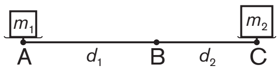
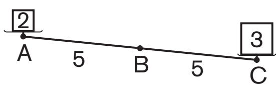
Mass Point Geometry Stretch

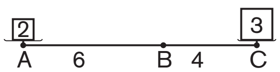
Center of Mass



Consider a seesaw, with a fulcrum at B, that has objects at A and C. As shown, the object at A has a mass of m_1, and its distance from B is d_1. The object at C has mass m_2, and its distance from B is d_2. These examples show how the position of the fulcrum determines whether the seesaw is balanced. The mass at B is $m_1 + m_2$, the sum of the masses at A and C.

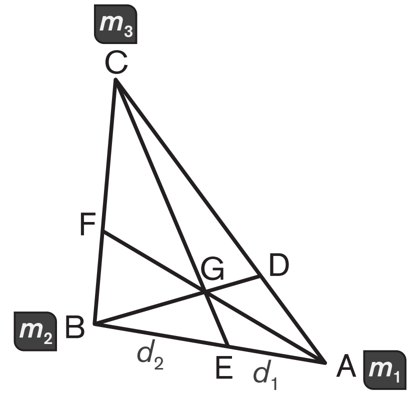


In this first example, B is positioned so that $d_1 = d_2 = 5$. Notice that $m_1 \times d_1 = 2 \times 5 = 10$ and $m_2 \times d_2 = 3 \times 5 = 15$. Although the objects at A and C are equidistant from B, the object at C is lower than the object at A because $m_1 \times d_1 < m_2 \times d_2$.



In this example, B is positioned so that $d_1 = 6$ and $d_2 = 4$. Here, $m_1 \times d_1 = 2 \times 6 = 12$ and $m_2 \times d_2 = 3 \times 4 = 12$. This time, the seesaw is balanced because $m_1 \times d_1 = m_2 \times d_2$. In this case, the position of B is known as the *center of mass*.

A *cevian* is a line segment that joins a vertex of a triangle with a point on the opposite side. Mass point geometry is a technique used to solve problems involving triangles and intersecting cevians by applying center of mass principles. Because triangle ABC, shown here, has cevians AF, BD and CE that intersect at point G, we can apply the center of mass principles presented. For example, side AB is balanced on point E when $m_1 \times d_1 = m_2 \times d_2$.



A *mass point*, denoted *m*P, consists of point P and its associated mass, *m*. Assume point G is the center of mass on which the entire triangle balances. Then the mass at G is the sum of the masses at the endpoints for each cevian and *m*G = *m*A + *m*F = *m*B + *m*D = *m*C + *m*E.

Suppose BF:CF = 3:4 and AD:CD = 2:5, and we are asked to determine the ratios AE:BE, AG:FG and BG:DG.

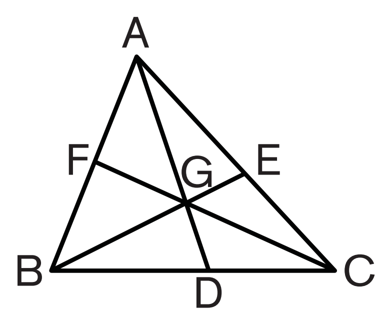
Start by finding *m*B and *m*C for side BC, which is balanced on point F. We know $m_2 \times 3 = m_3 \times 4$. We can let $m_2 = 4$ and $m_3 = 3$, so 4B + 3C = (4 + 3)F = 7F.

Next, find *m*A for side AC, which is balanced on point D. We know $m_1 \times 2 = m_3 \times 5$. Since $m_3 = 3$, it follows that $m_1 \times 2 = 3 \times 5$ and $m_1 = 15/2$. Rather than having mass point (15/2)A, we can multiply 4B, (15/2)A, 3C and 7F by 2 to get the following mass points: 8B, 15A, 6C and 14F. Now the mass at each point is of integer value.

Now, there is enough information to find *m*D and *m*E, since 15A + 6C = (15+ 6)D = 21D and 15A + 8B = (8 + 15)E = 23E. Therefore, given mass points 15A and 8B, it follows that side AB is balanced on point E when AE:BE = 8:15. In addition, given mass points 21D and 14F, we see that cevians AF and BD both are balanced on point G when AG:FG = 14:15 and BG:DG =21:8.

Solve the following problems by using mass point geometry. Express ratio answers as common fractions.

Triangle ABC, shown here, has cevians AD, BE and CF intersecting at point G, with AF:BF = 3:2 and BD:CD = 5:3.

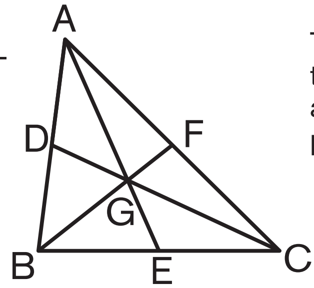


291.__________ What is the ratio of AE to CE?

292.__________ What is the ratio of BG to EG?

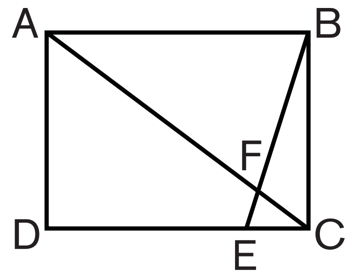
293.__________ What is the ratio of DG to AG?

294.__________

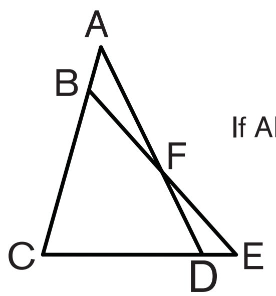


The medians of a triangle intersect at a point in the interior of the triangle as shown. What is the ratio of the lengths of the shorter and longer segments into which each median is divided at the point of intersection?

295.__________ In rectangle ABCD, point E is on side DC such that BC = 8, BE = 10 and AC = 17. If segments AC and BE intersect at F, what is the ratio of the area of triangle CFE to the area of triangle AFB?

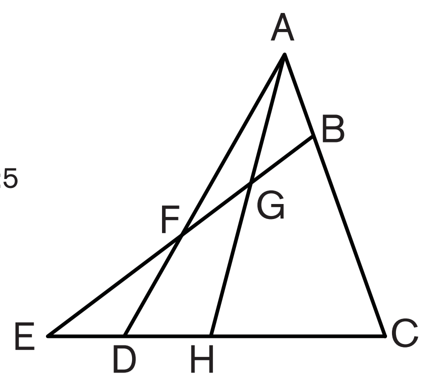


296.__________

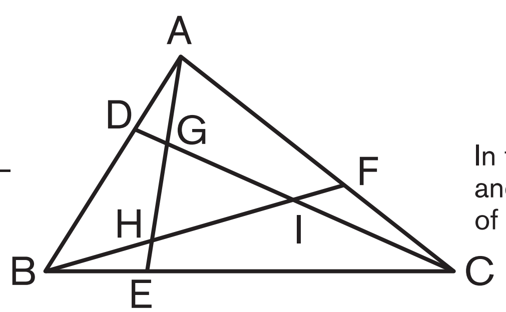


If AB:BC = 1:4 and AF:DF = 5:4, what is the ratio of DE to CD?

297.__________ For integers x, y and z, if AB:BC = 1:4, AG:GH = 3:5 and AF:DF = 5:4, then CH:DH:DE = x:y:z. What is the value of $x + y + z$?



298.__________



In triangle ABC, AD:BD = 1:2, BE:EC = 1:3 and AF:CF = 3:2. What is the ratio of the area of triangle GHI to the area of triangle ABC?

Triangle ABC, shown here, has cevian AD and transversal EF intersecting at G, with AE:CE = 1:2, AF:BF = 5:4 and BD:CD = 3:2.

299.__________ What is the ratio of AG to DG?

300.__________ What is the ratio of EG to FG?