Mathematics Curriculum Guide

Honors Geometry

2017-18

Topic 4: Congruent Triangles

To begin this unit, students will combine visual and deductive skills to determine triangle congruence. They will build their deductive arguments to prove consequences of congruent triangles, such as two congruent triangles having congruent third sides or altitudes. Students will continue with proving right triangles congruent by using deductive reasoning and following a more advanced argument involving three triangles, bringing back a more intricate use of the transitive property, and using an auxiliary line. Finally, students will discover properties of perpendicular bisectors and angle bisectors to find lengths of lengths of segments.

Common Misconceptions and/or Errors:

- **Proving Triangles Congruent:** Students will often try to use Side-Side-Angle and Angle-Angle-Angle corresponding parts to prove that two triangles are congruent. These are not valid methods.
- **Proving Right Triangles Congruent:** Students cannot use the Hypotenuse-Leg (HL) Theorem if they know only that the legs are congruent. In this case they can still prove that the triangles are congruent by Side-Angle-Side (SAS).
- **Special Segments in Triangles:** Special segments can often be difficult to distinguish from each other. Students should use angle and segment congruency marks and right angle marks to help make clear the purpose of a given segment.

Topic 4: Congruent Triangles

Transfer Goals	
1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution. 2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience. 3) Construct viable arguments and critique the reasoning of others using precise mathematical language.	**Timeframe: 4 weeks/18 days** **Start Date: October 20, 2017** **Assessment Dates: November 14-15, 2017**

Standards

G-SRT 5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G-CO 12 Make formal geometric constructions with a variety of tools and methods.

G-CO 10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

G-CO 9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Meaning-Making

Understandings
Students will understand that…
- Comparing the corresponding parts of two figures can show whether the figures are congruent.
- Two triangles can be proven to be congruent without having to show that all corresponding parts are congruent.
- Triangles can be proven to be congruent by SSS, SAS, ASA, AAS, and HL.
- If two triangles are congruent, then every pair of their corresponding parts is also congruent.
- The angles and sides of isosceles and equilateral triangles have special relationships.
- Another way triangles can be proven to be congruent is by using one pair of right angles, a pair of hypotenuses, and a pair of legs.
- All points on the perpendicular bisector are equidistant from the endpoints of the segment.
- Any point on the bisector of an angle is equidistant from the sides of the angle.

Essential Questions
Students will keep considering…
- How do you identify corresponding parts of congruent triangles?
- How do you prove two triangles are congruent?
- How can we tell if a triangle is isosceles or equilateral?
- What are the properties of a perpendicular bisector in a triangle?
- What are the properties of an angle bisector in a triangle?

Acquisition

Knowledge
Students will know…
Vocabulary: congruent polygons, congruent parts, CPCTC, legs of, base of, vertex of, and base angles of an isosceles triangle, equilateral, equiangular, corollary, hypotenuse, legs of a right triangle, equidistant, distance form a point to a line

Theorems & Postulates: SSS Postulate, SAS Postulate, ASA Postulate, AAS Theorem, Isosceles Triangle Theorem, Converse of the Isosceles Triangle Theorem, Hypotenuse-Leg (HL) Theorem, Conditions for HL Theorem, Perpendicular Bisector Theorem, Converse of the Perpendicular Bisector Theorem, Angle Bisector Theorem, Converse of the Angle Bisector Theorem

Skills
Students will be skilled at and able to do the following…
- Recognize congruent figures and their corresponding parts.
- Use the statement of congruence to find angle and segment lengths.
- Prove two triangles are congruent using the SSS, SAS, ASA, or AAS.
- Use the definition of congruent polygons (abbreviated CPCTC), to find the measures of corresponding parts and to prove corresponding parts congruent.
- Use and apply properties of isosceles and equilateral triangles to find perimeter, area, and missing measures.
- Prove right triangles congruent using the Hypotenuse-Leg Theorem.
- Use properties of perpendicular and angle bisectors to find segment lengths.

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Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

Transfer goals highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

1) **Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.**
2) **Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.**
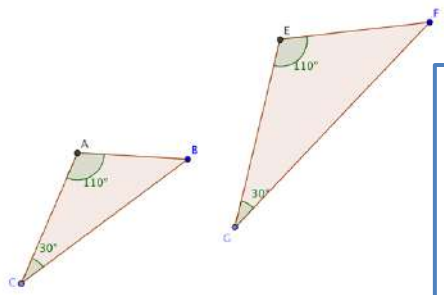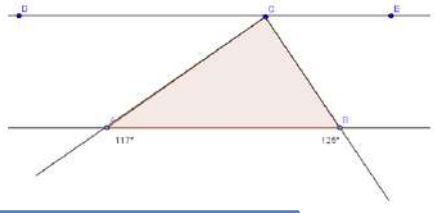3) **Construct viable arguments and critique the reasoning of others using precise mathematical language.**

Multiple measures will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

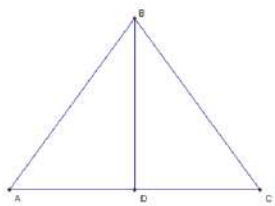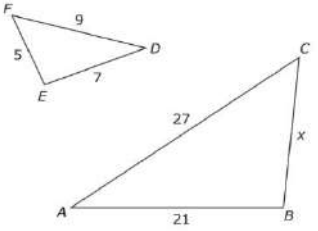| Formative Assessment	Summative Assessment
Aligning Assessment to Stage One	
- What constitutes evidence of understanding for this lesson? - Through what other evidence during the lesson (e.g. response to questions, observations, journals, etc.) will students demonstrate achievement of the desired results? - How will students reflect upon, self-assess, and set goals for their future learning?	- What evidence must be collected and assessed, given the desired results defined in stage one? - What is evidence of understanding (as opposed to recall)? - Through what task(s) will students demonstrate the desired understandings?
Opportunities	
- Discussions and student presentations - Checking for understanding (using response boards) - Ticket out the door, Cornell note summary, and error analysis *Performance Tasks* within a Unit - Teacher-created assessments/quizzes	- Unit assessments - Teacher-created quizzes and/or mid-unit assessments *Illustrative Mathematics* tasks (https://www.illustrativemathematics.org/) - Performance tasks

Topic 4: Congruent Triangles

The following pages address how a given skill may be assessed. Assessment guidelines, examples and possible question types have been provided to assist teachers in developing formative and summative assessments that reflect the rigor of the standards. *These exact examples cannot be used for instruction or assessment, but can be modified by teachers.*

Unit Skills	SBAC Targets (DOK)	Standards	Examples
• Recognize congruent figures and their corresponding parts. • Use the statement of congruence to find angle and segment lengths. • Prove two triangles are congruent using the SSS, SAS, ASA, or AAS. • Use the definition of congruent polygons (abbreviated CPCTC), to find the measures of corresponding parts and to prove corresponding parts congruent. • Use and apply properties of isosceles and equilateral triangles to find perimeter, area, and missing measures. • Prove right triangles congruent using the Hypotenuse-Leg Theorem. • Use properties of perpendicular and angle bisectors to find segment lengths.	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. (3,4) State logical assumptions being used. (2,3) Use the technique of breaking an argument into cases. (2,3) Distinguish correct logic or reasoning form that which is flawed and – if there is a flaw in the argument – explain what it is. (2,3,4) Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (2,3)	**G-SRT 5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **G-CO 12** Make formal geometric constructions with a variety of tools and methods. **G-CO 10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. **G-CO 9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	**G.CO.C.10 Item 4** Look at the two triangles shown below. 1. Make a conjecture about ∠B and ∠F. 2. Can you prove your conjecture? 3. If you changed one of the congruent angle pairs to different measure (but keeping them congruent), what would change in your answer to part 1? What would stay the same? 4. If you changed the measures of both congruent angle pairs (but keeping the measures in each pair congruent) what would change in your answer to part 1? What would stay the same? 5. Make a conjecture about two triangles that have two pairs of congruent angles. What will always be true about the two triangles? **G.CO.C.10 Item 1** Line DE ∥ line AB. Find the measures of the angles in the triangle: **G.CO.C.10 Item 6** Donald wants to build a triangular fence within a plot of land he has purchased for future development. He will build a new fence along the entire driveway, which is 80 feet in length. The previous owners left behind materials for 100 feet of fencing, which Donald will use as a second side. What is the minimum amount of fencing Donald will need to purchase to create the triangular space? If, instead, Donald would like a larger fenced area, what would be the maximum amount of fencing he could use? Solution: Donald must purchase more than 20 feet of fencing, but he cannot purchase over 180 feet of fencing.

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Standards: G-SRT 5, G-CO 12, G-CO 9, G-CO 10

Timeframe: 4 weeks/18 days
Start Date: October 20, 2017
Assessment Dates: November 14-15, 2017

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Resources
1 Day (Oct. 20th)	**Topic Opener** pg. 217 *Planning the Paths for a Park*					
2 Days (Oct. 21, 24)	**Lesson 4-1: Congruent Figures** SMP: 1,3,4,7 (pp. 218-224) **Prepares for G-SRT 5**	**Focus Question:** • What do we know is true about congruent figures? • How can we use the statement of congruence to identify corresponding parts of figures? **Inquiry Question:** Pg. 218 Solve It! Pg. 219 Problem 2	• Comparing the corresponding parts of two figures can show whether the figures are congruent.	**Vocabulary:** congruent polygons, congruent parts	• Recognize congruent figures and their corresponding parts. • Use the statement of congruence to find angle and segment lengths.	**CC Problems:** #5,6,7, 8, 9, 32, **34**, **39**, **43**, 44, 45, **46**, 47-49 **Thinking Map:** Circle Map to record what students know about congruent figures.

Common Core Practices

☐ Instruction in the Standards for Mathematical Practices
☐ Use of Talk Moves
☐ Note-taking

☐ Use of Manipulatives
☐ Use of Technology
☐ Use of Real-world Scenarios

☐ Project-based Learning
☐ Thinking Maps

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
4 Days (Oct. 25-28)	**Lesson 4-2: Triangle Congruence by SSS and SAS** SMP: 1,3,4, 7 (pp. 226-233) **G-SRT 5** **Lesson 4-3: Triangle Congruence by ASA and AAS** SMP: 1,3,7 (pp. 234-241) **G-SRT 5**	**Focus Question:** • What is the least amount of information needed to determine if two triangles are congruent? **Inquiry Question:** Pg. 226 Solve It!	• Two triangles can be proven to be congruent without having to show that all corresponding parts are congruent. • Ways triangles can be proven to be congruent: ○ by using three pairs of corresponding sides, or ○ by using two pairs of corresponding sides and one pair of corresponding angles, or ○ by using one pair of corresponding sides and two pairs of corresponding angles.	**Vocabulary:** SSS Postulate, SAS Postulate, ASA Postulate, AAS Theorem	• Prove two triangles are congruent using the SSS, SAS, ASA, or AAS.	**4-2 CC Problems:** #5,6,7, **8**, 9, 10, **15**, 16,17, **21**, 22, 23, **27**, 28, 29, 30, 31, 32, 33, **34** **4-3 CC Problems:** #5,6,7, 10, 11,12, 13, 14,15,19, 20, 21, 23,24, 25,26, 28 **Thinking Map:** Tree Map for SSS, SAS, ASA, and AAS with definitions, statements, and examples.
2 Days (Oct. 31, Nov. 1)	**Lesson 4-4: Using Corresponding Parts of Congruent Triangles** SMP: 1,3 (pp. 244-248) **G-SRT 5**, **G-CO 12**	**Focus Question:** • How can you use congruent triangles to find measures of other parts of triangles? **Inquiry Question:** Pg. 245 Solve It!	• If two triangles are congruent, then every pair of their corresponding parts is also congruent.	**Vocabulary:** CPCTC	• Use the definition of congruent polygons (abbreviated CPCTC), to find the measures of corresponding parts and to prove corresponding parts congruent.	**4-4 CC Problems:** #3,4, 5,6, 7, **8**, 9, 10, 11-13, **14**, 15, 16, 18, 19, 20 **Thinking Map:** Flow Map to show the logical sequence of proving CPCTC.
2 Days (Nov. 2-3)	**Lesson 4-5: Isosceles and Equilateral Triangles** SMP: 1,3,4 (pp. 250-256) **G-CO 10**, **G-SRT 5**	**Focus Question:** • What is the relation between the angles and sides of an isosceles triangle? • What is the relation of the angles of an equilateral triangle? **Inquiry Question:** Pg. 250 Solve It!	• The angles and sides of isosceles and equilateral triangles have special relationships.	**Vocabulary:** legs of, base of, vertex of, and base angles of an isosceles triangle, equilateral, equiangular, corollary **Concepts:** • Isosceles Triangle Theorem • Converse of the Isosceles Triangle Theorem	• Use and apply properties of isosceles and equilateral triangles to find perimeter, area, and missing measures.	**4-5 CC Problems:** #4, 5, **15**, **20**, **21**, 22, 23, **24**, 25, 26, 28, 36 **STEM:** #14, 27 **Thinking Map:** Double-Bubble Map to compare and contrast isosceles and equilateral triangles.

Time	Lesson/Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 Day (Nov. 4)	**Topic 4 Performance Task (p. 272)**					
1 Day (Nov. 7)	**Lesson 4-6: Congruence in Right Triangles** SMP: 1,3 (pp. 258-264) G-SRT 5	**Focus Questions:** • How can we show that two right triangles are congruent? **Inquiry Questions:** Pg. 259 Solve It!	• Two triangles can be proven to be congruent without having to show that all corresponding parts are congruent. • Another way triangles can be proven to be congruent is by using one pair of right angles, a pair of hypotenuses, and a pair of legs.	**Vocabulary:** hypotenuse, legs of a right triangle **Concepts:** • Hypotenuse-Leg (HL) Theorem • Conditions for HL Theorem	• Prove right triangles congruent using the Hypotenuse-Leg Theorem.	**Common Core Problems:** #5,6,7, **8**, **9**, 10, 11, **15**, **16**, 17, 18, 23, 24, **25**, 26, 27, 28 **Thinking Map:** Double-Bubble Map to compare and contrast the HL Theorem and the SAS Postulate.
2 Days (Nov. 8-9)	**Lesson 5-2: Perpendicular and Angle Bisectors** SMP: 1,3,4,5,8 (pp. 292-299) G-CO 9, G-SRT 5	**Focus Questions:** • What is the relationship between any point on the perpendicular bisector of a segment and the endpoints of the segment? • What is the relationship between any point on the bisector of an angle and the sides of the angle? **Inquiry Questions:** Pg. 292 Solve It!	• All points on the perpendicular bisector are equidistant from the endpoints of the segment. • Any point on the bisector of an angle is equidistant from the sides of the angle.	**Vocabulary:** equidistant, distance form a point to a line **Concepts:** • Perpendicular Bisector Theorem • Converse of the Perpendicular Bisector Theorem • Angle Bisector Theorem • Converse of the Angle Bisector Theorem	• Use properties of perpendicular and angle bisectors to find segment lengths.	**Common Core Problems:** #4,5, 9, 10, 11, 12-15, 16, 27, **23**, **24**, **25**, **28**, 32-35, 36, 37, 38 5-2 p. 294 Prob. #2
1 day (Nov. 10th)	**Review Topic 4 Concepts & Skills** Use Textbook Resources and/or Teacher Created Items					
2 days (Nov. 14-15)	**Topic 4 Assessment** (Created and provided by PUSD)					

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