

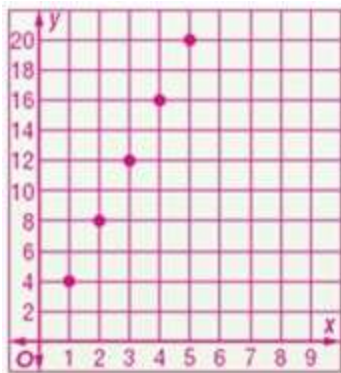
3-6 Proportional and Nonproportional Relationships

1. **GEOMETRY** The table shows the perimeter of a square with sides of a given length.

Side Length (in.)	1	2	3	4	5
Perimeter (in.)	4	8	12	16	20

- Graph the data.
- Write an equation to describe the relationship.
- What conclusion can you make regarding the relationship between the side and the perimeter?

SOLUTION:

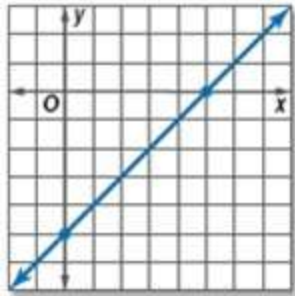


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- The difference in y -values is four times the difference in x -values. This suggests $y = 4x$.
- The perimeter is 4 times the length of the side.

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Write an equation in function notation for each relation.

So the equation for the relation in function notation is $f(x) = x - 5$.



2.

SOLUTION:

Make a table of ordered pairs for several points on the graph.

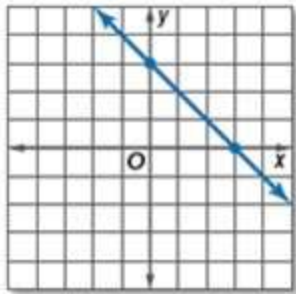
		+1	+1	+1	+1	+1
		↘	↘	↘	↘	↘
x	0	1	2	3	4	5
y	-5	-4	-3	-2	-1	0
		↗	↗	↗	↗	↗
		+1	+1	+1	+1	+1

The difference in y -values is equal to the difference in x -values. This suggests $y = x$, with no coefficient before the x .

Compare the differences of the y and x values.

x	0	1	2	3	4	5
y	-5	-4	-3	-2	-1	0
y - x	-5 - 0	-4 - 1	-3 - 2	-2 - 3	-1 - 4	0 - 5
y - x	-5	-5	-5	-5	-5	-5

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3.

SOLUTION:

Make a table of ordered pairs for several points on the graph.

		+1	+1	+1	+1	+1
		↘	↘	↘	↘	↘
x	-1	0	1	2	3	4
y	4	3	2	1	0	-1
		↖	↖	↖	↖	↖
		-1	-1	-1	-1	-1

The difference in y -values is equal to the negative difference in x -values. This suggests $y = -x$ since the ratio of y to x is -1 .

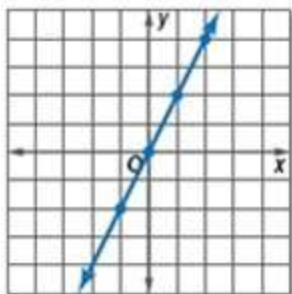
Compare the differences of the y and $-x$ values.

x	-1	0	1	2	3	4
y	4	3	2	1	0	-1
$y - (-x)$	4 - (-1)	3 - 0	2 - 1	1 - 2	0 - 3	-1 - 4
	(-1)	0	(-1)	(-2)	3	(-4)
$y - (-x)$	3	3	3	3	3	3

So the equation for the relation in function notation is $f(x) = -x + 3$.

3-6 Proportional and Nonproportional Relationships

Write an equation in function notation for each relation.



5.

SOLUTION:

Make a table of ordered pairs for several points on the graph.

		+1	+1	+1	
		↖	↖	↖	
x	-1	0	1	2	
y	-2	0	2	4	
		↗	↗	↗	
		+2	+2	+2	

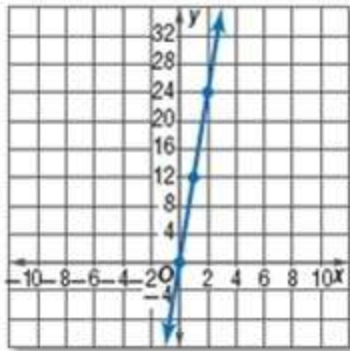
The difference in y -values is double the difference in x -values. This suggests $y = 2x$ since the ratio of y to x is 2.

Compare the differences of the y and x values.

x	-1	0	1	2
y	-2	0	2	4
$y - 2x$	$-2 - 2(-1)$	$0 - 2(0)$	$2 - 2(1)$	$4 - 2(2)$
$y - 2x$	0	0	0	0

Since the difference is 0, there is no constant term. So the equation for the relation in function notation is $f(x) = 2x$.

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6.

SOLUTION:

Make a table of ordered pairs for several points on the graph.

x	0	1	2
y	0	12	24

Diagram illustrating the relationship between the points in the table. Blue arrows show the change in x values: from 0 to 1 (+1) and from 1 to 2 (+1). Blue arrows show the change in y values: from 0 to 12 (+12) and from 12 to 24 (+12).

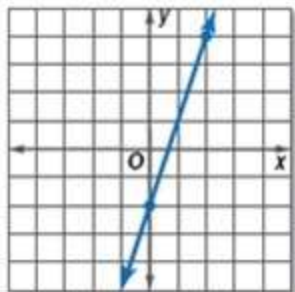
The difference in y -values is twelve times the difference in x -values. This suggests $y = 12x$ since the ratio of y to x is 12.

Find the differences of the y and x values.

x	0	1	2
y	0	12	24
$y - 12x$	$0 - 12(0)$	$12 - 12(1)$	$24 - 2(12)$
$y - 12x$	0	0	0

Since the differences are 0, there is no constant term. So the equation for the relation in function notation is $f(x) = 12x$.

3-6 Proportional and Nonproportional Relationships



7.

SOLUTION:

Make a table of ordered pairs for several points on the graph.

x	0	1	2
y	-2	1	4

Diagram showing the relationship between the table and the graph. Blue arrows indicate the change in x-values: from 0 to 1 (+1) and from 1 to 2 (+1). Blue arrows indicate the change in y-values: from -2 to 1 (+3) and from 1 to 4 (+3).

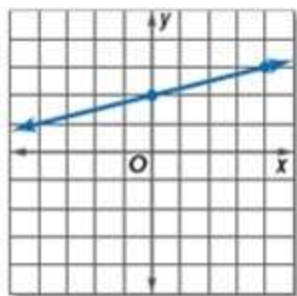
The difference in y -values is three times the difference in x -values. This suggests $y = 3x$ since the ratio of y to x is 3.

Find the differences of the y and x values.

x	0	1	2
y	-2	1	4
$y - 3x$	$-2 - 3(0)$	$1 - 3(1)$	$4 - 3(2)$
$y - 3x$	-2	-2	-2

The differences are -2 . So the equation for the relation in function notation is $f(x) = 3x - 2$.

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8.

SOLUTION:

Make a table of ordered pairs for several points on the graph.

		+4		+4
		↗ ↘		
x	-4	0	4	
y	1	2	3	
		↖ ↗		
		+1		+1

The difference in y -values is one-fourth the difference in x -values. This suggests $y = \frac{1}{4}x$ since the ratio of y to x is $\frac{1}{4}$.

Find the difference in y - and x -values.

x	-4	0	4
y	1	2	3
$y - \frac{1}{4}x$	$1 - \frac{1}{4}(-4)$	$2 - \frac{1}{4}(0)$	$3 - \frac{1}{4}(4)$
$y - \frac{1}{4}x$	2	2	2

The difference is 2. So the equation for the relation in function notation is $f(x) = \frac{1}{4}x + 2$.

3-6 Proportional and Nonproportional Relationships

For each arithmetic sequence, determine the related function. Then determine if the function is proportional or nonproportional. Explain.

9. 0, 3, 6, ...

SOLUTION:

The n th term of an arithmetic sequence with first term a_1 and common difference d is given by $f_n = a_1 + (n - 1)d$, where n is a positive integer.

$$f_n = a_1 + (n - 1)d$$

$$f_n = 0 + (n - 1)3$$

$$f_n = 3n - 3$$

This function is nonproportional.

Check to see if $(0, 0)$ is on the graph.

$$f_n = 3n - 3$$

$$0 \stackrel{?}{=} 3(0) - 3$$

$$0 \stackrel{?}{=} 0 - 3$$

$$0 \neq -3$$

So, the line does not contain $(0, 0)$.

10. -4, 0, 4, ...

SOLUTION:

The n th term of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

$$a_n = a_1 + (n - 1)d$$

$$a_n = -4 + (n - 1)(4)$$

$$a_n = -4 + 4n - 4$$

$$a_n = 4n - 8$$

This function is nonproportional.

Check to see if $(0, 0)$ is on the graph.

$$4(0) - 8 = -8$$

So, the line does not contain $(0, 0)$.

3-6 Proportional and Nonproportional Relationships

11. **BOWLING** Marielle is bowling with her friends. The table shows prices for renting a pair of shoes and bowling. Write an equation to represent the total price y if Marielle buys x games.

SOLUTION:

Find the difference in the x - and y -values.

	+2	+2	+2	
	↘	↘	↘	
Games Bowled	2	4	6	8
Total Price (\$)	7.00	11.50	16.00	20.50
	↗	↗	↗	
	+4.5	+4.5	+4.5	

The difference in the y -values is equal to 4.5 and the difference in the x -values is 2. This suggests that $y = 2.25x$, since the ratio of y and x difference is $\frac{4.5}{2} = 2.25$.

If $x = 2$, $y = 2.25(2)$ or 4.5. But the y -value of $x = 2$ is 7.

x	2	4	6	8
$2.25x$	4.50	9.00	13.50	18.00
y	7.00	11.50	16.00	20.50

The pattern shows that 2.5 should be added to one side of the equation. Thus, the equation is $y = 2.25x + 2.50$.

12. **SNOWFALL** The total snowfall each hour of a winter snowstorm is shown in the table below.

Hour	1	2	3	4
Inches of Snowfall	1.65	3.30	4.95	6.60

- Write an equation to fit the data in the table.
- Describe the relationship between the hour and inches of snowfall.

SOLUTION:

a. $a_n = a_1 + (n-1)d$

$$a_n = 1.65 + (n-1)1.65$$

$$a_n = 1.65 + 1.65n - 1.65$$

$$a_n = 1.65n$$

- As the number of hours increases from hour to hour, the inches of snowfall also increases at a constant rate. This means that the relation is proportional.

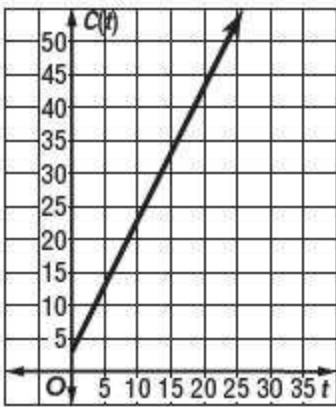
3-6 Proportional and Nonproportional Relationships

13. **FUNDRAISER** The Cougar Pep Squad wants to sell T-shirts in the bookstore for the spring dance. The cost in dollars to order T-shirts in their school colors is represented by the equation $C = 2t + 3$.
- Make a table of values that represents this relationship.
 - Rewrite the equation in function notation.
 - Graph the function.
 - Describe the relationship between the number of T-shirts and the cost.

SOLUTION:

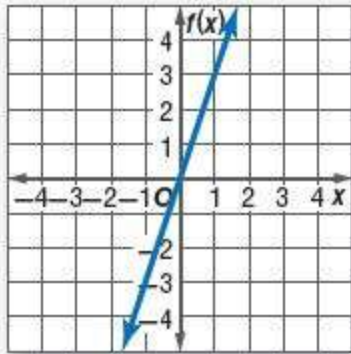
Number of T-Shirts Ordered	5	10	15	20	25
Cost (\$)	13	23	33	43	53

- Students' answers may vary.
- $C(t) = 2t + 3$
- To graph the function, use the table of values from part a.



- This relation is nonproportional. $C(0) = 2(0) + 3 = 3$.

14. **CCSS CRITIQUE** Quentin thinks that $f(x)$ and $g(x)$ are both proportional. Claudia thinks they are not proportional. Is either of them correct? Explain your reasoning.



x	$g(x)$
-2	-7
-1	-4
0	-1
1	2
2	5

SOLUTION:

A proportional equation is of the form $f(x) = kx$. Proportional equation, pass through the origin. The graph represents $f(x) = 3x$ is proportional. However, the table represents $g(x) = 3x - 1$. Since $g(x)$ does not pass through the origin, it is not proportional. Therefore there is one proportional equation and one nonproportional equation. Thus, neither Quentin or Claudia is correct.

3-6 Proportional and Nonproportional Relationships

15. **OPEN ENDED** Create an arithmetic sequence in which the first term is 4. Explain the pattern that you used. Write an equation that represents your sequence.

SOLUTION:

Consider the arithmetic sequence : 4, 7, 10, 13.

To find the common difference, subtract two consecutive numbers in the sequence.

$$\begin{aligned}7 - 4 &= 3 \\10 - 7 &= 3 \\13 - 10 &= 3\end{aligned}$$

The difference between terms is constant, so the sequence is an arithmetic sequence and the common difference is 3.

Compare $3x$ to the series values.

x	1	2	3	4
$y = 3x$	3	6	9	12
Actual	4	7	10	13
Difference	4-3	7-6	10-9	13-12
Difference	1	1	1	1

The difference is on. Thus $a_n = 3n + 1$.

16. **CHALLENGE** Describe how inductive reasoning can be used to write an equation from a pattern.

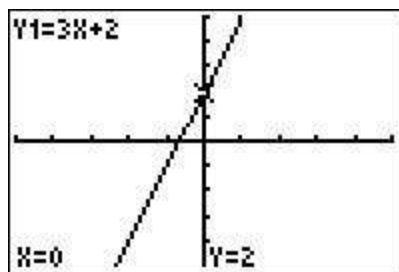
SOLUTION:

Once you recognize a pattern, you can find a general rule that can be written as an algebraic expression.

For example, the sequence 12, 19, 26, 33, ... follows a pattern of adding 7 to each term. Therefore, our common difference is 7. The first term is 12, and $5 + 7 = 12$, so $a_0 = 5$. The algebraic expression is $a_n = 5 + 7n$ where n is the term number.

17. **REASONING** A **counterexample** is a specific case that shows that a statement is false. Provide a counterexample to the following statement. *The related function of an arithmetic sequence is always proportional.* Explain why the counterexample is true.

SOLUTION:



$f(n) = 3n + 2$ is the related function for the arithmetic sequence 5, 8, 11, 14, ..., but it is not proportional. The line through $(1, 5)$ and $(2, 8)$ does not pass through $(0, 0)$. Any linear equation that has a constant term is not proportional. However $f(n) = 3n$ is proportion.

3-6 Proportional and Nonproportional Relationships

18. **WRITING IN MATH** Compare and contrast proportional relationships with nonproportional relationships.

SOLUTION:

In a proportional relationship, the ratio of $\frac{y}{x}$ is the same for each ordered pair in the line for which $x \neq 0$. However, this is not the case in a nonproportional relationship. Both can be represented by a linear equation.

This is an example of a proportional relation. The equation $y = 3x$ is represented in the table. The difference in the x -values is 1, the difference in the y -values, so the ratio of the y to x is 3.

x	1	2	3	4	5
y	3	6	9	12	15

Consider the difference of y and x .

x	1	2	3	4	5
y	3	6	9	12	15
$y - 3x$	0	0	0	0	0

Since the difference is 0, it is a proportion.

This is an example of a nonproportional relation. The equation $y = 3x + 1$ is represented in the table. The difference in the x -values is 1, the difference in the y -values, so the ratio of the y to x is 3.

x	1	2	3	4	5
y	4	7	10	13	16

x	1	2	3	4	5
y	4	7	10	13	16
$y - 3x$	1	1	1	1	1

Since the difference is 1, it is a nonproportional.

3-6 Proportional and Nonproportional Relationships

19. What is the slope of a line that contains the point $(1, -5)$ and has the same y -intercept as $2x - y = 9$?

- A -9
- B -7
- C 2
- D 4

SOLUTION:

First, find the y -intercept of $2x - y = 9$.

$$2x - y = 9 \quad \text{Original equation}$$

$$2(0) - y = 9 \quad \text{Replace } x \text{ with } 0.$$

$$-y = 9 \quad \text{Simplify.}$$

$$\frac{-y}{-1} = \frac{-9}{-1} \quad \text{Divide each side by } -1.$$

$$y = -9 \quad \text{Simplify.}$$

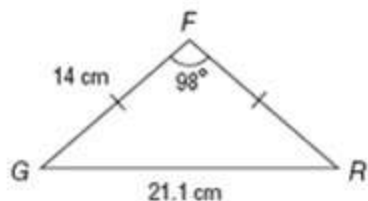
If the equation has the same intercept, then it has the point $(0, -9)$. Use points $(0, -9)$ and $(1, -5)$ to calculate the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - (-9)}{1 - 0} \\ &= \frac{-5 + 9}{1} \\ &= 4 \end{aligned}$$

The slope is 4, so the correct choice is D.

3-6 Proportional and Nonproportional Relationships

20. **SHORT RESPONSE** $\triangle FGR$ is an isosceles triangle. What is the measure of $\angle G$?



SOLUTION:

If $\triangle FGR$ is an isosceles triangle, then $\angle G$ is equal to $\angle R$. The three angles in a triangle add up to 180° .

$$98 + x + x = 180$$

$$98 + 2x = 180$$

$$98 - 98 + 2x = 180 - 98$$

$$2x = 82$$

$$\frac{2x}{2} = \frac{82}{2}$$

$$x = 41$$

So $\angle G = 41^\circ$.

21. Luis deposits \$25 each week into a savings account from his part-time job. If he has \$350 in savings now, how much will he have in 12 weeks?
- F** \$600
G \$625
H \$650
J \$675

SOLUTION:

Let w represent the number of weeks Luis deposits money.

$$25w + 350 = 25(12) + 350$$

$$= 300 + 350$$

$$= 650$$

Luis will have \$650 after 12 weeks, so the correct choice is H.

3-6 Proportional and Nonproportional Relationships

22. **GEOMETRY** Omar and Mackenzie want to build a pulley system by attaching one end of a rope to their 8-foot-tall tree house and anchoring the other end to the ground 28 feet away from the base of the tree house. How long, to the nearest foot, does the piece of rope need to be?
- A** 26 ft
B 27 ft
C 28 ft
D 29 ft

SOLUTION:

This creates a right triangle, so use the Pythagorean Theorem.

$$\begin{aligned}a^2 + b^2 &= c^2 \\8^2 + 28^2 &= c^2 \\64 + 784 &= c^2 \\848 &= c^2 \\\sqrt{848} &= \sqrt{c^2} \\29.12 &= c\end{aligned}$$

The piece of rope needs to be about 29 feet long, so the correct choice is D.

Find the next three terms in each sequence.

23. 3, 13, 23, 33, ...

SOLUTION:

Find the common difference by subtracting two consecutive terms.

$$13 - 3 = 10$$

The common difference is 10. Add 10 to the last term of the sequence until three terms are found; the next three terms are 43, 53, and 63.

24. -2, -1.4, -0.8, -0.2, ...

SOLUTION:

Find the common difference by subtracting two consecutive terms.

$$-1.4 - (-2) = 0.6$$

The common difference is 0.6.

Adding 0.6 to the last term of the sequence until three terms are found; the next three terms are 0.4, 1, and 1.6.

3-6 Proportional and Nonproportional Relationships

25. $\frac{3}{4}, \frac{7}{8}, 1, \frac{9}{8}, \dots$

SOLUTION:

Find the common difference by subtracting two consecutive terms.

$$1 - \frac{7}{8} = \frac{1}{8}$$

The common difference is $\frac{1}{8}$.

Add $\frac{1}{8}$ to the last term of the sequence until three terms are found; the next three terms are $\frac{5}{4}, \frac{11}{8}$, and $\frac{3}{2}$.

Suppose y varies directly as x . Write a direct variation equation that relates x and y . Then solve.

26. If $y = 45$ when $x = 9$, find y when $x = 7$.

SOLUTION:

$$y = kx$$

$$45 = k(9)$$

$$\frac{45}{9} = \frac{k(9)}{9}$$

$$5 = k$$

So, the direct variation equation is $y = 5x$. Substitute 7 for x and find y .

$$y = 5x$$

$$y = 5(7)$$

$$y = 35$$

So, $y = 35$ when $x = 7$.

3-6 Proportional and Nonproportional Relationships

27. If $y = -7$ when $x = -1$, find x when $y = -84$.

SOLUTION:

$$\begin{aligned}y &= kx \\-7 &= k(-1) \\ \frac{-7}{-1} &= \frac{k(-1)}{-1} \\ 7 &= k\end{aligned}$$

So, the direct variation equation is $y = 7x$. Substitute -84 for y and find x .

$$\begin{aligned}y &= 7x \\-84 &= 7x \\ \frac{-84}{7} &= \frac{7x}{7} \\ -12 &= x\end{aligned}$$

So, $x = -12$ when $y = -84$.

28. **GENETICS** About $\frac{2}{25}$ of the male population in the world cannot distinguish red from green. If there are 14 boys in the ninth grade who cannot distinguish red from green, about how many ninth-grade boys are there in all? Write and solve an equation to find the answer.

SOLUTION:

Let b represent the number of boys.

$$\begin{aligned}14 &= \frac{2}{25}b \\ \frac{25}{2}(14) &= \frac{25}{2}\left(\frac{2}{25}b\right) \\ 175 &= b\end{aligned}$$

So, there are 175 boys.

3-6 Proportional and Nonproportional Relationships

29. **GEOMETRY** The volume V of a cone equals one third times the product of π , the square of the radius r of the base, and the height h .

a. Write the formula for the volume of a cone.

b. Find the volume of a cone if r is 10 centimeters and h is 30 centimeters.

SOLUTION:

a. Rewrite the verbal sentence so it is easier to translate. *The volume equals one-third times the product of π , the square of the radius r of the base, and the height.*

$$\begin{array}{l} V \quad \text{equals} \quad \frac{1}{3} \quad \text{times} \quad \pi \quad \text{times} \quad r \quad \text{squared} \quad \text{times} \quad h \\ V \quad = \quad \frac{1}{3} \quad \cdot \quad \pi \quad \cdot \quad r \quad ^2 \quad \cdot \quad h \end{array}$$

The equation is $V = \frac{1}{3}\pi r^2 h$

$$\begin{aligned} \text{b. } V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(10)^2(30) \\ &= \frac{1}{3}\pi(100)(30) \\ &= \frac{1}{3}\pi(3000) \\ &= 1000\pi \\ &\approx 3141.59265 \end{aligned}$$

So the volume is about 3142 cm^3 .

Solve each equation for y .

30. $3x = y + 7$

SOLUTION:

$$3x = y + 7 \quad \text{Original equation}$$

$$3x - 7 = y + 7 - 7 \quad \text{Subtract 7 from each side.}$$

$$3x - 7 = y \quad \text{Simplify.}$$

31. $2y = 6x - 10$

SOLUTION:

$$2y = 6x - 10 \quad \text{Original equation}$$

$$\frac{2y}{2} = \frac{6x - 10}{2} \quad \text{Divide each side by 2.}$$

$$y = \frac{6x}{2} - \frac{10}{2} \quad \text{Rewrite fractions.}$$

$$y = 3x - 5 \quad \text{Simplify.}$$

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32. $9y + 2x = 12$

SOLUTION:

$$9y + 2x = 12 \quad \text{Original equation}$$

$$9y + 2x - 2x = 12 - 2x \quad \text{Subtract } x \text{ from each side.}$$

$$9y = -2x + 12 \quad \text{Simplify.}$$

$$\frac{9y}{9} = \frac{-2x + 12}{9} \quad \text{Divide each side by 9.}$$

$$y = -\frac{2}{9}x + \frac{12}{9} \quad \text{Simplify.}$$

$$y = -\frac{2}{9}x + \frac{4}{3} \quad \text{Simplify.}$$

Graph each equation.

33. $y = x - 8$

SOLUTION:

To graph the equation, find the x - and y -intercepts. Plot these two points. Then draw a line through them.

To find the x -intercept, let $y = 0$.

$$y = x - 8$$

$$0 = x - 8$$

$$0 + 8 = x - 8 + 8$$

$$8 = x$$

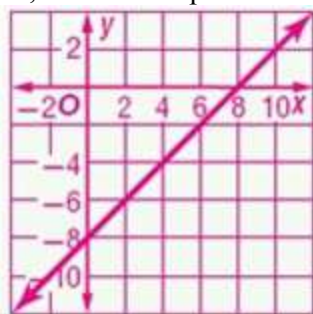
To find the y -intercept, let $x = 0$.

$$y = x - 8$$

$$y = 0 - 8$$

$$y = -8$$

So, the x -intercept is 8 and the y -intercept is -8.



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34. $x - y = -4$

SOLUTION:

To graph the equation, find the x - and y -intercepts. Plot these two points. Then draw a line through them.

To find the x -intercept, let $y = 0$.

$$x - y = -4$$

$$x - 0 = -4$$

$$x = -4$$

To find the y -intercept, let $x = 0$.

$$x - y = -4$$

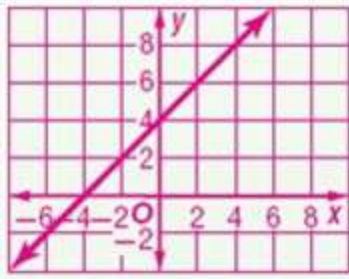
$$0 - y = -4$$

$$-y = -4$$

$$\frac{-y}{-1} = \frac{-4}{-1}$$

$$y = 4$$

So, the x -intercept is -4 , and the y -intercept is 4 .



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35. $2x + 4y = 8$

SOLUTION:

To graph the equation, find the x - and y -intercepts. Plot these two points. Then draw a line through them.

To find the x -intercept, let $y = 0$.

$$2x + 4y = 8$$

$$2x + 4(0) = 8$$

$$2x + 0 = 8$$

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

To find the y -intercept, let $x = 0$.

$$2x + 4y = 8$$

$$2(0) + 4y = 8$$

$$0 + 4y = 8$$

$$4y = 8$$

$$\frac{4y}{4} = \frac{8}{4}$$

$$y = 2$$

So, the x -intercept is 4, and the y -intercept is 2.

