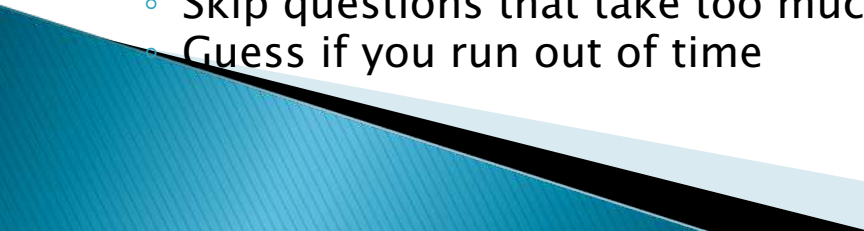


Darlington High School  
ACT Workshop (Math)  
Presented by  
Mr. Goodwin

# Before we start

- ▶ Get a good night's rest. Eat what you always eat for breakfast.
  - ▶ Use the test booklet for scratch paper. You can't bring your own.
  - ▶ Remember your formulas. You will not get them on the test.
  - ▶ Turn word problems into equations or equations into word problems -- whichever is easiest for you!
  - ▶ You can use a calculator.
  - ▶ Don't be afraid! Self-doubt lowers scores.
  - ▶ Hard questions vs. easy questions
    - Must answer all easy questions
    - Go back and guess on hard ones if you run out of time
  - ▶ One minute per question
    - Faster on easy questions
    - Skip questions that take too much time
    - Guess if you run out of time
- 

# 60 questions in 60 minutes

Content	Percent of Test	Number of Questions
Pre-Algebra	23%	14
Elementary Algebra	17%	10
Intermediate Algebra	15%	9
Coordinate Geometry	15%	9
Plane Geometry	23%	14
Trigonometry	7%	4
<b>TOTAL</b>	<b>100%</b>	<b>60</b>

Scores reported:

Total Mathematics Test score based on all 60 questions.

Pre-Algebra/Elementary Algebra Subscore

Intermediate Algebra/Coordinate Geometry Subscore

Plane Geometry/Trigonometry Subscore

Source: The Real ACT Prep Guide. ACT. 2<sup>nd</sup> Ed.

# Pre-Algebra

- ▶ Operations using whole numbers, fractions, and decimals.
  - PEMDAS
  - $2 \times 3 = ?$
  - $4/2 \times 6/2 = ?$
  - $1/5 \times .5 = ?$
  - $4/.5 = ?$
- ▶ Numbers raised to powers and square roots.
  - $2^2$
  - $4^{1/2}$
- ▶ Simple linear equations with one variable.
  - $3x + 7 = 16$ . Solve for X.
- ▶ Simple probability and counting the number of ways something can happen.
  - On a six sided die, what are the chances of rolling a five?

# Pre-Algebra

- ▶ Ratio, proportion, and percent.
  - 3 is what percent of 6? What is 50% of 6?
- ▶ Absolute value.
  - What is the absolute value of  $-3$ ?
  - $|-3| = ?$
- ▶ Ordering numbers from least to greatest.
- ▶ Reading information from charts and graphs.
- ▶ Simple stats
  - Mean: add all terms together and divide by number of terms.
  - Median: order terms from lowest to highest. Eliminate high and low terms till you've reached the middle. If two terms are left, take the mean.
  - Mode: most frequent term.

# Elementary Algebra

- ▶ Substituting the value of a variable in an expression.
  - Add like terms. Separate different terms.
  - $2x+2x+7y=15$ .
  - $Y=2$ . Solve for X.
- ▶ Performing basic operations on polynomials and factoring polynomials.
  - FOIL
  - $(x-3)(x+7) = ?$
  - $x^2+8x+12=0$ . Solve for X.
  - Factor  $x^2-11+30$ .
- ▶ Solving linear inequalities with one variable.
  - $X+7<12$ . What do we know about x?
  - $X+6>19$  and  $x-8<6$ . What do we know about x?

# Intermediate Algebra

## ▶ Quadratic Formula

- When you can't factor a polynomial cleanly. You can always use the quadratic formula
- In  $x^2 + 7x + 15 = 0$ , what is a, b, and c?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Intermediate algebra

*|Powers, Exponents, Roots*

$$x^a \cdot x^b = x^{a+b}$$

$$x^a / x^b = x^{a-b}$$

$$1/x^b = x^{-b}$$

$$(x^a)^b = x^{a \cdot b}$$

$$(xy)^a = x^a \cdot y^a$$

$$(-1)^n = \begin{cases} +1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd.} \end{cases}$$

$$x^0 = 1$$

$$\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

## *Complex Numbers*

A complex number is of the form  $a + bi$  where  $i^2 = -1$ . When multiplying complex numbers, treat  $i$  just like any other variable (letter), except remember to replace powers of  $i$  with  $-1$  or  $1$  as follows (the pattern repeats after the first four):

$$i^0 = 1 \quad i^1 = i \quad i^2 = -1 \quad i^3 = -i$$

$$i^4 = 1 \quad i^5 = i \quad i^6 = -1 \quad i^7 = -i$$

For example, using “FOIL” and  $i^2 = -1$ :  $(1 + 3i)(5 - 2i) = 5 - 2i + 15i - 6i^2 = 11 + 13i$ .

Source: <http://www.erikthered.com/tutor/act-facts-and-formulas.pdf>



# Intermediate Algebra

## ▶ Multiplying Matrices

- Scalar multiplication
- A number times everything inside the matrix.

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

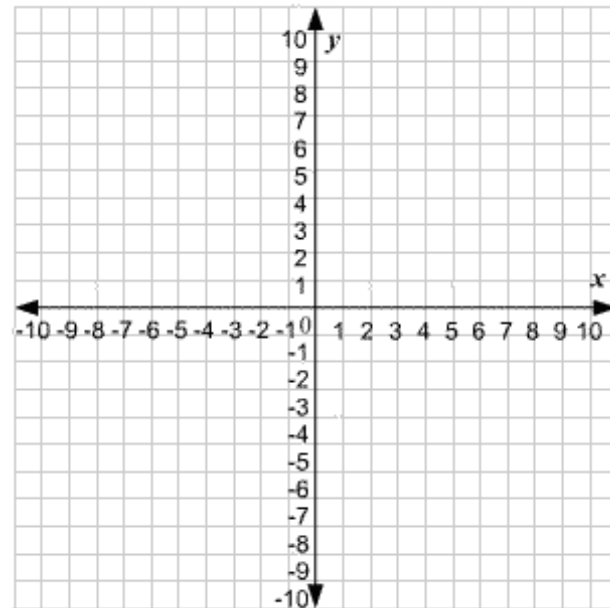
These are the calculations:

$2 \times 4 = 8$	$2 \times 0 = 0$
$2 \times 1 = 2$	$2 \times -9 = -18$

Source: <http://www.mathsisfun.com/algebra/matrix-multiplying.html>

# Coordinate Geometry

- ▶ Graphs of lines, curves, points, polynomials, circles in an  $(x,y)$  plane.
- ▶ Relationship between equations and graphs, slope, parallel and perpendicular lines, distance, midpoints, transformations, and conics.
- ▶ It's coordinate, so draw it on the graph!



# Coordinate Geometry

## ▶ Lines

- A line goes through points A(2, 3) and B(4, 5). You should be able to find the following:

Distance from  $A$  to  $B$ :  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

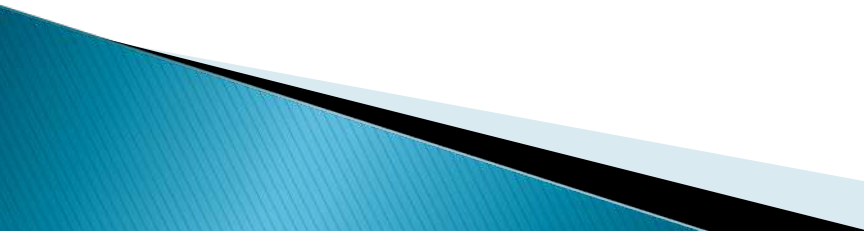
Mid-point of the segment  $\overline{AB}$ :  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope of the line:  $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$

- Parallel lines have the same slope. Perpendicular lines have inverted (Negative inverse) slopes.

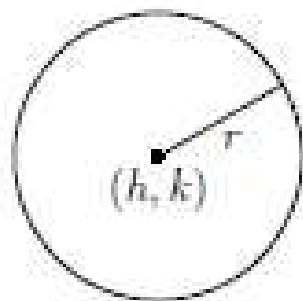
Source: <http://www.erikthered.com/tutor/act-facts-and-formulas.pdf>

# Plane Geometry

- ▶ Relations and properties of shapes (triangles, rectangles, parallelograms, trapezoids, and circles), angles, parallel lines, and perpendicular lines.
  - ▶ What happens when you move or change these shapes?
    - Translations, rotations, reflections
  - ▶ Proofs
    - Justification, logic.
  - ▶ Three-dimensional geometry
  - ▶ Measurements: perimeter, area, and volume.
- 

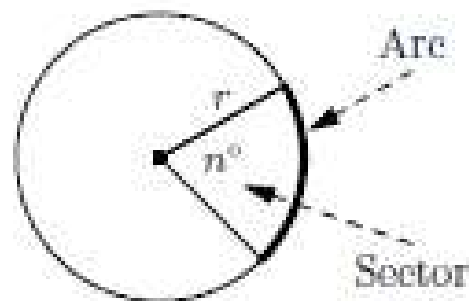
# Plane Geometry

## ► Circles



$$\begin{aligned}\text{Area} &= \pi r^2 \\ \text{Circumference} &= 2\pi r \\ \text{Full circle} &= 360^\circ\end{aligned}$$

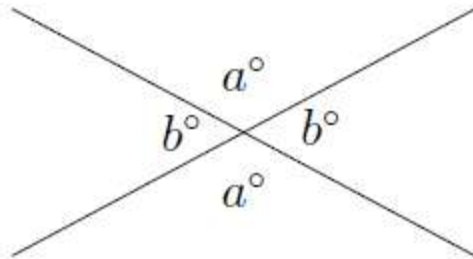
Equation of the circle (above left figure):  $(x - h)^2 + (y - k)^2 = r^2$ .



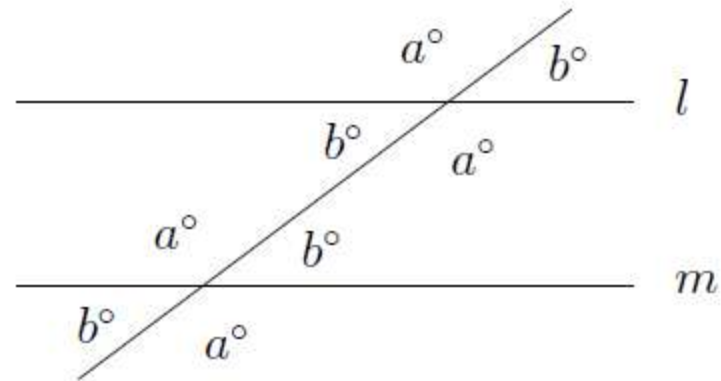
$$\begin{aligned}\text{Length Of Arc} &= (n^\circ/360^\circ) \cdot 2\pi r \\ \text{Area Of Sector} &= (n^\circ/360^\circ) \cdot \pi r^2\end{aligned}$$

# Plane geometry

## ▶ Lines in a plane



Intersecting Lines




Parallel Lines ( $l \parallel m$ )


## ▶ What do we know about $a$ and $b$ in both of these cases?

Source: <http://www.erikthered.com/tutor/act-facts-and-formulas.pdf>

# Plane Geometry

- ▶ Other shape areas and perimeters.

Area of  :  $\frac{1}{2}bh$

Area of  :  $S^2$

Area of  :  $L \times W$

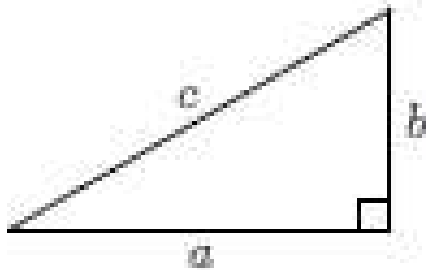
P of , ,  : the sum of all sides

- ▶ If an angle is greater than 90, it is obtuse.
- ▶ If an angle is less than 90, it is acute.
- ▶ If an angle is 90, it is a right angle.
- ▶ TRIANGLE: SUM OF ALL ANGLES = 180
- ▶ SQUARE AND RECTANGLE: SUM OF ALL ANGLES = 360

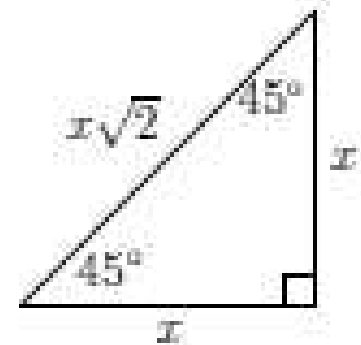
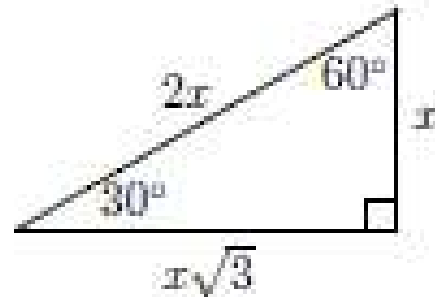
# Plane Geometry

## ▶ Right Triangles

- How do you find the length of a side in a right triangle? Pythagorean Theorem.



$$a^2 + b^2 = c^2$$



Special Right Triangles

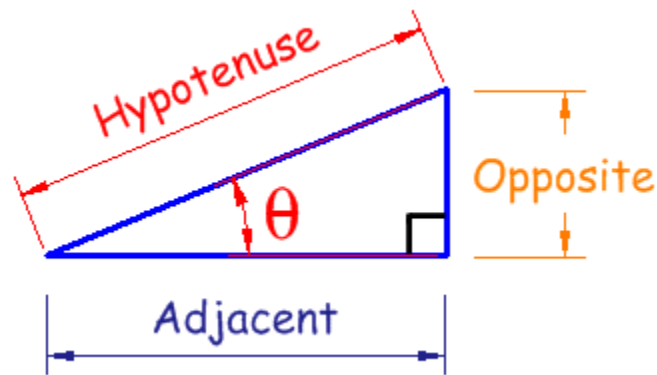
- ▶ Other Triangles: Equilateral (all three sides are equal), Isosceles (two equal sides), and Similar (corresponding angles are equal and sides are in proportion).

Source: <http://www.erikthered.com/tutor/act-facts-and-formulas.pdf>



# Trigonometry

- ▶ Trigonometric functions for right triangles:
  - SINE
  - COSINE
  - TANGENT



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

“SOH”

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

“CAH”

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

“TOA”

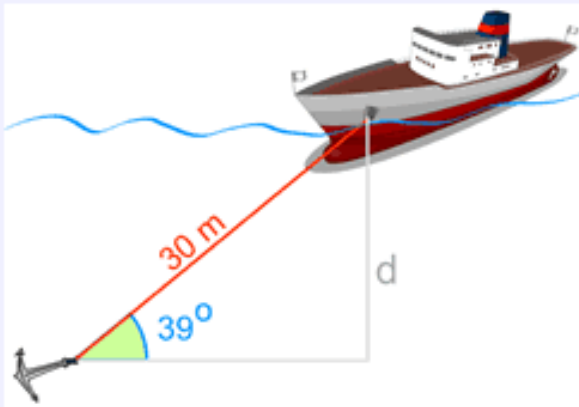
Source: <http://www.erikthered.com/tutor/act-facts-and-formulas.pdf> Source: <http://www.mathsisfun.com>

# Trigonometry

## Why?

Why are these functions important?

- Because they let you work out angles when you know sides
- And they let you work out sides when you know angles



**Example: Use the sine function to find "d"**

We know

- \* The angle the cable makes with the seabed is  $39^\circ$
- \* The cable's length is 30 m.

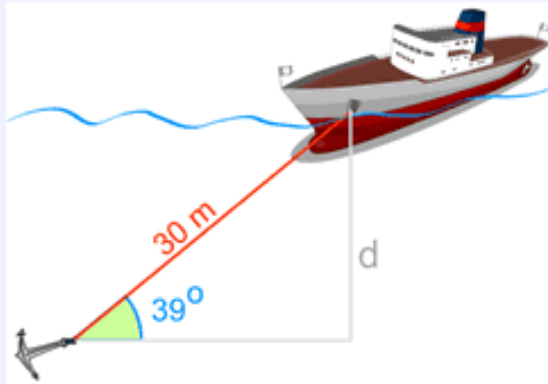
And we want to know "d" (the distance down).

# Trigonometry

## Why?

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- Because they let you work out angles when you know sides
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**Example: Use the sine function to find "d"**

We know

- \* The angle the cable makes with the seabed is  $39^\circ$
- \* The cable's length is 30 m.

And we want to know "d" (the distance down).

Start with:  $\sin 39^\circ = \text{opposite/hypotenuse} = d/30$

Swap Sides:  $d/30 = \sin 39^\circ$

Use a calculator to find  $\sin 39^\circ$ :  $d/30 = 0.6293\dots$

Multiply both sides by 30:  $d = 0.6293\dots \times 30 = \mathbf{18.88}$  to 2 decimal places.

The depth "d" is 18.88 m

# Don't be overwhelmed!

▶ For more Practice, go to:

[www.usatestprep.com](http://www.usatestprep.com)

[www.march2success.com](http://www.march2success.com)

Also go to ACT.org to set up your profile and develop a plan for taking your next ACT test.