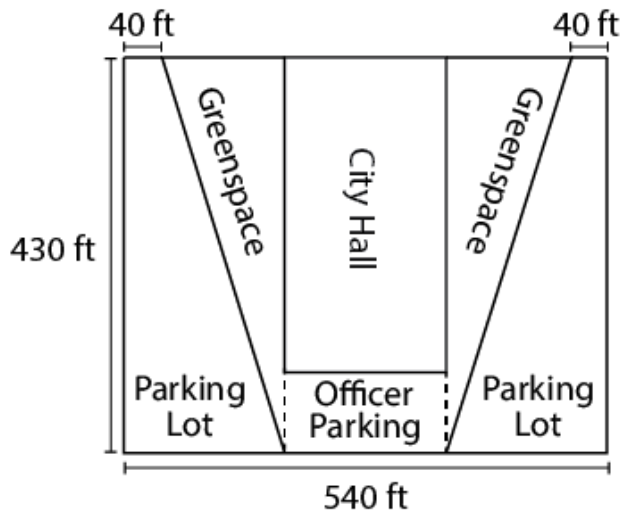
Many cities try to work "greenspaces" into their city planning because living plants help filter the city's air, reducing the effects of pollution. The figure above shows the plans for a new greenspace around City Hall, which will be created by converting portions of the existing parking lots. If the width of each parking lot is the same as the width of the City Hall building, how many thousands of square feet of greenspace will there be after the conversion? Round to the nearest thousand and enter your answer in terms of thousands. (For example, enter 14,000 as 14.)

Difficulty: Medium

Category: Additional Topics in Math / Geometry

Strategic Advice: Whenever a question asks about the amount of space something covers (here, the greenspace), you are looking for area. In this question, the area that you're looking for takes on the shape of a right triangle (actually, two of them), so use the formula $A = \frac{1}{2}bh$.

Getting to the Answer: The key to answering this question is in labeling the diagram. The calculations are very straightforward once you have the correct dimensions of the triangles. You're given that the width of each parking lot is equal to the width of the City Hall building, so each parking lot is $540 \div 3 = 180$ feet wide. This means the base of each triangle (at the top of the diagram) is $180 - 40 = 140$ feet.

The height of each triangle is the same as the length of the parking lot, which is 430 feet.
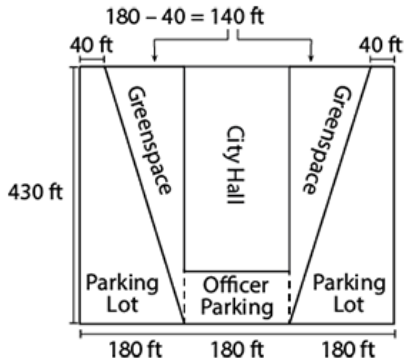
You now have all the numbers you need. The area of each triangle is $\frac{1}{2}(140)(430) = 30{,}100$, so both triangles together result in a greenspace that covers 60,200 square feet. Rounded to the nearest thousand, this is 60,000, which should be gridded in as 60.

11.

If x is an angle such that $0 < x < 90°$, which of the following statements is not always true?

Ⓐ $\cos(x) > 0$
Ⓑ $\cos(-x) > 0$
Ⓒ $\cos(x + 90°) < 0$
Ⓓ $\boxed{\cos(2x) < 0}$

Difficulty: Hard

Category: Additional Topics in Math / Trigonometry

Strategic Advice: When a trig question involves signs (+ and −) rather than numbers, using your knowledge of how the *unit circle* works will get you to the answer. Before you begin analyzing the answer choices, translate the inequality symbols to signs (> 0 means +, and < 0 means −).

Getting to the Answer: Draw a quick sketch of a unit circle and add the angle x. Because $0 < x < 90°$, you can draw the angle anywhere in the first quadrant. Drawing a small angle is usually the safest way to go. Then, sketch in each of the angles described in the answers. Keep in mind that the cosine of the angle is equal to the x-coordinate of the point where the angle hits the unit circle, so it's positive in Quadrants I and IV, and negative in Quadrants II and III.
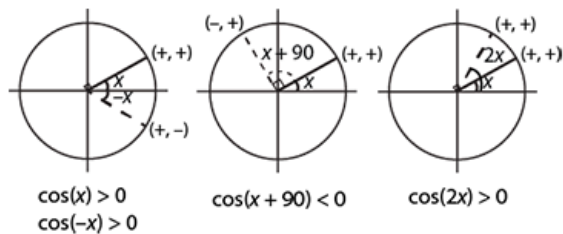
After sketching in each of the angles, compare them with the answer choices. The only one that is not always true is (D). When the angle is small, twice the angle still lands in Quadrant I, so the cosine is still positive (and therefore > 0, not < 0). Note that had you drawn a larger angle in Quadrant I, such as a 60°-degree angle, $2x$ would have landed in Quadrant II, in which case the cosine would have been negative. However, the question asks for the statement that is not *always* true, so you must consider both large and small angles in Quadrant I.

1.

If $g(x) = 2x^3 - 5x^2 + 4x + 6$, and P is the point on the graph of $g(x)$ that has an x-coordinate of 1, then what is the y-coordinate of the corresponding point on the graph of $g(x - 3) + 4$?

Difficulty: Hard

Category: Passport to Advanced Math / Functions

Strategic Advice: This question is, for the most part, conceptual. Start by finding the y-coordinate of P in the original equation. Then, perform the transformation on the coordinates (rather than the function) to save yourself valuable time.

Getting to the Answer: Substitute 1 for x in the original equation. Graphically, the resulting value of $g(1)$ is the y-coordinate of the point.

$g(x) = 2x^3 - 5x^2 + 4x + 6$

$g(1) = 2(1)^3 - 5(1)^2 + 4(1) + 6$

$ = 2 - 5 + 4 + 6$

$ = 7$

The point on the graph of $g(x)$ is (1, 7). Now, the question asks for the y-coordinate of the corresponding point on the transformed graph. When performing transformations, the operations grouped with the x are performed on the x-coordinate, and the operations *not* grouped with the x are performed on the y-coordinate. So, add 4 to 7 to find that the y-coordinate of the point on the transformed graph is 11.

2.

$$\frac{2}{i+6} + (2 + 5i)$$

Which of the following expressions is equivalent to the complex number given above?

Note that $\sqrt{-1} = i$.

Ⓐ $\boxed{\dfrac{32i + 9}{i + 6}}$

Ⓑ $\dfrac{34i + 7}{i + 6}$

Ⓒ $\dfrac{32i + 19}{i + 6}$

Ⓓ $\dfrac{37i + 14}{i + 6}$

Difficulty: Hard

Category: Additional Topics in Math / Imaginary Numbers

Strategic Advice: Fractions with complex numbers are no different from any other fraction. You must find a common denominator before adding them.

Getting to the Answer: Find a common denominator by multiplying the second term by $i + 6$. You're given that $\sqrt{-1} = i$, but a more useful fact is that $i^2 = -1$, so be sure to make this substitution as you go. Once you have found the common denominator, you can simply add like terms.

$$
\begin{aligned}
\frac{2}{i+6} + (2+5i) &= \frac{2}{i+6} + \frac{2+5i}{1} \\
&= \frac{2}{i+6} + \frac{2+5i}{1}\left(\frac{i+6}{i+6}\right) \\
&= \frac{2}{i+6} + \frac{2i+12+5(-1)+30i}{i+6} \\
&= \frac{2}{i+6} + \frac{32i+7}{i+6} \\
&= \frac{32i+9}{i+6}
\end{aligned}
$$

3. Calculator

Given that $\sqrt{-1} = i$, which of the following is equivalent to the sum $i^{125} + i^{125}$?

(A) i^{14}
(B) i^{250}
(C) $\boxed{2i^{45}}$
(D) $2i^{250}$

Difficulty: Medium

Category: Additional Topics in Math / Imaginary Numbers

Strategic Advice: To evaluate a high power of i, look for patterns and use the definition $\sqrt{-1} = i$, which, when written in a more useful form, is $i^2 = -1$.

Getting to the Answer: Write out enough powers of i for you to see the pattern:

$i^1 = i$
$i^2 = -1$ (by definition)
$i^3 = i \times i^2 = i \times -1 = -i$
$i^4 = i^2 \times i^2 = -1 \times -1 = 1$
$i^5 = i^4 \times i = 1 \times i = i$
$i^6 = i^4 \times i^2 = 1 \times -1 = -1$
$i^7 = i^6 \times i = -1 \times i = -i$
$i^8 = i^4 \times i^4 = 1 \times 1 = 1$

Notice that the pattern (i, -1, $-i$, 1, i, -1, $-i$, 1) repeats on a cycle of 4. To evaluate i^{125}, divide 125 by 4. The result is 31 remainder 1, which means 31 full cycles, and then back to i^1. This means i^{125} is equivalent to i^1, which is i. Because $i + i = 2i$, you are looking for the answer choice that is also equivalent to $2i$. Choices (C) and D look tempting (because of the 2), so start with them: (C) is correct because $45 \div 4 = 11$, remainder 1, which means i^{45} is equivalent to i and $2i^{45}$ is equal to $2i$.

4.
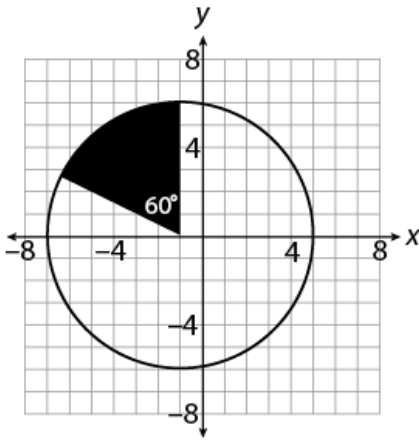
What is the area of the shaded sector of the circle shown in the figure above?

(A) 2π
(B) 6π
(C) 12π
(D) 36π

Difficulty: Medium

Category: Additional Topics in Math / Geometry

Strategic Advice: The area of a sector is equal to the area of the circle times the fraction of the circle represented by the sector.

Getting to the Answer: Start by finding the area of the whole circle: The diameter of the circle extends along the *x*-axis from −7 to 5, which is 12 units, which means the radius is 6. Substitute this into the area formula:

$$A = \pi r^2$$
$$= \pi(6)^2$$
$$= 36\pi$$

5.
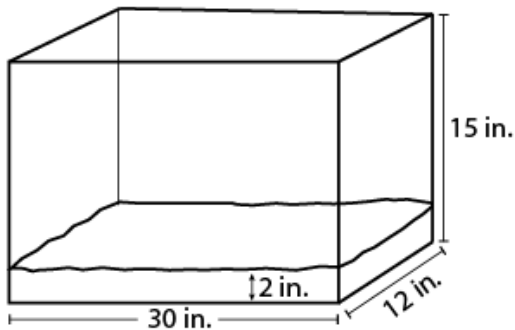
The figure above shows a fish tank with sand in the bottom. If the water level is to be 3 inches below the top, how many cubic inches of water are needed to fill the tank?

Difficulty: Medium

Category: Additional Topics in Math / Geometry

Strategic Advice: Determine the dimensions of the tank in which there will be water. Then, use the formula for finding the volume of a rectangular prism: Volume = length × width × height.

Getting to the Answer: The tank is 30 inches long, 15 inches tall, and 12 inches wide. The sand and the space left at the top of the tank do not affect the length or the width, only the height of the water. There are 2 inches of sand in the bottom and 3 inches of space left at the top, which means the height of the water is 15 − 2 − 3 = 10 inches. Use the formula Volume = *l* ×*w* ×*h* = 30 × 12 × 10.

To multiply the numbers without a calculator, multiply 3 × 1 × 12 = 36 and then add two zeros to get 3,600 cubic inches of water.

6.
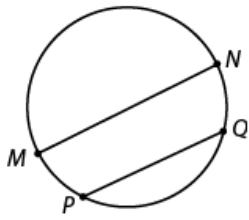
The circle shown has a radius of r centimeters. If chord PQ is parallel to diameter MN, and the length of chord PQ is $\frac{3}{4}$ of the length of the diameter, what is the distance in centimeters between chords MN and PQ in terms of r?
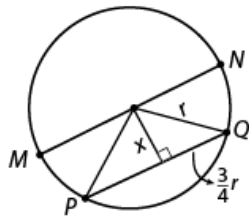
(A) $\boxed{\dfrac{\sqrt{7}}{4}r}$

(B) $\dfrac{\sqrt{3}}{2}r$

(C) $\dfrac{1}{4}\pi r$

(D) $\dfrac{3}{4}\pi r$

Difficulty: Hard

Category: Additional Topics in Math / Geometry

Strategic Advice: Drawing in a radius or two is usually a good way to start a circle question, especially when there doesn't seem to be a lot of information given. This question asks about the distance between the chord and the diameter, so start by drawing that in. Then, see if drawing a radius will help.

Getting to the Answer: After you've drawn in anything that you think might help you answer the question, go back and label wherever possible. The radius has length r, so add that to the diagram. The chord is $\frac{3}{4}$ as long as the diameter, which means half the chord is $\frac{3}{4}$ as long as the radius, so add that to the diagram. You are looking for the distance between the chord and the diameter, so call that x.

You now have a right triangle with enough labels to use the Pythagorean theorem.

$$a^2 + b^2 = c^2$$

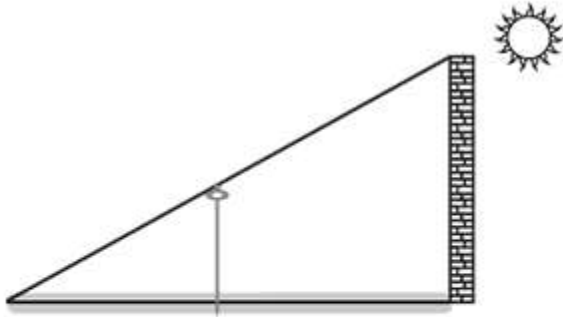$$x^2 + \left(\frac{3}{4}r\right)^2 = r^2$$

$$x^2 + \frac{9}{16}r^2 = r^2$$

$$x^2 = \frac{16}{16}r^2 - \frac{9}{16}r^2$$

$$x^2 = \frac{7}{16}r^2$$

$$x = \sqrt{\frac{7}{16}r^2} = \frac{\sqrt{7}}{4}r$$

7.

<u>Note:</u> Figure not drawn to scale.

A toy saber is stuck at a right angle into the ground 4 inches deep. It casts a shadow that is 4 feet long. The brick wall casts a shadow three times that long. If the wall is 7 feet 6 inches tall, how many inches long is the toy saber?

Difficulty: Hard

Category: Additional Topics in Math / Geometry

Strategic Advice: Drawing on the diagram is a great strategy to get started on a question like this. There are two right triangles—the smaller one formed by the saber, the path of the sun's rays, and the ground; and the larger one formed by the brick wall, the path of the sun's rays, and the ground. The two triangles share one angle (the small angle on the left side), and each has a 90-degree angle (where the saber and the brick wall each meet the ground), making the third pair of corresponding angles also congruent. This means the triangles are similar by AAA, and the sides of the triangles are proportional.

Getting to the Answer: Add information from the question to the diagram. You'll need to convert the height of the wall to inches because the question asks for the length of the saber in inches. (You could also convert the base lengths to inches, but it is not necessary because you can compare feet to feet in that ratio.)
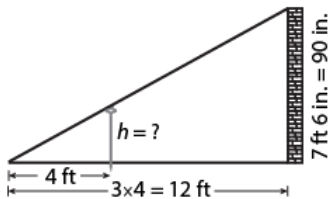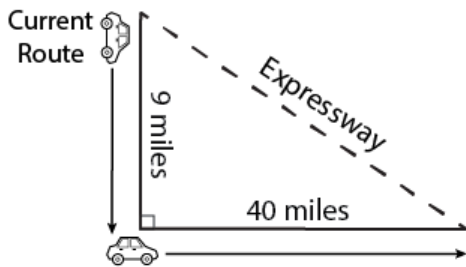

Now that you have a more detailed drawing, set up and solve a proportion:

$$\frac{\text{base of small triangle}}{\text{base of large triangle}} = \frac{\text{length of saber (in inches)}}{\text{height of wall (in inches)}}$$

$$\frac{4}{12} = \frac{h}{90}$$
$$4(90) = 12h$$
$$360 = 12h$$
$$30 = h$$

Don't forget to add the 4 inches that are stuck in the ground to find that the length of the saber is 30 + 4 = 34 inches.

8. Calculator

Current Route

9 miles

Expressway

40 miles

Note: Figure not drawn to scale.

The figure above shows the route that Max currently takes to work and back home every day. The city is planning to build an expressway that would cross through the city to help alleviate commuter traffic. Assuming an average gas consumption of 20 miles per gallon and a 5-day workweek, how many gallons of gas will Max save per week by taking the expressway to and from work each day instead of using his current route?

Ⓐ 2
Ⓑ 4
Ⓒ 8
Ⓓ 10.25

Difficulty: Medium

Category: Additional Topics / Geometry

Strategic Advice: It will save valuable time on Test Day if you can recognize the Pythagorean triple in this problem. If not, just use the Pythagorean theorem to find the length of the expressway.

Getting to the Answer: The roads form a right triangle with the expressway as the hypotenuse. The two legs are Max's current route. He travels on one road for 9 miles and the other for 40. You might recognize this as a Pythagorean triple, 9, 40, 41. Even if you don't, you can always use the Pythagorean theorem to solve for the length of the hypotenuse.

$$a^2 + b^2 = c^2$$
$$(9)^2 + (40)^2 = c^2$$
$$81 + 1,600 = c^2$$
$$1,681 = c^2$$
$$41 = c$$

Now that you know the length of the expressway, it's time to analyze what the question is actually asking.

The question asks how much gas he will save given that his car gets 20 miles per gallon. His current *round-trip* route is 2(9 + 40) = 2(49) = 98 miles, which will use 98 ÷ 20 = 4.9 gallons of gas per day, which is equal to 5(4.9) = 24.5 gallons per workweek. The *round-trip* expressway route is 2(41) = 82 miles, which will use 82 ÷ 20 = 4.1 gallons of gas per day, which is equal to 5(4.1) = 20.5 gallons per workweek. Thus, he will save 24.5 − 20.5 = 4 gallons of gas per week by taking the expressway.

9. Calculator
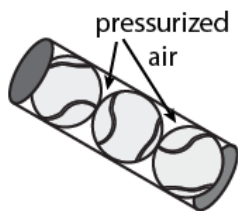
pressurized air

Higher-quality tennis balls are typically packaged in cylindrical cans, as shown above, which are pressurized with air to keep them fresh. If the can and the tennis balls have the same diameter, 2.6 inches, what is the volume in cubic inches of the air inside the can around the tennis balls? Assume that each tennis ball is tangent to the next and that the top and bottom tennis balls are tangent to the top and bottom of the can.

Ⓐ 4.4π
Ⓑ 8.1π
Ⓒ 10.3π
Ⓓ 29.3π

Difficulty: Hard

Category: Additional Topics / Geometry

Strategic Advice: Don't forget to refer to the formula page on Test Day. The can is a cylinder and a tennis ball is a sphere, so you'll need to use both equations.

Getting to the Answer: Make a plan before you start plugging values into the formulas: The volume of the air is equal to the volume of the can minus the volume of the three tennis balls. For both formulas, you will need the radius. The diameters of the cylinder and the balls are the same, 2.6, but you need the radius, so divide by 2 to get $2.6 \div 2 = 1.3$. For the cylinder you also need the height. Because there are 3 tennis balls and the top and bottom balls are tangent to the top and bottom of the can, the height is simply the diameter multiplied by 3, which is $2.6 \times 3 = 7.8$. Now you're ready to use the formulas. A quick peek at the answer choices will tell you that you don't need to simplify completely.
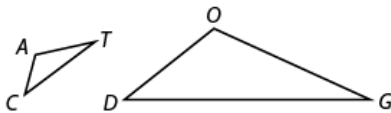
First, find the volume of the whole can:

$V = \pi r^2 h$
$V = \pi (1.3)^2 (7.8)$
$V = \pi (1.69)(7.8)$
$V = 13.182\pi$

Next, find the volume of three tennis balls:

$V = 3\left(\frac{4}{3}\pi r^3\right)$
$V = 3\left(\frac{4}{3}\pi (1.3)^3\right)$
$V = 4\pi (2.197)$
$V = 8.788\pi$

Finally, subtract to get $13.182\pi - 8.788\pi = 4.394\pi$, or about 4.4π cubic inches of air.

10.

Note: Figure not drawn to scale.

Note: Figure not drawn to scale. If triangle *CAT* shown above is similar to triangle *DOG*, and the ratio of the length of side *TC* to side *GD* is 2:7, which of the following ratios must also be equal to 2:7?

(A) $\overline{CA} : \overline{DG}$

(B) $m\angle C : m\angle D$

(C) area of $\triangle CAT$: area of $\triangle DOG$

(D) perimeter of $\triangle CAT$: perimeter of $\triangle DOG$

Difficulty: Medium

Category: Additional Topics in Math / Geometry

Strategic Advice: Corresponding sides of similar triangles are proportional, and corresponding angles are congruent.

Getting to the Answer: You can eliminate B immediately because corresponding angles of similar triangles are congruent, so they are always in a 1:1 ratio. You can also eliminate A because side *CA* does not correspond to side *DG* (*CA* corresponds to *DO*), so you cannot say that they will be in the same ratio. Because the side lengths are proportional, when you add the lengths of all the side lengths (the perimeter), this number will be in the same proportion, so (D) is correct. You can check this by assigning numbers that are in the ratio 2:7 and finding the perimeter of each triangle:

TC = 2 and *GD* = 7

CA = 4 and *DO* = 14

AT = 6 and *OG* = 21

Perimeter of triangle *CAT* = 2 + 4 + 6 = 12

Perimeter of triangle *DOG* = 7 + 14 + 21 = 42

12:42 = 2:7