

Mathematics Curriculum Guide

Honors Geometry

2017-18



Paramount Unified School District Educational Services

Topic 11: Circles

In this unit, students will explore concepts related to circles. Students will begin the unit by acquiring a large collection of terms related to circles. Then they will develop relationships between angles in and around circles by building on exterior angles, using isosceles triangles, and algebraic substitution. They will use inscribed angles to expand their knowledge of angle-arc relationships. Finally, students will develop algebraic equations for circles from the Pythagorean Theorem, and will apply their understanding of radius to vary the problems they can solve.

Common Misconceptions and/or Errors:

- Segments Related to Circles: Students sometimes get confused identifying segments of a circle. Have students create a Tree Map that includes definitions and diagrams of each type of segment. Discuss the relationship between the segments as well. For example, is a diameter a chord? [Yes, but a chord is not necessarily a diameter.]
- Angles Formed by Special Segments of Circles: Students might try to apply the Inscribed Angle Theorem, $m \angle B = \frac{1}{2}m\widehat{AB}$, when the vertex is not on the circle.
- Equations with Circles: When asked to give the center of the circle represented by the equation below, students might not take into account the subtraction inside the parentheses. For the equation $(x 3)^2 + (y + 7)^2 = 64$, students may write the center as (-3,7). The center is actually (3, -7).



Educational Services

Topic 11: Circles

Transfer Goals					
 Demonstrate perseverance Effectively communicate or Construct viable arguments 	Timeframe: 3 weeks/14 days Start Date: May 10, 2018 Assessment Dates: NO TEST				
Standards		Meaning-Making			
G-CO 1 Know precise definitions of angles, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G-C-2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the	Understand Students will understand that A radius of a circle and the tangent that intersects the end relationship. A circle has a special relationship to a triangle whose sides Information about congruent parts of a circle (or congruend other parts of the circle (or circles). Angles formed by intersecting lines have a special relation Arcs intercepted by chords that form inscribed angles are Angles formed by intersecting lines have a special relation Arcs formed by intersecting lines have a special relation Arcs formed by lines intersecting either within a circle or of the lines. There are special relationships between intersecting chord secant and tangent that intersect.	ings dpoint of the radius on the circle have a special s are tangent to the circle. Int circles) can be used to find information about aship to the arcs the intersecting lines intercept. related to the inscribed angles. Iship to the arcs the intersecting lines intercept. butside a circle are related to the angles formed by ds, intersecting chords, intersecting secants, or a	 Essential Questions Students will keep considering How can you prove relationships between angles and arcs in a circle? When lines intersect a circle, or within a circle, how do you find the measures of resulting angles, arcs, and segments? How do you find the equation of a circle in the equation of a circle in the coordinate plane? 		
perpendicular to the tangent	The equation of a circle can be written if its center and radius are known.				
where the radius intersects the circle.	Acquisition				
 G-C 3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G-C 4 Construct a tangent line from a point outside a given circle to a circle. G-GPE 1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. 	KnowledgeStudents will knowVocabulary: circle, center, diameter, radius, central angle, semicircle, minor arc, major arc, chord, inscribed angle, intercepted arc, locus, point of tangency, secant, standard form of an equation of a circle, tangent to a circle• Theorems: Tangent Lines, Chords and Arcs, Inscribed Angles, Angle Measures, Segment Lengths, Equation of a Circle, Chord-Chord, Secant-Tangent, Tangent-Secant• Key Concepts: \circ Standard Form of an Equation for a Circle: $(x - h)^2 + (y - k)^2 = r^2$	Skills Students will be skilled at and able to do the following • Use properties of tangent lines to find the lengths of tangent segments and to set up and solve right triangles. • Use the relationships between central angles, chords and arc length to solve for unknown angles, segments or arcs. • Given the measure of an inscribed angle, find the measure of its intercepted arc, and vice versa. • Find the measure of the angle formed by two intersecting chords, secants, or tangents. • Use the power theorems to find the lengths of segments within or around a circle. • Find the equation of a circle when given its radius and center • Find the equation of a circle when given its center and a point on the circle. • Graph a circle when given its equation.			



Topic 11: Circles

Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

Transfer goals highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

- 1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.
- 2) Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.
- 3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Multiple measures will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

Formative Assessment	Summative Assessment
Aligning Assessm	nent to Stage One
 What constitutes evidence of understanding for this lesson? 	What evidence must be collected and assessed, given the desired results
 Through what other evidence during the lesson (e.g. response to questions, 	defined in stage one?
observations, journals, etc.) will students demonstrate achievement of the	 What is evidence of understanding (as opposed to recall)?
desired results?	• Through what task(s) will students demonstrate the desired understandings?
 How will students reflect upon, self-assess, and set goals for their future 	
learning?	
Oppor	tunities
Discussions and student presentations	Unit assessments
 Checking for understanding (using response boards) 	 Teacher-created quizzes and/or mid-unit assessments
 Ticket out the door, Cornell note summary, and error analysis 	 Illustrative Mathematics tasks (<u>https://www.illustrativemathematics.org/</u>)
Performance Tasks within a Unit	Performance tasks
Teacher-created assessments/quizzes	



Plane Geometry (West) – Topic 11 Stage Two – Evidence of Learning

Educational Services

Topic 11: Circles

Unit Skills	SBAC Targets (DOK)	Standards	Examples
 Use properties of tangent lines to find the lengths of tangent segments and to set up and solve right triangles. Use the relationships between central angles, chords and arc length to solve for unknown angles, segments or arcs. Given the measure of an inscribed angle, find the measure of its intercepted arc, and vice versa. Find the measure of the angle formed by two intersecting chords, secants, or tangents. Use the power theorems to find the lengths of segments within or around a circle. Find the equation of a circle when given its radius and center Find the equation of a circle when given its center and a point on the circle. Graph a circle when given its equation. 	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. (3,4) State logical assumptions being used. (2,3) Distinguish correct logic or reasoning from that which is flawed and – if there is a flaw in the argument – explain what it is. (2,3,4) Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (2,3) Determine conditions under which an argument does and does not apply. (3,4)	 G-CO 1 Know precise definitions of angles, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G-C-2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G-C 3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G-C 4 Construct a tangent line from a point outside a given circle to a circle. G-GPE 1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. 	ECALITENT Prove the circles made by the bike's wheels and the gear are similar by taking measurements, then using transformations and/or algebraically. Possible Solution: One wheel can be translated or reflected onto the other, showing radius and diameter having a 1:1 ratio, or congruence. The gear's and wheel's radii and diameters can be measured as <i>r</i> , <i>R</i> , d, and D, then the ratios can be found to be the same. Alternatively, the gear's radius and/or diameter can be multiplied by R/r, which will give the radius or diameter of the wheel. ECALITENT ECALITENT James' math class compares the diameters and circumferences of multiple circular objects. They create a table of values and a graphical representation of the data in order to investigate the relationship between the two measurements. What will notice that, for every one unit of increase in diameter, there will be an increase in circumference of approximately 3.14 units. They will be able to calculate this rate of change from the table and to see it on the graph. A graph of diameter resus circumference will be a straight line, starting at (0,0) with a slope of approximately 3.14. This relates to what we already know about circumference of a circle is $C = \pi d$ (could be rewritten as approximately $y = 3.14$, where x represents diameter and y is the circumference.



Plane Geometry (West) – Topic 11 Stage Two – Evidence of Learning

Educational Services

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Unit Skills	SBAC Targets (DOK)	Standards	Examples
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Plane Geometry (West) – Topic 11 Stage Two – Evidence of Learning

Educational Services

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Unit Skills	SBAC Targets (DOK)	Standards	Examples
 Use properties of tangent lines to find the lengths of tangent segments and to set up and solve right triangles. Use the relationships between central angles, chords and arc length to solve for unknown angles, segments or arcs. Given the measure of an inscribed angle, find the measure of its intercepted arc, and vice versa. Find the measure of the angle formed by two intersecting chords, secants, or tangents. Use the power theorems to find the lengths of segments within or around a circle. Find the equation of a circle when given its radius and center Find the equation of a circle when given its center and a point on the circle. Graph a circle when given its equation. 	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. (3,4) State logical assumptions being used. (2,3) Distinguish correct logic or reasoning from that which is flawed and – if there is a flaw in the argument – explain what it is. (2,3,4) Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (2,3) Determine conditions under which an argument does and does not apply. (3,4)	 G-CO 1 Know precise definitions of angles, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G-C-2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G-C 3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G-C 4 Construct a tangent line from a point outside a given circle to a circle. G-GPE 1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. 	 14. The equation x² - 8x + y² = 9 defines a circle in the xy-coordinate plane. What is the radius of the circle? Enter your answer in the box. Use the information provided to answer Part A and Part B for question 6. The equation x² + y² - 4x + 2y = b describes a circle. 6. Part A Determine the y-coordinate of the center of the circle. Enter your answer in the box. Determine the y-coordinate of the center of the circle. Enter your answer in the box. Determine the y-coordinate of the center of the circle. Enter your answer in the box. Determine the y-coordinate of the center of the circle. Enter your answer in the box. Determine the y-coordinate of the center of the circle. Enter your answer in the box. Determine the y-coordinate of the center of the circle. Enter your answer in the box. Determine the y-coordinate of the center of the circle. Enter your answer in the box. Determine the y-coordinate of the circle is 7 units. What is the value of b in the equation? Enter your answer in the box.



Plane Geometry (West) – Topic 11 Stage Two – Evidence of Learning

Educational Services

Topic 11: Circles

Unit Skills	SBAC Targets (DOK)	Standards	Exa	mples
 Use properties of tangent lines to find the lengths of tangent segments and to set up and solve right triangles. Use the relationships between central angles, chords and arc length to solve for unknown angles, segments or arcs. Given the measure of an inscribed angle, find the measure of its intercepted arc, and vice versa. Find the measure of the angle formed by two intersecting chords, secants, or tangents. Use the power theorems to find the lengths of segments within or around a circle. Find the equation of a circle when given its radius and center Find the equation of a circle when given its center and a point on the circle. Graph a circle when given its equation. 	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. (3,4) State logical assumptions being used. (2,3) Distinguish correct logic or reasoning from that which is flawed and – if there is a flaw in the argument – explain what it is. (2,3,4) Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (2,3) Determine conditions under which an argument does and does not apply. (3,4)	 G-CO 1 Know precise definitions of angles, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G-C-2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G-C 3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G-C 4 Construct a tangent line from a point outside a given circle to a circle. G-C 9 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. 	Use the information provided to answer Part A and The figure shows a circle with center <i>P</i> , a diameter <i>BD</i> , length of <i>PC</i> is 10. Let $m \angle CBD = (x)^\circ$ and $m \angle BCD = (x)^\circ$ <i>B</i> <i>D</i> <i>B</i> <i>D</i> <i>B</i> <i>D</i> <i>D</i> <i>D</i> <i>D</i> <i>D</i> <i>D</i> <i>D</i> <i>D</i>	Part B for question 24. and inscribed $\triangle BCD$. The x + 54)°. Part B Identify each of the true statements about the figure. Select all that apply. (a) The length of \overline{CD} is less than 10. (b) The length of \overline{CD} is equal to 10. (c) The length of \overline{CD} is greater than 10. (c) $\triangle CPD$ is equilateral. (c) The measure of $\angle CPD$ is less than 60°. (c) The measure of $\angle CPD$ is greater than 60°. (c) The measure of $\angle CPD$ is greater than 60°. (c) The measure of $\angle CPD$ is greater than 60°. (c) The measure of $\angle CPD$ is greater than 60°. (c) The measure of $\angle CPD$ is greater than 60°. (c) The measure of $\angle CPD$ is greater than 60°. (c) The measure of $\angle CPD$ is greater than 60°.



Plane Geometry (West) – Topic 11 Stage Three –Learning Experiences & Instruction

Educational Services

Topic 11: Circles

			Transfer Go	als			
 Dem Effect Constant 	 Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution. Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience. Construct viable arguments and critique the reasoning of others using precise mathematical language. 						
 Solution of a circle in the equation of a circle in the equation of a circle in the coordinate plane? Essential Questions: How can you prove relationships between angles and arcs in a circle? When lines intersect a circle, or within a circle, how do you find the measures of resulting angles, arcs, and segments? How do you find the equation of a circle in the equation of a circle in the coordinate plane? 			Standards: G-CO 1, G-C G-GPE 1 Timeframe: 3 weeks/14 Start Date: May 10, 2013 Assessment Dates: NO be included on the final	<mark>days</mark> B TEST (content will			
Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Resources	
1 day (May 11 th)	1 day Opening Activity: (May Determining the Dimensions of a Logo p. 761						
2 days (May 12,15)	Lesson 12-1: Tangent Lines (pp. 762-769) SMP: 1,3,4 G-C 2, G-CO 1	 Focus Question: What are the properties of a tangent line to a circle? How can the properties of tangent lines to a circle help us solve for missing lengths? Inquiry Question: p. 762 Solve It! 	 If a line is tangent to a circle, then it is perpendicular to the radius at the point of tangency. If two tangent segments have the same external endpoint, then they are congruent. 	 Vocab: tangent to a circle, radius, point of tangency, perpendicular, inscribed, circumscribed Concepts: Tangent Lines Theorems Pythagorean Theorem 	 Use properties of tangents to find the lengths of tangent segments. Use properties of tangents to set up and solve right triangles. 	Common Core Problems: #4,5, 21, 22, 23, 24, 25, 30 STEM: #9-10, 20 Thinking Maps: Create a Tree Map to represent the Theorems in this lesson.	

Common Core Practices

- □ Instruction in the Standards for Mathematical Practices
- □ Use of Manipulatives

- Use of Talk Moves
- Note-taking

- Use of Technology
- □ Use of Real-world Scenarios

- □ Project-based Learning
- □ Thinking Maps

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 Days (May 16-17)	Lesson 12-2: Chords and Arcs SMP: 1,3 (pp. 771-779) G-C 2, G-CO 1	 Focus Question: What is the relationship between a chord, its intercepted arc and its central angle? Inquiry Question: p. 771 Solve It! 	 Chords equidistant from the center of a circle are congruent. Central angles that are congruent intercept congruent chords and arcs. A diameter that is perpendicular to a chord bisects the chord and its arc. 	 Vocab: chord, arc, central angle, intercept, bisect, equidistant, diameter, perpendicular bisector Concepts: Theorems regarding congruent central angles, arcs, chords, and centers Pythagorean Theorem Triangle Sum Theorem 	Use the relationships between central angles, chords and arc length to solve for unknown angles, segments or arcs.	Common Core Problems: #4,5, 18, 19, 20, 21, 22, 29, 33, 34, 35-39 Thinking Maps: Create a Tree Map to represent the Theorems in this lesson.
2 Days (May 18-19)	Lesson 12-3: Inscribed Angles SMP: 1,3,4,6 (pp. 780-787) G-C 2, G-C 3, G-C 4, G-CO 1	 Focus Question: What is the relationship between an inscribed angle and its intercepted arc? What is the measure of an angle formed by a chord and a tangent? Inquiry Question: p. 780 Solve It! 	 The measure of an inscribed angle is half the measure of its intercepted arc. The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc. An angle inscribed in a semicircle is a right angle. The opposite angles of a quadrilateral inscribed in a circle are supplementary. 	 Vocab: inscribed angle, chord, tangent, quadrilateral, diameter, intercepted arc, supplementary Concepts: Inscribed Angle Theorem & Corollaries Pythagorean Theorem Triangle Sum Theorem Sum of Interior Angles of a Quadrilateral 	 Given the measure of an inscribed angle, find the measure of its intercepted arc, and vice versa. 	Common Core Problems: #4,5, 19, 22, 26, 27, 28, 29, 31, 32, 33, 34 Thinking Maps: Create a Tree Map to represent the Theorems in this lesson.
1 Day (May 22 nd)	Performance Task: Revisit "Determining the Dimensions of a Logo" Pages 769 and 787					

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 Days (May 23-24)	Lesson 12-4: Angle Measures and Segment Lengths SMP: 1,3,4 (pp. 790-797) G-C 2, G-CO 1	 Focus Question: What are the measures of angles formed by intersecting chords, secants or tangents? What is the relationship between the segments formed by intersecting tangents, chords, or secants? Inquiry Question: p. 790 Solve It! 	 The measure of an angle formed by two intersecting chords is half the sum of their intercepted arcs. The measure of an angle formed by two lines that intersect outside a circle is half the difference of their intercepted arcs. 	 Vocab: secant, chord, tangent, intercepted arc Concepts: Chord-Chord Power Theorem Secant-Tangent Power Theorem Secant-Secant Power Theorem Triangle Sum Theorem Pythagorean Theorem 	 Find the measure of the angle formed by two intersecting chords, secants, or tangents. Use the power theorems to find the lengths of segments within or around a circle. 	Common Core Problems: #5, 7, 14, 29, 31, 35, 36, 37, 40, 41, 42, 43 STEM: #28 Thinking Maps: Create a Tree Map to represent the Theorems in this lesson.
2 Days (May 25-26)	Lesson 12-5: Circles in the Coordinate Plane SMP: 1,3,4,7 (pp. 798-803) G-GPE 1, G-CO 1	 Focus Question: How do you write the equation of a circle? Given the equation of a circle, what is the circle's center and radius? Inquiry Question: p. 798 Solve It! 	 The equation of a circle with center (h, k) and radius r is: (x - h)² + (y - k)² = r² You can write the equation of a circle if you know its center and radius. 	Vocab: radius, diameter, center, distance, standard form of an equation of a circle Concepts: • Distance Formula • Pythagorean Theorem • $(x - h)^2 + (y - k)^2 = r^2$	 Find the equation of a circle when given its radius and center Find the equation of a circle when given its center and a point on the circle. Graph a circle when given its equation. 	Common Core Problems: #7, 40, 44, 55, 56, 57 Thinking Maps: Create a Circle Map to record ideas in this lesson.
2 Days (May 30-31)		Use Textb	Review Topic 11 C book Resources and/or Tea	oncepts and Skills cher Created Items for Ass	essment	

Common Core Practices

Instruction in the Standards for Mathematical Practices	
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Use of Manipulatives

Use of Talk Moves

Note-taking

Use of Technology

□ Use of Real-world Scenarios

- Project-based Learning
- Thinking Maps

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