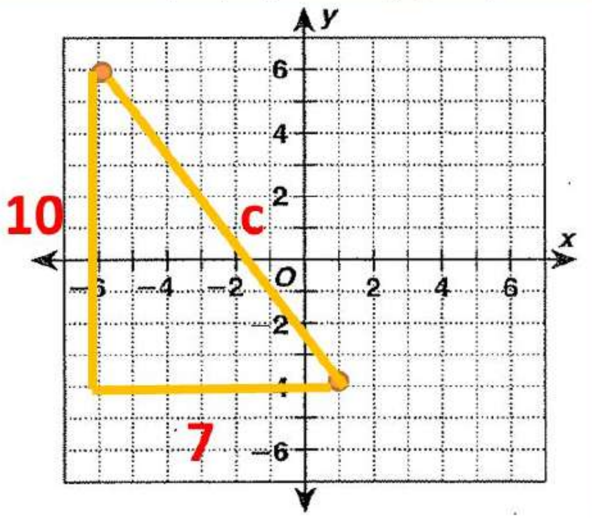
Geometry Basics

Concepts you must know!

How to find the distance between two points (this is the same as finding the length of a segment).

What is the distance between (-6, 6) and (1, -4)?



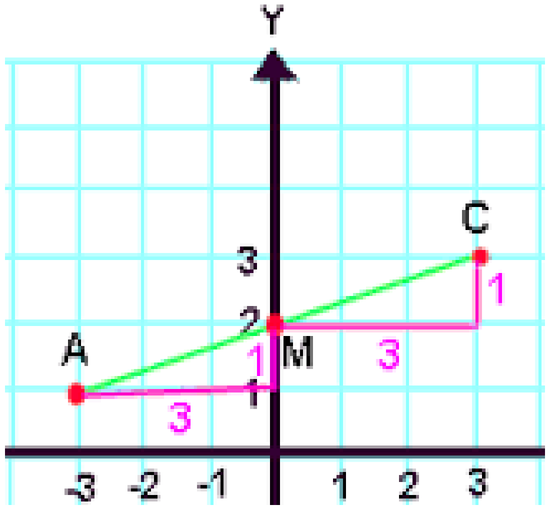
- Use the Pythagorean Theorem to find the missing side.
- $a^2 + b^2 = c^2$
- $10^2 + 7^2 = c^2$
- $100 + 49 = c^2$
- $149 = c^2$

$$\sqrt{149} = c$$

$\sqrt{149}$ Is between 12 and 13.

[Watch this video to review how to find the distance between two points.]

How to find the midpoint of a segment



In order to find the midpoint of segment AC, first find the slope from A to C.

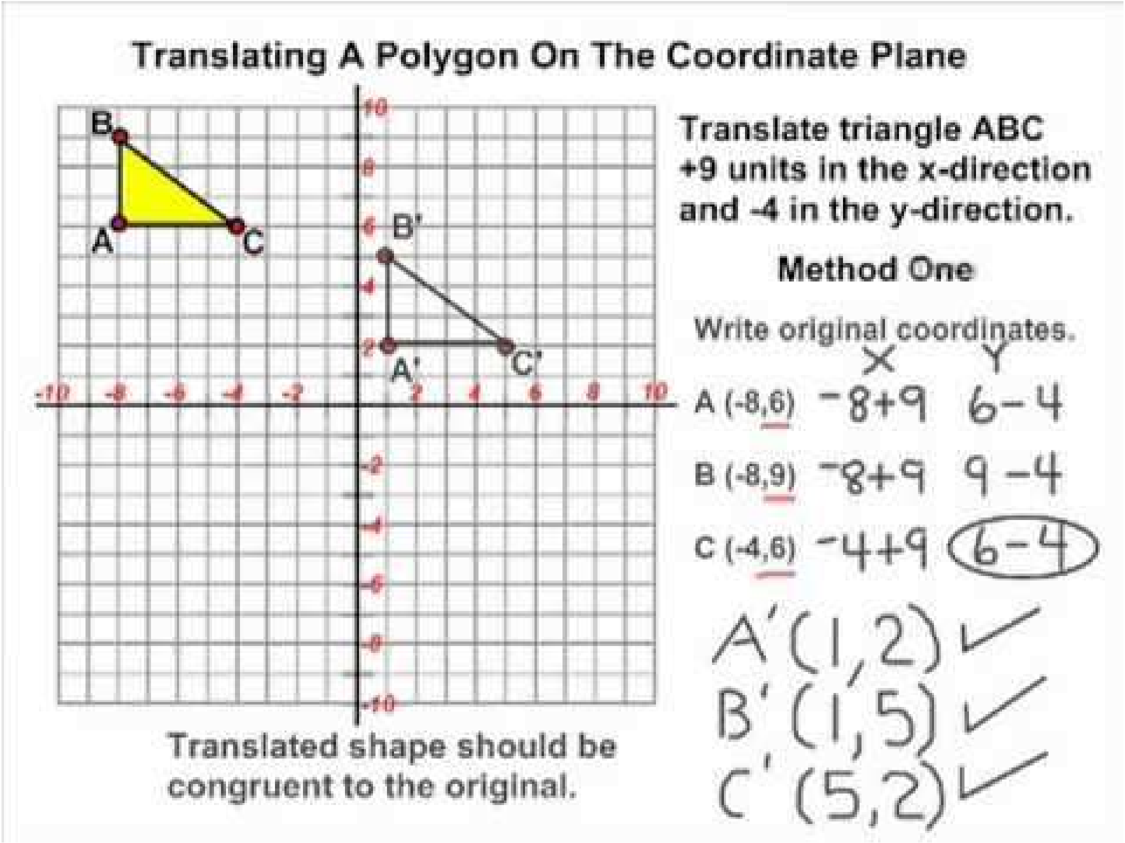
Slope = $\frac{rise}{run} = \frac{up\ 2}{right\ 6}$

Since we want to find the point in the middle, divide the rise and the run by two, to find the new slope that will take you to the midpoint.

Slope to find midpoint= $\frac{up\ 1}{right\ 3}$

The midpoint of $\overline{AC}$ is at (0,2).

Translations



This transformation could be described in the following three ways:

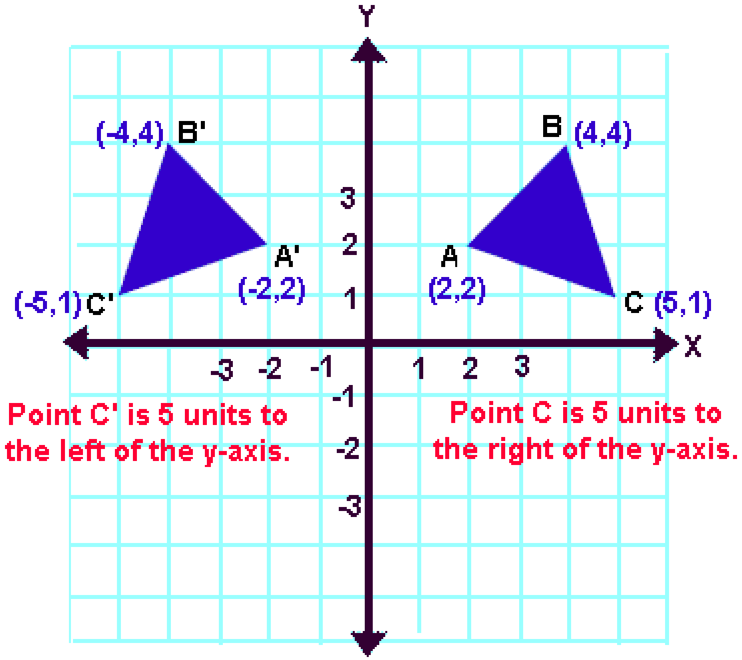
Using the rule (formula): $(x+9, y-4)$

In words: translate the shape 9 units to the right and 4 units down.

Using translation vector $\langle 9, -4 \rangle$

Watch video explaining this translation.

Reflection over the y axis

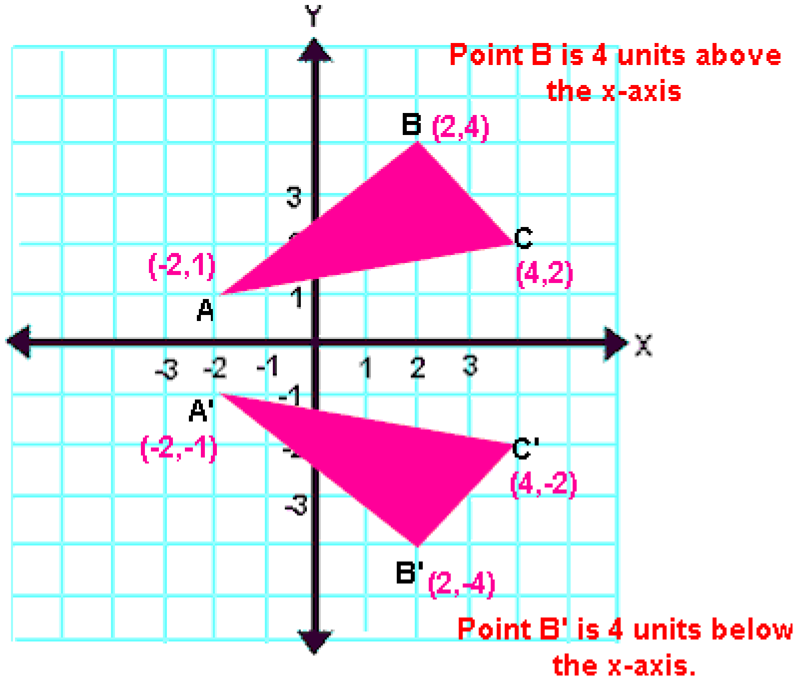


To reflect a point over the y axis, measure the distance from the point to the y axis and find the point on the other side of the y axis that is located that same distance from the y axis.

So point C is 5 units to the right of the y axis, notice that its image, C' is 5 units to the left of the y axis.

[Video explaining reflection over the y axis or over the x axis]

Reflection over the x axis

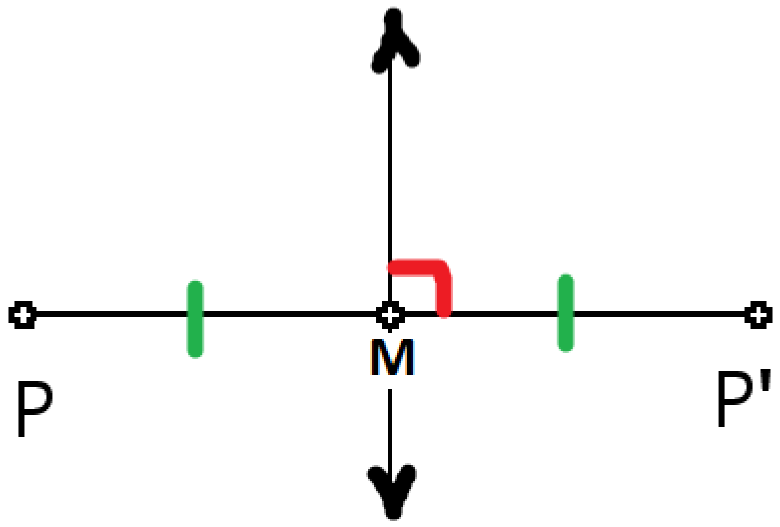


To reflect over the x axis, find the distance from the point to the x axis and count that same distance on the other side of the x axis to find the location of the image.

As you can see point B is 4 units above the x axis, and its image, B' is also 4 units from the x axis but under it.

[Video explaining reflecting over the y axis or over the x axis.]

Perpendicular bisector

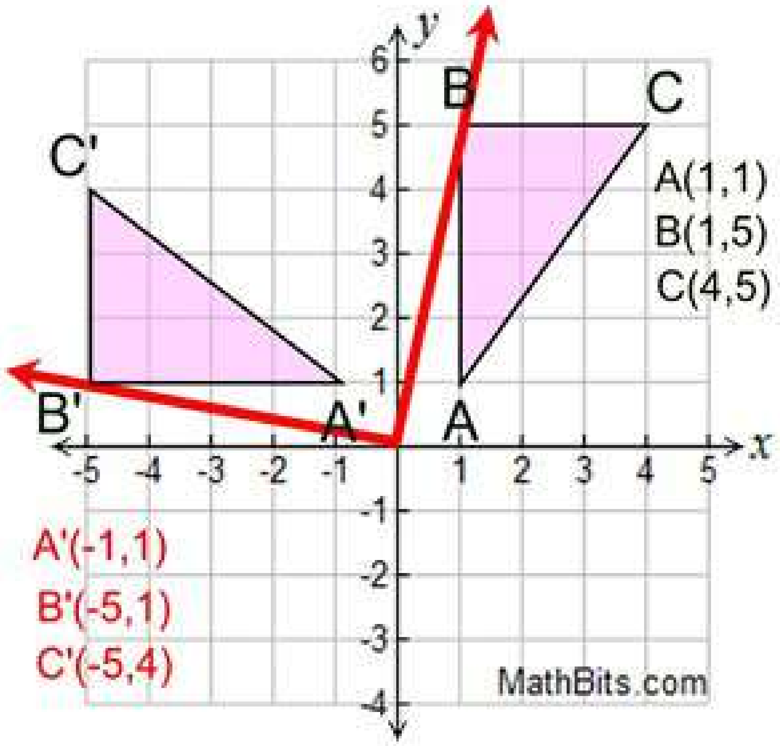


When a point is reflected over a line, the line of reflection is the perpendicular bisector of the segment connecting the preimage (point P) to the image (point P').

So, $\overline{PM} \cong \overline{MP'}$

And right angles are formed at the intersection.

90° CCW Rotation



Apply the formula to each point

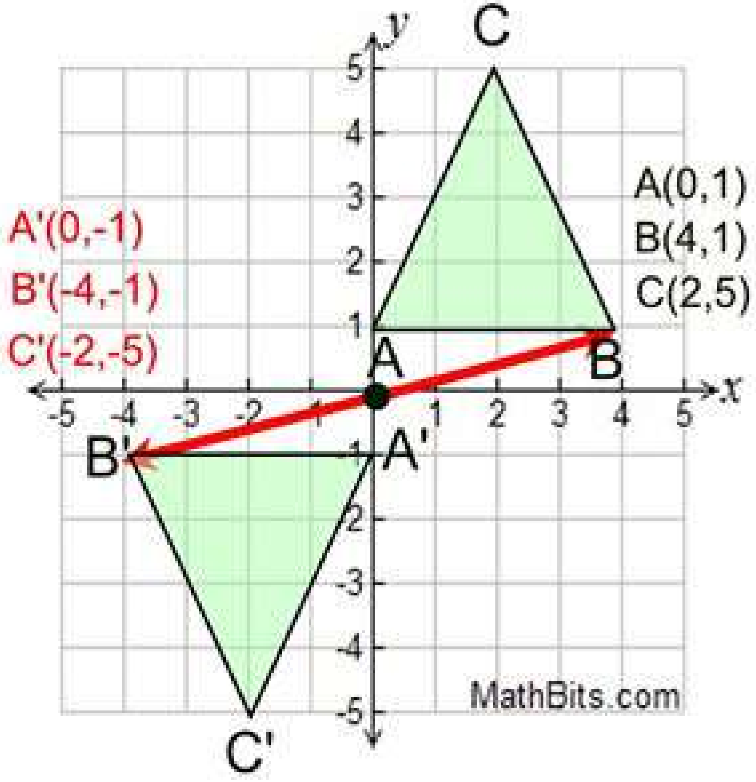
$$(x, y) \rightarrow (-y, x)$$

The formula shows that the x value becomes the new y, and the opposite of the y becomes the new x value.

So for example, the image of point $C(4,5)$ is $C'(-5,4)$

[Video explaining how to rotate 90 CCW]

180° Rotation



Apply the 90° rotation formula to each point **twice**. The x value becomes the new y, and the opposite of the y becomes the new x value.

(x, y) $(2, 5)$

$(-y, x)$ $(-5, 2)$

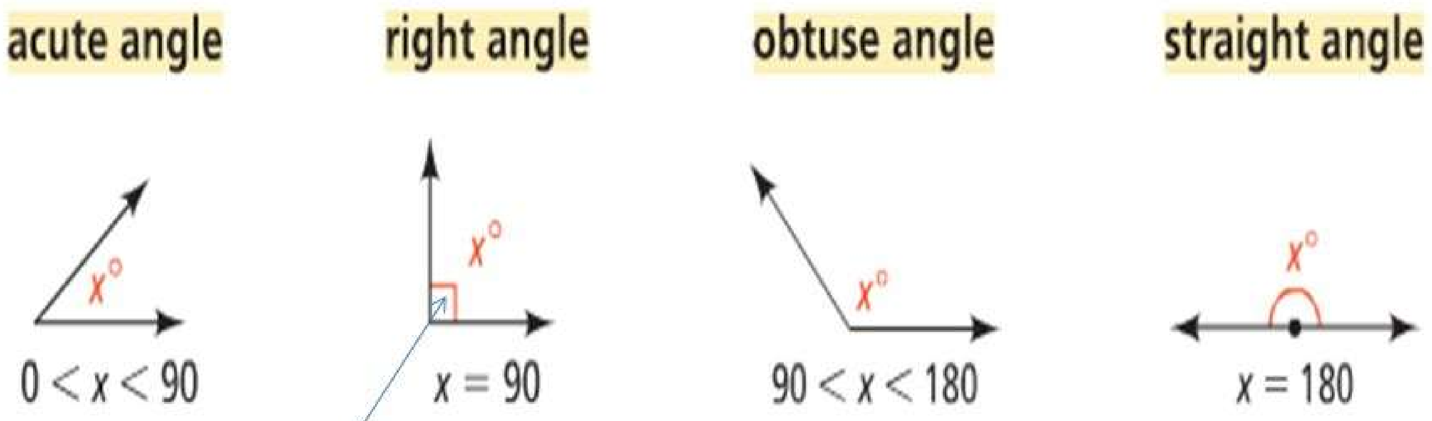
$(-x, -y)$ $(-2, -5)$

So the image of point $C(2,5)$ is $C'(-2,-5)$

Angles

Types of Angles

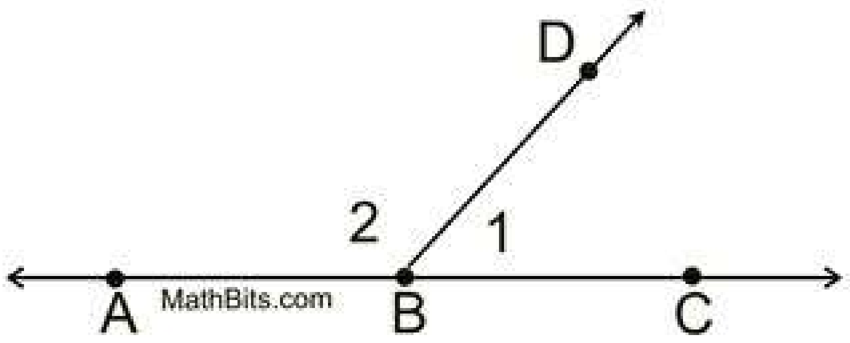
- You can classify angles according to their measures.



This symbol indicates a right angle.

Linear pair angles

Linear pair angles are adjacent angles (next to each other) that together form a straight angle.



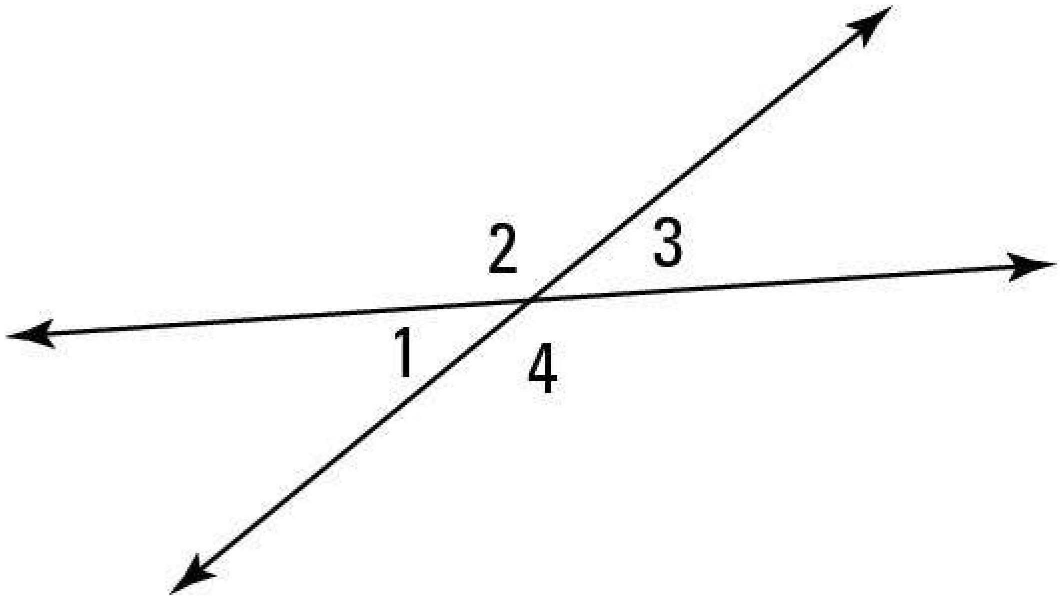
These two angles are supplementary (their sum is 180°).

So:

$$m\measuredangle 2 + m\measuredangle 1 = 180°$$

[Video explaining linear pair and vertical angles]

Vertical angles



When two lines intersect, four angles are formed.

The two angles that are opposite to each other are called vertical angles and they measure the same.

So,

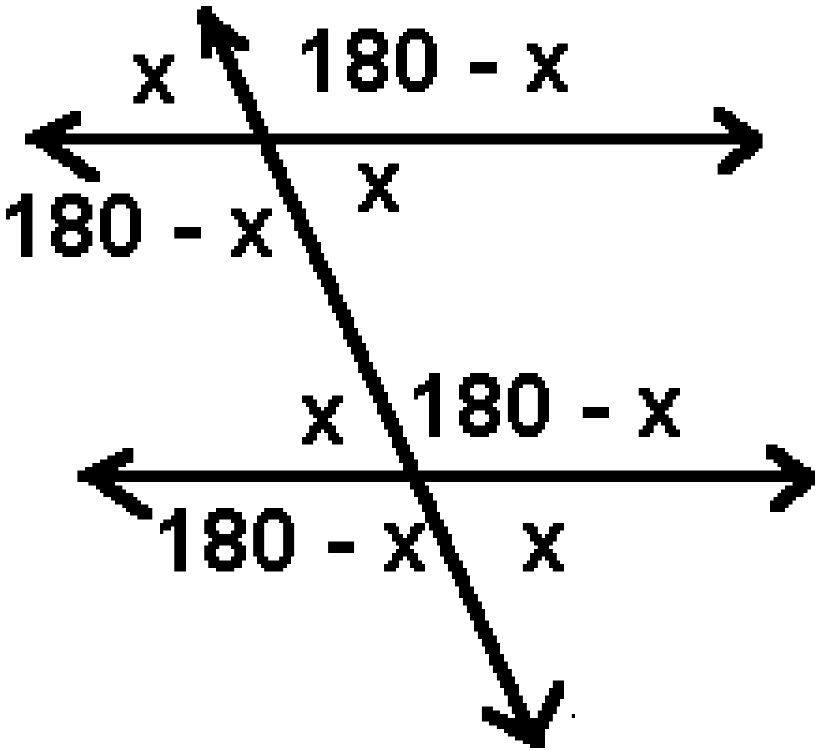
$$\measuredangle 1 = \measuredangle 3$$

And

$$\measuredangle 2 = \measuredangle 4$$

Video explaining linear pair and vertical angles

Angles formed by parallel lines



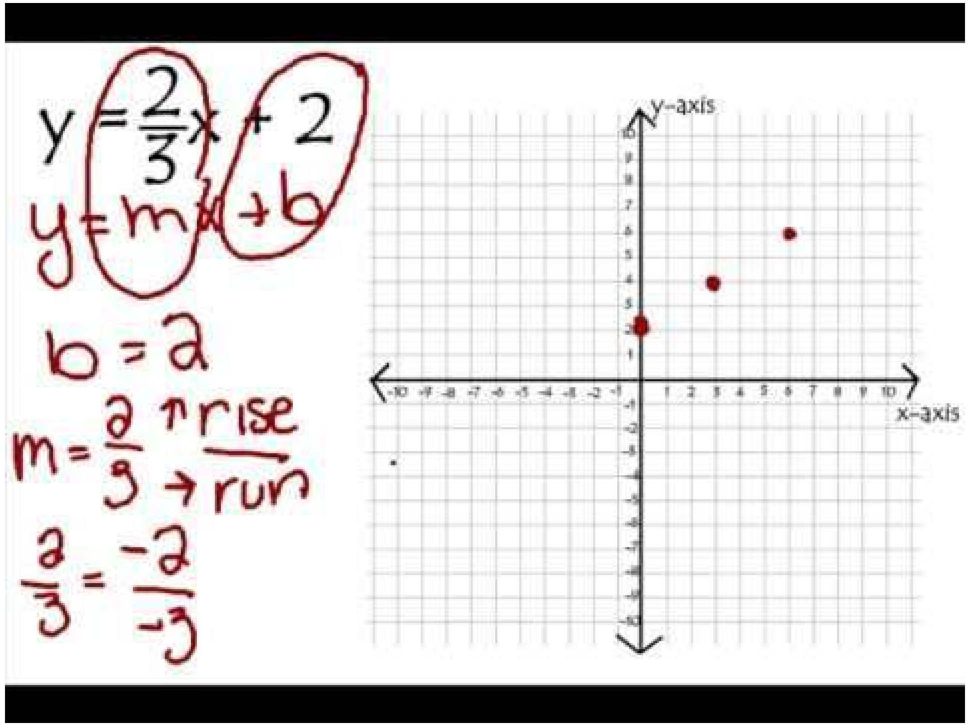
When two parallel lines are intersected by a transversal, the angles formed are equal.

Although these angles have specific names, the most important fact to know is that all the acute angles will be equal and all the obtuse angles will be equal.

In the diagram you can see that all the acute angles measure *x* and all the obtuse angles measure **180-*x***.

[Video explaining angles formed by parallel lines and a transversal]

Graphing lines



To graph an equation of the form $y = mx + b$:

1) graph the y intercept, in this example the y intercept is 2, so put a point at 2 on the y axis

2) Find other points by using the slope, in this case you find them by going up 2, to the right 3 or by going down 2 and to the left 3.

Video explaining how to graph $y = mx + b$

Writing the equation of a line

[Video showing how to find equation of a line]

If you know a point on a line and the slope of the line, you can find the equation of the line by using the point-slope formula.

Point	*Slope*
$(6,-1)$	$m = \frac{1}{2}$

$$y - y_1 = m(x - x_1) \qquad \textit{Point-slope form}$$

$$y - (-1) = \frac{1}{2}(x - 6)$$

$$y + 1 = \frac{1}{2}x - 3$$

$$y + 1 - 1 = \frac{1}{2}x - 3 - 1$$

$$y = \frac{1}{2}x - 4$$

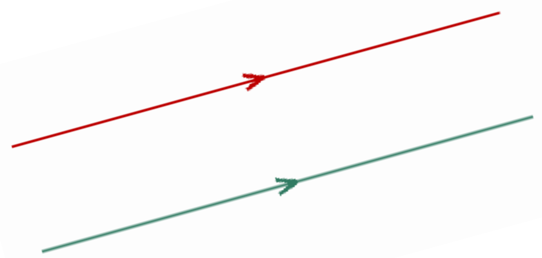
Definitions

Plane: a flat surface

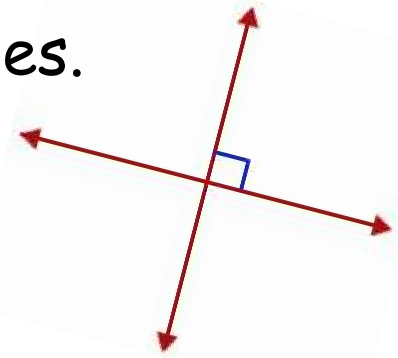
Collinear: points on the same line

Coplanar: points or shapes on the same plane

Parallel lines: two **coplanar** lines that never intersect.
The symbol for parallel is ∥



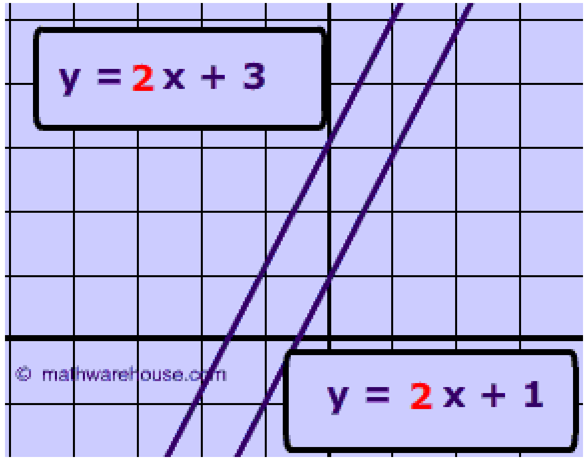
Perpendicular lines: two lines that intersect forming right angles.
The symbol for perpendicular is ⊥



Slopes of parallel, perpendicular lines

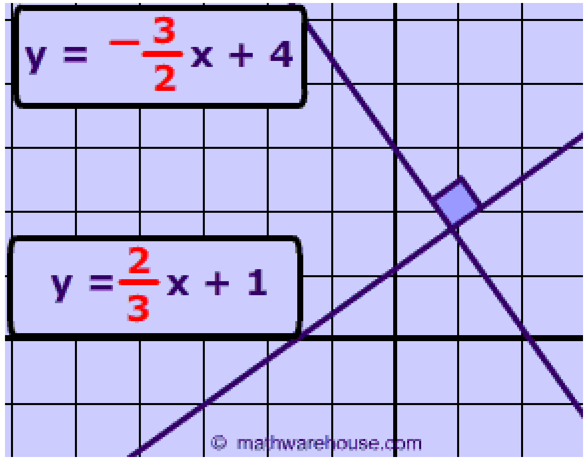
If two lines are parallel, then their slopes are equal.

Here you can see the slope of each line is 2.



If two lines are perpendicular, their slopes are opposite reciprocals.

Here you can see the slope of one line is $\frac{2}{3}$ while the other is $-\frac{3}{2}$



How to find the equation of a line parallel

[Video showing how to find equation of parallel or perpendicular line]

Write the slope-intercept form of an equation of the line that passes through the point (-2, 5) and is **parallel to** the graph of the equation $y = -4x + 2$.

What will the slope of the line be if it's **parallel** to the line $y = -4x + 2$?

-4

We have a point and a slope, which is enough information to find the equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -4(x - (-2))$$

$$y - 5 = -4(x + 2)$$

$$y - 5 = -4x - 8$$

$$y = -4x - 3$$

Parallel lines have the same slope, but different y-intercepts.

How to find the equation of a line perpendicular

Video showing how to find equation of parallel or perpendicular line

Write an equation in slope-intercept form for the line that passes through (3, 2) and is perpendicular to the line described by $y = 3x - 1$.

Step 1 Find the slope of the line.

$y = 3x - 1$ *The slope is 3.*

The perpendicular line has a slope of $-\frac{1}{3}$

Step 2 Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ *Use point-slope form.*

$y - 2 = -\frac{1}{3}(x - 3)$ *Substitute $-\frac{1}{3}$ for m, 3 for x_1, and 2 for y_1.*

Step 3 Write the equation in slope-intercept form.

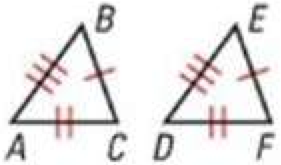
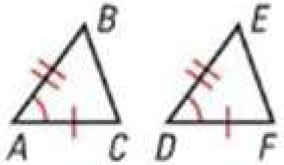
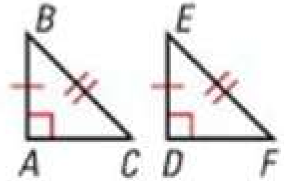
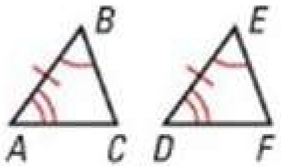
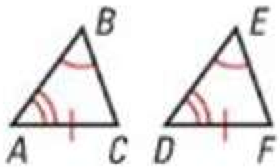
$y - 2 = -\frac{1}{3}(x - 3)$

$y - 2 = -\frac{1}{3}x + 1$ *Distribute $-\frac{1}{3}$ on the right side.*

$y = -\frac{1}{3}x + 3$ *Add 2 to both sides.*

Shortcuts to prove two triangles are congruent.

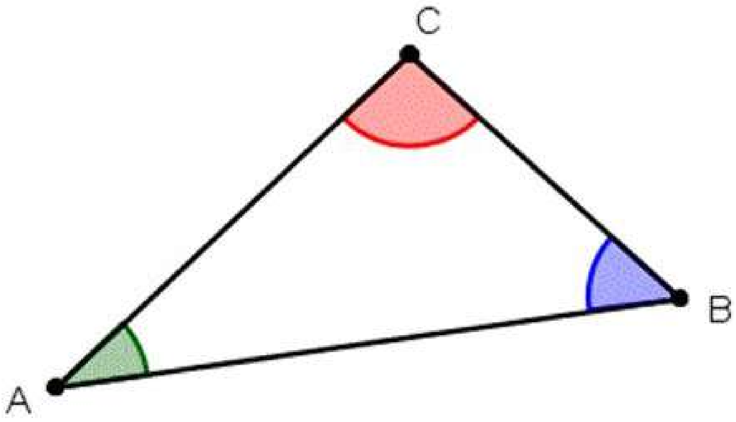
These are the only 5 ways to prove triangles are congruent

SSS	SAS	HL (right △ only)	ASA	AAS
				
All three sides are congruent.	Two sides and the included angle are congruent.	The hypotenuse and one of the legs are congruent.	Two angles and the included side are congruent.	Two angles and a (non-included) side are congruent.

Remember that AAA or SSA (the stinky one) cannot be used to prove that two triangles are congruent.

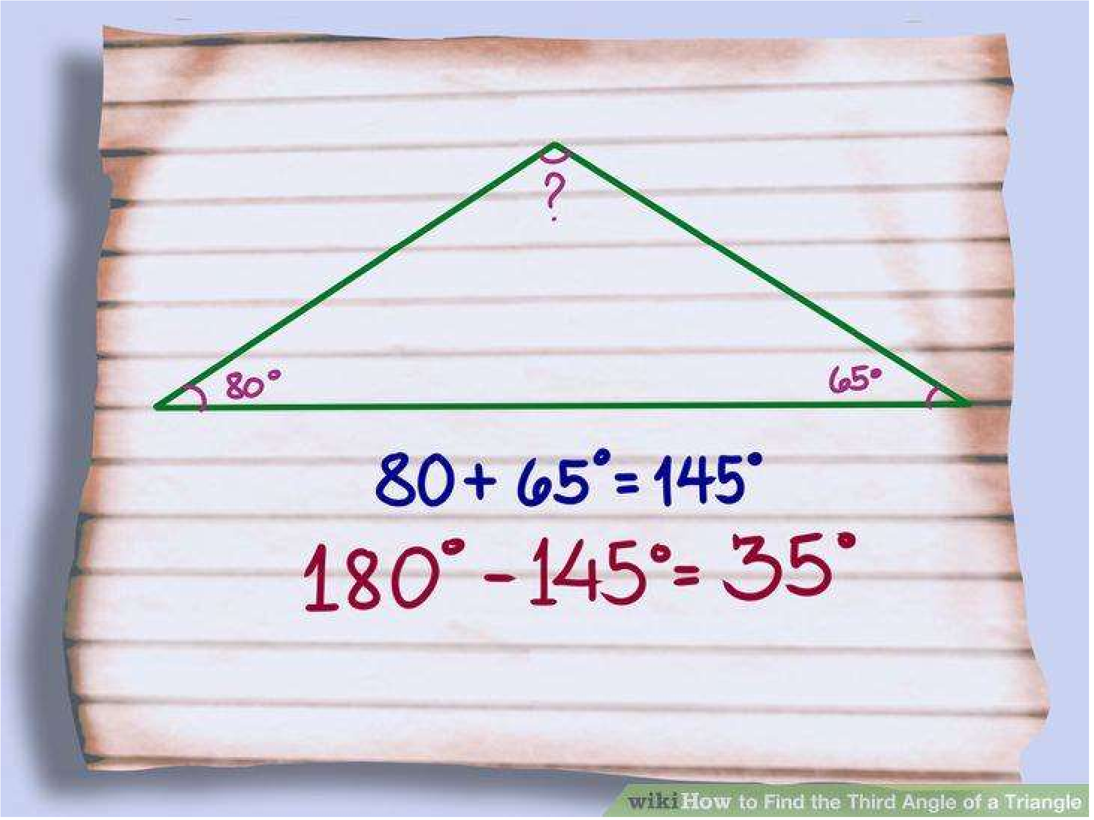
Triangle Angle Sum

The sum of the measures of the angles of a triangle is 180



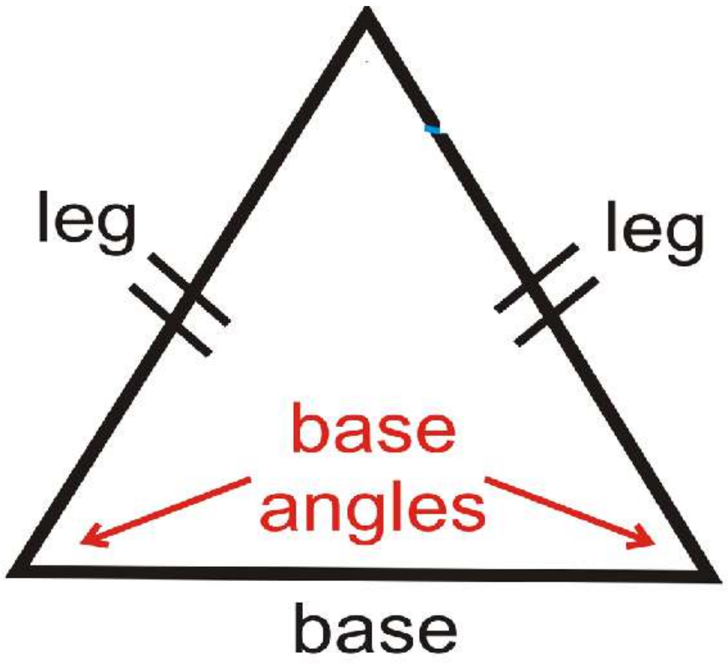
$m\angle A + m\angle B + m\angle C = 180$

Example:



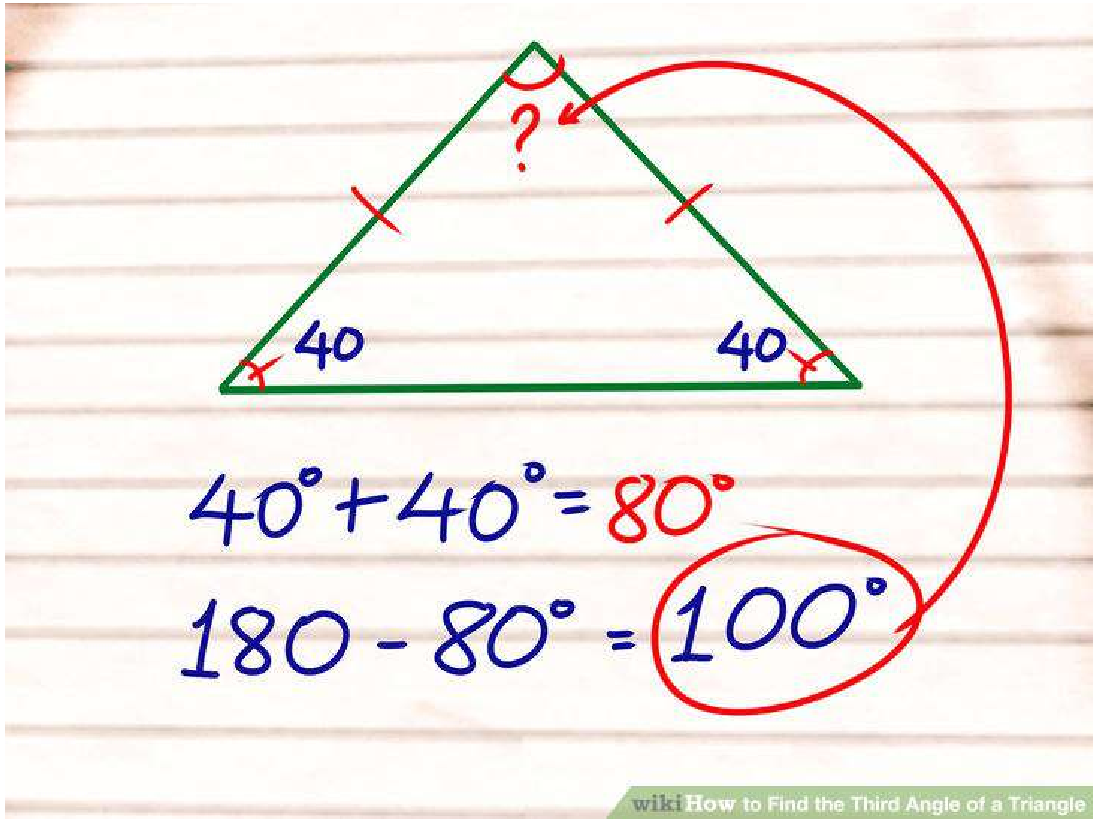
Isosceles Triangles

An isosceles triangles has two equal sides called the legs. The side that is not equal is called the base.



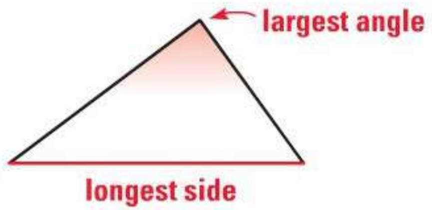
The base angles of an isosceles triangle are equal.

Example:

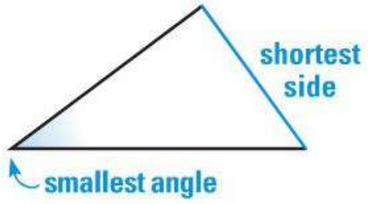


Relationship between sides and angles

- In every triangle, the longest side is opposite the largest angle.



- In every triangle, the shortest side is opposite the smallest angle.

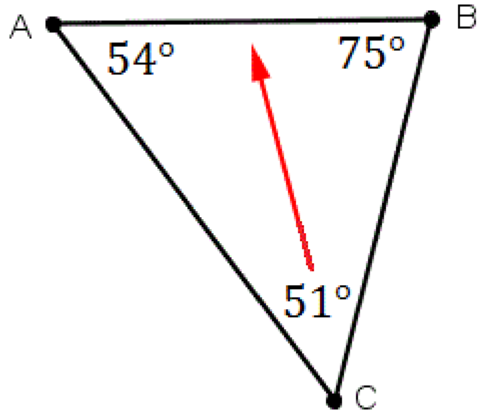


Example:

Which is the smallest side in the triangle below?

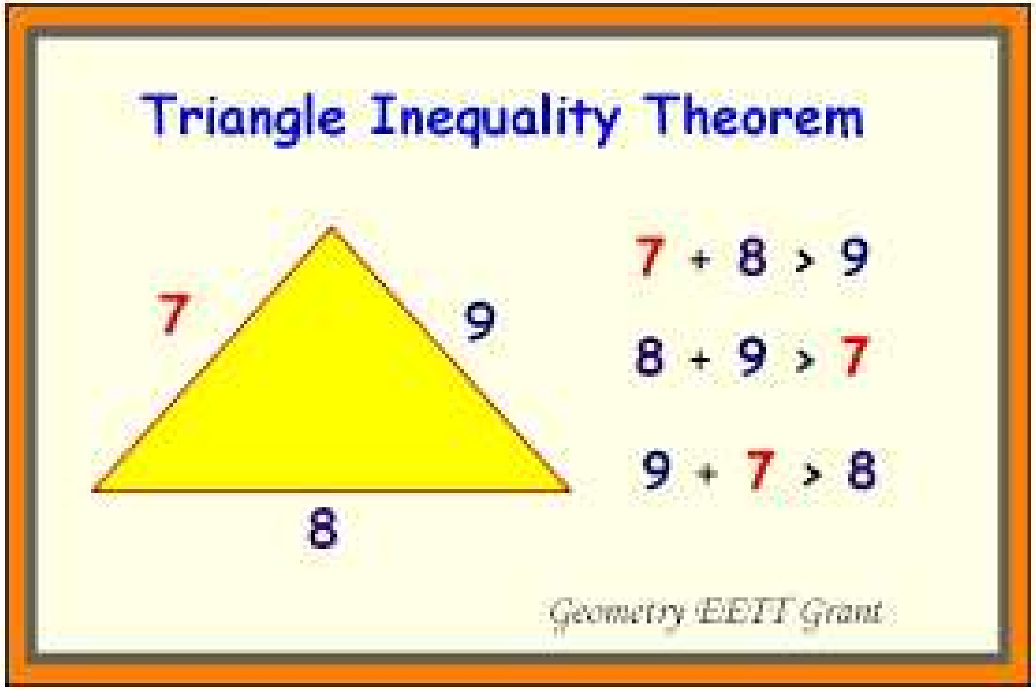
Since angle C is the smallest, the side opposite to it would be the shortest.

So the answer is side AB.

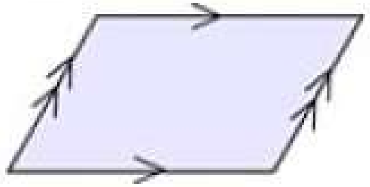
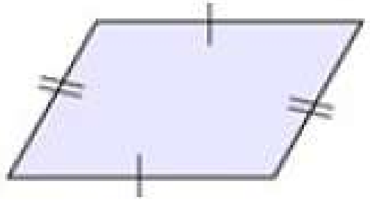


Triangle Inequality

- The sum of any two sides must be greater than third side or else the three sides cannot form a triangle.



Parallelogram Properties

DEFINITION: A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.	
THEOREM: If a quadrilateral is a parallelogram, it has 2 sets of opposite sides congruent.	
THEOREM: If a quadrilateral is a parallelogram, it has 2 sets of opposite angles congruent.	
THEOREM: If a quadrilateral is a parallelogram, it has consecutive angles which are supplementary.	
THEOREM: If a quadrilateral is a parallelogram, it has diagonals which bisect each other.