Mathematics Curriculum Guide

Honors Geometry

2017-18

Topic 10: Surface Area & Volume

In this unit students will develop formulas for surface area of three-dimensional figures by building on their understanding of area in the last unit. They will also learn about the volume of various figures, and examine a derivation involving a large number of pyramids inside a sphere to find the sphere's volume.

Common Misconceptions and/or Errors:

- **Surface Area:** The surface areas of pyramids and cones are computed using slant height (l), not height (h). If the problem gives the height and not the slant height, they need to use the Pythagorean Theorem to find the slant height.
- **Volume:** The surface area and volume of spheres look similar. Students who have worked to memorize the formulas of surface area and volume of spheres might accidentally transpose the 4 and $\frac{4}{3}$ or the r^2 and r^3.

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Transfer Goals

1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.
2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience.
3) Construct viable arguments and critique the reasoning of others using precise mathematical language.

Timeframe: 3 weeks/15 days
Start Date: April 19, 2018
Assessment Dates: May 8-9, 2018

Standards

G-MG 1 Use geometric shapes, their measures, and their properties to describe objects.

G-MG-2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

G-MG 3 Apply geometric methods to solve design problems.

G-GMD 1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

G-GMD 3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

G-GMD 4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Meaning-Making

Understandings

Students will understand that…

- A three-dimensional figure can be analyzed by describing the relationships among its vertices, edges, and faces.
- The surface area of a three-dimensional figure is equal to the sum of the areas of each surface of the figure.
- The volume of a prism and a cylinder can be found when its height and the area of its base are known.
- The volume of a pyramid is related to the volume of a prism with the same base and height.
- The surface area and the volume of a sphere can be found when its radius is known.

Essential Questions

Students will keep considering…

- How can you determine the intersection of a solid and a plane?
- How do you find the surface area and volume of three-dimensional figures such as: a prism, a cylinder, a pyramid, a cone, and a sphere?
- How do the surface areas and volumes of similar solids compare?
- How do you find the density of an object?

Acquisition

Knowledge

Students will know…

Vocabulary: polyhedron, face, edge, vertex, cross section, net, prism, cylinder, lateral area, base area, surface area, height, radius, diameter, pi, cone, pyramid, slant height, altitude, volume, cubic units, sphere

- **Formulas for:** Lateral and Surface Area Formulas for various three-dimensional figures (including a prism, a cylinder, a pyramid, a cone, and a sphere), Volume Formulas for various three-dimensional figures
- **Key Concepts:** Euler's Formula

Skills

Students will be skilled at and able to do the following…

- Examine and describe cross sections of polyhedra and recognize their parts and cross sections.
- Apply Euler's Formula to verify two-dimensional nets of three-dimensional figures.
- Use formulas to find surface areas and volumes of various solids such as: prisms, cylinders, pyramids, cones, and spheres.

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Transfer is a student's ability to independently apply understanding in a novel or unfamiliar situation. In mathematics, this requires that students use reasoning and strategy, not merely plug in numbers in a familiar-looking exercise, via a memorized algorithm.

Transfer goals highlight the effective uses of understanding, knowledge, and skills we seek in the long run – that is, what we want students to be able to do when they confront new challenges, both in and outside school, beyond the current lessons and unit. These goals were developed so all students can apply their learning to mathematical or real-world problems while simultaneously engaging in the Standards for Mathematical Practices. In the mathematics classroom, assessment opportunities should reflect student progress towards meeting the transfer goals.

With this in mind, the revised **PUSD transfer goals** are:

1) **Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution.**
2) **Effectively communicate orally, in writing, and by using models (e.g., concrete, representational, abstract) for a given purpose and audience.**
3) **Construct viable arguments and critique the reasoning of others using precise mathematical language.**

Multiple measures will be used to evaluate student acquisition, meaning-making and transfer. Formative and summative assessments play an important role in determining the extent to which students achieve the desired results in stage one.

Formative Assessment	Summative Assessment
Aligning Assessment to Stage One	
• What constitutes evidence of understanding for this lesson? • Through what other evidence during the lesson (e.g. response to questions, observations, journals, etc.) will students demonstrate achievement of the desired results? • How will students reflect upon, self-assess, and set goals for their future learning?	• What evidence must be collected and assessed, given the desired results defined in stage one? • What is evidence of understanding (as opposed to recall)? • Through what task(s) will students demonstrate the desired understandings?
Opportunities	
• Discussions and student presentations • Checking for understanding (using response boards) • Ticket out the door, Cornell note summary, and error analysis • *Performance Tasks* within a Unit • Teacher-created assessments/quizzes	• Unit assessments • Teacher-created quizzes and/or mid-unit assessments • *Illustrative Mathematics* tasks (https://www.illustrativemathematics.org/) • Performance tasks

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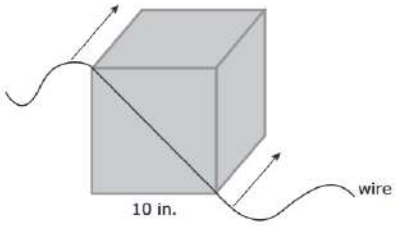
The following pages address how a given skill may be assessed. Assessment guidelines, examples and possible question types have been provided to assist teachers in developing formative and summative assessments that reflect the rigor of the standards. *These exact examples cannot be used for instruction or assessment, but can be modified by teachers.*

Unit Skills	SBAC Targets (DOK)	Standards	Examples
• Examine and describe cross sections of polyhedra and recognize their parts and cross sections. • Apply Euler's Formula to verify two-dimensional nets of three-dimensional figures. • Use formulas to find surface areas and volumes of various solids such as: prisms, cylinders, pyramids, cones, and spheres.	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. (3,4) State logical assumptions being used. (2,3) Distinguish correct logic or reasoning form that which is flawed and – if there is a flaw in the argument – explain what it is. (2,3,4) Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (2,3)	**G-MG 1** Use geometric shapes, their measures, and their properties to describe objects. **G-MG-2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). **G-MG 3** Apply geometric methods to solve design problems. **G-GMD 1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. **G-GMD 3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. **G-GMD 4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**13. Part A** Daniel buys a block of clay for an art project. The block is shaped like a cube with edge lengths of 10 inches. Daniel decides to cut the block of clay into two pieces. He places a wire across the diagonal of one face of the cube, as shown in the figure. Then he pulls the wire straight back to create two congruent chunks of clay.  Daniel wants to keep one chunk of the clay for later use. To keep that chunk from drying out, he wants to place a piece of plastic sheeting on the surface he exposed when he cut through the cube. Describe this newly exposed two-dimensional cross section, and find its area. Round your answer to the nearest whole square inch. Show your work. **Part B** Daniel wants to reshape the other chunk of clay to make a set of clay spheres. He wants each sphere to have a diameter of 4 inches. Find the maximum number of spheres that Daniel can make from the chunk of clay. Show your work. **15.** The table shows the approximate measurements of the Great Pyramid of Giza in Egypt and the Pyramid of Kukulcan in Mexico. **Pyramid** / **Height (meters)** / **Area of Base (square meters)** Great Pyramid of Giza / 147 / 52,900 Pyramid of Kukulcan / 30 / 3,025 Approximately what is the difference between the volume of the Great Pyramid of Giza and the volume of the Pyramid of Kukulcan? Ⓐ 1,945,000 cubic meters Ⓑ 2,562,000 cubic meters Ⓒ 5,835,000 cubic meters Ⓓ 7,686,000 cubic meters

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Transfer Goals						
1) Demonstrate perseverance by making sense of a never-before-seen problem, developing a plan, and evaluating a strategy and solution. 2) Effectively communicate orally, in writing, and using models (e.g., concrete, representational, abstract) for a given purpose and audience. 3) Construct viable arguments and critique the reasoning of others using precise mathematical language.						
Essential Questions: • Examine and describe cross sections of polyhedra and recognize their parts and cross sections. • Apply Euler's Formula to verify two-dimensional nets of three-dimensional figures. • Use formulas to find surface areas and volumes of various solids such as: prisms, cylinders, pyramids, cones, and spheres.					**Standards:** G-MG 1, G-MG 2, G-MG 3, G-GMD 1, G-GMD 3, G-GMD 4 **Timeframe:** 3 weeks/15 days **Start Date:** April 19, 2018 **Assessment Dates:** May 8-9, 2018	
Time	**Lesson/ Activity**	**Focus Questions for Lessons**	**Understandings**	**Knowledge**	**Skills**	**Resources**
1 Day (Apr. 13th)	**Opening Activity:** Coffee Cup http://mrmeyer.com/threeacts/hotcoffee/					
1 Day (Apr. 14th)	**Lesson 11-1: Space Figures and Cross Sections** SMP: 1,2,3,4,5,7 (pp. 688-695) **G-GMD 4**	**Focus Question:** • How are the number of faces, F, vertices, V, and edges, E, of a polyhedron related to each other? • What is a cross section? **Inquiry Question:** p. 688 Solve It!	• A three-dimensional figure can be analyzed by describing the relationships among its vertices, edges, and faces. • A cross section is the intersection of a three-dimensional figure and a plane.	**Vocab:** polyhedron, face, edge, vertex, cross section, net **Concepts:** • Euler's Formula	• Examine and describe cross sections of polyhedra and recognize their parts and cross sections. • Apply Euler's Formula to verify two-dimensional nets of three-dimensional figures.	**Common Core Problems:** #4, 5, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40 **Thinking Maps:** Create a Bridge Map to analyze the relationships in this lesson.

Common Core Practices

- ☐ Instruction in the Standards for Mathematical Practices
- ☐ Use of Talk Moves
- ☐ Note-taking
- ☐ Use of Manipulatives
- ☐ Use of Technology
- ☐ Use of Real-world Scenarios
- ☐ Project-based Learning
- ☐ Thinking Maps

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
2 Days (Apr. 24-25)	**Lesson 11-2: Surface Areas of Prisms and Cylinders** SMP: 1,3,4,6,7,8 (pp. 699-707) **G-MG 1**	**Focus Question:** • What shapes make up the net of a prism or cylinder? • How do you find the lateral and surface areas of a prism or cylinder? **Inquiry Question:** p. 699 Solve It!	• The area of a three-dimensional figure is equal to the sum of the areas of each surface of the figure. • The surface area of a prism is found using the formula: $\boldsymbol{S.A.} = \boldsymbol{L.A.} + \boldsymbol{2B}$ $(L.A. = lateral\ area, B = base\ area)$ • The surface area of a cylinder is found using the formula: $\boldsymbol{S.A.} = \boldsymbol{L.A.} + \boldsymbol{2B}$ **or** $\boldsymbol{S.A.} = \boldsymbol{2\pi rh} + \boldsymbol{2\pi r^2}$ $(r = radius, h = height)$	**Vocab:** prism, cylinder, lateral area, base area, surface area, height, radius, diameter, pi, net **Concepts:** • Area of a circle • Area of a rectangle • Area of a triangle • Area of a regular pentagon or hexagon • Special Right Triangles • Pythagorean Theorem • Radius of a circle is half the diameter	• Find the lateral and surface area of a prism or cylinder by adding up the areas of the individual faces. • Find the lateral and surface area of a prism or cylinder by using the formula: $\boldsymbol{S.A.} = \boldsymbol{L.A.} + \boldsymbol{2B}$	**Common Core Problems:** #5,6, 21, 22, 23, 24, 25, 26, 27, 28, 37 **Thinking Maps:** Create a Tree Map to add to for surface area for lessons 11-2 11-3, and 11-6. **STEM:** #20, 29
2 Days (Apr. 26-27)	**Lesson 11-3: Surface Areas of Pyramids and Cones** SMP: 1,3,4,6,7 (pp. 708-715) **G-MG 1**	**Focus Question:** • What shapes make up the net of pyramid or a cone? • What is the difference between an altitude and slant height? • How do you find the surface area of a pyramid or cone? **Inquiry Question:** p. 708 Solve It!	• The surface area of a pyramid is found using the formula: $\boldsymbol{S.A.} = \boldsymbol{L.A.} + \boldsymbol{B}$ **or** $\boldsymbol{S.A.} = \frac{\boldsymbol{1}}{\boldsymbol{2}}\boldsymbol{pl} + \boldsymbol{B}$ $(p = perimeter,\ l = slant\ height)$ • The surface area of a cone is found using the formula $\boldsymbol{S.A.} = \boldsymbol{L.A.} + \boldsymbol{B}$ **or** $\boldsymbol{S.A.} = \boldsymbol{\pi rl} + \boldsymbol{\pi r^2}$ $(l = slant\ height)$	**Vocab:** cone, pyramid, surface area, base area, lateral area, slant height, altitude, vertex, radius, diameter **Concepts:** • Pythagorean Theorem • Special Right Triangles • Area of a circle • Slant height vs altitude • Area of Isosceles Triangles • Radius of a circle is half the diameter	• Find the surface area of a pyramid by adding up the areas of the individual faces and base. • Find the surface area of a pyramid by using the formula: $\boldsymbol{S.A.} = \frac{\boldsymbol{1}}{\boldsymbol{2}}\boldsymbol{pl} + \boldsymbol{B}$ • Find the surface area of a cone by using the formula: $\boldsymbol{S.A.} = \boldsymbol{\pi rl} + \boldsymbol{\pi r^2}$	**Common Core Problems:** #5, 6, 7, 8, 15, 22, 24, 25, 29, 31, 32, 41 **Thinking Maps:** Add to the Tree Map for surface area started in lesson 11-2. **STEM:** #30

Common Core Practices

- ☐ Instruction in the Standards for Mathematical Practices
- ☐ Use of Talk Moves
- ☐ Note-taking
- ☐ Use of Manipulatives
- ☐ Use of Technology
- ☐ Use of Real-world Scenarios
- ☐ Project-based Learning
- ☐ Thinking Maps

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 Day (Apr. 28th)	**Performance Task:** Revisit Coffee Cup http://mrmeyer.com/threeacts/hotcoffee/					
2 Days (May 1-2)	**Lesson 11-4: Volumes of Prisms and Cylinders** SMP: 1,3,4,6,7 (pp. 717-724) **G-MG 1, G-GMD 1, G-GMD 3**	**Focus Question:** • What does volume measure? • How do you find the volume of a prism or cylinder? • How are the formulas similar? How are they different? **Inquiry Question:** p. 717 Solve It!	• The volume of a prism is found using the formula: $V = Bh$; $B = base\ area$ • The volume of a cylinder is found using the formula: $V = Bh$; $B = base\ area$ or $V = \pi r^2 h$	**Vocab:** volume, base area, height, cubic units **Concepts:** • Area of a circle • Area of a triangle • Area of a rectangle • Area of a parallelogram • Radius of a circle is half the diameter • Slant height vs altitude • Volume of a Prism and Cylinder	• Find the volume of a prism using the formula: $V = Bh$; $B = base\ area$ • Find the volume of a cylinder using the formula: $V = \pi r^2 h$	**Common Core Problems:** #3,4,5, 21, 22, 29, 31, 35, 39-42, 43, 45 **Thinking Maps:** Create a Tree Map to add to for volume for lessons 11-4, 11-5, and 11-6. **STEM:** #28, 44
1 Day (May 3rd)	**Lesson 11-5: Volumes of Pyramids and Cones** SMP: 1,3,4,7 (pp. 726-732) **G-MG 1, G-GMD 3**	**Focus Question:** • How do you find the volume of a pyramid or cone? • How do the volumes of right solids compare to the volume of oblique solids with the same dimensions? **Inquiry Question:** p. 726 Solve It!	• The volume of a pyramid is found using the formula: $V = \frac{1}{3}Bh$; $B = base\ area$ • The volume of a cone is found using the formula: $V = \frac{1}{3}Bh$; $B = base\ area$ or $V = \frac{1}{3}\pi r^2 h$	**Vocab:** volume, slant height, altitude, base area, radius, diameter, cubic units **Concepts:** • Area of a circle • Radius of a circle is half the diameter. • Area of a rectangle • Area of triangle • Special Right Triangles • Slant height vs altitude • Volume of a Pyramid and Cone	• Find the volume of a pyramid using the formula: $V = \frac{1}{3}Bh$; $B = base\ area$ • Find the volume of a cone using the formula: $V = \frac{1}{3}\pi r^2 h$	**Common Core Problems:** #3, 4, 20, 21, 22, 26, 29, 33-36, 37, 38 **Thinking Maps:** Add to the Tree Map for volume started in lesson 11-4. **STEM:** #15, 16, 27, 28

Common Core Practices

- ☐ Instruction in the Standards for Mathematical Practices
- ☐ Use of Talk Moves
- ☐ Note-taking
- ☐ Use of Manipulatives
- ☐ Use of Technology
- ☐ Use of Real-world Scenarios
- ☐ Project-based Learning
- ☐ Thinking Maps

Time	Lesson/ Activity	Focus Questions for Lessons	Understandings	Knowledge	Skills	Additional Resources
1 Day (May 4th)	**Lesson 11-6: Surface Areas and Volumes of Spheres** SMP: 1,3,4,6,7,8 (pp. 733-740) **G-MG 1, G-GMD 3**	**Focus Question:** • How do you find the surface area and volume of a sphere? **Inquiry Question:** p. 733 Solve It!	• The surface area of a sphere is found using the formula: $S.A. = 4\pi r^2$ • The volume of a sphere is found using the formula: $V = \frac{4}{3}\pi r^3$	**Vocab:** sphere, radius, diameter **Concepts:** • Radius of a circle is half the diameter • Surface Area Formula for a Sphere • Volume Formula for a Sphere	• Find the volume of a sphere using the formula: $V = \frac{4}{3}\pi r^3$ • Find the surface area of a sphere using the formula: $S.A. = 4\pi r^2$	**Common Core Problems:** # 4,5, 26, 27, 28, 29, 32, 33, 43, 48, 54, 57, 59 **Thinking Maps:** Add to the Tree Maps started in 11-2 and 11-4. **STEM:** #31, 52, 53
2 Days (May 5 & 8)	**Review Topic 10 Concepts & Skills** Use Textbook Resources and/or Teacher Created Items					
2 Days (May 9-10)	**Topic 10 Assessment** (Created and provided by PUSD)					

Common Core Practices

- ☐ Instruction in the Standards for Mathematical Practices
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- ☐ Project-based Learning
- ☐ Thinking Maps

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