2.6 Prove Statements About Segments and Angles

Goal: Write proofs using geometric theorems.

Given: $3x - 9 = 0$, Prove: $x = 3$

Statements	Reasons
1. $3x - 9 = 0$	1. Given
2. $3x = 9$	2. Addition Postulate
3. $x = 3$	3. Division Postulate

The two-column proof is a formal way to organize your reasons to show a statement is true. The two-column proof has numbered statements on the left side, which are made one at a time until you reach the conclusion and corresponding reasons on the right side (which include definitions, properties, postulates, and theorems-statements that can be proven) that show an argument in logical order.

Use a property of equality to complete the statement.

1. If $m\angle 1 = m\angle 3$, then $m\angle 3 =$ ___?___. ∠1
2. If $AB = CD$ and $CD = TU$, then ___?___. AB=TU
3. If $RS = WX$, then ___?___ $+ AB =$ ___?___ $+ AB$. RS, WX
4. If $m\angle EFG = 28°$ and $m\angle GFH = 62°$, then ___?___ $+ 62° = m\angle EFG + m\angle GFH$. 28, Substitution

Solve. Give a reason for each step.

1. $-5x + 18 = 3x - 38$

2. $-3(x - 5) = 2(x + 10)$

Identify the property of equality.

3. If $m\angle 3 = m\angle 5$ and $m\angle 5 = m\angle 8$, then $m\angle 3 = m\angle 8$. T

4. If $CD = EF$, then $EF = CD$. S

Proof - an argument that uses logic to show that a statement is true

Two - column proof - a form of proof in which the statements are written in the left hand column and the reasons are written in the right hand column

(Conclusions) (Justifications)

Statements	Reasons
Given	Definitions Properties Theorems Postulates
Prove	

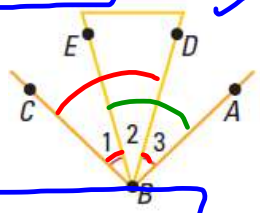
4 Key Parts to a Proof -

Given, To Prove, Picture or Drawing, & Argument

Write a two-column proof

GIVEN ▶ $m\angle 1 = m\angle 3$

PROVE ▶ $m\angle EBA = m\angle DBC$



STATEMENTS	REASONS
$m\angle 1 = m\angle 3$	GIVEN
$m\angle 2 + m\angle 3 = m\angle EBA$	Angle Addition Postulate
$m\angle 1 + m\angle 2 = m\angle DBC$	Angle Addition Postulate
$m\angle 3 + m\angle 2 = m\angle DBC$	Substitution Prop
$m\angle EBA = m\angle DBC$	Transitive Prop

Congruence of Segments

Segment congruence is reflexive, symmetric, and transitive.

Reflexive	For any segment AB, $\overline{AB} \cong \overline{AB}$.
Symmetric	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Congruence of Angles

Angle congruence is reflexive, symmetric, and transitive.

Reflexive	For any angle A, $\angle A \cong \angle A$.
Symmetric	If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
Transitive	If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Name the property illustrated by the statement.

1 If $\angle R \cong \angle T$ and $\angle T \cong \angle P$, then $\angle R \cong \angle P$. Transitive

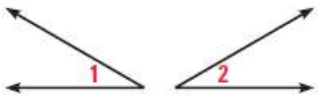
2 If $\overline{NK} \cong \overline{BD}$, then $\overline{BD} \cong \overline{NK}$. Symmetric

3 $\overline{CD} \cong \overline{CD}$ Reflexive

4 If $\angle Q \cong \angle V$, then $\angle V \cong \angle Q$. Symmetric

Writing a Two-Column Proof

In a proof, you make one statement at a time, until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs.

Proof of the Symmetric Property of Angle Congruence

GIVEN ▶ $\angle 1 \cong \angle 2$

PROVE ▶ $\angle 2 \cong \angle 1$

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

Statements based on facts that you know or on conclusions from deductive reasoning

Definitions, postulates, or proven theorems that allow you to state the corresponding statement

The number of statements will vary.

Remember to give a reason for the last statement.

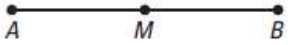
Use properties of equality

Prove this property of midpoints: If you know that M is the midpoint of $\overline{AB}$, prove that AB is two times AM and AM is one half of AB.

GIVEN ▶ M is the midpoint of $\overline{AB}$.

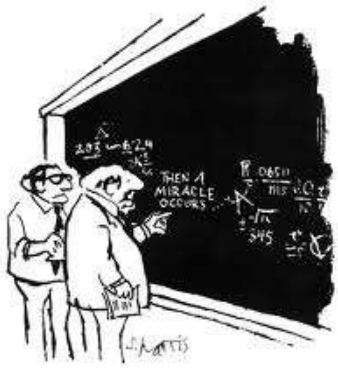
PROVE ▶ ~~a. $AB = 2 \cdot AM$~~

b. $AM = \frac{1}{2}AB$



STATEMENTS	REASONS
M is midpoint of $\overline{AB}$	Given
$\overline{AM} = \overline{MB}$	Definition of Midpoint
$\overline{AM} + \overline{MB} = \overline{AB}$	Segment Addition Postulate
$\overline{AM} + \overline{AM} = \overline{AB}$	Substitution Prop
$2\overline{AM} = \overline{AB}$	Simplify/Combine Like Terms
$AM = \frac{1}{2}AB$	Division Prop

HW: PG 108 #'s 3, 5-7, 8-12, 17, 18, 21, 24, 26



"I think you should be more explicit here in step two."