PUYALLUP SCHOOL DISTRICT
A Tradition of Excellence

PSD MATH: GEOMETRY (Ch9 – Ch12)

This packet is a general review of important concepts (Chapters 9- 12) in Geometry.

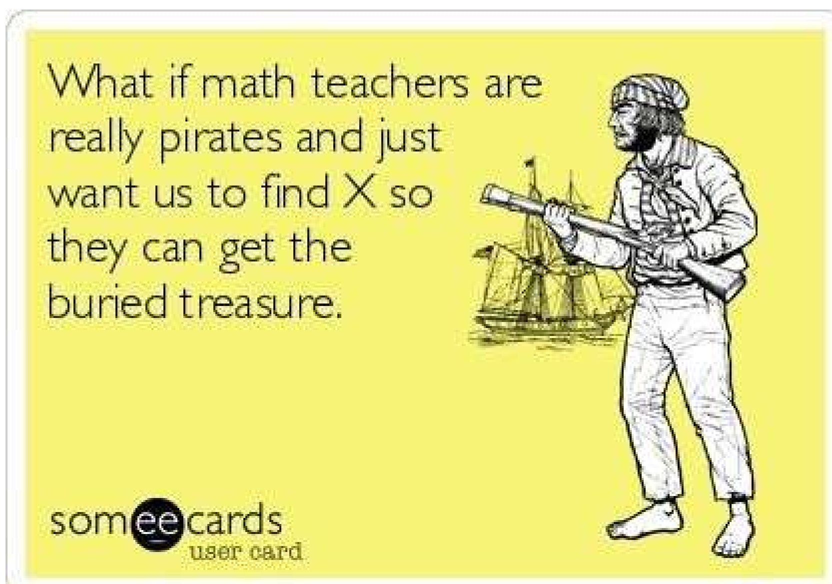
In this packet, you'll find:

A) Pearson re-teaching lessons broken down by chapter so students can see examples from each chapter if useful

B) Need math help 24/7? Click on this link to Khan Academy. Search by topic to see examples done on video. For example, students could search "factoring quadratics" or "exponential growth and decay" or "rules of logarithms."

These videos can be found at: https://www.khanacademy.org/

C) Contact your math teacher directly via e-mail or Schoology for questions, help & support. Reach out to your teachers!

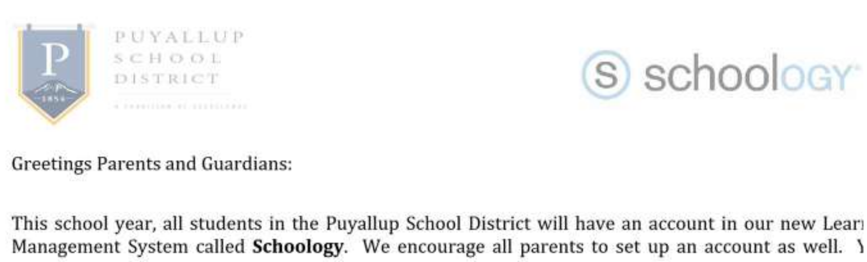


Puyallup School District Virtual Learning Resources

Virtual Learning Opportunities – Puyallup Teachers will communicate lessons and activity resources through your child's Schoology Course or Group. Your child's teacher is ready to support your student through virtual learning!

Clever- a platform that makes it easier for schools to use many popular educational technology products. Essentially, it is a "bookmark" bar for the educational system- curriculum, support, and accessible links are housed in one location. You can access through PSD Favorites folder in the internet browser on a district issued device.

Schoology- The Puyallup School District platform teachers use to communicate, send course updates, collect assignments and assessments, host Schoology conferences (audio and video) and is the electronic gradebook.


PUYALLUP SCHOOL DISTRICT

schoology

Greetings Parents and Guardians:

This school year, all students in the Puyallup School District will have an account in our new Lear
Management System called **Schoology**. We encourage all parents to set up an account as well. Y

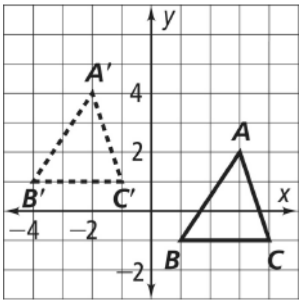
Name______________________________________Class _____________ Date____________

9-1 Reteaching

Translations

A *translation* is a type of *transformation.* In a translation, a geometric figure changes position, but does not change shape or size. The original figure is called the *preimage* and the figure following transformation is the *image.*

The diagram at the right shows a translation in the coordinate plane. The preimage is ΔABC. The image is $\Delta A'B'C'$.

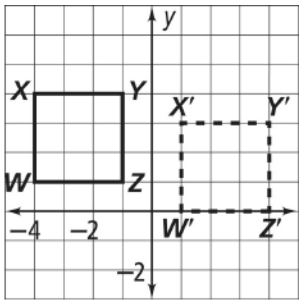


Each point of ΔABC has moved 5 units left and 2 units up. Moving left is in the negative x direction, and moving up is in the positive y direction. So, the rule for the translation is $(x, y) \rightarrow (x - 5, y + 2)$.

All translations are *isometries* because the image and the preimage are congruent. In this case, $\Delta ABC \cong \Delta A'B'C'$.

Problem

What are the images of the vertices of *WXYZ* for the translation $(x, y) \rightarrow (x + 5, y - 1)$? Graph the image of *WXYZ.*



$W(-4, 1) \rightarrow (-4 + 5, 1 - 1)$, or $W'(1, 0)$

$X(-4, 4) \rightarrow (-4 + 5, 4 - 1)$, or $X'(1, 3)$

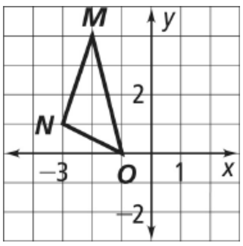
$Y(-1, 4) \rightarrow (-1 + 5, 4 - 1)$, or $Y'(4, 3)$

$Z(-1, 1) \rightarrow (-1 + 5, 1 - 1)$, or $Z'(4, 0)$

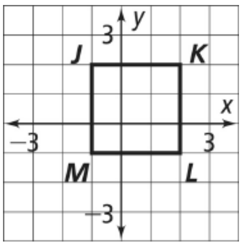
Exercises

Use the rule to find the images of the vertices for the translation.

1. ΔMNO $(x, y) \rightarrow (x + 2, y - 3)$



2. square *JKLM* $(x, y) \rightarrow (x - 1, y)$

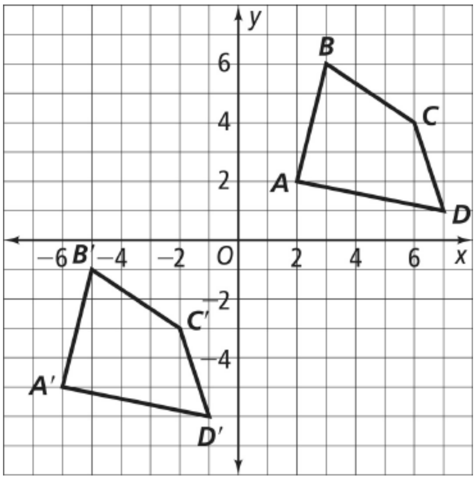


9-1 Reteaching (continued)

Translations

Problem

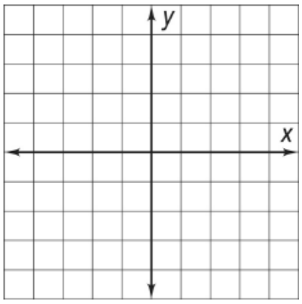
What rule describes the translation of *ABCD* to *A'B'C'D'*?



To get from *A* to *A′* (or from *B* to *B′* or *C* to *C′* or *D* to ☐), you move 8 units left and 7 units down.
The rule that describes this translation is $(x, y) \rightarrow (x - 8, y - 7)$.

Exercises

- On graph paper, draw the *x*- and *y*-axes, and label Quadrants I–IV.
- Draw a quadrilateral in Quadrant I. Make sure that the vertices are on the intersection of grid lines.
- Trace the quadrilateral, and cut out the copy.
- Use the cutout to translate the figure to each of the other three quadrants.



Name the rule that describes each translation of your quadrilateral.

3. from Quadrant I to Quadrant II

4. from Quadrant I to Quadrant III

5. from Quadrant I to Quadrant IV

6. from Quadrant II to Quadrant III

7. from Quadrant III to Quadrant I

Refer to *ABCD* in the problem above.

8. Give the image of the vertices of *ABCD* under the translation $(x, y) \rightarrow (x - 2, y - 5)$.

9. Give the image of the vertices of *ABCD* under the translation $(x, y) \rightarrow (x + 2, y - 4)$.

10. Give the image of the vertices of *ABCD* under the translation $(x, y) \rightarrow (x + 1, y + 3)$.

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9-2 Reteaching

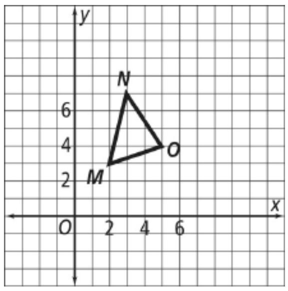
Reflections

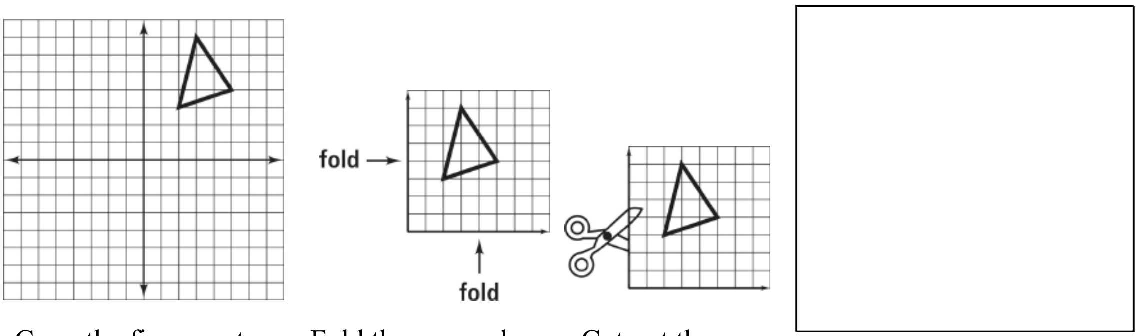
A *reflection* is a type of transformation in which a geometric figure is flipped over a *line of reflection.*

In a reflection, a *preimage* and an *image* have opposite orientations, but are the same shape and size. Because the preimage and image are congruent, a reflection is an *isometry.*

Problem

What are the reflection images of Δ*MNO* across *x*- and *y*-axes? Give the coordinates of the vertices of the images.





Copy the figure onto a piece of paper. | Fold the paper along the *x*-axis and *y*-axis. | Cut out the triangle. | Unfold the paper.

From the graph you can see that the reflection image of Δ*MNO* across the *x*-axis has vertices at (2, –3), (3, –7), and (5, –4). The reflection image of Δ*MNO* across the *y*-axis has vertices at (–2, 3), (–3, 7), and (–5, 4).

Exercises

Use a sheet of graph paper to complete Exercises 1–5.

1. Draw and label the *x*- and *y*-axes on a sheet of graph paper.

2. Draw a scalene triangle in one of the four quadrants. Make sure that the vertices are on the intersection of grid lines.

3. Fold the paper along the axes.

4. Cut out the triangle, and unfold the paper.

5. Label the coordinates of the vertices of the reflection images across the *x*- and *y*-axes.

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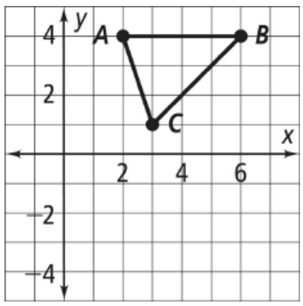
9-2 Reteaching (continued)

Reflections

To graph a reflection image on a coordinate plane, graph the images of each vertex. Each vertex in the image must be the same distance from the line of reflection as the corresponding vertex in the preimage.

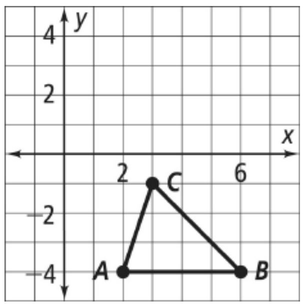
Reflection across the *x*-axis:	Reflection across the *y*-axis:
$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (-x, y)$
The *x*-coordinate does not change.	The *y*-coordinate does not change.
The *y*-coordinate tells the distance from the *x*-axis.	The *x*-coordinate tells the distance from the *y*-axis

Problem



ΔABC has vertices at $A(2, 4)$, $B(6, 4)$, and $C(3, 1)$. What is the image of ΔABC reflected over the *x*-axis?

Step 1: Graph A', the image of *A*. It is the same distance from the *x*-axis as *A*. The distance from the *y*-axis has not changed. The coordinates for A' are $(2, -4)$.



Step 2: Graph B'. It is the same distance from the *x*-axis as *B*. The distance from the *y*-axis has not changed. The coordinates for B' are $(6, -4)$.

Step 3: Graph C'. It is the same distance from the *x*-axis as *C*. The coordinates for C' are $(3, -1)$.

Each figure is reflected across the line indicated. Find the coordinates of the vertices for each image.

6. ΔFGH with vertices $F(-1, 3)$, $G(-5, 1)$, and $H(-3, 5)$ reflected across *x*–axis

7. ΔCDE with vertices $C(2, 4)$, $D(5, 2)$, and $E(6, 3)$ reflected across *x*–axis

8. ΔJKL with vertices $J(-1, -5)$, $K(-2, -3)$, and $L(-4, -6)$ reflected across *y*–axis

9. Quadrilateral *WXYZ* with vertices $W(-3, 4)$, $X(-4, 6)$, $Y(-2, 6)$, and $Z(-1, 4)$ reflected across *y*–axis

Name______________________________________Class _____________ Date____________

9-3 Reteaching

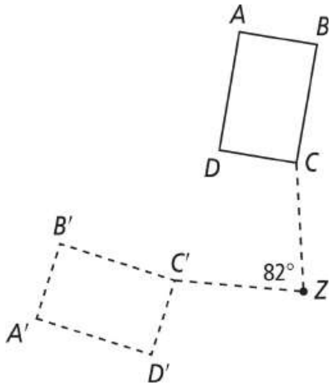
Rotations

A turning of a geometric figure about a point is a *rotation.* The *center of rotation* is the point about which the figure is turned. The number of degrees the figure turns is the *angle of rotation.* (In this chapter, rotations are counterclockwise unless otherwise noted.)

A rotation is an isometry. The image and preimage are congruent.

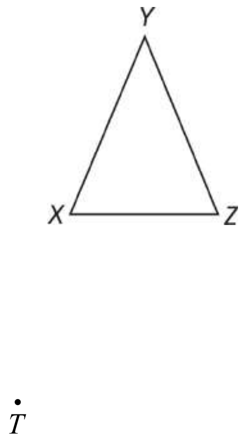
ABCD is rotated about *Z*. *ABCD* is the preimage and $A'B'C'D'$ is the image. The center of rotation is point *Z*. The angle of rotation is 82°.

A *composition of rotations* is two or more rotations in combination. If the center of rotation is the same, the measures for the angles of rotation can be added to find the total rotation of the combination.



Exercises

Complete the following steps to draw the image of $\triangle XYZ$ under an 80° rotation about point *T*.



1. Draw $\angle XTX'$ so that $m\angle XTX' = 80$ and $\overline{TX} \cong \overline{TX'}$.

2. Draw $\angle ZTZ'$ so that $m\angle ZTZ' = 80$ and $\overline{TZ} \cong \overline{TZ'}$.

3. Draw $\angle YTY'$ so that $m\angle YTY' = 80$ and $\overline{TY} \cong \overline{TY'}$.

4. Draw $\overline{X'Z'}$, $\overline{X'Y'}$, and $\overline{Y'Z'}$ to complete $\triangle X'Y'Z'$.

Copy $\triangle XYZ$ to complete Exercises 5-7.

5. Draw the image of $\triangle XYZ$ under a 120° rotation about *T*.

6. Draw a point *S* inside $\triangle XYZ$. Draw the image of $\triangle XYZ$ under a 135° rotation about *S*.

7. Draw the image of $\triangle XYZ$ under a 90° rotation about *Y*.

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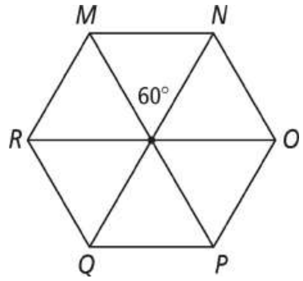
9-3 Reteaching (continued)

Rotations

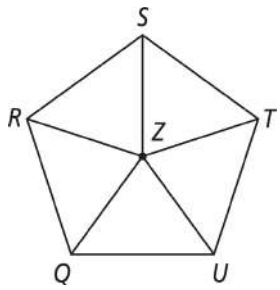
In a regular polygon, the center is the same distance from every vertex. Regular polygons can be divided into a number of congruent triangles. The number of triangles is the same as the number of sides of the polygon. The measure of each central angle (formed by one vertex, the center, and an adjacent vertex) is $\dfrac{360}{\text{number of congruent triangles}}$.



In regular hexagon *MNOPQR,* the center and the vertices can be used to divide the hexagon into six congruent triangles. The measure of each central angle is $\frac{360}{6}$, or 60.

Problem

In regular pentagon *QRSTU,* what is the image of point *Q* for a rotation of 144° about point *Z?*



First find the measure of the central angle of a regular pentagon.

$$m\angle RZS = \frac{360}{5} = 72$$

When *Q* rotates 72°, it moves one vertex counterclockwise. When *Q* rotates 144°, it moves two vertices counterclockwise. So, for a rotation of 144°, the image of *Q* is point *T.*

Exercises

Point *Z* is the center of regular pentagon *QRSTU.* Find the image of the given point or segment for the given rotation.

8. 216° rotation of *S* about *Z*

9. 144° rotation of $\overline{TU}$ about *Z*

10. 360° rotation of *Q* about *Z*

11. 288° rotation of *R* about *Z*

12. is the measure of the angle of rotation that maps *T* onto *U?*
(*Hint:* How many vertices away from *T* is *U,* counterclockwise?)

13. What is the measure of the angle of rotation for the regular hexagon *ABCDEF* that maps *A* onto *C?*

14. What is the measure of the angle of rotation for the regular octagon *DEFGHIJK* that maps *F* onto *K?*

Name ______________________________ Class ____________ Date____________

9-4 Reteaching

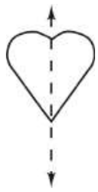
Symmetry

A figure with *symmetry* can be reflected onto itself or rotated onto itself.

Line Symmetry

Line symmetry is also called *reflectional symmetry.* If a figure has line symmetry, it has a *line of symmetry* that divides the figure into two congruent halves. A figure may have one or more than one line of symmetry.

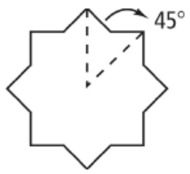
The heart has line symmetry. The dashed line is its one line of symmetry.



Rotational and Point Symmetry

If a figure has *rotational symmetry,* there is a rotation of one-half turn (180°) or less for which the figure is its own image.

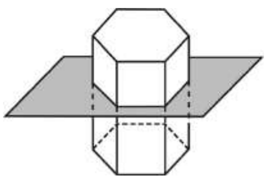
The star at the right has rotational symmetry. The smallest angle needed for the star to rotate onto itself is 45°. This is the angle of rotation.



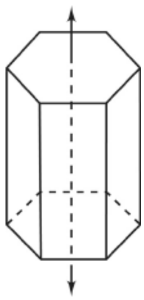
A figure has *point symmetry* if a 180° rotation maps the figure onto itself. The letter S has point symmetry.

Three-Dimensional Symmetry

The regular hexagonal prism has reflectional symmetry in a plane; the plane divides the prism into two congruent halves.



The regular hexagonal prism also has rotational symmetry. Any rotation that is a multiple of 60° around the dashed line will map the prism onto itself.



Name ______________________________ Class ___________ Date___________

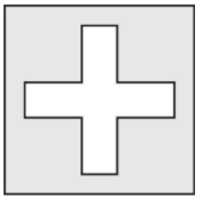
9-4 Reteaching (continued)

Symmetry

Consider the following types of symmetry: rotational, point, and (line) reflectional.

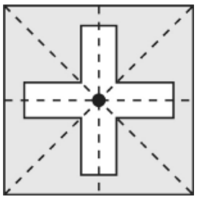
Problem

What types of symmetry does the flag have?



Switzerland

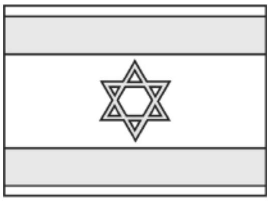
The flag has four lines of symmetry shown by the dotted lines.
It has 90° rotational symmetry and point symmetry about its center.



Exercises

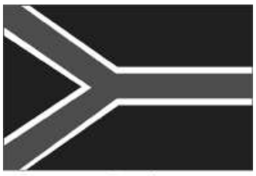
Describe the symmetries in each flag.

1.



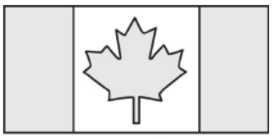
Israel

2.



South Africa

3.



Canada

4.

United Kingdom

5.

Honduras

6.

Somalia

Tell whether each three-dimensional object has reflectional symmetry about a plane, rotational symmetry about a line, or both.

7. a pen

8. a juice box

9. a candle

10. a sofa

Name______________________________ Class __________ Date__________

9-5 Reteaching

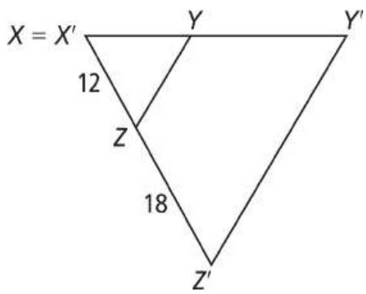
Dilations

A *dilation* is a transformation in which a figure changes size. The preimage and image of a dilation are similar. The scale factor of the dilation is the same as the scale factor of these similar figures.

To find the scale factor, use the ratio of lengths of corresponding sides. If the scale factor of a dilation is greater than 1, the dilation is an *enlargement*. If it is less than 1, the dilation is a *reduction*.

Example

$\Delta X'Y'Z'$ is the dilation image of ΔXYZ. The center of dilation is X. The image of the center is itself, so $X' = X$.



The scale factor, n, is the ratio of the lengths of corresponding sides.

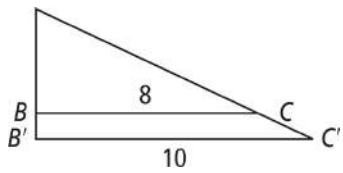
$$n = \frac{X'Z'}{XZ} = \frac{30}{12} = \frac{5}{2}$$

This dilation is an enlargement with a scale factor of $\frac{5}{2}$.

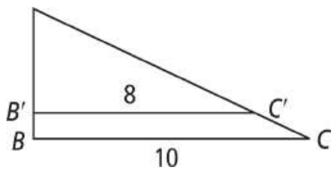
Exercises

For each of the dilations below, *A* is the center of dilation. Tell whether the dilation is a reduction or an enlargement. Then find the scale factor of the dilation.

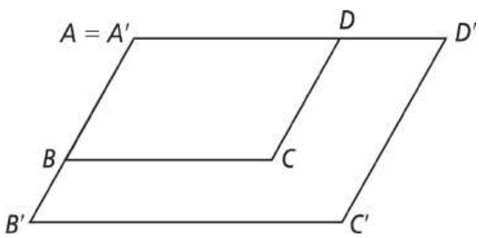
1. $A = A'$



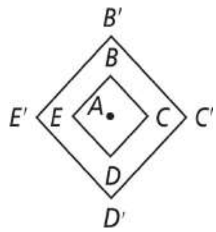
2. $A = A'$



3. $AB = 2$; $A'B' = 3$



4. $DE = 3$; $D'E' = 6$



5. The image of a mosquito under a magnifying glass is six times the mosquito's actual size and has a length of 3 cm. Find the actual length of the mosquito.

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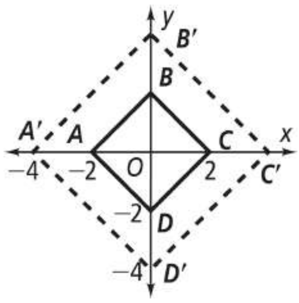
9-5 Reteaching (continued)

Dilations

Problem

Quadrilateral $ABCD$ has vertices $A(-2, 0)$, $B(0, 2)$, $C(2, 0)$, and $D(0, -2)$. Find the image of $ABCD$ under the dilation centered at the origin with scale factor 2. Then graph $ABCD$ and its image.



To find the image of the vertices of $ABCD$, multiply the x-coordinates and y-coordinates by 2.

$A(-2, 0) \rightarrow A'(-4, 0)$ $B(0, 2) \rightarrow B'(0, 4)$

$C(2, 0) \rightarrow C'(4, 0)$ $D(0, -2) \rightarrow D'(0, -4)$

Exercises

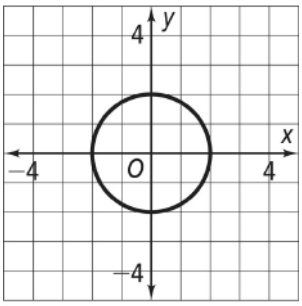
Use graph paper to complete Exercise 6.

6. a. Draw a quadrilateral in the coordinate plane.

b. Draw the image of the quadrilateral under dilations centered at the origin with scale factors $\frac{1}{2}$, 2, and 4.

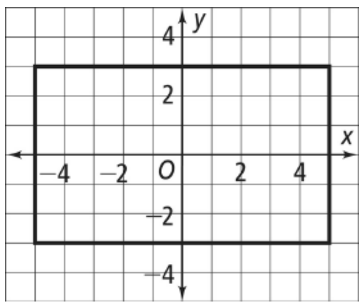
Graph the image of each figure under a dilation centered at the origin with the given scale factor.

7.



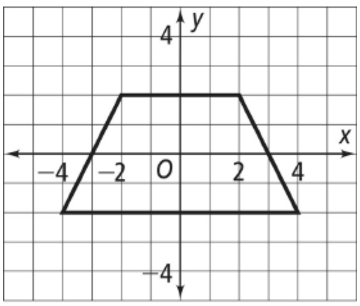
scale factor 2

8.



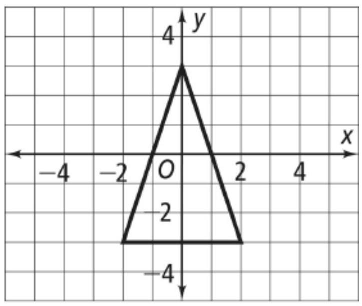
scale factor $\frac{1}{2}$

9.



scale factor $\frac{2}{3}$

10.



scale factor $\frac{3}{2}$

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9-6 Reteaching

Compositions of Reflections

Two congruent figures in a plane can be mapped onto one another by a single reflection, compositions of reflections, or *glide reflections.*

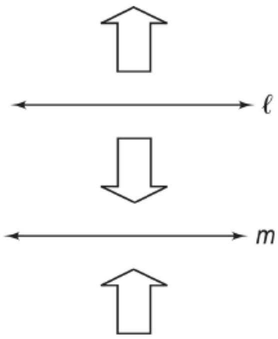
Compositions of two reflections may be either translations or rotations.

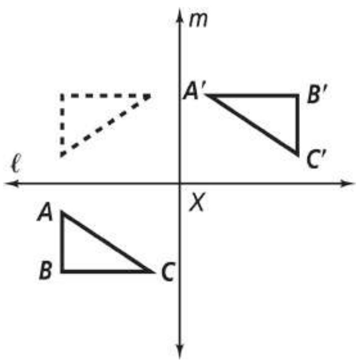
If a figure is reflected across two parallel lines, it is a translation.

If a figure is reflected across intersecting lines, it is a rotation.

For both translations and rotations, the preimage and the image have the same orientation.

The arrow is reflected first across line ℓ and then across line *m.* Lines ℓ and *m* are parallel. These two reflections are equivalent to translation of the arrow downward.

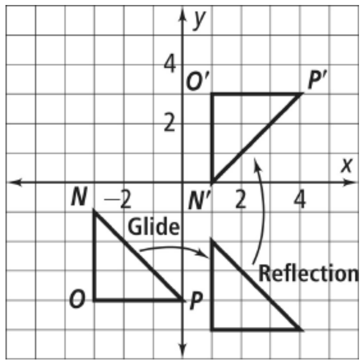




The triangle is reflected first across line ℓ and then across line *m.* Lines ℓ and *m* intersect at point *X. These* two reflections are equivalent to a rotation. The center of rotation is *X* and the angle of rotation is twice the angle of intersection, in this case 2 × 90°, or 180°.

A composition of a translation and a reflection is a glide reflection. For both reflections and glide reflections, the image and the preimage have opposite orientations.

$\Delta N'O'P'$ is the image of ΔNOP, for a glide reflection where the translation is $(x, y) \rightarrow (x + 4, y - 1)$ and the line of reflection is $y = -1$.



Name______________________________________Class _____________ Date_____________

9-6 Reteaching (continued)

Compositions of Reflections

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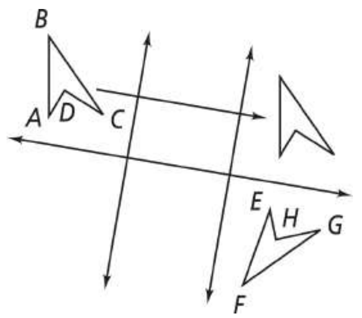
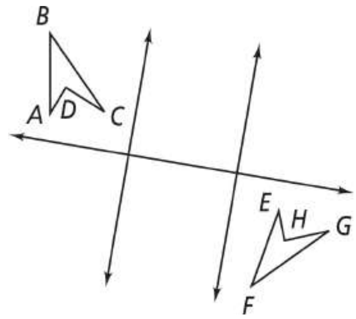
Problem

What transformation maps the figure *ABCD* onto the figure *EFGH* shown at the right?

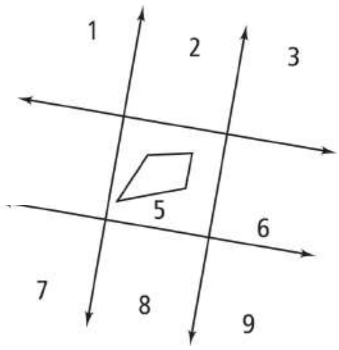




The transformation is a glide reflection. It involves a translation, or glide, followed by a reflection in a line parallel to the translation vector.

Exercises

- Draw two pairs of parallel lines that intersect as shown at the right.
- Draw a nonregular quadrilateral in the center of the four lines.
- Use paper folding and tracing to reflect the figure and its images so that there is a figure in each of the nine sections.
- Label the figures 1 through 9 as shown.



Describe a transformation that maps each of the following.

1. figure 4 onto figure 6

2. figure 1 onto figure 2

3. figure 7 onto figure 5

4. figure 2 onto figure 9

5. figure 1 onto figure 5

6. figure 6 onto figure 7

7. figure 8 onto figure 9

8. figure 2 onto figure 8

***P*(2, 3) → *P′* by a glide reflection with the given translation and line of reflection. What are the coordinates of *P′*? (*Hint:* it may help to graph the transformations.)**

9. $(x, y) \to (x + 3, y - 2)$; $y = 0$

10. $(x, y) \to (x - 4, y + 2)$; $x = 0$

11. $(x, y) \to (x, y - 3)$; $y = x$

12. $(x, y) \to (x - 2, y - 3)$; $y = 4$

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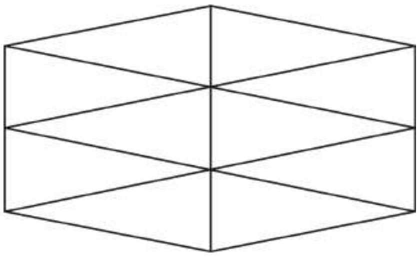
9-7 Reteaching

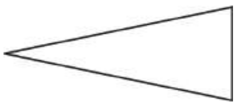
Tessellations

Tessellations are repeating patterns of figures that completely cover a plane without overlaps or gaps. A tessellation can be made by congruent copies of one figure or by congruent copies of multiple shapes.

If the figures in a tessellation are polygons, the sum of the measures of the angles around any vertex is 360°. To determine if a regular polygon will tessellate, find the measure of each angle of the polygon. If 360 is a multiple of this measure, the figure will tessellate.

A tessellation is shown at the right. The repeating figure in the tessellation is shown below.





Problem

Will a regular 24-gon tessellate a plane?

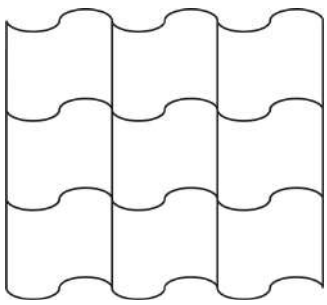
a = the measure of one angle of a 24-gon (all angles of a regular polygon are congruent)

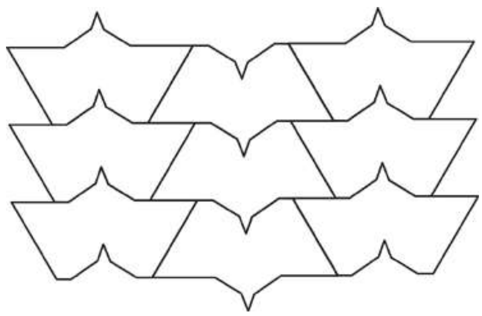
$a = \dfrac{180(n-2)}{n}$	Polygon Angle-Sum Theorem
$a = \dfrac{180(24-2)}{24}$	Substitute 24 for *n*.
$a = 165$	Solve.

No multiple of 165 is equal to 360. A regular 24-gon will not tessellate.

Exercises

Find the repeating figure for each tessellation.

1. 

2. 

3. Will a regular 22-gon tessellate a plane? Explain.

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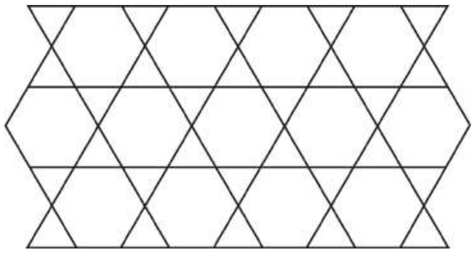
9-7 Reteaching (continued)

Tessellations

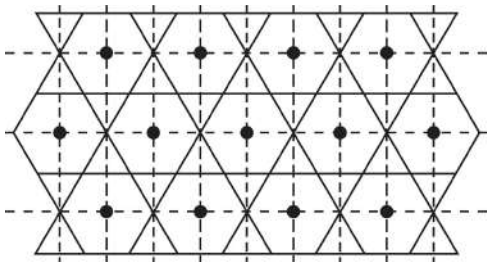
Tessellations have symmetry. The types of possible symmetry are reflectional (or line) symmetry, rotational symmetry, translational symmetry, and glide reflectional symmetry. A tessellation may have more than one type of symmetry.

Problem

What are the symmetries of the tessellation below?



The tessellation has line symmetry as shown by the dotted lines. It has rotational symmetry about the points shown. It has translational symmetry and glide reflection symmetry.



Exercises

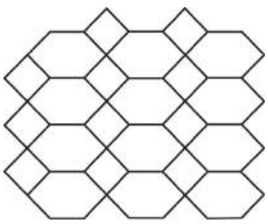
Copy the figure at the right onto stiff paper or cardboard. Then cut it out.

4. Use the cutout to make a tessellation.

5. List the symmetries of the tessellation.

List the symmetries of each tessellation.

6.



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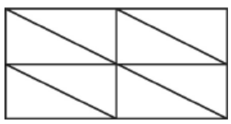

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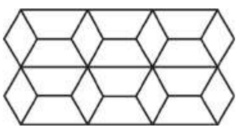


Name______________________________________Class______________ Date_____________

10-1 Reteaching

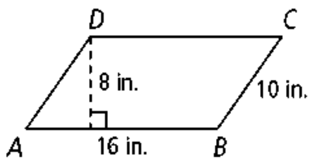
Areas of Parallelograms and Triangles

The area of a parallelogram is base × height. The base can be any side of the parallelogram. The height is the length of the corresponding altitude.

Problem

What is the area of ▱*ABCD?*

$\overline{AB}$ is the correct base to use for the given altitude.

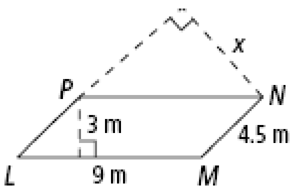


$A = bh$ Substitute and simplify.

$A = 8(16) = 128$ in.^{2}

Problem

What is the value of *x?*



Step 1: Find the area of the parallelogram using the altitude perpendicular to $\overline{LM}$.

$A = bh$ Substitute and simplify.

$A = 9(3) = 27$ m^2

Step 2: Use the area of the parallelogram to find the value of *x*.

$A = bh$ Substitute.

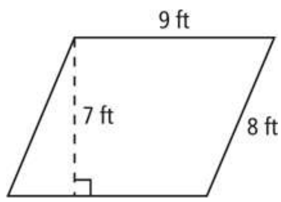
$27 = 4.5x$ Simplify.

$x = 6$ m

Exercises

Find the area of each parallelogram.

1.



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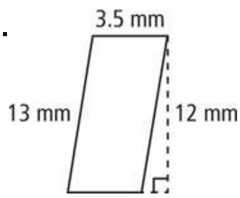

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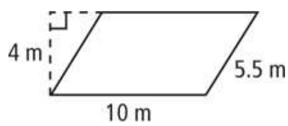
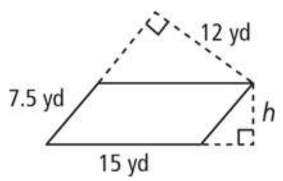


Find the value of *h* for each parallelogram.

4.



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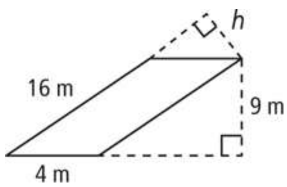

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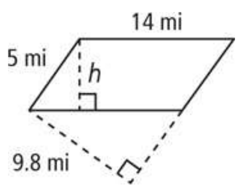


10-1 Reteaching (continued)

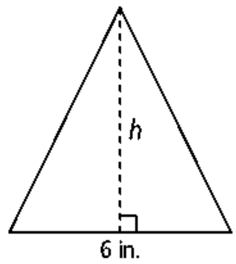
Areas of Parallelograms and Triangles

The area of a triangle is $\frac{1}{2}$ × base × height. The base is any side of the triangle. The height is the length of the corresponding altitude.

Problem

A triangle has an area of 18 in.^{2}. The length of its base is 6 in. What is the corresponding height?

Draw a sketch. Then substitute into the area formula, and solve for *h*.



$A = \frac{1}{2}bh$ Substitute.

$18 = \frac{1}{2}(6)h = 3h$ Simplify.

$h = 6$ in.

Exercises

7. Use graph paper. Draw an obtuse, an acute, and a right triangle, each with an area of 12 square units. Label the base and height of each triangle.

8. Draw a different obtuse, acute, and right triangle, each with an area of 12 square units. Label the base and height of each triangle.

9. A triangle has height 5 cm and base length 8 cm. Find its area.

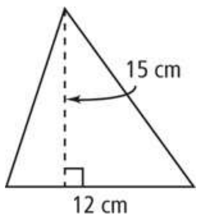
10. A triangle has height 11 in. and base length 10 in. Find its area.

11. A triangle has area 24 m^2 and base length 8 m. Find its height.

12. A triangle has area 16 ft^2 and height 4 ft. Find its base.

Find the area of each triangle.

13.



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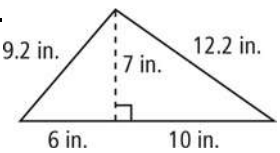
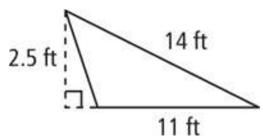
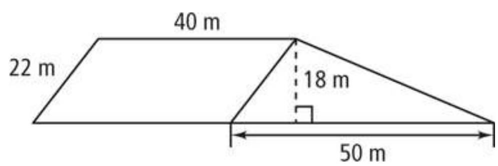


15.



16. The figure at the right consists of a parallelogram and a triangle. What is the area of the figure?



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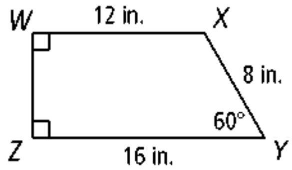
10-2 Reteaching

Areas of Trapezoids, Rhombuses, and Kites

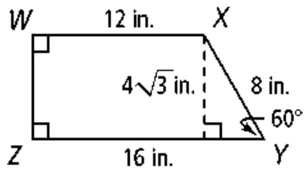
The area of a trapezoid is $\frac{1}{2}h(b_1 + b_2)$, where h is the length of the height and b_1 and b_2 are the lengths of the two parallel bases.

Problem



What is the area of trapezoid *WXYZ?*

Draw an altitude to divide the trapezoid into a rectangle and a 30°-60°-90° triangle. In a 30°-60°-90° triangle, the length of the longer leg is $\frac{\sqrt{3}}{2}$ times the length of the hypotenuse.



$$h = \frac{\sqrt{3}}{2}(8) = 4\sqrt{3} \text{ in.}$$

Use the formula for the area of a trapezoid.

$$A = \frac{1}{2}h(b1 + b2) \qquad \text{Substitute.}$$

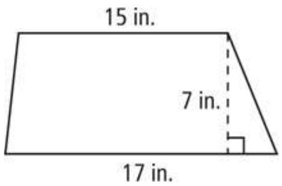
$$= \frac{1}{2}\left(4\sqrt{3}\right)\left(12+16\right) \qquad \text{Simplify.}$$

$$= 56\sqrt{3} \text{ in.}^2$$

The area of trapezoid *WXYZ* is $56\sqrt{3}$ in.^{2}

Find the area of each trapezoid. If necessary, leave your answer in simplest radical form.

1.



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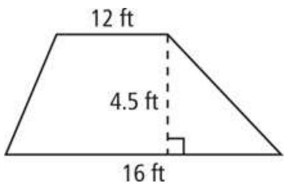

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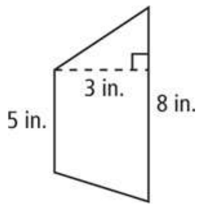

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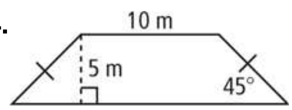

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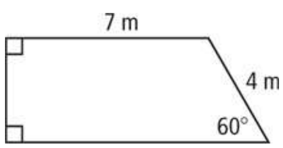

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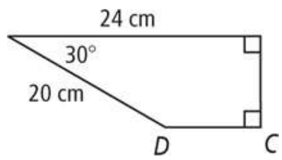

7. Find the area of a trapezoid with bases 9 m and 12 m and height 6 m.

8. Find the area of a trapezoid with bases 7 in. and 11 in. and height 3 in.

9. Find the area of a trapezoid with bases 14 in. and 5 in. and height 11 in.

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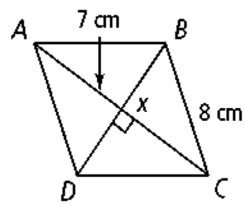
10-2 Reteaching (continued)

Areas of Trapezoids, Rhombuses, and Kites

The area of a rhombus or a kite is $\frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of the diagonals. The diagonals bisect each other and intersect at right angles.

Problem

What is the area of rhombus *ABCD?* First find the length of each diagonal.



$d_1 = 7 + 7 = 14$	Diagonals of a rhombus bisect each other.
$8^2 = 7^2 + x^2$	Pythagorean Theorem
$64 = 49 + x^2$	Simplify.
$x = \sqrt{15}$	
$d_2 = 2x = 2\sqrt{15}$ cm	

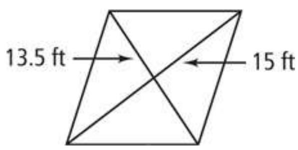
Use the formula for the area of a rhombus.

$A = \frac{1}{2}d_1d_2$	Substitute.
$= \frac{1}{2}(14)(2\sqrt{15})$	Simplify.
$= 14\sqrt{15}$ cm^2	

The area of rhombus *ABCD* is $14\sqrt{15}$ cm^2.

Find the area of each rhombus. Leave your answer in simplest radical form.

10.



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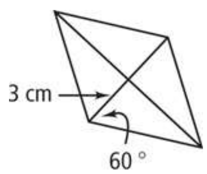

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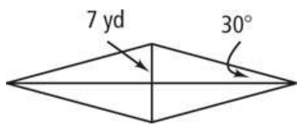
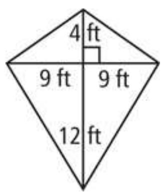


Find the area of each kite. Leave your answer in simplest radical form.

13.



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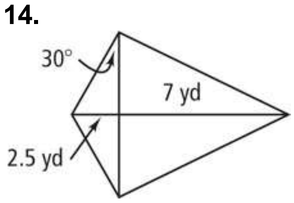

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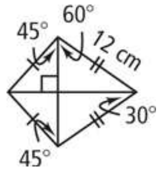

16. Find the area of a kite with diagonals 13 cm and 14 cm.

17. Find the area of a rhombus with diagonals 16 ft and 21 ft.

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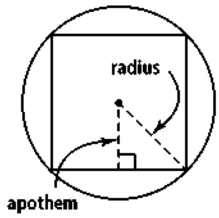
10-3 Reteaching

Areas of Regular Polygons

If you circumscribe a circle around a regular polygon:

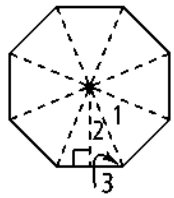
The *radius* of the circle is also the radius of the polygon, that is, the distance from the center to any vertex of the polygon.

The *apothem* is the perpendicular distance from the center to any side of the polygon.



Problem

What are $m\angle 1$, $m\angle 2$, and $m\angle 3$?



To find $m\angle 1$, divide the number of degrees in a circle by the number of sides.

$$m\angle 1 = \frac{360}{8} = 45$$

The apothem bisects the vertex angle, so you can find $m\angle 2$ using $m\angle 1$.

$$m\angle 2 = \frac{1}{2} m\angle 1 = \frac{1}{2}(45) = 22.5$$

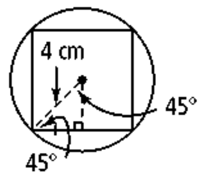
Find $\angle 3$ using the fact that $\angle 3$ and $\angle 2$ are complementary angles.

$$m\angle 3 = 90 - 22.5 = 67.5$$

For any regular polygon, the area is $A = \frac{1}{2}ap$. You can use the properties of special triangles to help you find the area of regular polygons.

Problem

What is the area of a regular quadrilateral (square) inscribed in a circle with radius 4 cm?



Draw one apothem to the base to form a 45°-45°-90° triangle. Using the 45°-45°-90° Triangle Theorem, find the length of the apothem.

$4 = \sqrt{2}a$	The hypotenuse = $\sqrt{2}$ · leg in a 45°-45°-90° triangle.
$a = \frac{4}{\sqrt{2}}$	Simplify.
$a = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$ cm	Rationalize the denominator.

The apothem has the same length as the other leg, which is half as long as a side. To find the square's area, use the formula for the area of a regular polygon.

$$A = \frac{1}{2}ap$$

$$= \frac{1}{2}\left(2\sqrt{2}\right)\left(16\sqrt{2}\right) \qquad \text{Substitute } p = 4(4\sqrt{2}) = 16\sqrt{2}.$$

$$= 32 \text{ cm}^2$$

The area of a square inscribed in a circle with radius 4 cm is 32 cm^2.

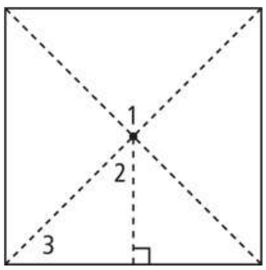
10-3 Reteaching (continued)

Areas of Regular Polygons

Exercises

Each regular polygon has radii and apothem as shown. Find the measure of each numbered angle.

1.



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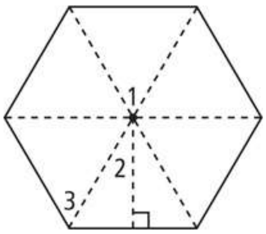

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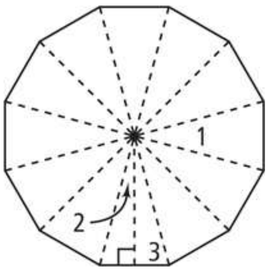


Find the area of each regular polygon with the given apothem *a* and side length *s*.

4. pentagon, $a = 4.1$ m, $s = 6$ m

5. hexagon, $a = 10.4$ in., $s = 12$ in.

6. 7-gon, $a = 8.1$ cm, $s = 7.8$ cm

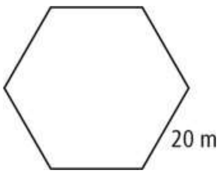
7. octagon, $a = 11.1$ ft, $s = 9.2$ ft

8. nonagon, $a = 13.2$ in., $s = 9.6$ in.

9. decagon, $a = 8.6$ m, $s = 5.6$ m

Find the area of each regular polygon. Round your answer to the nearest tenth.

10.



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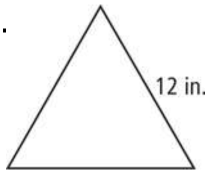

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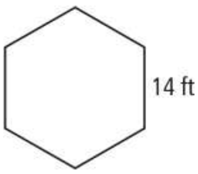
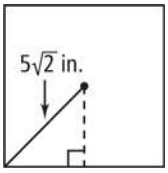


13. Reasoning How does the area of an equilateral triangle with sides 24 ft compare to the area of a regular hexagon with sides 24 ft? Explain.

Find the area of each regular polygon with the given radius or apothem. If necessary, leave your answer in simplest radical form.

14.



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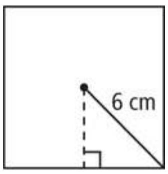

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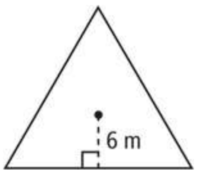

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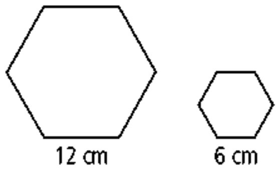
10-4 Reteaching

Perimeters and Areas of Similar Figures

Corresponding sides of *similar figures* are in proportion. The relationship between the lengths of corresponding sides in the two figures is called the *scale factor.* The perimeters and areas are related by the scale factor.

Scale Factor	Ratio of Perimeters	Ratio of Areas
$\frac{a}{b}$	$\frac{a}{b}$	$\frac{a^2}{b^2}$

Problem



The hexagons at the right are similar. What is the ratio (smaller to larger) of their perimeters and their areas?

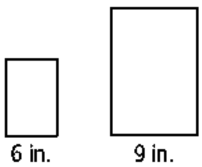
The ratio of the corresponding sides is $\frac{6}{12}$.

$$\frac{P_{\text{smaller}}}{P_{\text{larger}}} = \frac{6}{12} = \frac{1}{2}$$ Simplify.

The ratio of the areas is the square of the ratio of the corresponding sides.

$$\frac{A_{\text{smaller}}}{A_{\text{larger}}} = \frac{1^2}{2_2} = \frac{1}{4}$$

Problem



The rectangles at the right are similar. The area of the smaller rectangle is 72 in.^{2}. What is the area of the larger rectangle?

The ratio of corresponding sides is $\frac{a}{b} = \frac{6}{9} = \frac{2}{3}$.

Set up a proportion and solve:

$$\frac{A_{\text{smaller}}}{A_{\text{larger}}} = \frac{a^2}{b^2}$$

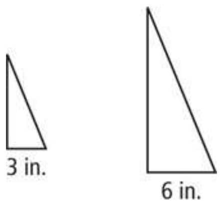
$$\frac{72}{A_{\text{larger}}} = \frac{2^2}{3^2}$$ Substitute.

$$A_{\text{larger}} = 72\left(\frac{9}{4}\right) = 162\ in.^2$$ Cross Products Property

Exercises

The figures in each pair are similar. Compare the first figure to the second. Give the ratio of the perimeters and the ratio of the areas.

1.



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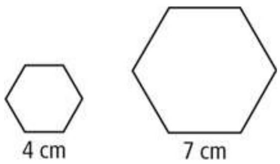

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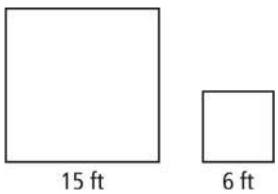


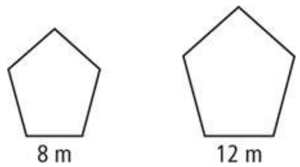
Name____________________________________Class _____________ Date____________

10-4 Reteaching (continued)

Perimeters and Areas of Similar Figures

The figures in each pair are similar. The area of one figure is given. Find the area of the other figure to the nearest whole number.

4.

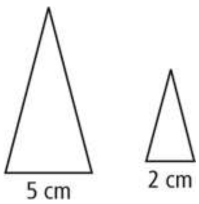


Area of smaller pentagon = 112 m^2

5.

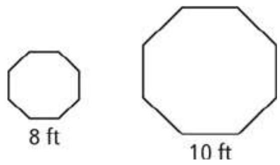
Area of smaller rectangle = 78 $in.^2$

6.



Area of larger triangle = 75 cm^2

7.



Area of smaller octagon = 288 ft^2

The scale factor of two similar polygons is given. Find the ratio of their perimeters and the ratio of their areas.

8. 4 : 3 **9.** 5 : 8 **10.** $\frac{3}{7}$ **11.** $\frac{9}{2}$

12. The area of a regular nonagon is 34 m^2. What is the area of a regular nonagon with sides five times the sides of the smaller nonagon?

13. A town is installing a sandbox in the park. The sandbox will be in the shape of a regular hexagon. On the plans for the sandbox, the sides are 4 in. and the area is about 42 $in.^2$. If the actual area of the sandbox will be 168 ft^2, what will be the length of one side of the sandbox?

14. The longer sides of a parallelogram are 6 ft. The longer sides of a similar parallelogram are 15 ft. The area of the smaller parallelogram is 27 ft^2. What is the area of the larger parallelogram?

15. The shortest side of a pentagon is 4 cm. The shortest side of a similar pentagon is 9 cm. The area of the larger pentagon is 243 cm^2. What is the area of the smaller pentagon?

16. The scale factor of two similar floors is 5:6. It costs $340 to tile the smaller floor. At that rate, how much would it cost to tile the larger floor?

10-5 Reteaching

Trigonometry and Area

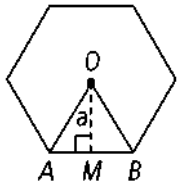
You can use the trigonometric ratios of sine, cosine, and tangent to help you find the area of regular figures.

$$\text{Sin } \angle A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{CB}{AB}$$

$$\cos \angle A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$\tan \angle A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{CB}{AC}$$

What is the area of a regular hexagon with side 12 cm?



Area = $\frac{1}{2}ap$, where a = apothem

p = perimeter

$p = 6 \cdot 12 = 72$ cm, because the figure is 6-sided.

Area = [ ]

To find a, examine ΔAOB above. The apothem is measured along $\overline{OM}$, which divides ΔAOB into congruent triangles.

$$AM = \frac{1}{2}AB = 6$$

$$m\angle AOM = \frac{1}{2}m\angle AOB$$

$$= \frac{1}{2}\left(\frac{360}{6}\right)$$

$$= 30$$

Divide 360 by 6 because there are six congruent central angles.

So, by trigonometry, $\tan 30° = \frac{AM}{a}$.

$$\tan 30^\circ = \frac{6}{a}$$

$$a = \frac{6}{\tan 30^\circ}$$

Finally, area $= \frac{1}{2}ap = \frac{1}{2}\left(\frac{6}{\tan 30^\circ}\right)(72) \approx 374.1\text{cm}^2$.

Exercises

Find the area of each regular polygon. Round your answers to the nearest tenth.

1. octagon with side 2 in.

2. decagon with side 4 cm

3. pentagon with side 10 in.

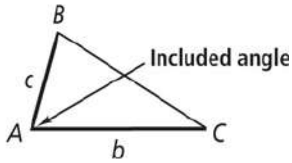
4. 20-gon with side 40 in.

10-5 Retaching (continued)

Trigonometry and Area

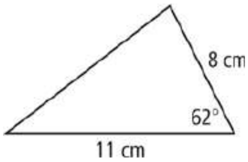
Theorem 10-8 Area of a Triangle Given SAS

$$A = \frac{1}{2}bc(\sin A)$$



Problem

What is the area of the triangle? Round your answer to the nearest tenth.



$$A = \frac{1}{2}bc(\sin A)$$

$= \frac{1}{2}(8)(11)(\sin 62^\circ)$ Substitute.

$\approx 38.84969 \text{ cm}^2$ Use a calculator.

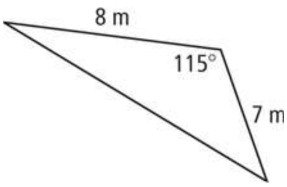
$\approx 38.8 \text{ cm}^2$ Round to the nearest tenth.

The area of the triangle is about 38.8 cm^2.

Exercises

Find the area of each triangle. Round your answers to the nearest tenth.

5.



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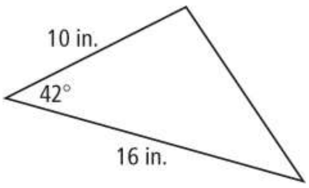

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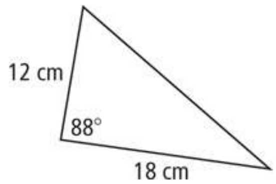

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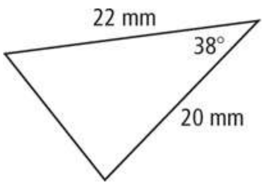

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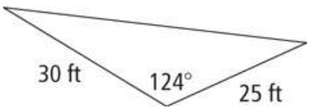

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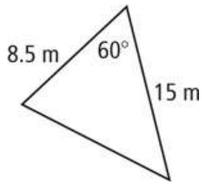
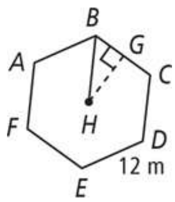


11. *ABCDEF* is a regular hexagon with center *H* and side 12 m. Find each measure. If necessary, round your answers to the nearest tenth.



a. $m\angle BHC$ **b.** $m\angle BHG$

c. BG **d.** HG

e. perimeter of *ABCDEF* **f.** area of *ABCDEF*

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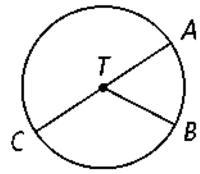
10-6 Reteaching

Circles and Arcs

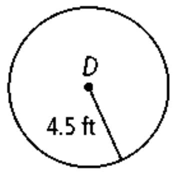
The circumference is the measure of the outside edge of a circle. Sections of the circumference are called arcs. There are three types of arcs.



The circumference of a circle is $C = 2\pi r$, or $C = \pi d$.

Problem

A circle has a 4.5-ft radius. What is the circumference of the circle?



$C = 2\pi r$

$C = 2(3.14)(4.5)$ Substitute 3.14 for π and 4.5 for r.

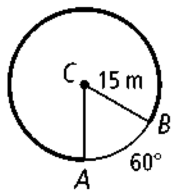
$C = 28.26$ ft Simplify.

The measure of an arc is in degrees. The arc's length depends on the size of the circle because it represents a fraction of the circumference.

Length of

Problem

What is the length of the darkened arc? Leave your answer in terms of π.



Length of

Substitute 300 for m and 15 for r.

$= 25\pi$ m Simplify.

The length of the darkened arc is 25π m.

10-6 Reteaching (continued)

Circles and Arcs

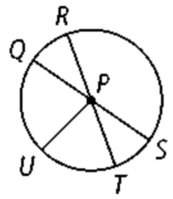
Exercises

Name the following in ⊙*P*.

1. the minor arcs

2. the major arcs

3. the semicircles

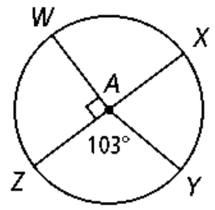


Find the measure of each arc in ⊙*A*.

4. 5. 6.

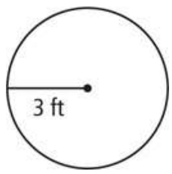
7. 8. 9.

10. 11. 12.



Find the circumference of each circle. Leave your answers in terms of π.

13.



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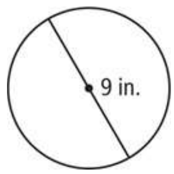
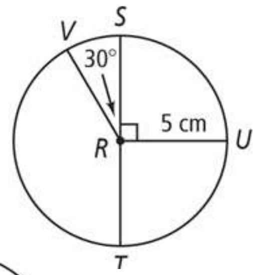


15.

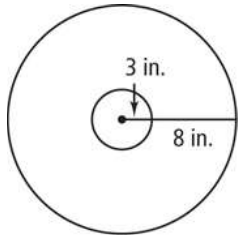
Find the length of each arc. Leave your answers in terms of π.

16. 17. 18.

19. 20. 21.



22. The wheel of a car is shown at the right. How far does the hubcap of the tire travel in one complete rotation? How far does the tire itself travel in one complete rotation?



23. How far does the tip of a minute hand on a clock travel in 20 minutes if the distance from the center to the tip is 9 cm? Leave your answer in terms of π. Then, round your answer to the nearest tenth.

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10-7 Reteaching

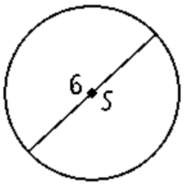
Areas of Circles and Sectors

Finding the Area of a Circle

The area of a circle is $A = \pi r^2$. So, to find the area of a circle, you need to know the radius, r. Sometimes you are given the radius directly. Sometimes you are given the diameter and have to divide by 2 to find the radius.

Problem

What is the area of $\odot S$?



$r = \frac{6}{2} = 3$	Find the radius.
$A = \pi(3)^2$	Substitute.
$A = 9\pi$	Simplify.
$A \approx 28.3$	Approximate.

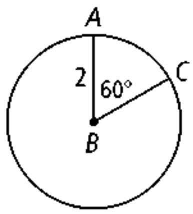
The area of $\odot S$ is about 28.3 units^2.

Finding the Area of a Sector

To find the area of a sector, find the area of the circle, then multiply by the fraction of the circumference covered by the arc of the sector.

Problem

$\odot B$ has a radius of 2 and $m\overset{\frown}{AC} = 60$. What is the area of sector ABC?



First, find the area of $\odot B$: $A = (2)^2 \cdot \pi = 4\pi$

Then, find the fraction of the circumference covered by the arc.

$$\frac{m\overset{\frown}{AC}}{360} = \frac{60}{360} = \frac{6}{36} = \frac{1}{6}$$

Last, multiply by the fraction.

Area of sector $ABC = \frac{1}{6} \cdot 4\pi = \frac{4}{6}\pi = \frac{2}{3}\pi \approx 2.1$

Exercises

Find the areas of circles and sectors described below. Write your answer in terms of π and round to the nearest tenth.

1. circle with radius 5

2. circle with diameter 16

3. sector ABC in $\odot B$ of radius 4 and $m\overset{\frown}{AC} = 90$

4. sector RST in $\odot S$ of diameter 12 and $m\overset{\frown}{RT} = 45$

10-7 Reteaching (continued)

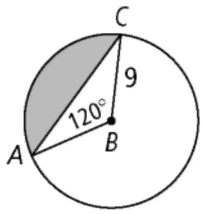
Areas of Circles and Sectors

Finding the Area of a Segment

To find the area of a segment, subtract the area of the triangle from the area of the sector. So, use your knowledge of how to find the area of a sector.

Problem

What is the area of the shaded segment?



a. Find the area of sector *ABC*.

Circle: $A = (9)^2 \bullet \pi = 81\pi$

Fraction: $\frac{m\widehat{AC}}{360} = \frac{120}{360} = \frac{1}{3}$

Multiply: Area of sector ABC $= \frac{1}{3} \cdot 81\pi = 27\pi$

b. Find the area of ΔABC.

Triangle: $A = \frac{1}{2}(AB)(BC)\sin\angle ABC$

$A = \frac{1}{2}(9)^2 \sin 120 = \frac{81\sqrt{3}}{4}$

c. Subtract to find the area of the segment.

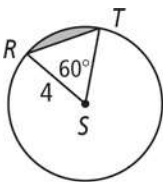
Segment: $A = 27\pi - \frac{81\sqrt{3}}{4} \approx 49.7$

The area of the shaded segment is about 49.7 units^2.

Exercises

Find the area of each shaded segment.

5.



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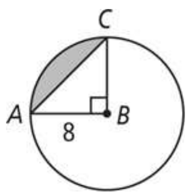

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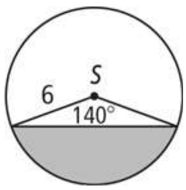
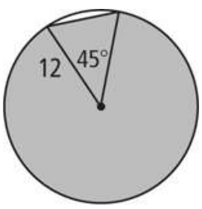


Find the area of the shaded region. Leave your answer in terms of π and in simplest radical form.

8.



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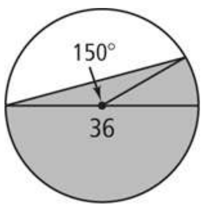

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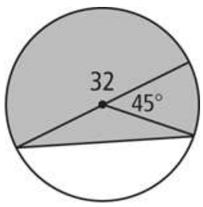


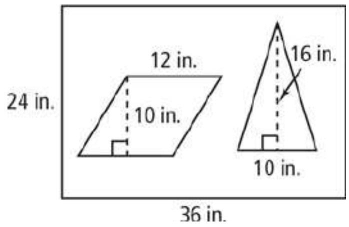
Name____Class______________ Date______________

10-8 Reteaching

Geometric Probability

Problem

If a dart lands at random on the poster at the right, what is the probability that the dart will land inside one the polygons?



Find the sum of the areas of the polygons.

area of polygons = area of parallelogram + area of triangle

$$= (12)(10) + \frac{1}{2}(10)(16)$$

$$= 120 + 80$$

$$= 200 \text{ in.}^2$$

Find the total area of the poster.

$A = (24)(36) = 864 \text{ in.}^2$

Calculate the probability.

$$\text{P(polygon)} = \frac{\text{area of polygons}}{\text{total area}}$$

$$= \frac{200}{864}$$

$$\approx 23\%$$

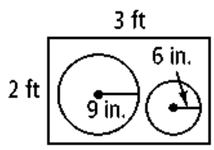
Exercises

Complete each exercise.

1. Use a compass to draw a circle with radius 1 in. on an index card.
2. Calculate the theoretical probability that if a tack is dropped on the card, its tip will land in the circle.
3. Lift a tack 12 in. above the index card and drop it. Repeat this 25 times. Record how many times the tip of the tack lands in the circle. (Ignore the times that the tack bounces of the card.) Calculate the experimental probability:

$$P = \frac{\text{number of times tip landed in circle}}{25}$$

4. How do the probabilities you found in Exercises 2 and 3 compare?



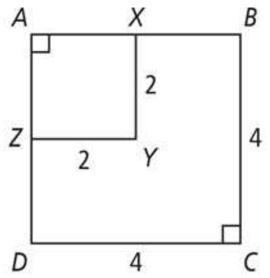
5. If you repeated the experiment 100 times, what would you expect the results to be?
6. If a dart lands at random on the poster at the right, what is the probability that the dart will land in a circle?

10-8 Reteaching (continued)

Geometric Probability

For Exercises 7–10 give your answer as a ratio and as a percent.

For Exercises 7 and 8 use square *ABCD* at the right.

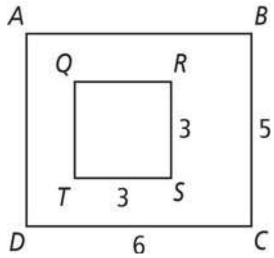


7. Point *P* in square *ABCD* is chosen at random.

Find the probability that *P* is in square *AXYZ.*

8. Find the probability that *P* is *not* in square *AXYZ.*

For Exercises 9 and 10 use rectangle *ABCD* at the right.

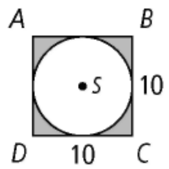


9. Point *P* in rectangle *ABCD* is chosen at random.

Find the probability that *P* is in square *QRST.*

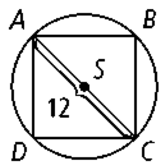
10. Find the probability that *P* is *not* in square *QRST.*

Give your answer in terms of π, then as a percent.



11. Point *P* in square *ABCD* is chosen at random.

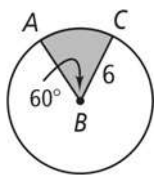
Find the probability that *P* is in *not* in ⊙*S.*



12. Point *P* in ⊙*S* is chosen at random. Find the probability that *P* is in *not* in square *ABCD.*

Point *P* in ⊙*S* is chosen at random. Find the probability that *P* is in sector *ABC.* Give your answer in terms of a ratio, then as a percent.

13.



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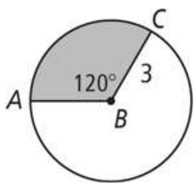
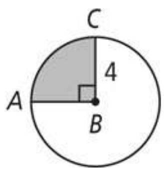


15.



16. The cycle of the light on George Street at the intersection of George Street and Main Street is 10 seconds green, 5 seconds yellow, and 60 seconds red. If you reach the intersection at a random time, what is the probability that the light is red?

Name ______________________________ Class____________ Date __________

11-1 Reteaching

Space Figures and Cross Sections

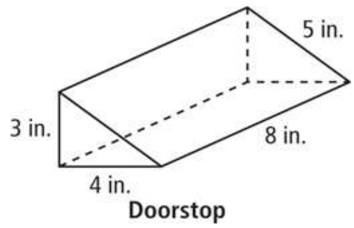
A *polyhedron* is a three-dimensional figure with *faces* that are polygons. Faces intersect at *edges,* and edges meet at vertices.

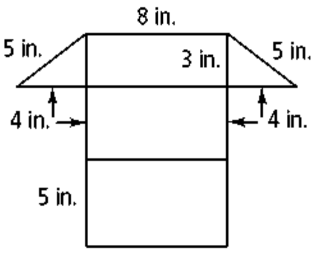
Faces, vertices, and edges are related by *Euler's Formula:* $F + V = E + 2$.

For two dimensions, such as a representation of a polyhedron by a *net,* Euler's Formula is $F + V = E + 1$. (F is the number of regions formed by V vertices linked by E segments.)

Problem

What does a net for the doorstop at the right look like? Label the net with its appropriate dimensions.





Exercises

Complete the following to verify Euler's Formula.

1. On graph paper, draw three other nets for the polyhedron shown above. Let each unit of length represent $\frac{1}{4}$ in.

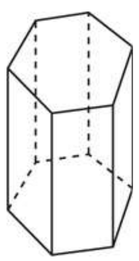
2. Cut out each net, and use tape to form the solid figure.

3. Count the number of vertices, faces, and edges of one of the figures.

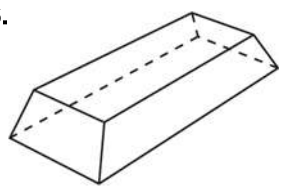
4. Verify that Euler's Formula, $F + V = E + 2$, is true for this polyhedron.

Draw a net for each three-dimensional figure.

5.



6.



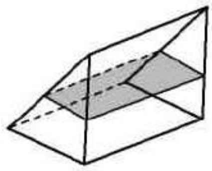
Name____________________________________Class _____________ Date_____________

11-1 Reteaching (continued)

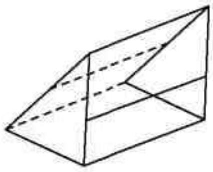
Space Figures and Cross Sections

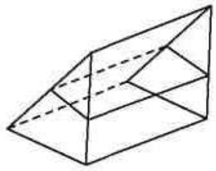
A *cross section* is the intersection of a solid and a plane. Cross sections can be many different shapes, including polygons and circles.

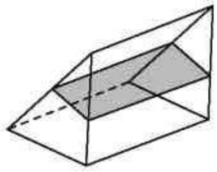
The cross section of this solid and this plane is a rectangle. This cross section is a horizontal plane.



To draw a cross section, visualize a plane intersecting one face at a time in parallel segments. Draw the parallel segments, then join their endpoints and shade the cross section.

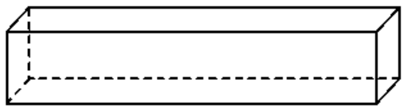






Exercises

Draw and describe the cross section formed by intersecting the rectangular prism with the plane described.

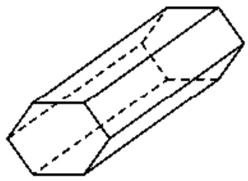


7. a plane that contains the vertical line of symmetry

8. a plane that contains the horizontal line of symmetry

9. a plane that passes through the midpoint of the top left edge, the midpoint of the top front edge, and the midpoint of the left front edge

10. What is the cross section formed by a plane that contains a vertical line of symmetry for the figure at the right?



11. Visualization What is the cross section formed by a plane that is tilted and intersects the front, bottom, and right faces of a cube?

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11-2 Reteaching

Surface Areas of Cylinders and Prisms

A *prism* is a polyhedron with two congruent parallel faces called *bases*. The non-base faces of a prism are *lateral faces*. The dimensions of a right prism can be used to calculate its lateral area and surface area.

The lateral area of a right prism is the product of the perimeter of the base and the height of the prism.

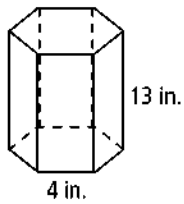
L.A. = ph

The surface area of a prism is the sum of the lateral area and the areas of the bases of the prism.

S.A. = L.A. + $2B$

Problem

What is the lateral area of the regular hexagonal prism?

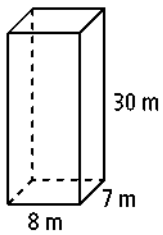


L.A. = ph	
$p = 6(4\text{ in.}) = 24\text{ in.}$	Calculate the perimeter.
L.A. = 24 in. × 13 in.	Substitute.
L.A. = 312 in.^{2}	Multiply.

The lateral area is 312 in^2

Problem

What is the surface area of the prism?



S.A. = L.A. + $2B$	
$p = 2(7\text{ m} + 8\text{ m})$	Calculate the perimeter.
$p = 30\text{ m}$	Simplify.
L.A. = ph	
L.A. = 30 m × 30 m	Substitute.
L.A. = 900 m^2	Multiply.
$B = 8\text{ m} \times 7\text{ m}$	Find base area.
$B = 56\text{ m}^2$	Multiply.
S.A. = L.A. + $2B$	
S.A. = 900 m^2 + 2 × 56 m^2	Substitute.
S.A. = 1012 m^2	Simplify.

The surface area of the prism is 1012 m^2.

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11-2 Reteaching (continued)

Surface Areas of Cylinders and Prisms

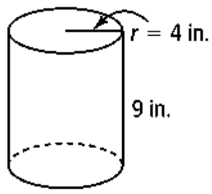
A *cylinder* is like a prism, but with circular bases. For a right cylinder, the radius of the base and the height of the cylinder can be used to calculate its *lateral area* and *surface area.*

Lateral area is the product of the circumference of the base ($2\pi r$) and the height of the cylinder. Surface area is the sum of the lateral area and the areas of the bases ($2\pi r^2$).

L.A. = $2\pi rh$ or πdh S.A. = $2\pi rh + 2\pi r^2$

Problem

The diagram at the right shows a right cylinder. What are the lateral area and surface area of the cylinder?



L.A. = $2\pi rh$ or πdh

L.A. = $2\pi \times 4$ in. $\times$ 9 in. Substitute for *r* and *h*.

L.A. = 72π in.^{2} Multiply.

The lateral area is 72π in.^{2}.

S.A. = $2\pi rh + 2\pi r^2$

S.A. = $2\pi \times 4$ in. $\times$ 9 in. + $2\pi \times (4 \text{ in.})^2$ Substitute for *r* and *h*.

S.A. = 72π in.^{2} + 32π in.^{2} Multiply.

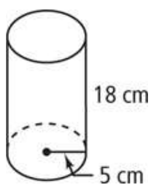
S.A. = 104π in.^{2} Add.

The surface area is 104π in.^{2}.

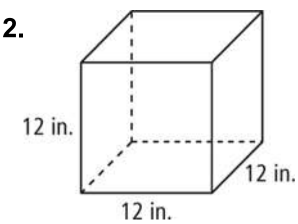
Exercises

Find the lateral area and surface area of each figure. Round your answers to the nearest tenth, if necessary.

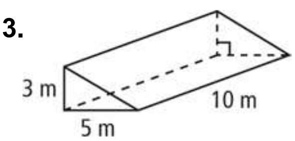
1.



2.



3.



4. A cylindrical carton of raisins with radius 4 cm is 25 cm tall. If all surfaces except the top are made of cardboard, how much cardboard is used to make the raisin carton? Round your answer to the nearest square centimeter.

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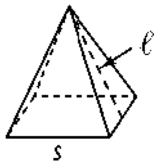
11-3 Reteaching

Surface Areas of Pyramids and Cones

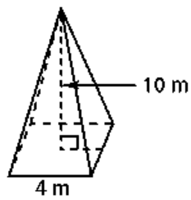
A *pyramid* is a polyhedron in which the *base* is any polygon and the *lateral faces* are triangles that meet at the *vertex.* In a *regular pyramid,* the base is a regular polygon. The *height* is the measure of the altitude of a pyramid, and the *slant height* is the measure of the altitude of a lateral face. The dimensions of a regular pyramid can be used to calculate its lateral area (L.A.) and surface area (S.A.).



, where *p* is the perimeter of the base and *l* is slant height of the pyramid.

S.A. = L.A. + *B,* where *B* is the area of the base.

What is the surface area of the square pyramid to the nearest tenth?



S.A. =L.A. + *B*

$\text{L.A.} = \frac{1}{2}p\ell$ — Find lateral area first.

$p = 4(4\text{ m}) = 16\text{ m}$ — Find the perimeter.

$\ell^2 = \sqrt{2^2 + 10^2} = \sqrt{104}$

$\ell \approx 10.2$

$\text{L.A} = \frac{1}{2}(16\text{m})(10.2) = 81.6\text{m}^2$ — Substitute to find L.A.

$B = (4\text{ m})(4\text{ m}) = 16\text{ m}^2$ — Find area of the base.

$\text{S.A.} = 81.6\text{ m}^2 + 16\text{ m}^2 = 97.6\text{ m}^2$ — Substitute to find S.A.

The surface area of the square pyramid is about 97.6 m^2.

Exercises

Use graph paper, scissors, and tape to complete the following.

1. Draw a net of a square pyramid on graph paper.
2. Cut it out, and tape it together.
3. Measure its base length and slant height.
4. Find the surface area of the pyramid.

In Exercises 5 and 6, round your answers to the nearest tenth, if necessary.

5. Find the surface area of a square pyramid with base length 16 cm and slant height 20 cm.
6. Find the surface area of a square pyramid with base length 10 in. and height 15 in.

Name_________________________ Class____________ Date__________

11-3 Reteaching (continued)

Surface Areas of Pyramids and Cones

A *cone* is like a pyramid, except that the base of a cone is a circle. The radius of the base and cylinder height can be used to calculate the lateral area and surface area of a right cone.

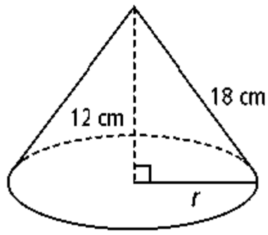
L.A. = $\pi r \ell$, where r is the radius of the base and ℓ is slant height of the cone.

S.A. = L.A. + B, where B is the area of the base ($B = \pi r^2$).

Problem

What is the surface area of a cone with slant height 18 cm and height 12 cm?

Begin by drawing a sketch.



Use the Pythagorean Theorem to find r, the radius of the base of the cone.

$$r^2 + 12^2 = 18^2$$

$$r^2 + 144 = 324$$

$$r^2 = 180$$

$$r \approx 13.4$$

Now substitute into the formula for the surface area of a cone.

$$\text{S.A.} = \text{L.A.} + B$$

$$= \pi r \ell + \pi r^2$$

$$= \pi(13.4)(18) + 180\pi$$

$$\approx 1323.2$$

The surface area of the cone is about 1323.2 cm^2.

In Exercises 7–10, round your answers to the nearest tenth, if necessary.

7. Find the surface area of a cone with radius 5 m and slant height 15 m.

8. Find the surface area of a cone with radius 6 ft and height 11 ft.

9. Find the surface area of a cone with radius 16 cm and slant height 20 cm.

10. Find the surface area of a cone with radius 10 in. and height 15 in.

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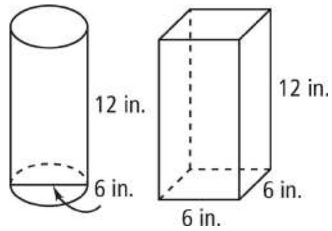
11-4 Reteaching

Volumes of Prisms and Cylinders

Problem

Which is greater: the volume of the cylinder or the volume of the prism?



Volume of the cylinder: $V = Bh$

$= \pi r^2 \cdot h$

$= \pi(3)^2 \cdot 12$

$\approx 339.3 \text{ in.}^3$

Volume of the prism: $V = Bh$

$= s^2 \cdot h$

$= 6^2 \cdot 12$

$= 432 \text{ in.}^3$

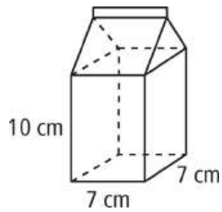
The volume of the prism is greater.

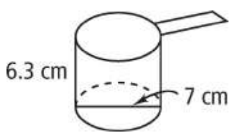
Exercises

Find the volume of each object.

1. the rectangular prism part of the milk container

2. the cylindrical part of the measuring cup





Find the volume of each of the following. Round your answers to the nearest tenth, if necessary.

3. a square prism with base length 7 m and height 15 m

4. a cylinder with radius 9 in. and height 10 in.

5. a triangular prism with height 14 ft and a right triangle base with legs measuring 9 ft and 12 ft

6. a cylinder with diameter 24 cm and height 5 cm

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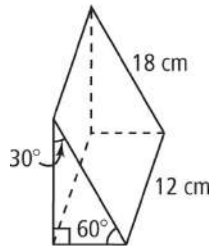
11-4 Reteaching (continued)

Volumes of Prisms and Cylinders

Problem

What is the volume of the triangular prism?

Sometimes the height of a triangular base in a triangular prism is not given. Use what you know about right triangles to find the missing value. Then calculate the volume as usual.



hypotenuse = 18 cm	Given
short leg = 9 cm	30°-60°-90° triangle theorem
long leg = 9 ☐ cm	30°-60°-90° triangle theorem

☐

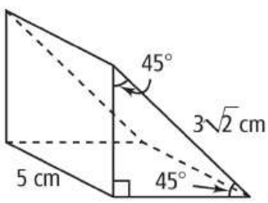
Volume of prism: $V = Bh$

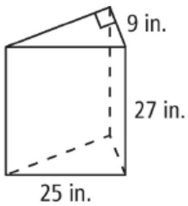
$V \approx 841.8\text{ cm}^3$

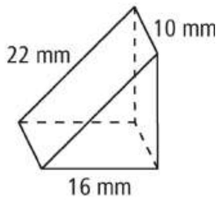
The volume of the triangular prism is about 841.8 cm^3.

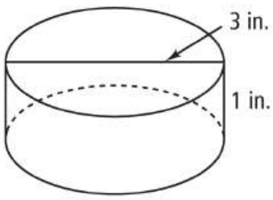
Exercises

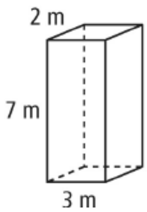
Find the volume of each prism. Round to the nearest tenth.

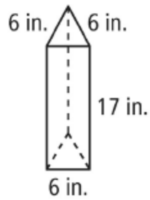
7. 

8. 

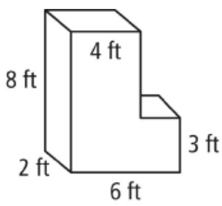
9. 

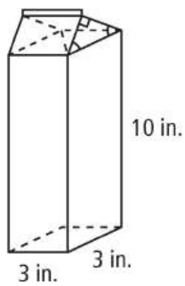
10. 

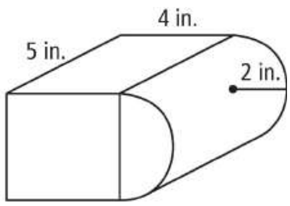
11. 

12. 

Find the volume of each composite figure to the nearest tenth.

13. 

14. 

15. 

Name ______________________ Class __________ Date __________

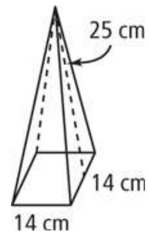
11-5 Reteaching

Volumes of Pyramids and Cones

Problem

What is the volume of the square pyramid?

Sometimes the height of a triangular face in a square pyramid is not given. Here the slant height and the lengths of the sides of the base are given. Use what you know about right triangles to find the missing value. Then calculate the volume as usual.



$7^2 + x^2 = 25^2$	Use the Pythagorean Theorem.
$49 + x^2 = 625$	Substitute.
$x^2 = 625 - 49$	Isolate the variable.
$x^2 = 576$	Simplify.
$x = 24$ cm	Find the square root of each side.

Volume of the pyramid:

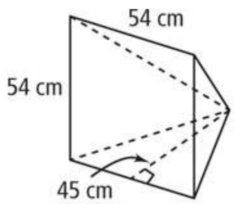
$V = \frac{1}{3}Bh$	Use the formula for volume of a pyramid.
$= \frac{1}{3}(14 \times 14)(24)$	Substitute.
$= 1568 \text{ cm}^3$	Simplify.

The volume of the square pyramid is 1568 cm^3.

Exercises

Find the volume of each pyramid. Round to the nearest whole number.

1.



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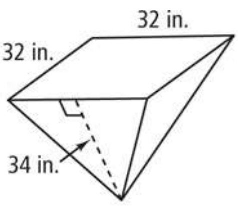

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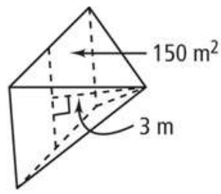

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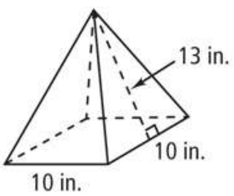

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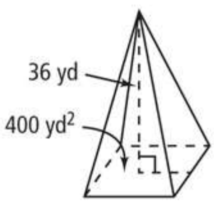

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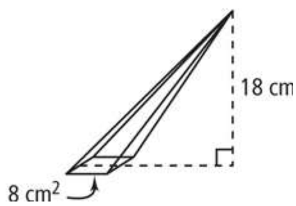


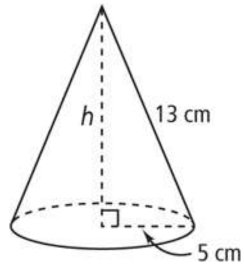
Name ________________ Class ________ Date ________

11-5 Reteaching

Volumes of Pyramids and Cones

Problem

What is the volume of the cone?



Find the height of the cone.

$13^2 = h^2 + 5^2$	Use the Pythagorean Theorem.
$169 = h^2 + 25$	Substitute.
$h^2 = 144$	Simplify.
$h = 12$	Take the square root of each side.

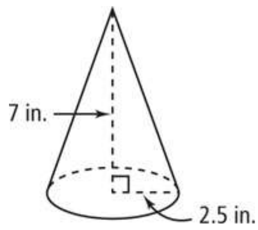
Find the height of the cone.

$V = \frac{1}{3}\pi r^2 h$	Use the formula for the volume of a cone.
$= \frac{1}{3}\pi (5)^2 \cdot 12$	Substitute.
5 100p	Simplify.
≈ 314.2	

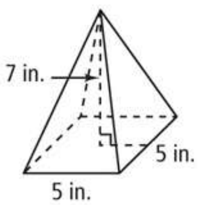
The volume of the cone is about 314.2 cm^2.

Exercises

7. From the figures shown below, choose the pyramid with volume closest to the volume of the cone at the right.



A.



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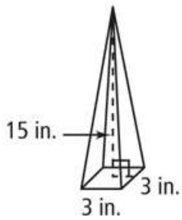
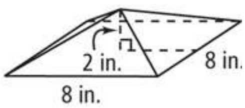
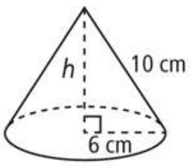


C.



Find the volume of each figure. Round your answers to the nearest tenth.

8.



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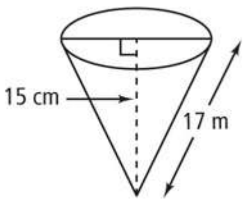

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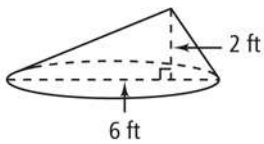

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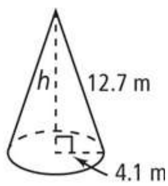


Name ______________________________ Class _______________ Date _______________

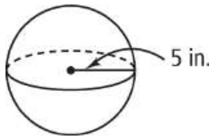
11-6 Reteaching

Surface Areas and Volumes of Spheres

Problem

What are the surface area and volume of the sphere?

Substitute $r = 5$ into each formula, and simplify.



$$\text{S.A.} = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$$

$$= 4\pi(5)^2 \qquad = \frac{4}{3}\pi(5)^3$$

$$= 100\pi \qquad = \frac{500\pi}{3}$$

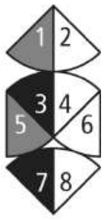
$$\approx 314.2 \qquad \approx 523.6$$

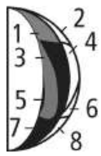
The surface area of the sphere is about 314.2 in.^{2}. The volume of the sphere is about 523.6 in.^{3}.

Exercises

Use the figures at the right to guide you in completing the following.

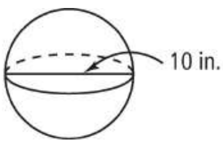
1. Use a compass to draw two circles, each with radius 3 in. Cut out each circle.
2. Fold one circle in half three successive times. Number the central angles 1 through 8.
3. Cut out the sectors, and tape them together as shown.
4. Take the other circle, fold it in half, and tape it to the rearranged circle so that they form a quadrant of a sphere.





5. The area of one circle has covered one quadrant of a sphere. How many circles would cover the entire sphere?
6. How is the radius of the sphere related to the radius of the circle?

Find the volume and surface area of a sphere with the given radius or diameter. Round your answers to the nearest tenth.

7.


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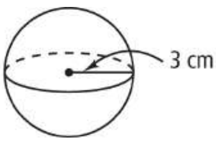

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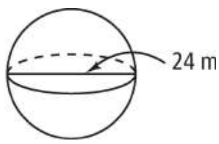
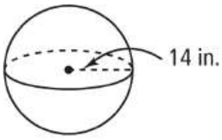


Name________________________________ Class ______________ Date _____________

11-6 Reteaching (continued)

Surface Areas and Volumes of Spheres

Find the volume and surface area of the sphere. Round to the nearest tenth.

10.


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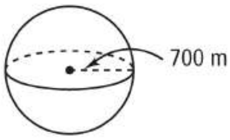

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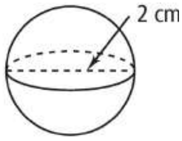

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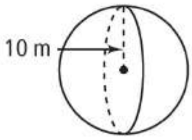

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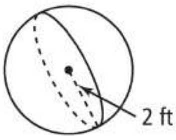

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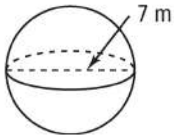


A sphere has the volume given. Find its surface area to the nearest whole number.

16. 1436.8 mi^3 **17.** 808 cm^3 **18.** 72 m^3

Find the volume of each sphere with the given surface area. Round to the nearest whole number.

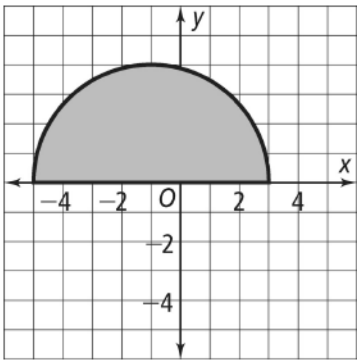
19. 435 yd^2 **20.** 907 cm^2 **21.** 28 m^2

22. Visualization The region enclosed by the semicircle at the right is revolved completely about the *x*-axis.

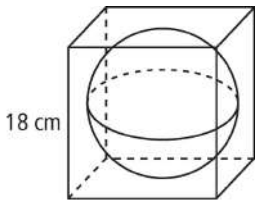
a. Describe the solid of revolution that is formed.

b. Find its volume in terms of π.

c. Find its surface area in terms of π.



23. The sphere at the right fits snugly inside a cube with 18 cm edges. What is the volume of the sphere? What is the surface area of the sphere? Leave your answers in terms of π.

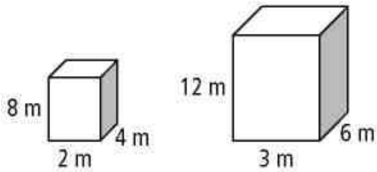


Name______________________________________Class _____________ Date_____________

11-7 Reteaching

Areas and Volumes of Similar Solids

When two solids are similar, their corresponding dimensions are proportional.



Rectangular prisms *A* and *B* are similar because the ratio of their corresponding dimensions is $\frac{2}{3}$

height: $\frac{8\text{m}}{12\text{m}} = \frac{2}{3}$ length: $\frac{2\text{m}}{3\text{m}} = \frac{2}{3}$ width: $\frac{4\text{m}}{6\text{m}} = \frac{2}{3}$

The ratio of the corresponding dimensions of similar solids is called the scale factor. All the linear dimensions (length, width, and height) of a solid must have the same scale factor for the solids to be similar.

Areas and Volumes of Similar Solids

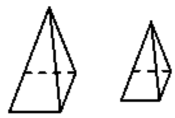
Area

- The ratio of corresponding areas of similar solids is the square of the scale factor.
- The ratio of the areas of prisms *A* and *B* is $\frac{2^2}{3^2}$, or $\frac{4}{9}$.

Volume

- The ratio of the volumes of similar solids is the cube of the scale factor.
- The ratio of the volumes of prisms *A* and *B* is $\frac{2^3}{3^3}$, or $\frac{8}{27}$.

Problem



The pyramids shown are similar, and they have volumes of 216 in.^{3} and 125 in.^{3} The larger pyramid has surface area 250 in.^{2}

What is the ratio of their surface areas?
What is the surface area of the smaller pyramid?

By Theorem 11-12, if similar solids have similarity ratio *a* : *b*, then the ratio of their volumes is $a^3 : b^3$.
So,

$$\frac{a^3}{b^3} = \frac{216}{125}$$

$$\frac{a}{b} = \frac{6}{5}$$ Take the cube root of both sides to get *a* : *b*.

$$\frac{a^2}{b^2} = \frac{36}{25}$$ Square both sides to get $a^2 : b^2$.

Ratio of surface areas = 36 : 25

If the larger pyramid has surface area 250 in.^{2}, let the smaller pyramid have surface area *x*.
Then,

$$\frac{250}{x} = \frac{36}{25}$$

$$36x = 6250$$

$$x \approx 173.6 \text{ in.}^2$$

The surface area of the smaller pyramid is about 173.6 in^2.

Name______________________________________Class _____________ Date_____________

11-7 Reteaching (continued)

Areas and Volumes of Similar Solids

Exercises

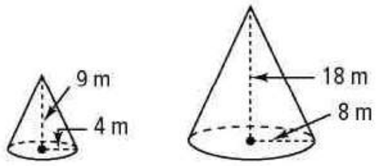
Find the scale factors.

1. Similar cylinders have volumes of 200π in.^{3} and $25\ \pi$ in.^{3}

2. Similar cylinders have surface areas of $45\ \pi$ in.^{2} and $20\ \pi$ in.^{2}

Are the two figures similar? If so, give the scale factor.

3.



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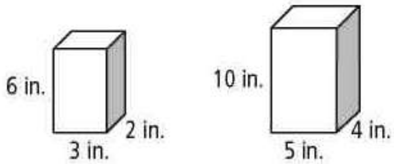
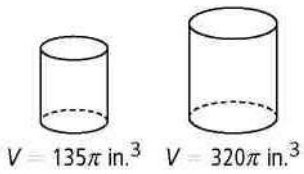


Each pair of figures is similar. Use the given information to find the scale factor of the smaller figure to the larger figure.

5.



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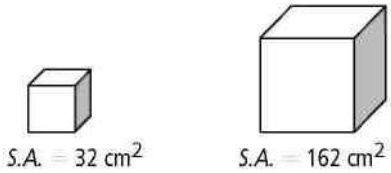

Find the ratio of volumes.

7. Two cubes have sides of length 4 cm and 5 cm.

8. Two cubes have surface areas of 64 in.^{2} and 49 in.^{2}

The surface areas of two similar figures are given. The volume of the larger figure is given. Find the volume of the smaller figure.

9. S.A. = 16 cm^2
S.A. = 100 cm^2
V = 500 cm^3

10. S.A. = 6 ft^2
S.A. = 294 ft^2
V = 3430 ft^3

11. S.A. = 45 m^2
S.A. = 80 m^2
V = 320 m^3

The volumes of two similar figures are given. The surface area of the smaller figure is given. Find the surface area of the larger figure.

12. V = 12 in.^{3}
V = 96 in.^{3}
S.A. = 12 in.^{2}

13. V = 6 cm^3
V = 384 cm^3
S.A. = 6 cm^2

14. V = 40 ft^3
V = 135 ft^3
S.A. = 20 ft^2

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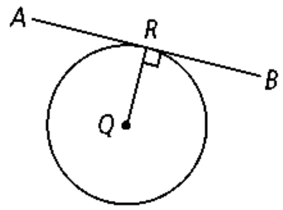
12-1 Reteaching

Tangent Lines



A tangent is a line that touches a circle at exactly one point. In the diagram, $\overline{AB}$ is tangent to $\odot Q$. You can apply theorems about tangents to solve problems.

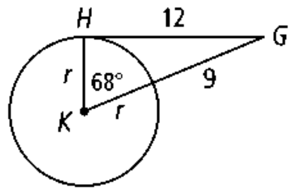
Theorem 12-1

If a line is tangent to a circle, then that line forms a right angle with the radius at the point where the line touches the circle.

Theorem 12-2

If a line in the same plane as a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

Problem



Use the diagram at the right to solve the problems below.

$\overline{GH}$ is tangent to $\odot K$.

What is the measure of $\angle G$?

Because $\overline{GH}$ is tangent to $\odot K$, it forms a right angle with the radius.

The sum of the angles of a triangle is always 180. Write an equation to find $m\angle G$.

$$m\angle G + m\angle H + m\angle K = 180$$

$$m\angle G + 90 + 68 = 180$$

$$m\angle G + 158 = 180$$

$$m\angle G = 22$$

What is the length of the radius? You can use the Pythagorean Theorem to find missing lengths.

$$HK^2 + HG^2 = GK^2$$

$$r^2 + 12^2 = (9 + r)^2$$

$$r^2 + 144 = (9 + r)(9 + r)$$

$$r^2 + 144 = 81 + 18r + r^2$$

$$63 = 18r$$

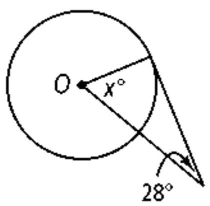
$$3.5 = r$$

So, the measure of $\angle G$ is 22 and the length of the radius is 3.5 units.

Exercises

In each circle, what is the value of x?

1.



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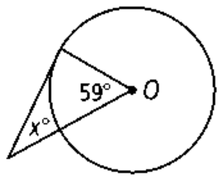

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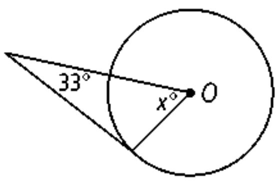
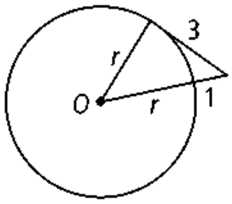


12-1 Reteaching (continued)

Tangent Lines

In each circle, what is the value of *r*?

4.



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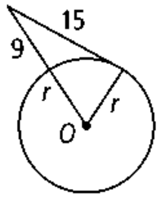

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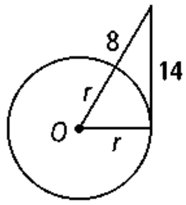
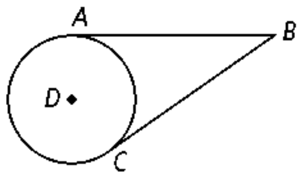


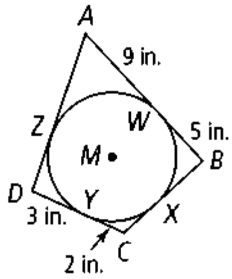
Theorem 12-3

If two segments are tangent to a circle from the same point outside the circle, then the two segments are equal in length.



In the diagram, $\overline{AB}$ and $\overline{BC}$ are both tangent to ⊙*D*. So, they are also congruent.

When circles are drawn inside a polygon so that the sides of the polygon are tangents, the circle is inscribed in the figure. You can apply Theorem 12-3 to find the perimeter, or distance around the polygon.



Problem

⊙*M* is inscribed in quadrilateral *ABCD*.
What is the perimeter of *ABCD?*

$ZA = AW = 9 \quad WB = BX = 5$

$CY = XC = 2 \quad YD = DZ = 3$

Now add to find the length of each side:

$AB = AW + WB = 9 + 5 = 14 \qquad BC = BX + CX = 5 + 2 = 7$

$CD = CY + YD = 2 + 3 = 5 \qquad DA = DZ + ZA = 3 + 9 = 12$

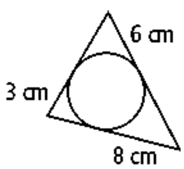
$14 + 7 + 5 + 12 = 38$

The perimeter is 38 in.

Exercises

Each polygon circumscribes a circle. What is the perimeter of each polygon?

7.



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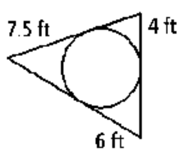
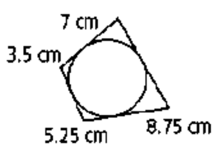


9.



Name________________________________ Class ____________ Date____________

12-2 Reteaching

Chords and Arcs

Several relationships between chords, arcs, and the central angles of a circle are listed below. The converses of these theorems are also true.

Theorem 12-4 Congruent central angles have congruent arcs.

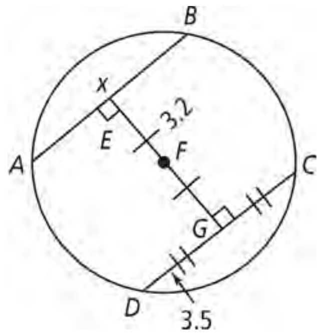
Theorem 12-5 Congruent central angles have congruent chords.

Theorem 12-6 Congruent chords have congruent arcs.

Theorem 12-7 Chords equidistant from the center are congruent.

Problem

What is the value of *x?*



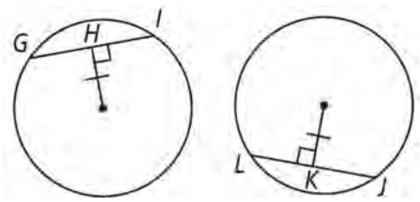
$EF = FG = 3.2$	Given
$\overline{AB} \cong \overline{DC}$	Chords equidistant from the center of a circle are congruent.
$DC = DG + GC$	Segment Addition Postulate
$AB = x + GC$	Substitution
$DG = GC = 3.5$	Given
$x = 3.5 + 3.5 = 7$	Substitution

The values of *x* is 7.

Exercises

In Exercises 1 and 2, the circles are congruent. What can you conclude?

1.



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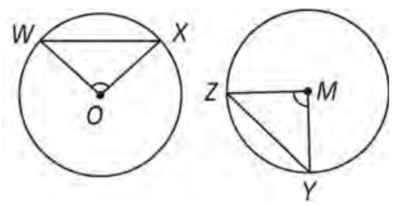
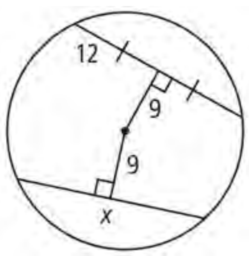


Find the value of *x*.

3.



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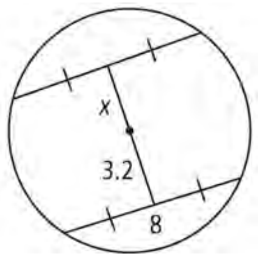

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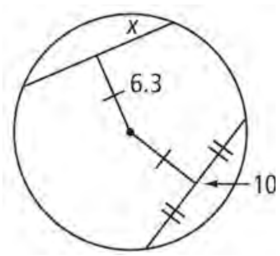

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12-2 Reteaching (continued)

Chords and Arcs

Useful relationships between diameters, chords, and arcs are listed below. To bisect a figure means to divide it exactly in half.

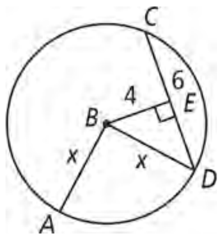
Theorem 12-8 In a circle, if a diameter is perpendicular to a chord, it bisects that chord and its arc.

Theorem 12-9 In a circle, if a diameter bisects a chord that is not a diameter of the circle, it is perpendicular to that chord.

Theorem 12-10 If a point is an equal distance from the endpoints of a line segment, then that point lies on the perpendicular bisector of the segment.

Problem

What is the value of x to the nearest tenth?



In this problem, x is the radius. To find its value draw radius $\overline{BD}$, which becomes the hypotenuse of right ΔBED. Then use the Pythagorean Theorem to solve.

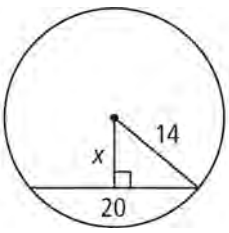
$ED = CE = 3$	A diameter perpendicular to a chord bisects the chord.
$x^2 = 3^2 + 4^2$	Use the Pythagorean Theorem.
$x^2 = 9 + 16 = 25$	Solve for x^2.
$x = 5$	Find the positive square root of each side.

The value of x is 5.

Exercises

Find the value of x to the nearest tenth.

6.



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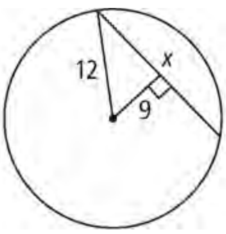

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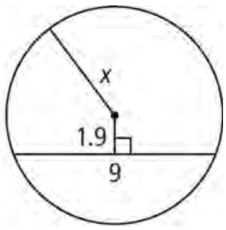
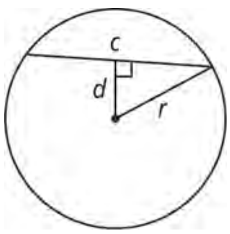


Find the measure of each segment to the nearest tenth.



9. Find c when $r = 6$ cm and $d = 1$ cm.

10. Find c when $r = 9$ cm and $d = 8$ cm.

11. Find d when $r = 10$ in. and $c = 10$ in.

12. Find d when $r = 8$ in. and $c = 15$ in.

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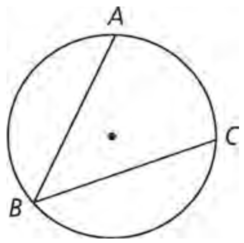
12-3 Reteaching

Inscribed Angles

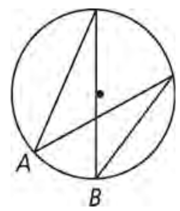
Two chords with a shared endpoint at the vertex of an angle form an inscribed angle. The intercepted arc is formed where the other ends of the chords intersect the circle.



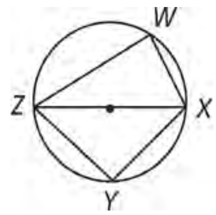
In the diagram at the right, chords $\overline{AB}$ and $\overline{BC}$ form inscribed $\angle ABC$. They also create intercepted arc $\widehat{AC}$.

The following theorems and corollaries relate to inscribed angles and their intercepted arcs.

Theorem 12-11: The measure of an inscribed angle is half the measure of its intercepted arc.



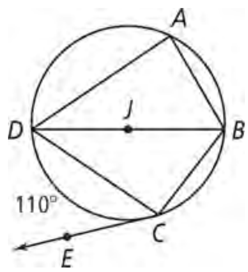
- *Corollary 1:* If two inscribed angles intercept the same arc, the angles are congruent. So, $m\angle A \cong m\angle B$.
- *Corollary 2:* An angle that is inscribed in a semicircle is always a right angle. So, $m\angle W = m\angle Y = 90$.
- *Corollary 3:* When a quadrilateral is inscribed in a circle, the opposite angles are supplementary. So, $m\angle X + m\angle Z = 180$.



Theorem 12-12: The measure of an angle formed by a tangent and a chord is half the measure of its intercepted arc.

Problem

Quadrilateral $ABCD$ is inscribed in $\odot J$.
$m\angle ADC = 68$; $\overrightarrow{CE}$ is tangent to $\odot J$
What is $m\angle ABC$? What is $m\widehat{CB}$? What is $m\angle DCE$?



$m\angle ABC + m\angle ADC = 180$	Corollary 3 of Theorem 12-11
$m\angle ABC + 68 = 180$	Substitution
$m\angle ABC = 112$	Subtraction Property
$m\widehat{DB} = m\widehat{DC} + m\widehat{CB}$	Arc Addition Postulate
$180 = 110 + m\widehat{CB}$	Substitution
$70 = m\widehat{CB}$	Simplify.
$m\widehat{CD} = 110$	Given
$m\angle DCE = \frac{1}{2}m\widehat{CD}$	Theorem 12-12
$m\angle DCE = \frac{1}{2}(110)$	Substitution
$m\angle DCE = 55$	Simplify.

So, $m\angle ABC = 112$, $m\widehat{CB} = 70$, and $m\angle DCE = 55$.

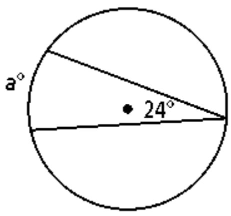
12-3 Reteaching (continued)

Inscribed Angles

Exercises

In Exercises 1–9, find the value of each variable.

1.



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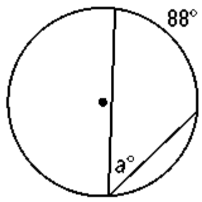

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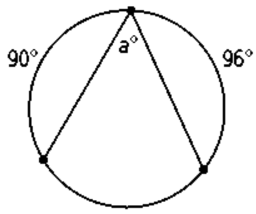

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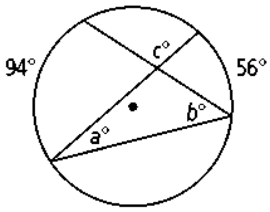

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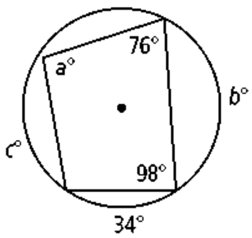

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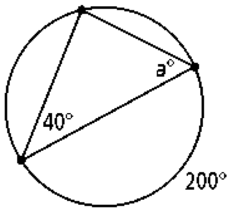

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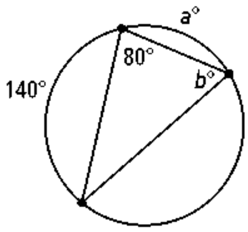

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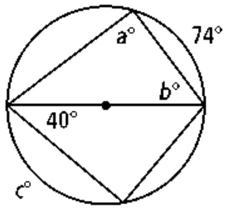

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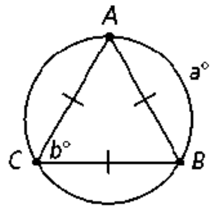
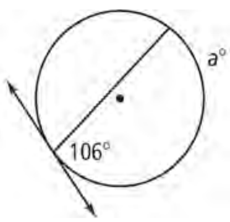


Find the value of each variable. Lines that appear to be tangent are tangent.

10.



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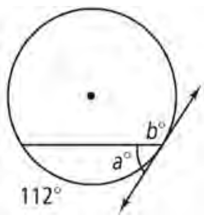

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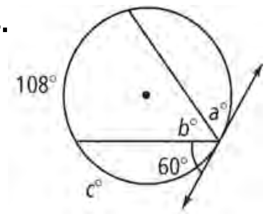

Points *A*, *B*, and *D* lie on ⊙*C*. $m\angle ACB = 40$. $m\widehat{AB} < m\widehat{AD}$. Find each measure.

13. $m\widehat{AB}$ **14.** $m\angle ADB$ **15.** $m\angle BAC$

16. A student inscribes a triangle inside a circle. The triangle divides the circle into arcs with the following measures: 46°, 102°, and 212°. What are the measures of the angles of the triangle?

17. A student inscribes *NOPQ* inside ⊙*Y*. The measure of $m\angle N = 68$ and $m\angle O = 94$. Find the measures of the other angles of the quadrilateral.

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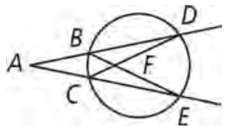
12-4 Reteaching

Angle Measures and Segment Lengths

Problem



In the circle shown, ____ = 15 and ____ = 35.

What are $m\angle A$ and $m\angle BFC$?

Because ____ and ____ are secants, $m\angle A$ can be found using Theorem 12-14.

____

Because ____ and ____ are chords, $m\angle BFC$ can be found using Theorem 12-13.

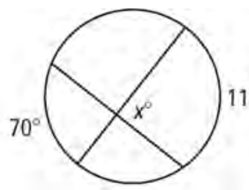
____

So, $m\angle A = 10$ and $m\angle BFC = 25$.

Exercises

Algebra Find the value of each variable.

1.



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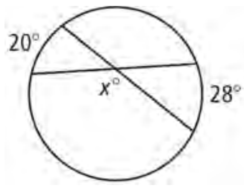

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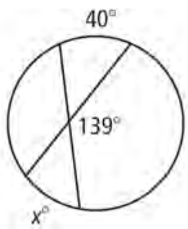

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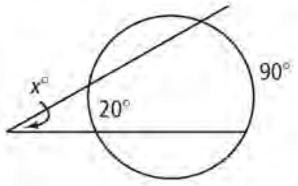

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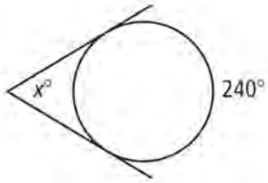

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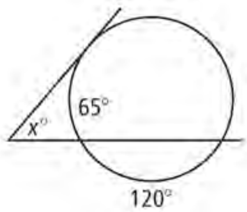

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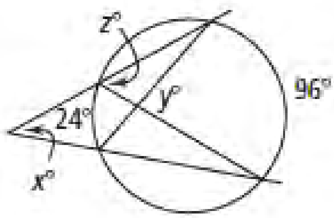

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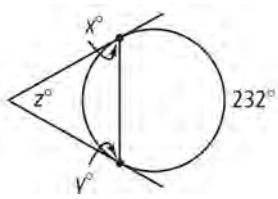

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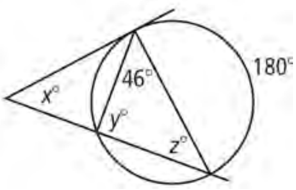

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12-4 Reteaching (continued)

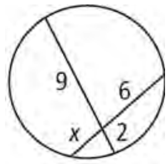
Angle Measures and Segment Lengths

Segment Lengths

Here are some examples of different cases of Theorem 12-15.

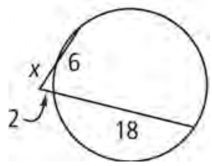
A. Chords intersecting inside a circle:
part • part = part • part

$$6x = 18$$



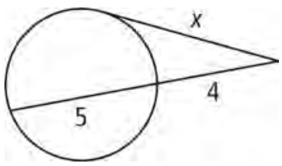
B. Secants intersecting outside a circle:
outside • whole = outside • whole

$$x(x + 6) = 2(18 + 2)$$
$$x^2 + 6x = 40$$
$$x^2 + 6x - 40 = 0$$
$$(x + 10)(x - 4) = 0$$
$$x = -10 \text{ or } x = 4$$



C. Tangent and secant intersecting outside a circle:
tangent • tangent = outside • whole

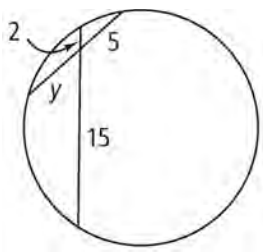
$$x(x) = 4(4 + 5)$$
$$x^2 = 4(9)$$
$$x^2 = 36$$
$$x = -6 \text{ or } x = 6$$



Exercises

Algebra Find the value of each missing variable.

10.



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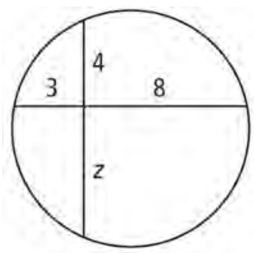

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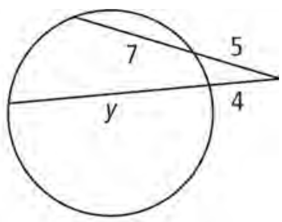

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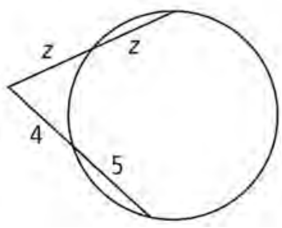

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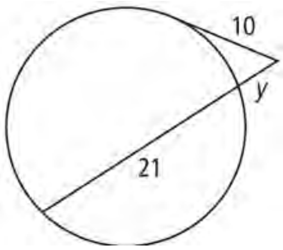

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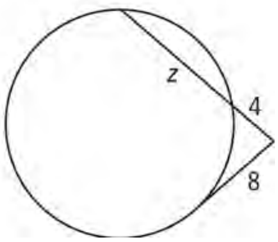

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12-5 Reteaching

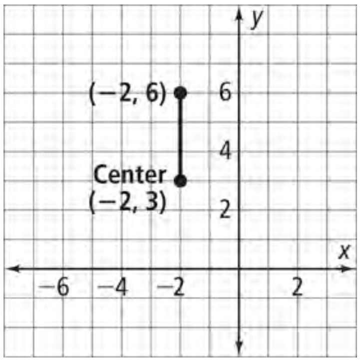
Circles in the Coordinate Plane

Writing the Equation of a Circle from a Description

The standard equation for a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. The *opposite* of the coordinates of the *center* appear in the equation. The *radius* is *squared* in the equation.

Problem

What is the standard equation of a circle with center (−2, 3) that passes through the point (−2, 6)?



Step 1 Graph the points.

Step 2 Find the radius using both given points. The radius is the *distance* from the center to a point on the circle, so $r = 3$.

Step 3 Use the radius and the coordinates of the center to write the equation.

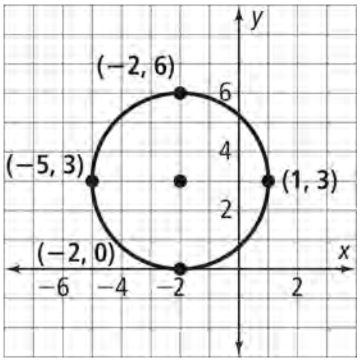
$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - (-2))^2 + (y - 3)^2 = 3^2$$
$$(x + 2)^2 + (y - 3)^2 = 9$$

Step 4 To check the equation, graph the circle. Check several points on the circle.

For (1, 3): $(1 + 2)^2 + (3 - 3)^2 = 3^2 + 0^2 = 9$

For (−5, 3): $(-5 + 2)^2 + (3 - 3)^2 = (-3)^2 + 0^2 = 9$

For (−2, 0): $(-2 + 2)^2 + (0 - 3)^2 = 0^2 + (-3)^2 = 9$



The standard equation of this circle is $(x + 2)^2 + (y - 3)^2 = 9$.

Exercises

Write the standard equation of the circle with the given center that passes through the given point. Check the point using your equation.

1. center (2, −4); point (6, −4)

2. center (0, 2); point (3, −2)

3. center (−1, 3); point (7, −3)

4. center (1, 0); point (0, 5)

5. center (−4, 1); point (2, −2)

6. center (8, −2); point (1, 4)

12-5 Reteaching (continued)

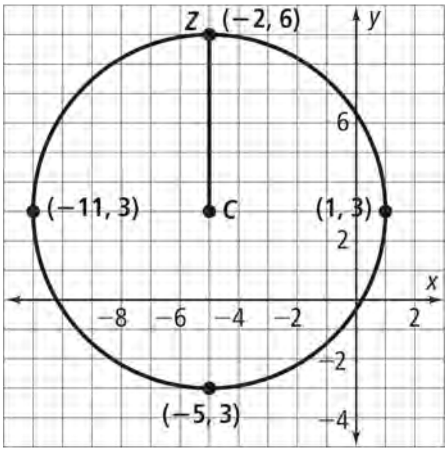
Circles in the Coordinate Plane

Writing the Equation of a Circle from a Graph

You can inspect a graph to find the coordinates of the circle's center. Use the center and a point on the circle to find the radius. It is easier if you use a horizontal or vertical radius.

Problem

What is the standard equation of the circle in the diagram at the right?



Step 1 Write the coordinates of the center.
The center is at $C(-5, 3)$.

Step 2 Find the radius. Choose a vertical radius: $\square$. The length is 6, so the radius is 6.

Step 3 Write the equation using the radius and the coordinates of the center.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - (-5))^2 + (y - 3)^2 = 6^2$$

$$(x + 5)^2 + (y - 3)^2 = 36$$

Step 4 Check two points on the circle.

For (1, 3): $(1 + 5)^2 + (3 - 3)^2 = 6^2 + 0^2 = 36$

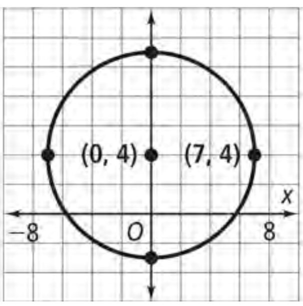
For (−11, 3): $(-11 + 5)^2 + (3 - 3)^2 = 6^2 + 0^2 = 36$

The standard equation of this circle is $(x + 5)^2 + (y - 3)^2 = 36$.

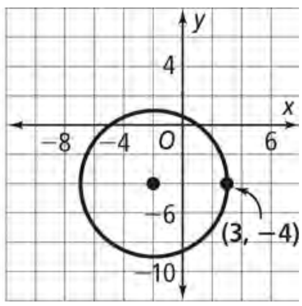
Exercises

Write the standard equation of each circle. Check two points using your equation.

7.



8.



Name____________________________________Class _____________ Date_____________

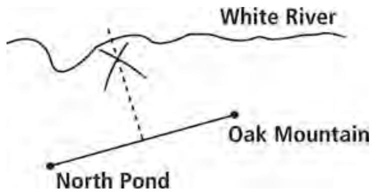
12-6 Reteaching

Locus: A Set of Points

A *locus* is a set of points that all meet a condition or conditions. Finding a locus is a strategy that can be used to solve a word problem.

Problem

A family on vacation wants to hike on Oak Mountain and fish at North Pond and along the White River. Where on the river should they fish to be equidistant from North Pond and Oak Mountain?



Draw a line segment joining North Pond and Oak Mountain.

Construct the perpendicular bisector of that segment.

The family should fish where the perpendicular bisector meets the White River.

Exercises

Describe each of the following, and then compare your answers with those of a partner.

1. the locus of points equidistant from your desk and your partner's desk

2. the locus of points on the floor equidistant from the two side walls of your classroom

3. the locus of points equidistant from a window and the door of your classroom

4. the locus of points equidistant from the front and back walls of your classroom

5. the locus of points equidistant from the floor and the ceiling of your classroom

Use points *A* and *B* to complete the following. ***A*•** **•*B***

6. Describe the locus of points in a plane equidistant from *A* and *B*.

7. How many points are equidistant from *A* and *B* and also lie on $\overleftrightarrow{AB}$? Explain your reasoning.

8. Describe the locus of points in space equidistant from *A* and *B*.

9. Draw $\overline{AB}$. Describe the locus of points in space 3 mm from $\overline{AB}$.

12-6 Reteaching (continued)

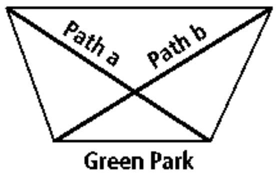
Locus: A Set of Points

10. Two students meet every Saturday afternoon to go running. Describe how they could use the map to find a variety of locations to meet that are equidistant from their homes.

Use what you know about geometric figures to answer the following questions.

11. Sam tells Tony to meet him in the northeast section of town, 1 mi from the town's center. Tony looks at his map of the town and picks up his cell phone to call Sam for more information. Why?

12. How can city planners place the water sprinklers at the park so they are always an equal distance from the two main paths of the park?



13. An old pirate scratches the following note into a piece of wood: "The treasure is 50 ft from a cedar tree and 75 ft from an oak." Under what conditions would this give you one point to dig? two? none?

14. A ski resort has cut a wide path through mountain trees. Skiers will be coming down the hill, but the resort also needs to install the chairlift in the same space. What design allows skiers to ski down the hill with the maximum amount of space between them and the trees and the huge poles that support the chairlift?

15. A telecommunications company is building a new cell phone tower and wants to cover three different villages. What location allows all three villages to get equal reception from the new tower