Palsson_Geometry_Circles_Week#3

Student Name:
Teacher Name: Palsson
Class Name/Subject: Geometry
Period:
Assignment Week #: 3
Due date: May 15 (take a photo and email me if you can!)

YOU ONLY NEED TO DO THIS PAPER VERSION WORK IF YOU DO NOT HAVE ACCESS TO INTERNET.
IF YOU DO HAVE ACCESS TO INTERNET, GO TO mpalsson.weebly.com EVERY DAY TO SEE WHAT YOU NEED TO DO.

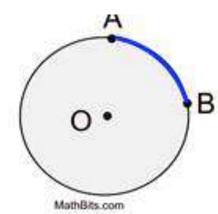
I RECOMMEND THAT WHEN YOU ARE DONE WITH THIS WORK, TAKE A PHOTO OF YOUR SOLUTIONS AND EMAIL IT TO ME AT mpalsson@tusd.net

THIS WAY YOU WILL GET YOUR GRADE MUCH FASTER THAN IF YOU TURN IN YOUR SOLUTIONS TO THE KHS OFFICE,

Also, feel free to email me if you have any questions.

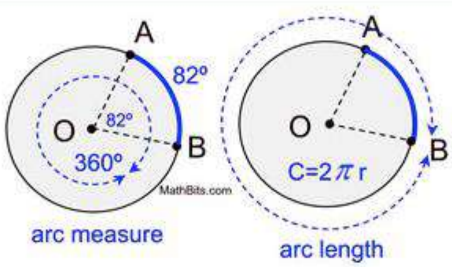
An arc of a circle is a "portion" of the circumference of the circle.

The length of an arc is simply the length of its "portion" of the circumference. The circumference itself can be considered a full circle arc length.



Definition:	Arc Measure: In a circle, the degree measure of an arc is equal to the measure of the central angle that intercepts the arc.

Definition:	Arc Length: In a circle, the length of an arc is a portion of the circumference. The letter "s" is used to represent arc length.



Consider the following proportion:

$$\frac{\text{arc measure}}{360^\circ} = \frac{\text{arc length}}{\text{circumference}}$$

If we solve the proportion for arc length, and replace "arc measure" with its equivalent "central angle", we can establish the formula:

$$\text{arc length} = \frac{\text{central angle}}{360^\circ} \bullet \text{circumference}$$

Notice that arc length is a fractional part of the circumference. For example, an arc measure of 60° is one-sixth of the circle (360°), so the length of that arc will be one-sixth of the circumference of the circle.

Example:

In circle O, the radius is 8 inches and minor arc $\overset{\frown}{AB}$ is intercepted by a central angle of 110 degrees. Find the length of minor arc $\overset{\frown}{AB}$ to the *nearest integer*.

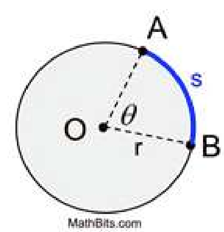
Answer: $\text{arc length} = \frac{\text{central angle}}{360^\circ} \bullet \text{circumference}$

$$\text{arc length} = \frac{110^\circ}{360^\circ} \bullet 2\pi(8) \approx 15.35889742 \approx 15 \text{ inches}$$

Radian Measure

As you progress in your study of mathematics and angles, you will see more references made to the term "radians" instead of "degrees". So, what is a "radian" ?

Definition: The radian measure, θ, of a central angle is defined as the ratio of the length of the arc the angle subtends, s, divided by the radius of the circle, r.



$\overset{\frown}{AB}$ subtends angle θ

subtend = "to be opposite to"

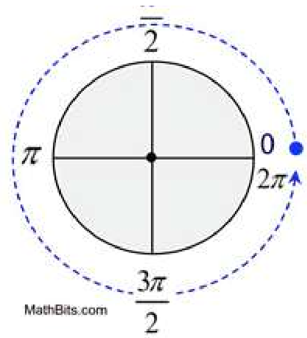
$$\theta = \frac{s}{r} = \frac{\text{length of subtended arc}}{\text{length of radius}}$$

which gives arc length, s: $\quad s = \theta r$

One radian is the central angle that subtends an arc length of one radius ($s = r$). Since all circles are similar, one radian is the same value for all circles.

Relationship between Degrees and Radians:

In a circle, the arc measure of the entire circle is 360º and the arc length of the entire circle is represented by the formula for circumference of the circle: $C = \pi d$ or $C = 2\pi r$.



Substituting C into the formula $s = \theta r$ shows:

$$C = \theta r \qquad 2\pi r = \theta r \qquad 2\pi = \theta$$

The arc measure of the central angle of an entire circle is 360º and the radian measure of the central angle of an entire circle = 2π.

360º (degrees) = 2π (radians)

$360^\circ = 2\pi$ (divide both sides by 2) $180^\circ = \pi$	$360^\circ = 2\pi$ (divide both sides by 4) $90^\circ = \frac{\pi}{2}$	$360^\circ = 2\pi$ (divide both sides by 360) $1^\circ = \frac{\pi}{180}$	$360^\circ = 2\pi$ (divide both sides by 2π) $\frac{180}{\pi} = 1$ radian

To visualize the size of one radian, 1 radian $= \frac{180}{\pi} \approx 57.296^\circ$, but this approximation is not used to convert radians to degrees.

To change **from degrees to radians,** multiply degrees by $\frac{\pi}{180}$	To change **from radians to degrees,** multiply radians by $\frac{180}{\pi}$

Examples:

1. Convert 60º to radians.

Answer: $60^\circ \times \frac{\pi}{180} = \frac{60\pi}{180} = \frac{\pi}{3}$

2. Convert 135º to radians.

Answer: $135^\circ \times \frac{\pi}{180} = \frac{135\pi}{180} = \frac{3\pi}{4}$

3. Convert $\frac{\pi}{6}$ to degrees.

Answer: $\frac{\pi}{6} \times \frac{180}{\pi} = \frac{180}{6} = 30^\circ$

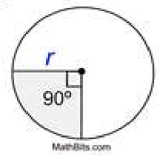
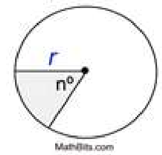
4. Convert $\frac{5\pi}{2}$ to degrees.

Answer: $\frac{5\pi}{2} \times \frac{180}{\pi} = \frac{5(180)}{2} = 450^\circ$

5. Find the length of an arc subtended by an angle of $\frac{7\pi}{4}$ radians in a circle of radius 20 centimeters.

Answer: $s = \theta r = \frac{7\pi}{4}(20) = 35\pi \approx 109.9557429$ cm

Area of a Sector of a Circle

Circle	Semi-circle	Quarter-circle	Any Sector
			
(full circle) $A_{\odot} = \pi r^2$ central $\angle = 360°$ fractional part: 360/360 = 1	(half circle = half of area) $A = \frac{1}{2}\pi r^2$ central $\angle = 180°$ fractional part: 180/360 = 1/2	(¼ of circle = ¼ of area) $A = \frac{1}{4}\pi r^2$ central $\angle = 90°$ fractional part: 90/360 = 1/4	(fractional part of circle) $A = \frac{n}{360}\pi r^2$ n = number of degrees in central angle of sector. $A = \frac{s}{2\pi r}\pi r^2$ where s is the arc length of the sector.

When finding the area of a sector, you are actually finding a fractional part of the area of the entire circle. The fraction is determined by the ratio of the central angle of the sector to the "entire central angle" of 360 degrees. [frational part = $\frac{n}{360}$; where n = central angle]

For the "Any Sector" only use the formula to the left A=n/360 * pi * r squared.

Examples:

1. Find the area of a sector with a central angle of 40° and a radius of 12 cm. Express answer to the *nearest tenth.*

Solution:

$$A = \frac{n}{360}\pi r^2$$

$$A = \frac{40}{360}\pi(12)^2$$

$$A \approx 50.26548246$$

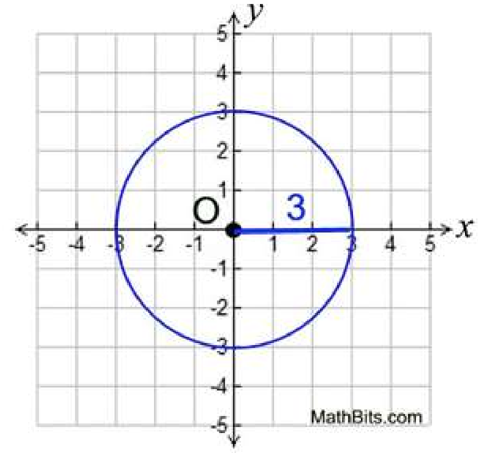
$$A \approx 50.3 \text{ sq.cm.}$$

Equations of Circles:

- **Circle with Center at Origin (0,0)**

$$x^2 + y^2 = r^2$$
with the center at (0,0)
and the radius r

In the example at the right, $x^2 + y^2 = 9$.
The center is (0,0) and the radius = 3.



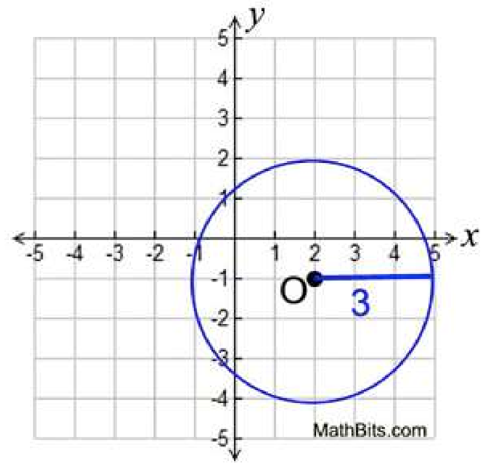
- **Circle with Center at Point (h, k)**

(Known as "center-radius form" or "standard form".)

$$(x - h)^2 + (y - k)^2 = r^2$$
with the center at (h, k)
and the radius r

In the example at the right,
the center is (2,-1) and the radius = 3.

$$(x - 2)^2 + (y - (-1))^2 = 9$$
$$(x - 2)^2 + (y + 1)^2 = 9.$$



Did you notice that the formula for the circle with the center at the origin is just a special case of the formula for a circle at point (h,k)?
When $h = 0$ and $k = 0$ (for the center at the origin), the formula becomes

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 0)^2 + (y - 0)^2 = r^2$$
$$x^2 + y^2 = r^2$$

Examples:

Example 1: Write the equation of a circle with center at (4,8) and a radius of 12.

ANSWER:

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-4)^2 + (y-8)^2 = 144$$

Example 2: Given the equation of a circle, $(x + 4)^2 + (y - 5)^2 = 50$, find the coordinates of the center and the radius.

ANSWER:

$$(x+4)^2 + (y-5)^2 = 50$$
$$(x-(-4))^2 + (y-5)^2 = 50$$

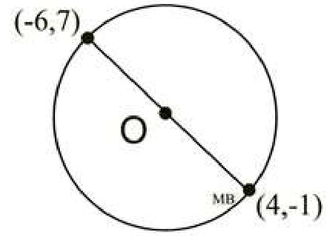
Remember that a + sign inside the parentheses means that the coordinate will be a negative value.
The **center** is (-4,5) and the **radius** is $\sqrt{50}$ or $5\sqrt{2}$.

Notice that the answer for the radius is left in radical form. Do not estimate (round) an answer unless told to do so, or unless working in a real world measurement situation. It is hard to measure a radical number of feet.

Example 4: Write the equation of a circle whose diameter has endpoints (4,-1) and (-6,7).

Find the center of the circle by using the midpoint formula.

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-6+4}{2},\frac{7+(-1)}{2}\right)=(-1,3)$$



Find the radius by using the distance formula.

Note: we are using the center (-1,3).

$$r=\sqrt{(-6-(-1))^2+(7-3)^2}=\sqrt{(-5)^2+(4)^2}$$
$$=\sqrt{25+16}=\sqrt{41}$$

Equation: $(x+1)^2+(y-3)^2=41$

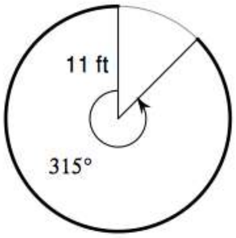
Example 5: Write the equation for the circle $x^2 + y^2 = 16$, if its center is translated by $(x, y)\rightarrow(x + 6, y - 3)$.

ANSWER: The center of the given circle is at the origin. The translated center will be shifted 6 units to the right and 3 units down, placing it at (6,-3). The circle has a radius of 4. The equation will be $(x - 6)^2 + (y + 3)^2 = 16$.

Look at the examples above when you solve the math problems below.

Find the length of each arc. Round your answers to the nearest tenth.

1)



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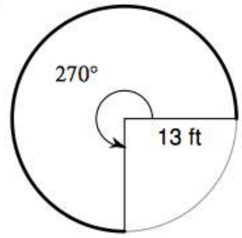

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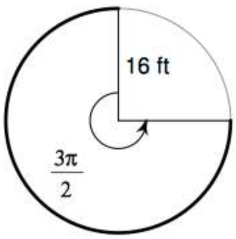

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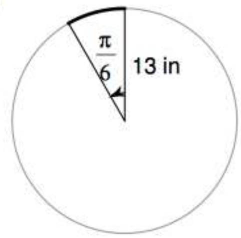
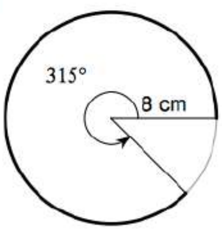
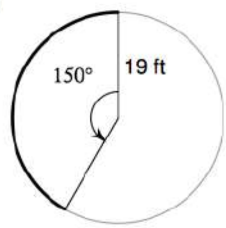


Find the length of each arc. Do not round.

9)



10)



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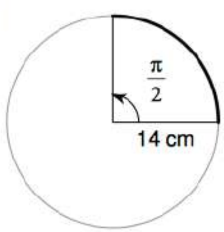
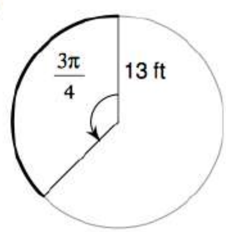
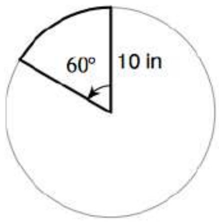


12)

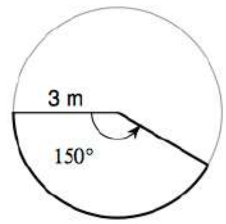


Find the area of each sector. Round your answers to the nearest tenth.

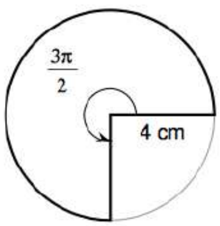
13)



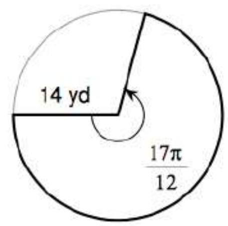
14)



15)



16)



Convert each degree measure into radians.

1) $-200°$

2) $-210°$

3) $765°$

4) $-570°$

5) $-675°$

6) $600°$

Convert each radian measure into degrees.

11) $\frac{19\pi}{18}$

12) $-\frac{9\pi}{4}$

13) $-\frac{13\pi}{12}$

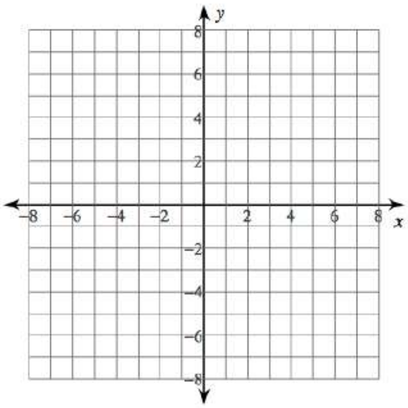
14) $\frac{11\pi}{4}$

15) $\frac{13\pi}{18}$

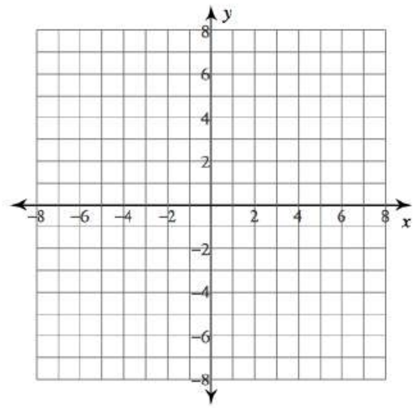
16) $-\frac{5\pi}{4}$

Identify the center and radius of each. Then sketch the graph.

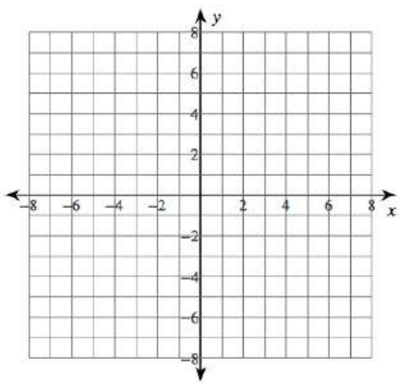
1) $(x-1)^2+(y+3)^2=4$



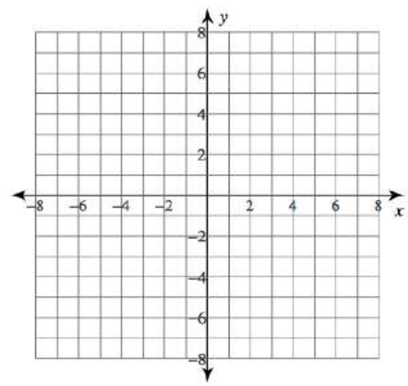
2) $(x-2)^2+(y+1)^2=16$



3) $(x-1)^2+(y+4)^2=9$



4) $x^2+(y-3)^2=14$



Use the information provided to write the equation of each circle.

9) Center: $(13, -13)$
Radius: 4

10) Center: $(-13, -16)$
Point on Circle: $(-10, -16)$

11) Ends of a diameter: $(18, -13)$ and $(4, -3)$

12) Center: $(10, -14)$
Tangent to $x = 13$

13) Center lies in the first quadrant
Tangent to $x = 8$, $y = 3$, and $x = 14$

14) Center: $(0, 13)$
Area: 25π