

# Preparing for Geometry



## Now

- **Chapter 0** contains lessons on topics from previous courses. You can use this chapter in various ways.
  - Begin the school year by taking the Pretest. If you need additional review, complete the lessons in this chapter. To verify that you have successfully reviewed the topics, take the Posttest.
  - As you work through the text, you may find that there are topics you need to review. When this happens, complete the individual lessons that you need.
  - Use this chapter for reference. When you have questions about any of these topics, flip back to this chapter to review definitions or key concepts.

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Vocabulary



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Foldables



Self-Check Practice



Worksheets



# Get Started on the Chapter

You will review several concepts, skills, and vocabulary terms as you study Chapter 0. To get ready, identify important terms and organize your resources.

## FOLDABLES® Study Organizer



Throughout this text, you will be invited to use Foldables to organize your notes.

**Why** should you use them?

- They help you organize, display, and arrange information.
- They make great study guides, specifically designed for you.
- You can use them as your math journal for recording main ideas, problem-solving strategies, examples, or questions you may have.
- They give you a chance to improve your math vocabulary.

**How** should you use them?

- Write general information — titles, vocabulary terms, concepts, questions, and main ideas — on the front tabs of your Foldable.
- Write specific information — ideas, your thoughts, answers to questions, steps, notes, and definitions — under the tabs.
- Use the tabs for:
  - math concepts in parts, like types of triangles,
  - steps to follow, or
  - parts of a problem, like *compare* and *contrast* (2 parts) or *what*, *where*, *when*, *why*, and *how* (5 parts).
- You may want to store your Foldables in a plastic zipper bag that you have three-hole punched to fit in your notebook.

**When** should you use them?

- Set up your Foldable as you begin a chapter, or when you start learning a new concept.
- Write in your Foldable every day.
- Use your Foldable to review for homework, quizzes, and tests.

## Review Vocabulary



| English                  |        | Español                   |
|--------------------------|--------|---------------------------|
| experiment               | p. P8  | experimento               |
| trial                    | p. P8  | prueba                    |
| outcome                  | p. P8  | resultado                 |
| event                    | p. P8  | evento                    |
| probability              | p. P8  | probabilidad              |
| theoretical probability  | p. P9  | probabilidad teórica      |
| experimental probability | p. P9  | probabilidad experimental |
| ordered pair             | p. P15 | par ordenado              |
| x-coordinate             | p. P15 | coordenada x              |
| y-coordinate             | p. P15 | coordenada y              |
| quadrant                 | p. P15 | cuadrante                 |
| origin                   | p. P15 | origen                    |
| system of equations      | p. P17 | sistema de ecuaciones     |
| substitution             | p. P17 | sustitución               |
| elimination              | p. P18 | eliminación               |
| Product Property         | p. P19 | Propiedad de Producto     |
| Quotient Property        | p. P19 | Propiedad de Cociente     |



State which metric unit you would probably use to measure each item.

- length of a computer keyboard
- mass of a large dog

Complete each sentence.

- $4 \text{ ft} = \underline{\quad} \text{ in.}$
- $21 \text{ ft} = \underline{\quad} \text{ yd}$
- $180 \text{ g} = \underline{\quad} \text{ kg}$
- $3 \text{ T} = \underline{\quad} \text{ lb}$
- $32 \text{ g} \approx \underline{\quad} \text{ oz}$
- $3 \text{ mi} \approx \underline{\quad} \text{ km}$
- $35 \text{ yd} \approx \underline{\quad} \text{ m}$
- $5.1 \text{ L} \approx \underline{\quad} \text{ qt}$

11. **TUNA** A can of tuna is 6 ounces. About how many grams is it?

12. **CRACKERS** A box of crackers is 453 grams. About how many pounds is it? Round to the nearest pound.

13. **DISTANCE** A road sign in Canada gives the distance to Toronto as 140 kilometers. What is this distance to the nearest mile?

**PROBABILITY** A bag contains 3 blue chips, 7 red chips, 4 yellow chips, and 5 green chips. A chip is randomly drawn from the bag. Find each probability.

- $P(\text{yellow})$
- $P(\text{green})$
- $P(\text{red or blue})$
- $P(\text{not red})$

Evaluate each expression if  $r = 3$ ,  $q = 1$ , and  $w = -2$ .

- $4r + q$
- $rw - 6$
- $\frac{r + 3q}{4r}$
- $\frac{5w}{3r + q}$
- $|2 - r| + 17$
- $8 + |q - 5|$

Solve each equation.

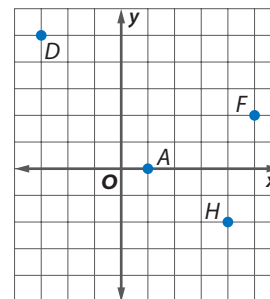
- $k + 3 = 14$
- $a - 7 = 9$
- $5c = 20$
- $n + 2 = -11$
- $6t - 18 = 30$
- $4x + 7 = -1$
- $\frac{r}{4} = -8$
- $\frac{3}{5}b = -2$
- $-\frac{w}{2} = -9$
- $3y - 15 = y + 1$
- $27 - 6d = 7 + 4d$
- $2(m - 16) = 44$

Solve each inequality.

- $y - 13 < 2$
- $t + 8 \geq 19$
- $\frac{n}{4} > -6$
- $9a \leq 45$
- $x + 12 > -14$
- $-2w < 24$
- $-\frac{n}{7} \geq 3$
- $-\frac{b}{5} \leq -6$

Write the ordered pair for each point shown.

- $F$
- $H$
- $A$
- $D$



Graph and label each point on the coordinate plane above.

- $B(4, 1)$
- $G(0, -3)$
- $R(-2, -4)$
- $P(-3, 3)$
- Graph the triangle with vertices  $J(1, -4)$ ,  $K(2, 3)$ , and  $L(-1, 2)$ .
- Graph four points that satisfy the equation  $y = 2x - 1$ .

Solve each system of equations.

- $y = 2x$   
 $y = -x + 6$
- $-3x - y = 4$   
 $4x + 2y = -8$
- $y = 2x + 1$   
 $y = 3x$
- $\frac{1}{2}x - y = -1$   
 $x - 2y = 5$
- $x + y = -6$   
 $2x - y = 3$
- $\frac{1}{3}x - 3y = -4$   
 $x - 9y = -12$

Simplify.

- $\sqrt{18}$
- $\sqrt{\frac{25}{49}}$
- $\sqrt{24x^2y^3}$
- $\frac{3}{4 - \sqrt{5}}$

# LESSON 0-1

## Changing Units of Measure Within Systems

### Objective

- Convert units of measure within the customary and metric systems.

#### Example 1 Choose Best Unit of Measure



State which metric unit you would use to measure the length of your pen.

A pen has a small length, but not very small. The *centimeter* is the appropriate unit of measure.

| Metric Units of Length             |
|------------------------------------|
| 1 kilometer (km) = 1000 meters (m) |
| 1 m = 100 centimeters (cm)         |
| 1 cm = 10 millimeters (mm)         |

| Customary Units of Length     |
|-------------------------------|
| 1 foot (ft) = 12 inches (in.) |
| 1 yard (yd) = 3 ft            |
| 1 mile (mi) = 5280 ft         |

- To convert from larger units to smaller units, multiply.
- To convert from smaller units to larger units, divide.
- To use dimensional analysis, multiply by the ratio of the units.

#### Example 2 Convert from Larger Units to Smaller Units of Length



Complete each sentence.

a.  $4.2 \text{ km} = \underline{\quad ? \quad} \text{ m}$

There are 1000 meters in a kilometer.  
 $4.2 \text{ km} \times 1000 = 4200 \text{ m}$

b.  $13 \text{ yd} = \underline{\quad ? \quad} \text{ ft}$

There are 3 feet in a yard.  
 $13 \text{ yd} \times 3 = 39 \text{ ft}$

#### Example 3 Convert from Smaller Units to Larger Units of Length



Complete each sentence.

a.  $17 \text{ mm} = \underline{\quad ? \quad} \text{ m}$

There are 100 centimeters in a meter. First change *millimeters* to *centimeters*.

$17 \text{ mm} = \underline{\quad ? \quad} \text{ cm}$       smaller unit  $\rightarrow$  larger unit

$17 \text{ mm} \div 10 = 1.7 \text{ cm}$       Since  $10 \text{ mm} = 1 \text{ cm}$ , divide by 10.

Then change *centimeters* to *meters*.

$1.7 \text{ cm} = \underline{\quad ? \quad} \text{ m}$       smaller unit  $\rightarrow$  larger unit

$1.7 \text{ cm} \div 100 = 0.017 \text{ m}$       Since  $100 \text{ cm} = 1 \text{ m}$ , divide by 100.

b.  $6600 \text{ yd} = \underline{\quad ? \quad} \text{ mi}$

Use dimensional analysis.

$6600 \cancel{\text{ yd}} \times \frac{3 \cancel{\text{ ft}}}{1 \cancel{\text{ yd}}} \times \frac{1 \text{ mi}}{5280 \cancel{\text{ ft}}} = 3.75 \text{ mi}$

| Metric Units of Capacity            |
|-------------------------------------|
| 1 liter (L) = 1000 milliliters (mL) |

| Customary Units of Capacity        |                       |
|------------------------------------|-----------------------|
| 1 cup (c) = 8 fluid ounces (fl oz) | 1 quart (qt) = 2 pt   |
| 1 pint (pt) = 2 c                  | 1 gallon (gal) = 4 qt |



**StudyTip**

**Dimensional Analysis** You can use dimensional analysis for any conversion in this lesson.

**Example 4** Convert Units of Capacity

Complete each sentence.

a.  $3.7 \text{ L} = \underline{\quad ? \quad} \text{ mL}$

There are 1000 milliliters in a liter.

$$3.7 \text{ L} \times 1000 = 3700 \text{ mL}$$

c.  $7 \text{ pt} = \underline{\quad ? \quad} \text{ fl oz}$

There are 8 fluid ounces in a cup.

First change *pints* to *cups*.

$$7 \text{ pt} = \underline{\quad ? \quad} \text{ c}$$

$$7 \text{ pt} \times 2 = 14 \text{ c}$$

Then change *cups* to *fluid ounces*.

$$14 \text{ c} = \underline{\quad ? \quad} \text{ fl oz}$$

$$14 \text{ c} \times 8 = 112 \text{ fl oz}$$

b.  $16 \text{ qt} = \underline{\quad ? \quad} \text{ gal}$

There are 4 quarts in a gallon.

$$16 \text{ qt} \div 4 = 4 \text{ gal}$$

d.  $4 \text{ gal} = \underline{\quad ? \quad} \text{ pt}$

There are 4 quarts in a gallon.

First change *gallons* to *quarts*.

$$4 \text{ gal} = \underline{\quad ? \quad} \text{ qt}$$

$$4 \text{ gal} \times 4 = 16 \text{ qt}$$

Then change *quarts* to *pints*.

$$16 \text{ qt} = \underline{\quad ? \quad} \text{ pt}$$

$$16 \text{ qt} \times 2 = 32 \text{ pt}$$

The mass of an object is the amount of matter that it contains.

| Metric Units of Mass             |
|----------------------------------|
| 1 kilogram (kg) = 1000 grams (g) |
| 1 g = 1000 milligrams (mg)       |

| Customary Units of Weight     |
|-------------------------------|
| 1 pound (lb) = 16 ounces (oz) |
| 1 ton (T) = 2000 lb           |

**Example 5** Convert Units of Mass

Complete each sentence.

a.  $5.47 \text{ kg} = \underline{\quad ? \quad} \text{ mg}$

There are 1000 milligrams in a gram.

Change *kilograms* to *grams*.

$$5.47 \text{ kg} = \underline{\quad ? \quad} \text{ g}$$

$$5.47 \text{ kg} \times 1000 = 5470 \text{ g}$$

Then change *grams* to *milligrams*.

$$5470 \text{ g} = \underline{\quad ? \quad} \text{ mg}$$

$$5470 \text{ g} \times 1000 = 5,470,000 \text{ mg}$$

b.  $5 \text{ T} = \underline{\quad ? \quad} \text{ oz}$

There are 16 ounces in a pound.

Change *tons* to *pounds*.

$$5 \text{ T} = \underline{\quad ? \quad} \text{ lb}$$

$$5 \text{ T} \times 2000 = 10,000 \text{ lb}$$

Then change *pounds* to *ounces*.

$$10,000 \text{ lb} = \underline{\quad ? \quad} \text{ oz}$$

$$10,000 \text{ lb} \times 16 = 160,000 \text{ oz}$$

**Exercises**

State which metric unit you would probably use to measure each item.

- radius of a tennis ball
- length of a notebook
- mass of a textbook
- mass of a beach ball
- liquid in a cup
- water in a bathtub

Complete each sentence.

7.  $120 \text{ in.} = \underline{\quad ? \quad} \text{ ft}$

8.  $18 \text{ ft} = \underline{\quad ? \quad} \text{ yd}$

9.  $10 \text{ km} = \underline{\quad ? \quad} \text{ m}$

10.  $210 \text{ mm} = \underline{\quad ? \quad} \text{ cm}$

11.  $180 \text{ mm} = \underline{\quad ? \quad} \text{ m}$

12.  $3100 \text{ m} = \underline{\quad ? \quad} \text{ km}$

13.  $90 \text{ in.} = \underline{\quad ? \quad} \text{ yd}$

14.  $5280 \text{ yd} = \underline{\quad ? \quad} \text{ mi}$

15.  $8 \text{ yd} = \underline{\quad ? \quad} \text{ ft}$

16.  $0.62 \text{ km} = \underline{\quad ? \quad} \text{ m}$

17.  $370 \text{ mL} = \underline{\quad ? \quad} \text{ L}$

18.  $12 \text{ L} = \underline{\quad ? \quad} \text{ mL}$

19.  $32 \text{ fl oz} = \underline{\quad ? \quad} \text{ c}$

20.  $5 \text{ qt} = \underline{\quad ? \quad} \text{ c}$

21.  $10 \text{ pt} = \underline{\quad ? \quad} \text{ qt}$

22.  $48 \text{ c} = \underline{\quad ? \quad} \text{ gal}$

23.  $4 \text{ gal} = \underline{\quad ? \quad} \text{ qt}$

24.  $36 \text{ mg} = \underline{\quad ? \quad} \text{ g}$

25.  $13 \text{ lb} = \underline{\quad ? \quad} \text{ oz}$

26.  $130 \text{ g} = \underline{\quad ? \quad} \text{ kg}$

27.  $9.05 \text{ kg} = \underline{\quad ? \quad} \text{ g}$



# LESSON 0-2 Changing Units of Measure Between Systems

## Objective

- Convert units of measure between the customary and metric systems.

The table below shows approximate equivalents between customary units of length and metric units of length.

| Units of Length        |                        |
|------------------------|------------------------|
| Customary → Metric     | Metric → Customary     |
| 1 in. $\approx$ 2.5 cm | 1 cm $\approx$ 0.4 in. |
| 1 yd $\approx$ 0.9 m   | 1 m $\approx$ 1.1 yd   |
| 1 mi $\approx$ 1.6 km  | 1 km $\approx$ 0.6 mi  |



### Example 1 Convert Units of Length Between Systems

Complete each sentence.

a. 30 in.  $\approx$    ?   cm

There are approximately 2.5 centimeters in an inch.

$$30 \text{ in.} \times 2.5 = 75 \text{ cm}$$

b. 5 km  $\approx$    ?   mi

There is approximately 0.6 mile in a kilometer.

$$5 \text{ km} \times 0.6 = 3 \text{ mi}$$



### Example 2 Convert Units of Length Between Systems

Complete: 2000 yd  $\approx$    ?   km.

There is approximately 0.9 meter in a yard. First find the number of meters in 2000 yards.

$$2000 \text{ yd} \times 0.9 = 1800 \text{ m}$$

Then change *meters* to *kilometers*. There are 1000 meters in a kilometer.

$$1800 \text{ m} \div 1000 = 1.8 \text{ km}$$

The table below shows approximate equivalents between customary units of capacity and metric units of capacity.

| Units of Capacity    |                      |
|----------------------|----------------------|
| Customary → Metric   | Metric → Customary   |
| 1 qt $\approx$ 0.9 L | 1 L $\approx$ 1.1 qt |
| 1 pt $\approx$ 0.5 L | 1 L $\approx$ 2.1 pt |



### Example 3 Convert Units of Capacity Between Systems

Complete each sentence.

a. 7 qt  $\approx$    ?   L

There is approximately 0.9 liter in a quart.

$$7 \text{ qt} \times 0.9 = 6.3 \text{ L}$$

b. 2 L  $\approx$    ?   pt

There are approximately 2.1 pints in a liter.

$$2 \text{ L} \times 2.1 = 4.2 \text{ pt}$$



**Example 4** Convert Units of Capacity Between Systems

**Complete:** 10 L  $\approx$  ? gal.

There are approximately 1.1 quarts in a liter. First find the number of quarts in 10 liters.

$$10 \text{ L} \times 1.1 = 11 \text{ qt}$$

Then change *quarts* to *gallons*. There are 4 quarts in a gallon.

$$11 \text{ qt} \div 4 = 2.75 \text{ gal}$$

You can also use dimensional analysis.

$$10 \cancel{\text{L}} \times \frac{1.1 \cancel{\text{qt}}}{1 \cancel{\text{L}}} \times \frac{1 \text{ gal}}{4 \cancel{\text{qt}}} = 2.75 \text{ gal}$$

**StudyTip**

**Dimensional Analysis** If the unit that you want to eliminate is in the numerator, make sure it is in the denominator of the ratio when you multiply. If it is in the denominator, make sure that it is in the numerator of the ratio.

The table below shows approximate equivalents between customary units of weight and metric units of mass.

| Units of Weight/Mass           |                                |
|--------------------------------|--------------------------------|
| Customary $\rightarrow$ Metric | Metric $\rightarrow$ Customary |
| 1 oz $\approx$ 28.3 g          | 1 g $\approx$ 0.04 oz          |
| 1 lb $\approx$ 0.5 kg          | 1 kg $\approx$ 2.2 lb          |

**Example 5** Convert Units of Mass Between Systems

**Complete each sentence.**

a. 58.5 kg  $\approx$  ? lb

There are approximately 2.2 pounds in a kilogram.

$$58.5 \text{ kg} \times 2.2 = 128.7 \text{ lb}$$

b. 14 oz  $\approx$  ? g

There are approximately 28.3 grams in an ounce.

$$14 \text{ oz} \times 28.3 = 396.2 \text{ g}$$

**Exercises**

**Complete each sentence.**

1. 8 in.  $\approx$  ? cm

2. 15 m  $\approx$  ? yd

3. 11 qt  $\approx$  ? L

4. 25 oz  $\approx$  ? g

5. 10 mi  $\approx$  ? km

6. 32 cm  $\approx$  ? in.

7. 20 km  $\approx$  ? mi

8. 9.5 L  $\approx$  ? qt

9. 6 yd  $\approx$  ? m

10. 4.3 kg  $\approx$  ? lb

11. 10.7 L  $\approx$  ? pt

12. 82.5 g  $\approx$  ? oz

13.  $2\frac{1}{4}$  lb  $\approx$  ? kg

14. 10 ft  $\approx$  ? m

15.  $1\frac{1}{2}$  gal  $\approx$  ? L

16. 350 g  $\approx$  ? lb

17. 600 in.  $\approx$  ? m

18. 2.1 km  $\approx$  ? yd

19. **CEREAL** A box of cereal is 13 ounces. About how many grams is it?

20. **FLOUR** A bag of flour is 2.26 kilograms. How much does it weigh? Round to the nearest pound.

21. **SAUCE** A jar of tomato sauce is 1 pound 10 ounces. About how many grams is it?



# LESSON 0-3 Simple Probability

## Objective

- Find the probability of simple events.



### New Vocabulary

experiment  
trial  
outcome  
event  
probability  
theoretical probability  
experimental probability



### Common Core State Standards

#### Content Standards

**S.MD.6 (+)** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

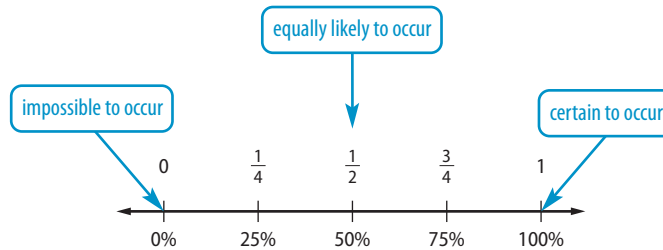
**S.MD.7 (+)** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

#### Mathematical Practices

4 Model with mathematics.

A situation involving chance such as flipping a coin or rolling a die is an **experiment**. A single performance of an experiment such as rolling a die one time is a **trial**. The result of a trial is called an **outcome**. An **event** is one or more outcomes of an experiment.

When each outcome is equally likely to happen, the **probability** of an event is the ratio of the number of favorable outcomes to the number of possible outcomes. The probability of an event is always between 0 and 1, inclusive.



### Example 1 Find Probability



**Suppose a die is rolled. What is the probability of rolling an odd number?**

There are 3 odd numbers on a die: 1, 3, and 5.

There are 6 possible outcomes: 1, 2, 3, 4, 5, and 6.

$$P(\text{odd}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$= \frac{3}{6} \text{ or } \frac{1}{2}$$

The probability of rolling an odd number is  $\frac{1}{2}$  or 50%.

For a given experiment, the sum of the probabilities of all possible outcomes must sum to 1.

### Example 2 Find Probability



**Suppose a bag contains 4 red, 3 green, 6 blue, and 2 yellow marbles. What is the probability a randomly chosen marble will not be yellow?**

Since the sum of the probabilities of all of the colors must sum to 1, subtract the probability that the marble will be yellow from 1.

The probability that the marble will be yellow is  $\frac{2}{15}$  because there are 2 yellow marbles and 15 total marbles.

$$P(\text{not yellow}) = 1 - P(\text{yellow})$$

$$= 1 - \frac{2}{15}$$

$$= \frac{13}{15}$$

The probability that the marble will not be yellow is  $\frac{13}{15}$  or about 87%.





The probabilities in Examples 1 and 2 are called theoretical probabilities. The **theoretical probability** is what *should* occur. The **experimental probability** is what *actually* occurs when a probability experiment is repeated many times.



### StudyTip

#### Experimental Probability

The experimental probability of an experiment is not necessarily the same as the theoretical probability, but when an experiment is repeated many times, the experimental probability should be close to the theoretical probability.

### Example 3 Find Experimental Probability

The table shows the results of an experiment in which a number cube was rolled. Find the experimental probability of rolling a 3.

$$P(3) = \frac{\text{number of times 3 occurs}}{\text{total number of outcomes}} \text{ or } \frac{7}{25}$$

The experimental probability for getting a 3 in this case is  $\frac{7}{25}$  or 28%.

| Outcome | Tally | Frequency |
|---------|-------|-----------|
| 1       |       | 6         |
| 2       |       | 4         |
| 3       |       | 7         |
| 4       |       | 3         |
| 5       |       | 4         |
| 6       |       | 1         |

## Exercises

A die is rolled. Find the probability of each outcome.

- $P(\text{less than } 3)$
- $P(\text{even})$
- $P(\text{greater than } 2)$
- $P(\text{prime})$
- $P(4 \text{ or } 2)$
- $P(\text{integer})$

A jar contains 65 pennies, 27 nickels, 30 dimes, and 18 quarters. A coin is randomly selected from the jar. Find each probability.

- $P(\text{penny})$
- $P(\text{quarter})$
- $P(\text{not dime})$
- $P(\text{penny or dime})$
- $P(\text{value greater than } \$0.15)$
- $P(\text{not nickel})$
- $P(\text{nickel or quarter})$
- $P(\text{value less than } \$0.20)$

**PRESENTATIONS** The students in a class are randomly drawing cards numbered 1 through 28 from a hat to determine the order in which they will give their presentations. Find each probability.

- $P(13)$
- $P(1 \text{ or } 28)$
- $P(\text{less than } 14)$
- $P(\text{not } 1)$
- $P(\text{not } 2 \text{ or } 17)$
- $P(\text{greater than } 16)$

The table shows the results of an experiment in which three coins were tossed.

| Outcome   | HHH | HHT | HTH | THH | TTH | THT | HTT | TTT |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| Tally     |     |     |     |     |     |     |     |     |
| Frequency | 5   | 5   | 6   | 6   | 7   | 5   | 8   | 8   |

- What is the experimental probability that all three of the coins will be heads? The theoretical probability?
- What is the experimental probability that at least two of the coins will be heads? The theoretical probability?
- DECISION MAKING** You and two of your friends have pooled your money to buy a new video game. Describe a method that could be used to make a fair decision as to who gets to play the game first.
- DECISION MAKING** A new study finds that the incidence of heart attack while taking a certain diabetes drug is less than 5%. Should a person with diabetes take this drug? Should they take the drug if the risk is less than 1%? Explain your reasoning.



## Algebraic Expressions

## Objective

- Use the order of operations to evaluate algebraic expressions.

An expression is an algebraic expression if it contains sums and/or products of variables and numbers. To evaluate an algebraic expression, replace the variable or variables with known values, and then use the order of operations.

| Order of Operations |   |
|---------------------|---|
| <b>Step 1</b>       | Evaluate expressions inside grouping symbols.               |
| <b>Step 2</b>       | Evaluate all powers.  |
| <b>Step 3</b>       | Do all multiplications and/or divisions from left to right. |
| <b>Step 4</b>       | Do all additions and/or subtractions from left to right.    |

**Example 1** Addition/Subtraction Algebraic Expressions

Evaluate  $x - 5 + y$  if  $x = 15$  and  $y = -7$ .

$$\begin{aligned} x - 5 + y &= 15 - 5 + (-7) && \text{Substitute.} \\ &= 10 + (-7) \text{ or } 3 && \text{Subtract.} \end{aligned}$$

**Example 2** Multiplication/Division Algebraic Expressions

Evaluate each expression if  $k = -2$ ,  $n = -4$ , and  $p = 5$ .

|   |   |
|---|---|
| <p>a. <math>\frac{2k + n}{p - 3}</math></p> $\begin{aligned} \frac{2k + n}{p - 3} &= \frac{2(-2) + (-4)}{5 - 3} && \text{Substitute.} \\ &= \frac{-4 - 4}{5 - 3} && \text{Multiply.} \\ &= \frac{-8}{2} \text{ or } -4 && \text{Subtract.} \end{aligned}$ | <p>b. <math>-3(k^2 + 2n)</math></p> $\begin{aligned} -3(k^2 + 2n) &= -3[(-2)^2 + 2(-4)] \\ &= -3[4 + (-8)] \\ &= -3(-4) \text{ or } 12 \end{aligned}$ |
|---|---|

**Example 3** Absolute Value Algebraic Expressions

Evaluate  $3|a - b| + 2|c - 5|$  if  $a = -2$ ,  $b = -4$ , and  $c = 3$ .

$$\begin{aligned} 3|a - b| + 2|c - 5| &= 3|-2 - (-4)| + 2|3 - 5| && \text{Substitute for } a, b, \text{ and } c. \\ &= 3|2| + 2|-2| && \text{Simplify.} \\ &= 3(2) + 2(2) \text{ or } 10 && \text{Find absolute values.} \end{aligned}$$

**Exercises**

Evaluate each expression if  $a = 2$ ,  $b = -3$ ,  $c = -1$ , and  $d = 4$ .

- |                        |                    |                       |                       |
|------------------------|--------------------|-----------------------|-----------------------|
| 1. $2a + c$            | 2. $\frac{bd}{2c}$ | 3. $\frac{2d - a}{b}$ | 4. $3d - c$           |
| 5. $\frac{3b}{5a + c}$ | 6. $5bc$           | 7. $2cd + 3ab$        | 8. $\frac{c - 2d}{a}$ |

Evaluate each expression if  $x = 2$ ,  $y = -3$ , and  $z = 1$ .

- |                   |                    |                    |                     |
|-------------------|--------------------|--------------------|---------------------|
| 9. $24 +  x - 4 $ | 10. $13 +  8 + y $ | 11. $ 5 - z  + 11$ | 12. $ 2y - 15  + 7$ |
|-------------------|--------------------|--------------------|---------------------|



## Objective

- Use algebra to solve linear equations.

If the same number is added to or subtracted from each side of an equation, the resulting equation is true.


**Example 1** Addition/Subtraction Linear Equations

Solve each equation.

a.  $x - 7 = 16$

$$x - 7 = 16$$

Original equation

$$x - 7 + 7 = 16 + 7$$

Add 7 to each side.

$$x = 23$$

Simplify.

b.  $m + 12 = -5$

$$m + 12 = -5$$

Original equation

$$m + 12 + (-12) = -5 + (-12)$$

Add  $-12$  to each side.

$$m = -17$$

Simplify.

c.  $k + 31 = 10$

$$k + 31 = 10$$

Original equation

$$k + 31 - 31 = 10 - 31$$

Subtract 31 from each side.

$$k = -21$$

Simplify.

If each side of an equation is multiplied or divided by the same number, the resulting equation is true.


**Example 2** Multiplication/Division Linear Equations

Solve each equation.

a.  $4d = 36$

$$4d = 36$$

Original equation

$$\frac{4d}{4} = \frac{36}{4}$$

Divide each side by 4.

$$d = 9$$

Simplify.

b.  $-\frac{t}{8} = -7$

$$-\frac{t}{8} = -7$$

Original equation

$$-8\left(-\frac{t}{8}\right) = -8(-7)$$

Multiply each side by  $-8$ .

$$t = 56$$

Simplify.

c.  $\frac{3}{5}x = -8$

$$\frac{3}{5}x = -8$$

Original equation

$$\frac{5}{3}\left(\frac{3}{5}\right)x = \frac{5}{3}(-8)$$

Multiply each side by  $\frac{5}{3}$ .

$$x = -\frac{40}{3}$$

Simplify.

To solve equations with more than one operation, often called *multi-step equations*, undo operations by working backward.





### Example 3 Multi-step Linear Equations

Solve each equation.

a.  $8q - 15 = 49$

$8q - 15 = 49$  Original equation

$8q = 64$  Add 15 to each side.

$q = 8$  Divide each side by 8.

b.  $12y + 8 = 6y - 5$

$12y + 8 = 6y - 5$  Original equation

$12y = 6y - 13$  Subtract 8 from each side.

$6y = -13$  Subtract 6y from each side.

$y = -\frac{13}{6}$  Divide each side by 6.

#### WatchOut!

##### Order of Operations

Remember that the order of operations applies when you are solving linear equations.

When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

### Example 4 Multi-step Linear Equations

Solve  $3(x - 5) = 13$ .

$3(x - 5) = 13$  Original equation

$3x - 15 = 13$  Distributive Property

$3x = 28$  Add 15 to each side.

$x = \frac{28}{3}$  Divide each side by 3.

## Exercises

Solve each equation.

1.  $r + 11 = 3$

2.  $n + 7 = 13$

3.  $d - 7 = 8$

4.  $\frac{8}{5}a = -6$

5.  $-\frac{p}{12} = 6$

6.  $\frac{x}{4} = 8$

7.  $\frac{12}{5}f = -18$

8.  $\frac{y}{7} = -11$

9.  $\frac{6}{7}y = 3$

10.  $c - 14 = -11$

11.  $t - 14 = -29$

12.  $p - 21 = 52$

13.  $b + 2 = -5$

14.  $q + 10 = 22$

15.  $-12q = 84$

16.  $5t = 30$

17.  $5c - 7 = 8c - 4$

18.  $2\ell + 6 = 6\ell - 10$

19.  $\frac{m}{10} + 15 = 21$

20.  $-\frac{m}{8} + 7 = 5$

21.  $8t + 1 = 3t - 19$

22.  $9n + 4 = 5n + 18$

23.  $5c - 24 = -4$

24.  $3n + 7 = 28$

25.  $-2y + 17 = -13$

26.  $-\frac{t}{13} - 2 = 3$

27.  $\frac{2}{9}x - 4 = \frac{2}{3}$

28.  $9 - 4g = -15$

29.  $-4 - p = -2$

30.  $21 - b = 11$

31.  $-2(n + 7) = 15$

32.  $5(m - 1) = -25$

33.  $-8a - 11 = 37$

34.  $\frac{7}{4}q - 2 = -5$

35.  $2(5 - n) = 8$

36.  $-3(d - 7) = 6$



## Objective

- Use algebra to solve linear inequalities.

Statements with greater than ( $>$ ), less than ( $<$ ), greater than or equal to ( $\geq$ ), or less than or equal to ( $\leq$ ) are inequalities. If any number is added or subtracted to each side of an inequality, the resulting inequality is true.


**Example 1** Addition/Subtraction Linear Inequalities

Solve each inequality.

a.  $x - 17 > 12$

$$x - 17 > 12 \quad \text{Original inequality}$$

$$x - 17 + 17 > 12 + 17 \quad \text{Add 17 to each side.}$$

$$x > 29 \quad \text{Simplify.}$$

The solution set is  $\{x | x > 29\}$ .

b.  $y + 11 \leq 5$

$$y + 11 \leq 5 \quad \text{Original inequality}$$

$$y + 11 - 11 \leq 5 - 11 \quad \text{Subtract 11 from each side.}$$

$$y \leq -6 \quad \text{Simplify.}$$

The solution set is  $\{y | y \leq -6\}$ .

If each side of an inequality is multiplied or divided by a positive number, the resulting inequality is true.


**Example 2** Multiplication/Division Linear Inequalities

Solve each inequality.

a.  $\frac{t}{6} \geq 11$

$$\frac{t}{6} \geq 11 \quad \text{Original inequality}$$

$$(6)\frac{t}{6} \geq (6)11 \quad \text{Multiply each side by 6.}$$

$$t \geq 66 \quad \text{Simplify.}$$

The solution set is  $\{t | t \geq 66\}$ .

b.  $8p < 72$

$$8p < 72 \quad \text{Original inequality}$$

$$\frac{8p}{8} < \frac{72}{8} \quad \text{Divide each side by 8.}$$

$$p < 9 \quad \text{Simplify.}$$

The solution set is  $\{p | p < 9\}$ .

If each side of an inequality is multiplied or divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is true.


**Example 3** Multiplication/Division Linear Inequalities

Solve each inequality.

a.  $-5c > 30$

$$-5c > 30 \quad \text{Original inequality}$$

$$\frac{-5c}{-5} < \frac{30}{-5} \quad \text{Divide each side by } -5. \text{ Change } > \text{ to } <.$$

$$c < -6 \quad \text{Simplify.}$$

The solution set is  $\{c | c < -6\}$ .

(continued on the next page)



$$\text{b. } -\frac{d}{13} \leq -4$$

$$-\frac{d}{13} \leq -4$$

Original inequality

$$(-13)\left(\frac{-d}{13}\right) \geq (-13)(-4)$$

Multiply each side by  $-13$ . Change  $\leq$  to  $\geq$ .

$$d \geq 52$$

Simplify.

The solution set is  $\{d \mid d \geq 52\}$ .

Inequalities involving more than one operation can be solved by undoing the operations in the same way you would solve an equation with more than one operation.



### Example 4 Multi-Step Linear Inequalities

Solve each inequality.

$$\text{a. } -6a + 13 < -7$$

$$-6a + 13 < -7$$

Original inequality

$$-6a + 13 - 13 < -7 - 13$$

Subtract 13 from each side.

$$-6a < -20$$

Simplify.

$$\frac{-6a}{-6} > \frac{-20}{-6}$$

Divide each side by  $-6$ . Change  $<$  to  $>$ .

$$a > \frac{10}{3}$$

Simplify.

The solution set is  $\left\{a \mid a > \frac{10}{3}\right\}$ .

$$\text{b. } 4z + 7 \geq 8z - 1$$

$$4z + 7 \geq 8z - 1$$

Original inequality

$$4z + 7 - 7 \geq 8z - 1 - 7$$

Subtract 7 from each side.

$$4z \geq 8z - 8$$

Simplify.

$$4z - 8z \geq 8z - 8 - 8z$$

Subtract  $8z$  from each side.

$$-4z \geq -8$$

Simplify.

$$\frac{-4z}{-4} \leq \frac{-8}{-4}$$

Divide each side by  $-4$ . Change  $\geq$  to  $\leq$ .

$$z \leq 2$$

Simplify.

The solution set is  $\{z \mid z \leq 2\}$ .

#### WatchOut!

##### Dividing by a Negative

Remember that any time you divide an inequality by a negative number you reverse the direction of the sign.

## Exercises

1.  $x - 7 < 6$

2.  $a + 7 \geq -5$

3.  $4y < 20$

4.  $-\frac{a}{8} < 5$

5.  $\frac{t}{6} > -7$

6.  $\frac{a}{11} \leq 8$

7.  $d + 8 \leq 12$

8.  $m + 14 > 10$

9.  $12k \geq -36$

10.  $6t - 10 \geq 4t$

11.  $3z + 8 < 2$

12.  $4c + 23 \leq -13$

13.  $m - 21 < 8$

14.  $x - 6 \geq 3$

15.  $-3b \leq 48$

16.  $\frac{p}{5} \geq 14$

17.  $2z - 9 < 7z + 1$

18.  $-4h > 36$

19.  $\frac{2}{5}b - 6 \leq -2$

20.  $\frac{8}{3}t + 1 > -5$

21.  $7q + 3 \geq -4q + 25$

22.  $-3n - 8 > 2n + 7$

23.  $-3w + 1 \leq 8$

24.  $-\frac{4}{5}k - 17 > 11$



## Objective

- Name and graph points in the coordinate plane.



## New Vocabulary

ordered pair  
 x-coordinate  
 y-coordinate  
 quadrant  
 origin

Points in the coordinate plane are named by **ordered pairs** of the form  $(x, y)$ . The first number, or **x-coordinate**, corresponds to a number on the  $x$ -axis. The second number, or **y-coordinate**, corresponds to a number on the  $y$ -axis.



## Example 1 Writing Ordered Pairs

Write the ordered pair for each point.

a.  $A$

The  $x$ -coordinate is 4.

The  $y$ -coordinate is  $-1$ .

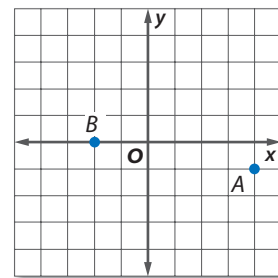
The ordered pair is  $(4, -1)$ .

b.  $B$

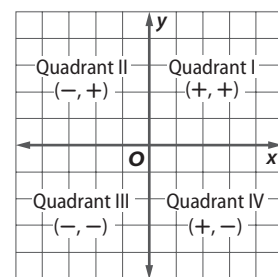
The  $x$ -coordinate is  $-2$ .

The point lies on the  $x$ -axis, so its  $y$ -coordinate is 0.

The ordered pair is  $(-2, 0)$ .



The  $x$ -axis and  $y$ -axis separate the coordinate plane into four regions, called **quadrants**. The point at which the axes intersect is called the **origin**. The axes and points on the axes are not located in any of the quadrants.



## Example 2 Graphing Ordered Pairs

Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.

a.  $G(2, 1)$

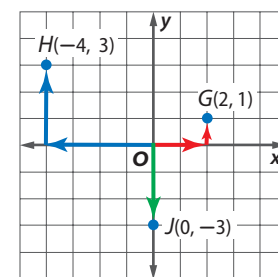
Start at the origin. Move 2 units right, since the  $x$ -coordinate is 2. Then move 1 unit up, since the  $y$ -coordinate is 1. Draw a dot, and label it  $G$ .  
 Point  $G(2, 1)$  is in Quadrant I.

b.  $H(-4, 3)$

Start at the origin. Move 4 units left, since the  $x$ -coordinate is  $-4$ . Then move 3 units up, since the  $y$ -coordinate is 3. Draw a dot, and label it  $H$ .  
 Point  $H(-4, 3)$  is in Quadrant II.

c.  $J(0, -3)$

Start at the origin. Since the  $x$ -coordinate is 0, the point lies on the  $y$ -axis. Move 3 units down, since the  $y$ -coordinate is  $-3$ . Draw a dot, and label it  $J$ .  
 Because it is on one of the axes, point  $J(0, -3)$  is not in any quadrant.

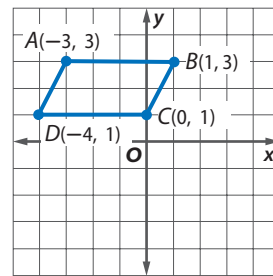




### Example 3 Graphing Multiple Ordered Pairs

Graph a polygon with vertices  $A(-3, 3)$ ,  $B(1, 3)$ ,  $C(0, 1)$ , and  $D(-4, 1)$ .

Graph the ordered pairs on a coordinate plane. Connect each pair of consecutive points. The polygon is a parallelogram.



### StudyTip

**Lines** There are infinitely many points on a line, so when you are asked to find points on a line, there are many answers.

### Example 4 Graphing and Solving for Ordered Pairs

Graph four points that satisfy the equation  $y = 4 - x$ .

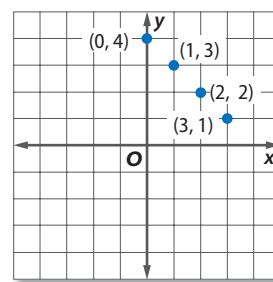
Make a table.

Choose four values for  $x$ .

Evaluate each value of  $x$  for  $4 - x$ .

| $x$ | $4 - x$ | $y$ | $(x, y)$ |
|-----|---------|-----|----------|
| 0   | $4 - 0$ | 4   | (0, 4)   |
| 1   | $4 - 1$ | 3   | (1, 3)   |
| 2   | $4 - 2$ | 2   | (2, 2)   |
| 3   | $4 - 3$ | 1   | (3, 1)   |

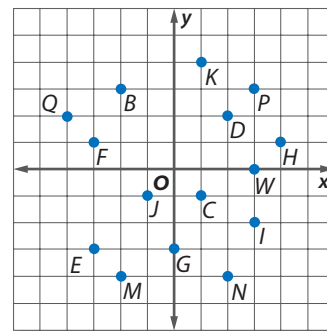
Plot the points.



## Exercises

Write the ordered pair for each point shown at the right.

- |         |         |         |
|---------|---------|---------|
| 1. $B$  | 2. $C$  | 3. $D$  |
| 4. $E$  | 5. $F$  | 6. $G$  |
| 7. $H$  | 8. $I$  | 9. $J$  |
| 10. $K$ | 11. $W$ | 12. $M$ |
| 13. $N$ | 14. $P$ | 15. $Q$ |



Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.

- |                |               |                 |                 |
|----------------|---------------|-----------------|-----------------|
| 16. $M(-1, 3)$ | 17. $S(2, 0)$ | 18. $R(-3, -2)$ | 19. $P(1, -4)$  |
| 20. $B(5, -1)$ | 21. $D(3, 4)$ | 22. $T(2, 5)$   | 23. $L(-4, -3)$ |

Graph the following geometric figures.

- a square with vertices  $W(-3, 3)$ ,  $X(-3, -1)$ ,  $Z(1, 3)$ , and  $Y(1, -1)$
- a polygon with vertices  $J(4, 2)$ ,  $K(1, -1)$ ,  $L(-2, 2)$ , and  $M(1, 5)$
- a triangle with vertices  $F(2, 4)$ ,  $G(-3, 2)$ , and  $H(-1, -3)$

Graph four points that satisfy each equation.

- |              |                 |                  |                 |
|--------------|-----------------|------------------|-----------------|
| 27. $y = 2x$ | 28. $y = 1 + x$ | 29. $y = 3x - 1$ | 30. $y = 2 - x$ |
|--------------|-----------------|------------------|-----------------|





## Objective

- Use graphing, substitution, and elimination to solve systems of linear equations.



## New Vocabulary

system of equations  
substitution  
elimination

Two or more equations that have common variables are called a **system of equations**. The solution of a system of equations in two variables is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have zero, one, or an infinite number of solutions. There are three methods by which systems of equations can be solved: graphing, elimination, and substitution.



## Example 1 Graphing Linear Equations

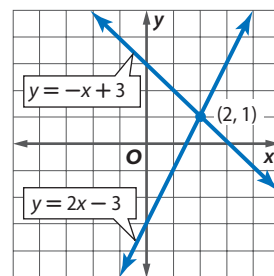
Solve each system of equations by graphing. Then determine whether each system has *no solution*, *one solution*, or *infinitely many solutions*.

a.  $y = -x + 3$   
 $y = 2x - 3$

The graphs appear to intersect at  $(2, 1)$ .  
Check this estimate by replacing  $x$  with 2 and  $y$  with 1 in each equation.

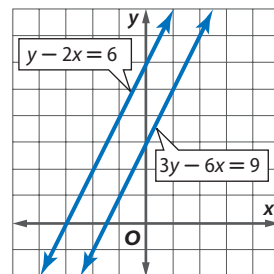
**CHECK**  $y = -x + 3$        $y = 2x - 3$   
 $1 \stackrel{?}{=} -2 + 3$        $1 \stackrel{?}{=} 2(2) - 3$   
 $1 = 1$  ✓       $1 = 1$  ✓

The system has one solution at  $(2, 1)$ .



b.  $y - 2x = 6$   
 $3y - 6x = 9$

The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions of this system of equations. Notice that the lines have the same slope but different  $y$ -intercepts. Equations with the same slope *and* the same  $y$ -intercepts have an infinite number of solutions.



It is difficult to determine the solution of a system when the two graphs intersect at noninteger values. There are algebraic methods by which an exact solution can be found. One such method is **substitution**.

## Example 2 Substitution

Use substitution to solve the system of equations.

$y = -4x$   
 $2y + 3x = 8$

Since  $y = -4x$ , substitute  $-4x$  for  $y$  in the second equation.

$$\begin{aligned} 2y + 3x &= 8 && \text{Second equation} \\ 2(-4x) + 3x &= 8 && y = -4x \\ -8x + 3x &= 8 && \text{Simplify.} \\ -5x &= 8 && \text{Combine like terms.} \\ \frac{-5x}{-5} &= \frac{8}{-5} && \text{Divide each side by } -5. \\ x &= -\frac{8}{5} && \text{Simplify.} \end{aligned}$$

Use  $y = -4x$  to find the value of  $y$ .

$$\begin{aligned} y &= -4x && \text{First equation} \\ &= -4\left(-\frac{8}{5}\right) && x = -\frac{8}{5} \\ &= \frac{32}{5} && \text{Simplify.} \end{aligned}$$

The solution is  $\left(-\frac{8}{5}, \frac{32}{5}\right)$ .



Sometimes adding or subtracting two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination**.



### Example 3 Elimination

Use elimination to solve the system of equations.

$$3x + 5y = 7$$

$$4x + 2y = 0$$

Either  $x$  or  $y$  can be eliminated. In this example, we will eliminate  $x$ .

$$\begin{array}{rcl} 3x + 5y = 7 & \xrightarrow{\text{Multiply by 4.}} & 12x + 20y = 28 \\ 4x + 2y = 0 & \xrightarrow{\text{Multiply by } -3.} & + (-12x) - 6y = 0 \\ \hline & & 14y = 28 & \text{Add the equations.} \\ & & \frac{14y}{14} = \frac{28}{14} & \text{Divide each side by 14.} \\ & & y = 2 & \text{Simplify.} \end{array}$$

Now substitute 2 for  $y$  in either equation to find the value of  $x$ .

$$4x + 2y = 0 \quad \text{Second equation}$$

$$4x + 2(2) = 0 \quad y = 2$$

$$4x + 4 = 0 \quad \text{Simplify.}$$

$$4x + 4 - 4 = 0 - 4 \quad \text{Subtract 4 from each side.}$$

$$4x = -4 \quad \text{Simplify.}$$

$$\frac{4x}{4} = \frac{-4}{4} \quad \text{Divide each side by 4.}$$

$$x = -1 \quad \text{Simplify.}$$

The solution is  $(-1, 2)$ .

#### StudyTip

**Checking Solutions** You can confirm that your solutions are correct by substituting the values into both of the original equations.

## Exercises

Solve by graphing.

1.  $y = -x + 2$

$$y = -\frac{1}{2}x + 1$$

2.  $y = 3x - 3$

$$y = x + 1$$

3.  $y - 2x = 1$

$$2y - 4x = 1$$

Solve by substitution.

4.  $-5x + 3y = 12$

$$x + 2y = 8$$

5.  $x - 4y = 22$

$$2x + 5y = -21$$

6.  $y + 5x = -3$

$$3y - 2x = 8$$

Solve by elimination.

7.  $-3x + y = 7$

$$3x + 2y = 2$$

8.  $3x + 4y = -1$

$$-9x - 4y = 13$$

9.  $-4x + 5y = -11$

$$2x + 3y = 11$$

Name an appropriate method to solve each system of equations. Then solve the system.

10.  $4x - y = 11$

$$2x - 3y = 3$$

11.  $4x + 6y = 3$

$$-10x - 15y = -4$$

12.  $3x - 2y = 6$

$$5x - 5y = 5$$

13.  $3y + x = 3$

$$-2y + 5x = 15$$

14.  $4x - 7y = 8$

$$-2x + 5y = -1$$

15.  $x + 3y = 6$

$$4x - 2y = -32$$



## Objective

- Evaluate square roots and simplify radical expressions.



## New Vocabulary

Product Property  
Quotient Property

A radical expression is an expression that contains a square root. The expression is in simplest form when the following three conditions have been met.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

The **Product Property** states that for two numbers  $a$  and  $b \geq 0$ ,  $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ .

## Example 1 Product Property

Simplify.

a.  $\sqrt{45}$

$$\begin{aligned}\sqrt{45} &= \sqrt{3 \cdot 3 \cdot 5} \\ &= \sqrt{3^2} \cdot \sqrt{5} \\ &= 3\sqrt{5}\end{aligned}$$

Prime factorization of 45

Product Property of Square Roots

Simplify.

b.  $\sqrt{6} \cdot \sqrt{15}$

$$\begin{aligned}\sqrt{6} \cdot \sqrt{15} &= \sqrt{6 \cdot 15} \\ &= \sqrt{3 \cdot 2 \cdot 3 \cdot 5} \\ &= \sqrt{3^2} \cdot \sqrt{10} \\ &= 3\sqrt{10}\end{aligned}$$

Product Property

Prime factorization

Product Property

Simplify.

For radical expressions in which the exponent of the variable inside the radical is *even* and the resulting simplified exponent is *odd*, you must use absolute value to ensure nonnegative results.

## Example 2 Product Property

Simplify  $\sqrt{20x^3y^5z^6}$ .

$$\begin{aligned}\sqrt{20x^3y^5z^6} &= \sqrt{2^2 \cdot 5 \cdot x^3 \cdot y^5 \cdot z^6} \\ &= \sqrt{2^2} \cdot \sqrt{5} \cdot \sqrt{x^3} \cdot \sqrt{y^5} \cdot \sqrt{z^6} \\ &= 2 \cdot \sqrt{5} \cdot x \cdot \sqrt{x} \cdot y^2 \cdot \sqrt{y} \cdot |z^3| \\ &= 2xy^2|z^3|\sqrt{5xy}\end{aligned}$$

Prime factorization

Product Property

Simplify.

Simplify.

The **Quotient Property** states that for any numbers  $a$  and  $b$ , where  $a \geq 0$  and  $b \geq 0$ ,  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ .

## Example 3 Quotient Property

Simplify  $\sqrt{\frac{25}{16}}$ .

$$\begin{aligned}\sqrt{\frac{25}{16}} &= \frac{\sqrt{25}}{\sqrt{16}} \\ &= \frac{5}{4}\end{aligned}$$

Quotient Property

Simplify.



Rationalizing the denominator of a radical expression is a method used to eliminate radicals from the denominator of a fraction. To rationalize the denominator, multiply the expression by a fraction equivalent to 1 such that the resulting denominator is a perfect square.



### Example 4 Rationalize the Denominator

Simplify.

a.  $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad \text{Multiply by } \frac{\sqrt{3}}{\sqrt{3}}.$$

$$= \frac{2\sqrt{3}}{3} \quad \text{Simplify.}$$

b.  $\frac{\sqrt{13y}}{\sqrt{18}}$

$$\frac{\sqrt{13y}}{\sqrt{18}} = \frac{\sqrt{13y}}{\sqrt{2 \cdot 3 \cdot 3}} \quad \text{Prime factorization}$$

$$= \frac{\sqrt{13y}}{3\sqrt{2}} \quad \text{Product Property}$$

$$= \frac{\sqrt{13y}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{Multiply by } \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{26y}}{6} \quad \text{Product Property}$$

#### WatchOut!

**Rationalizing the Denominator** Don't forget to multiply both the numerator and denominator by the radical when you rationalize the denominator.

Sometimes, conjugates are used to simplify radical expressions. Conjugates are binomials of the form  $p\sqrt{q} + r\sqrt{t}$  and  $p\sqrt{q} - r\sqrt{t}$ .



### Example 5 Conjugates

Simplify  $\frac{3}{5 - \sqrt{2}}$ .

$$\frac{3}{5 - \sqrt{2}} = \frac{3}{5 - \sqrt{2}} \cdot \frac{5 + \sqrt{2}}{5 + \sqrt{2}} \quad \frac{5 + \sqrt{2}}{5 + \sqrt{2}} = 1$$

$$= \frac{3(5 + \sqrt{2})}{5^2 - (\sqrt{2})^2} \quad (a - b)(a + b) = a^2 - b^2$$

$$= \frac{15 + 3\sqrt{2}}{25 - 2} \quad \text{Multiply. } (\sqrt{2})^2 = 2$$

$$= \frac{15 + 3\sqrt{2}}{23} \quad \text{Simplify.}$$

## Exercises

Simplify.

- |                                      |                                      |                                      |                                       |
|--------------------------------------|--------------------------------------|--------------------------------------|---------------------------------------|
| 1. $\sqrt{32}$                       | 2. $\sqrt{75}$                       | 3. $\sqrt{50} \cdot \sqrt{10}$       | 4. $\sqrt{12} \cdot \sqrt{20}$        |
| 5. $\sqrt{6} \cdot \sqrt{6}$         | 6. $\sqrt{16} \cdot \sqrt{25}$       | 7. $\sqrt{98x^3y^6}$                 | 8. $\sqrt{56a^2b^4c^5}$               |
| 9. $\sqrt{\frac{81}{49}}$            | 10. $\sqrt{\frac{121}{16}}$          | 11. $\sqrt{\frac{63}{8}}$            | 12. $\sqrt{\frac{288}{147}}$          |
| 13. $\frac{\sqrt{10p^3}}{\sqrt{27}}$ | 14. $\frac{\sqrt{108}}{\sqrt{2q^6}}$ | 15. $\frac{4}{5 - 2\sqrt{3}}$        | 16. $\frac{7\sqrt{3}}{5 - 2\sqrt{6}}$ |
| 17. $\frac{3}{\sqrt{48}}$            | 18. $\frac{\sqrt{24}}{\sqrt{125}}$   | 19. $\frac{3\sqrt{5}}{2 - \sqrt{2}}$ | 20. $\frac{3}{-2 + \sqrt{13}}$        |



State which metric unit you would probably use to measure each item.

- mass of a book
- length of a highway

Complete each sentence.

- 8 in. = ? ft
- 24 fl oz = ? pt
- 4.2 km = ? m
- 0.75 kg = ? mg
- 6 yd = ? ft
- 3.7 kg = ? lb
- 285 g = ? kg
- 1.9 L = ? qt

11. **PROBABILITY** The table shows the results of an experiment in which a number cube was rolled. Find the experimental probability of rolling a 4.

| Outcome | Tally | Frequency |
|---------|-------|-----------|
| 1       |       | 4         |
| 2       |       | 6         |
| 3       |       | 5         |
| 4       |       | 3         |
| 5       |       | 7         |

**CANDY** A bag of candy contains 3 lollipops, 8 peanut butter cups, and 4 chocolate bars. A piece of candy is randomly drawn from the bag. Find each probability.

- $P(\text{peanut butter cup})$
- $P(\text{lollipop or peanut butter cup})$
- $P(\text{not chocolate bar})$
- $P(\text{chocolate bar or lollipop})$

Evaluate each expression if  $x = 2$ ,  $y = -3$ , and  $z = 4$ .

- $6x - z$
- $3yz$
- $\frac{y + 2x}{10z}$
- $6y + xz$
- $\frac{6z}{xy}$
- $7 + |y - 11|$

Solve each equation.

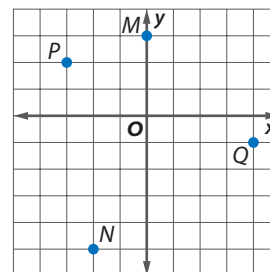
- $9 + s = 21$
- $\frac{4m}{14} = 18$
- $3(20 - b) = 36$
- $\frac{x}{6} = 7$
- $h - 8 = 12$
- $\frac{2}{9}d = 10$
- $37 + w = 5w - 27$
- $\frac{1}{4}(n + 5) = 16$

Solve each inequality.

- $4y - 9 > 1$
- $3r + 7 < r - 8$
- $-3(b - 4) > 33$
- $8 \leq r - 14$
- $-2z + 15 \geq 4$
- $-\frac{2}{5}k - 20 \leq 10$
- $2 - m \leq 6m - 12$
- $\frac{2}{3}n < \frac{3}{9}n - 5$

Write the ordered pair for each point shown.

- $M$
- $N$
- $P$
- $Q$



Graph and label each point on the coordinate plane above.

- $A(-2, 0)$
- $D(-4, -4)$
- $C(1, 3)$
- $F(3, -5)$
- Graph the quadrilateral with vertices  $R(2, 0)$ ,  $S(4, -2)$ ,  $T(4, 3)$ , and  $W(2, 5)$ .
- Graph three points that satisfy the equation  $y = \frac{1}{2}x - 5$ .

Solve each system of equations.

- $2r + m = 11$   
 $6r - 2m = -2$
- $2c + 6d = 14$   
 $-\frac{7}{3} + \frac{1}{3}c = -d$
- $6d + 3f = 12$   
 $2d = 8 - f$
- $2x + 4y = 6$   
 $7x = 4 + 3y$
- $5a - b = 17$   
 $3a + 2b = 5$
- $4x - 5y = 17$   
 $3x + 4y = 5$

Simplify.

- $\sqrt{80}$
- $\sqrt{36} \cdot \sqrt{81}$
- $\sqrt{\frac{5}{81}}$
- $\sqrt{\frac{128}{5}}$
- $\sqrt{\frac{7x^3}{3}}$
- $\sqrt{12x^5y^2}$