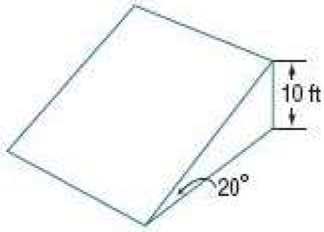
10-5 Angles of Elevation and Depression

1. **BIKING** Lenora wants to build the bike ramp shown. Find the length of the base of the ramp.



SOLUTION:

We are given a 20-degree reference angle and are finding the side adjacent to it and know the opposite side. Therefore, we want to use the tangent function:

$$\text{Tan} = \frac{\text{opposite}}{\text{adjacent}}$$

Let *x* be the length of the base of the ramp.

$$\tan 20° = \frac{10}{x}$$
$$x\tan 20° = 10$$
$$x = \frac{10}{\tan 20°}$$

Use a calculator, in degree mode.

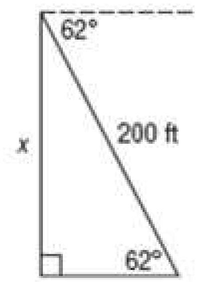
$x \approx 27.5$

The length of the base of the ramp is about 27.5 ft.

2. **BASEBALL** A fan is seated in the upper deck of a stadium 200 feet away from home plate. If the angle of depression to the field is $62°$, at what height is the fan sitting?

SOLUTION:

Make a sketch of the situation.



Since the horizontal and vertical line are parallel, alternate interior angles are congruent. We are finding the side opposite the 62 degree angle of reference and know the hypotenuse. Therefore, we will use the sine function to solve for *x*.

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$$

Let *x* be the unknown.

$$\sin 62° = \frac{x}{200}$$
$$x = 200(\sin 62°)$$

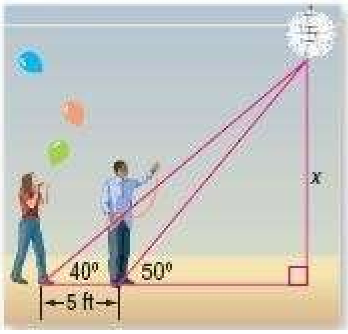
Use a calculator, in degree mode.

$x \approx 176.6$

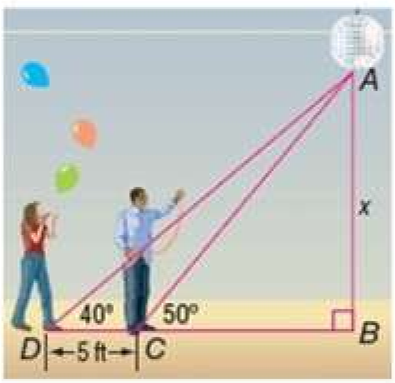
The fan is sitting at the height of about 176.6 feet.

3. **CCSS MODELING** Annabelle and Rich are setting up decorations for their school dance. Rich is standing 5 feet directly in front of Annabelle under a disco ball. If the angle of elevation from Annabelle to

the ball is $40°$ and Rich to the ball is $50°$, how high is the disco ball?



SOLUTION:
Label the triangle with the vertices, as seen below:



We have two right triangles. We want to find the value of *x*, but before we do this, we need to find the length of *BC*.
In ΔABC, the side opposite of $\angle ACB$ is $\overline{AB}$ (or *x*) and the adjacent side is BC, therefore we will choose the tangent function.

$$\tan 50 = \frac{AB}{BC}$$

Substitute and solve for *x*.

$$\tan 50° = \frac{x}{BC}$$
$$x = BC\tan 50°$$

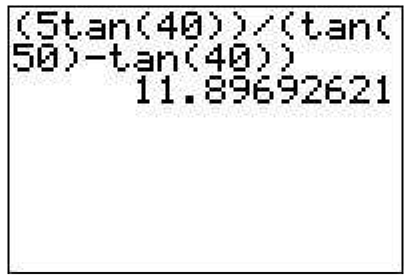
Solve for *x* in ΔABD. The side opposite the given 40 degree angle is *x* and the adjacent side is *BC+5*, therefore we will again use the tangent function to solve for *x*.

$$\tan 40° = \frac{x}{BC+5}$$
$$(BC+5)\tan 40 = x$$

Use substitution to find *BC*.

$$\tan 40°(BC+5) = BC\tan 50°$$
$$BC\tan 40° + 5\tan 40° = BC\tan 50°$$
$$5\tan 40° = BC\tan 50° - BC\tan 40°$$
$$5\tan 40° = BC(\tan 50° - \tan 40°)$$
$$\frac{5\tan 40°}{(\tan 50° - \tan 40°)} = BC$$

Use a calculator, in degree mode.



Substitute the value of *BC* in one of the original equations to find *x*.

.
$$x \approx (11.90)\tan 50°$$
$$x \approx 14.2$$

The height of the disco ball is about 14.2 feet.

4. **HOCKEY** A hockey player takes a shot 20 feet away from a 5-foot goal. If the puck travels at a $15°$ angle of elevation toward the center of the goal, will the player score?

SOLUTION:

Since we are finding the side opposite the given angle (height of puck at goal) and know the side adjacent (distance from the goal), we can use the tangent function.

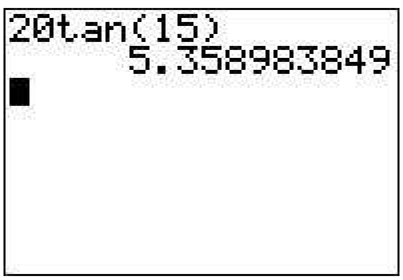
$$\text{Tan} = \frac{\text{opposite}}{\text{adjacent}}$$

Let x be the height of the puck at the goal.

$$\tan 15° = \frac{x}{20}$$

$$x = 20(\tan 15°)$$

Use a calculator, in degree mode.

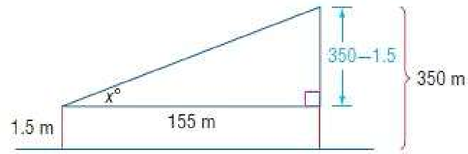


$x \approx 5.4$

The player will not score, because 5.4 > 5.

5. **MOUNTAINS** Find the angle of elevation to the peak of a mountain for an observer who is 155 meters from the mountain if the observer's eye is 1.5 meters above the ground and the mountain is 350 meters tall.

SOLUTION:



Since we know the side opposite the angle of elevation, as well as the side adjacent to it, we will use the tangent function.

$$\text{Tan} = \frac{\text{opposite}}{\text{adjacent}}$$

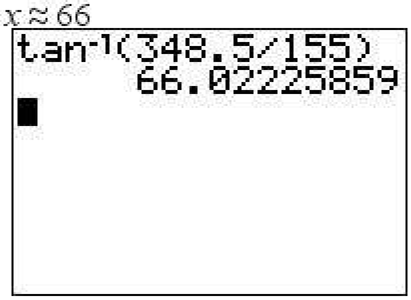
Let x be the angle of elevation.

$$\tan x = \frac{350-1.5}{155}$$

$$\tan x = \frac{348.5}{155}$$

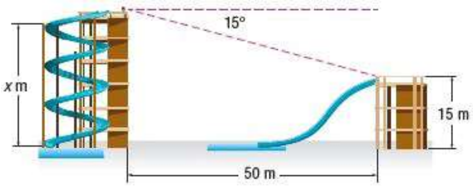
$$x = \tan^{-1}\left(\frac{348.5}{155}\right)$$

$$x \approx 66$$



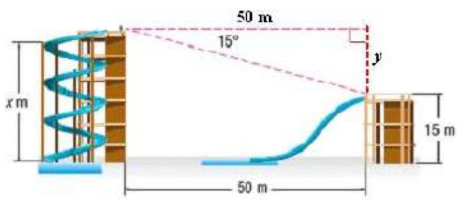
The angle of elevation is about 66 degrees.

6. **WATERPARK** Two water slides are 50 meters apart on level ground. From the top of the taller slide, you can see the top of the shorter slide at an angle of depression of 15°. If you know that the top of the other slide is approximately 15 meters above the ground, about how far above the ground are you? Round to the nearest tenth of a meter.



SOLUTION:

Since the slides are 50 meters apart, the top side of the triangle has a length of 50 meters. Let y represent the length of the vertical leg of the right triangle. The height of the taller slide x is equal to the sum of 15 and y.



$$\tan 15 = \frac{y}{50} \qquad \tan = \frac{\text{opposite}}{\text{adjacent}}$$
$$50\tan 15 = y \qquad \text{Multiply each side by 50.}$$
$$13.4 \approx y \qquad \text{Use a calculator.}$$

$$x = 15 + y$$
$$= 15 + 13.4$$
$$= 28.4$$

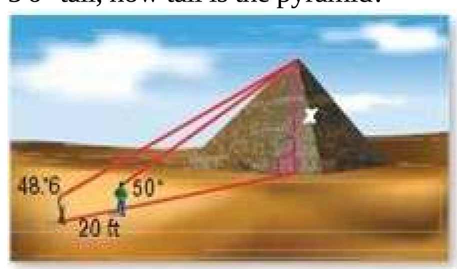
Therefore, from the top of the taller slide you are about 28.4 meters above the ground.

7. **AVIATION** Due to a storm, a pilot flying at an altitude of 528 feet has to land early. If he has a horizontal distance of 2000 feet to land, at what angle of depression should he land?

SOLUTION:

$$\text{Tan} = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan x = \frac{528}{2000}$$
$$\tan x = 0.264$$
$$x \approx 14.8°$$

8. **PYRAMIDS** Miko and Tyler are visiting the Great Pyramid in Egypt. From where Miko is standing, the angle of elevation to the top of the pyramid is $48.6°$. From Tyler's position, the angle of elevation is $50°$. If they are standing 20 feet apart, and both boys are 5'6" tall, how tall is the pyramid?



SOLUTION:

Let y be the horizontal distance between Tyler and the Pyramid. Since we are finding the opposite side from the given angle and the adjacent side is y units, we can set up an equation using the tangent function:

$$\tan 50° = \frac{x}{y}$$
$$x = y\tan 50° \Rightarrow (1)$$

Similarly,

$$\tan 48.6° = \frac{x}{y+20}.$$
$$x = \tan 48.6°(y+20) \Rightarrow (2)$$

Substitute and solve for y:

$$\tan 48.6°(y+20) = y\tan 50°$$
$$\tan 48.6°y + 20\tan 48.6° = y\tan 50°$$

Use a calculator, in degree mode.

```
tan(48.6)
     1.134277349
20tan(48.6)
     22.68554699
tan(50)
     1.191753593
```

Substitute these values and solve for y:

$$1.13427y + 22.68554 = 1.19175y$$
$$0.0574835y = 22.68554$$
$$y \approx 394.6443762$$

Substitute the value of y in (1).

$$x \approx (394.6443762)\tan 50°$$
$$x \approx 470$$

So, the height of the pyramid is about 470 feet +5.5 feet = 475.5 or 475' 6" tall.

9. **DIVING** Austin is standing on the high dive at the local pool. Two of his friends are in the water on the opposite side of the pool. If the angle of depression to one of his friends is $40°$, and $30°$ to his other friend who is 5 feet beyond the first, how tall is the platform?

SOLUTION:

Let y be the horizontal distance between one of Austin's friends (closest to the platform) and the platform. Since we are finding the height of the platform (opposite) and know the horizontal distance to the platform (adjacent), we can use the tangent function to solve for x and y:

$$\tan 40° = \frac{x}{y}$$
$$x = y\tan 40° \Rightarrow (1)$$

$$\tan 30° = \frac{x}{y+5}.$$
$$x = \tan 30°(y+5) \Rightarrow (2)$$

Substitute and solve for y:

$$\tan 30°(y+5) = y\tan 40°$$
$$\tan 30° y + 5\tan 30° = y\tan 40°$$

Use a calculator, in degree mode.

```
tan(30)
     .5773502692
5tan(30)
     2.886751346
tan(40)
     .8390996312
```

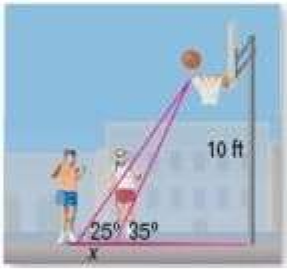
$$0.57735y + 2.88675 = 0.83910y$$
$$-0.26175y = -2.88675$$
$$y \approx 11.02865$$

Substitute the value of y in (1).

$$x \approx (11.02865)\tan 40$$
$$x \approx 9.3 \text{ ft}$$

Therefore, the height of the platform is about 9.3 feet.

10. **BASKETBALL** Claire and Marisa are both waiting to get a rebound during a basketball game. If the height of the basketball hoop is 10 feet, the angle of elevation between Claire and the goal is $35°$, and the angle of elevation between Marisa and the goal is $25°$, how far apart are they standing?



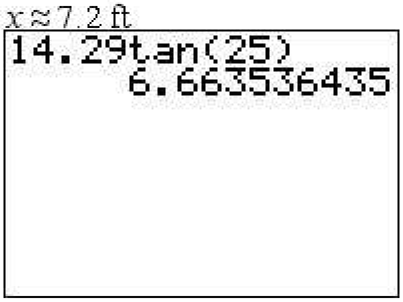
SOLUTION:

Let y be the horizontal distance between Claire and the goal. Since we are finding the horizontal (adjacent) side and know the side opposite the given angles, we can use the tangent function to find the distance from Claire to the goal.

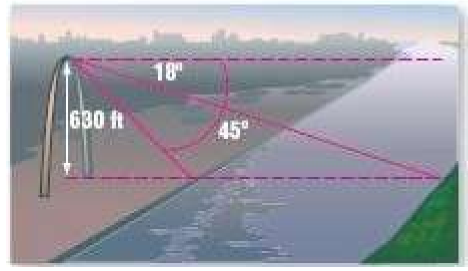
$$\tan 35° = \frac{10}{y}$$
$$y = \frac{10}{\tan 35°}$$
$$y \approx 14.29$$

We can similarly use the tangent function to find the distance from Marisa to the goal.

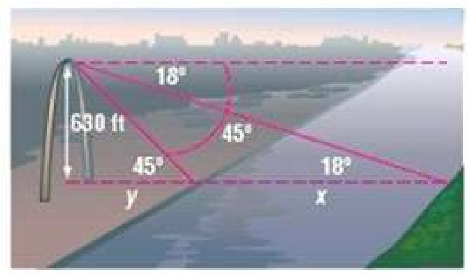
$$\tan 25 = \frac{10}{x + 14.29}$$
$$10 = \tan 25°(x + 14.29)$$
$$x\tan 25° = 10 - 14.29\tan 25°$$
$$x\tan 25° = 10 - 14.29\tan 25°$$
$$x\tan 25° = 10 - 6.66$$
$$x\tan 25° = 3.34$$
$$x \approx 7.2 \text{ ft}$$



11. **RIVERS** Hugo is standing in the top of St. Louis' Gateway Arch, looking down on the Mississippi River. The angle of depression to the closer bank is $45°$ and the angle of depression to the farther bank is $18°$. The arch is 630 feet tall. Estimate the width of the river at that point.



SOLUTION:



We know the side opposite the given angles (Height of arch) and are finding the adjacent side (horizontal distance of the river), therefore we can use the tangent function.

$$\tan 45° = \frac{630}{y}$$
$$y = \frac{630}{\tan 45°}$$
$$y = 630$$

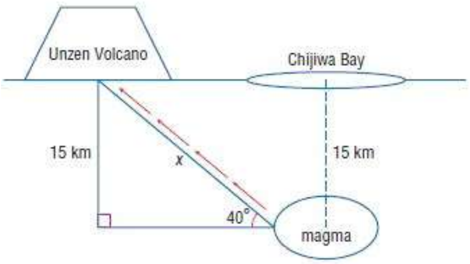
$$\tan 18° = \frac{630}{x + 630}$$
$$x\tan 18° = 630 - 630\tan 18°$$
$$x\tan 18° = 630 - 204.699$$
$$x\tan 18° = 425.301$$
$$x \approx 1309$$

The river is approximately 1309 ft wide at this point.

12. **CCSS MODELING** The Unzen Volcano in Japan has a magma reservoir that is located 15 kilometers beneath the Chijiwa Bay, located east of the volcano. A magma channel, which connects the reservoir to the volcano, rises at a $40°$ angle of elevation toward the volcano. What length of magma channel is below sea level?

SOLUTION:

Let *x* be the length of the magma channel.



$$\sin 40° = \frac{15}{x}$$

$$x = \frac{15}{\sin 40°}$$

$$x \approx 23.3\text{ km}$$

13. **BRIDGES** Suppose you are standing in the middle of the platform of the world's longest suspension bridge, the Akashi Kaikyo Bridge. If the height from the top of the platform holding the suspension cables is 297 meters, and the length from the platform to the center of the bridge is 995 meters, what is the angle of depression from the center of the bridge to the platform?

SOLUTION:

Since we know the side opposite the angle of depression (height of the bridge) as well as the side adjacent to it (the distance from the platform to the center of the bridge), we can use the tangent function.

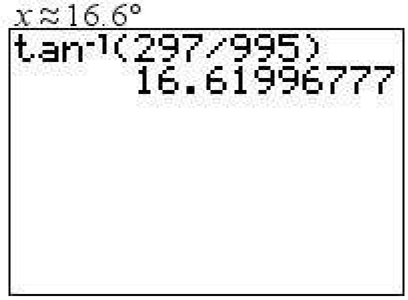
$$\text{Tan} = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute and solve for *x*:

$$\tan x = \frac{297}{995}$$

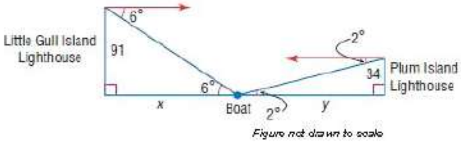
$$x = \tan^{-1}\left(\frac{297}{995}\right)$$

$$x \approx 16.6°$$



14. **LIGHTHOUSES** To aid in navigation, Little Gull Island Lighthouse shines a light from a height of 91 feet with a $6°$ angle of depression. Plum Island Lighthouse, 1800 feet away, shines a light from a height of 34 feet with a $2°$ angle of depression. Which light will reach a boat that sits exactly between Little Gull Island Lighthouse and Plum Island Lighthouse?

SOLUTION:



The angles of depression are congruent to the angles of elevation since they are alternate interior angles formed by parallel lines.

Let *x* be the horizontal range of the light from Little Gull Island Lighthouse. Set up an equation and solve for *x*:

$$\tan = \frac{opposite}{adjacent}$$

$$\tan(6) = \frac{91}{x}$$

$$x\tan(6) = 91$$

$$x = \frac{91}{\tan(6)}$$

$x \approx 866.8\,ft$

Since the boat is half of 1800 feet away from Little Gull Lighthouse, or 900 ft, the light from Little Gull Island won't reach the boat.

Let y be the horizontal range of the light from Plum Island Lighthouse. Set up an equation and solve for y:

$$\tan = \frac{opposite}{adjacent}$$

$$\tan(2) = \frac{34}{x}$$

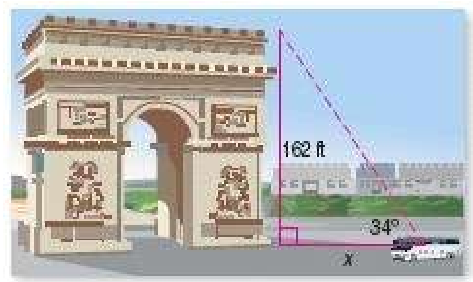
$$x\tan(2) = 34$$

$$x = \frac{34}{\tan(2)}$$

$x \approx 973.6\,ft$

Since the boat is half of 1800 feet away from Little Gull Lighthouse, or 900 ft, the light from Little Gull Island will reach the boat.

15. **TOURISM** From the position of the bus on the street, the L'arc de Triomphe is at a $34°$ angle of elevation. If the arc is 162 feet tall, how far away is the bus? Round to the nearest tenth.



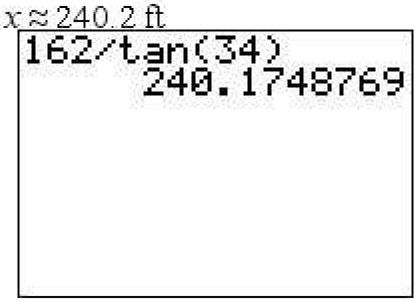
SOLUTION:

$$\tan = \frac{opposite}{adjacent}$$

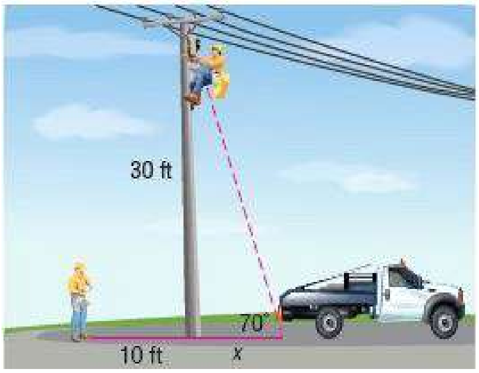
$$\tan 34° = \frac{162}{x}$$

$$x = \frac{162}{\tan 34°}$$

$x \approx 240.2$ ft



16. **MAINTENANCE** Two telephone repair workers arrive at a location to restore electricity after a power outage. One of the workers climbs up the telephone pole while the other worker stands 10 feet to left of the pole. If the terminal box is located 30 feet above ground on the pole and the angle of elevation from the truck to the repair worker is 70 degrees, how far is the worker on the ground standing from the truck?



SOLUTION:

To find the distance from the worker to the truck, we need to find the distance from the pole to the truck, using trigonometric ratios, and then add this answer to 10 ft, the distance from the worker to the pole.

$$\tan = \frac{opposite}{adjacent}$$

$$\tan 70° = \frac{30}{x}$$

$$x = \frac{30}{\tan 70°}$$

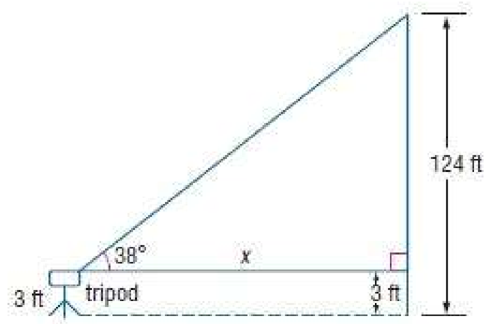
$$x \approx 10.9 \text{ ft}$$

So, the distance between the worker and the truck is about 10.9 +10 or 20.9 ft.

17. **PHOTOGRAPHY** A digital camera with a panoramic lens is described as having a view with an angle of elevation of $38°$. If the camera is on a 3-foot tripod aimed directly at a 124-foot-tall monument, how far from the monument should you place the tripod to see the entire monument in your photograph?

SOLUTION:

Let *x* be the distance from the tripod to the monument. To create a right triangle with the camera and the monument, we need to subtract three feet from the height of the monument, since the height of the tripod is 3 feet high.



$$\tan = \frac{opposite}{adjacent}$$

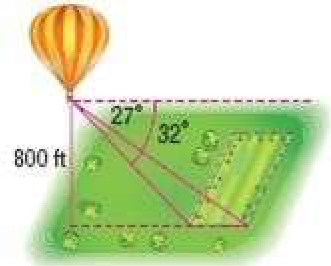
$$\tan 38° = \frac{124 - 3}{x}$$

$$\tan 38° = \frac{121}{x}$$

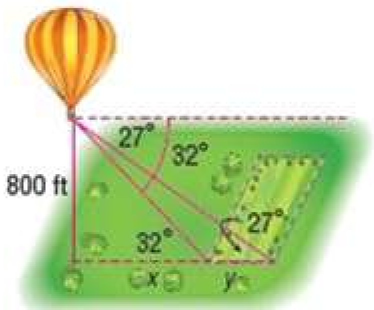
$$x = \frac{121}{\tan 38°}$$

$$x \approx 154.9 \text{ ft}$$

18. **CCSS MODELING** As a part of their weather unit, Anoki's science class took a hot air balloon ride. As they passed over a fenced field, the angle of depression of the closer side of the fence was $32°$, and the angle of depression of the farther side of the fence was $27°$. If the height of the balloon was 800 feet, estimate the width of the field.



SOLUTION:



Two triangles are formed. Find x in the small triangle. Find $x + y$ in the large triangle. Then subtract x from $x + y$ to determine y, the length of the field.

$$\tan 32° = \frac{800}{x}$$
$$\tan 32°(x) = 800$$
$$x = \frac{800}{\tan 32} \approx 1280.3$$

$$\tan 27° = \frac{800}{x+y}$$
$$\tan 27°(x+y) = 800$$
$$x + y = \frac{800}{\tan 27°} \approx 1570.1$$

Since $x+y$ = 1570.1 and x = 1280.3, then y = *1570.1-1280.3=289.8.*

Therefore, the width of the field is about 289.8 ft.

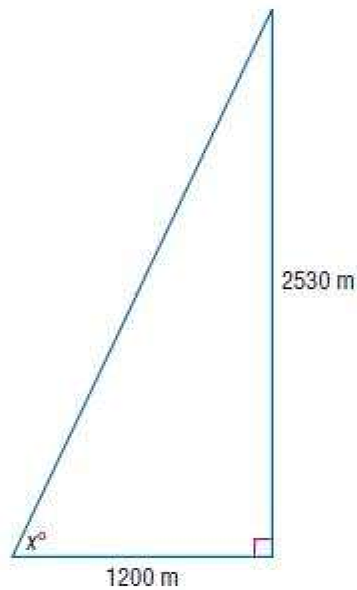
19. **MARATHONS** The Badwater Ultramarathon is a race that begins at the lowest point in California, Death Valley, and ends the highest point of the state, Mount Whitney. The race starts at a depth of 86 meters below sea level and ends 2,530 meters above sea level.

a. Refer to the photo in the text. Determine the angle of elevation to Mount Whitney if the horizontal distance from the base to the peak is 1,200 meters.

b. Refer to the photo in the text. If the angle of depression to Death Valley is $38°$, what is the horizontal distance from sea level?

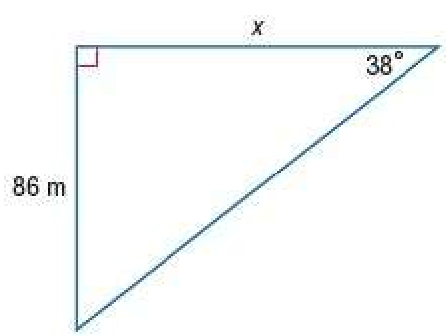
SOLUTION:

a.



$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan x = \frac{2530}{1200}$$
$$\tan x = 2.108\overline{3}$$
$$x \approx 64.6°$$

b.



$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 38° = \frac{86}{x}$$

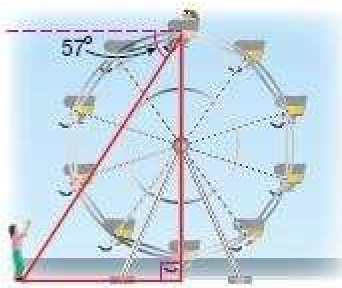
$$\tan 38°(x) = 86$$

$$x = \frac{86}{\tan 38°} \approx 110.1$$

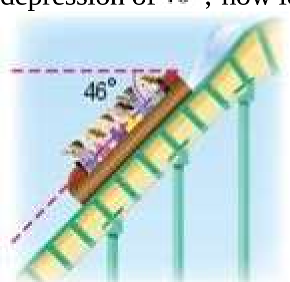
Therefore, the horizontal distance from the sea level is about 110.1 m.

20. **AMUSEMENT PARKS** India, Enrique, and Trina went to an amusement park while visiting Japan. They went on a Ferris wheel that was 100 meters in diameter and on an 80-meter cliff-dropping slide.

a. When Enrique and Tina are at the topmost point on the Ferris wheel as shown, how far are they from India?

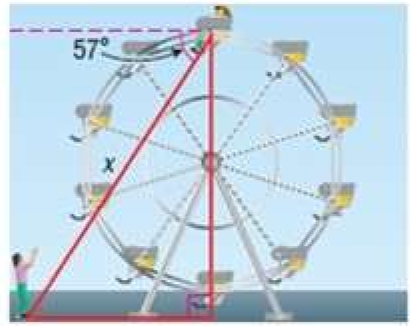


b. If the cliff-dropping ride has an angle of depression of $46°$, how long is the slide?



SOLUTION:

a. Since the horizontal lines are parallel to each other, the alternate interior angles formed are congruent. Therefore, the angle of elevation of the triangle is 57 degrees. Also, the height of the wheel is the same as its diameter, which is 100 ft. .



Let x be the distance between India and the others.

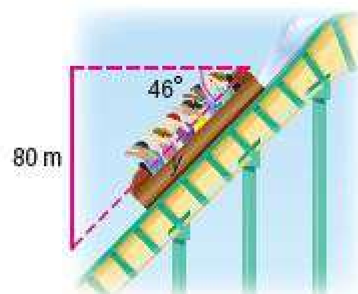
$$\sin = \frac{opposite}{hypotenuse}$$

$$\sin 57° = \frac{100}{x}$$

$$\sin 57°(x) = 100$$

$$x = \frac{100}{\sin 57°} \approx 119.2 \text{ m}$$

b. Let y be the length of the slide. The slide has a 80 meter drop, so that is its height.



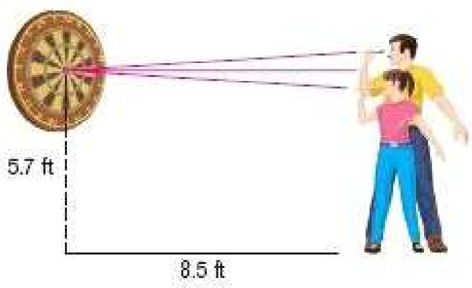
$$\sin = \frac{opposite}{hypotenuse}$$

$$\sin 46° = \frac{80}{y}$$

$$\sin 46° y = 80$$

$$y = \frac{80}{\sin 46°} \approx 111.2 \text{ m}$$

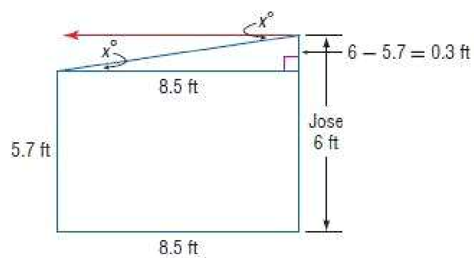
21. **DARTS** Kelsey and José are throwing darts from a distance of 8.5 feet. The center of the bull's-eye on the dartboard is 5.7 feet from the floor. José throws from a height of 6 feet, and Kelsey throws 5 feet. What are the angles of elevation or depression from which each must throw to get a bull's-eye? Ignore other factors such as air resistance, velocity, and gravity.



SOLUTION:

Jose:

Jose is taller than the target, therefore, he will be throwing the dart at an angle of depression. The side opposite the angle of depression is 6-5.7=0.3 feet. Let *x* be the angle of depression.

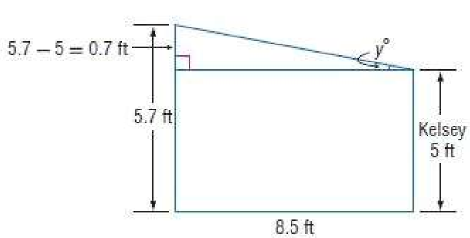


$$\tan = \frac{opposite}{adjacent}$$
$$\tan x = \frac{6-5.7}{8.5}$$
$$\tan x = \frac{0.3}{8.5}$$
$$x \approx 2.02°$$

Therefore, Jose throws at an angle of depression of $2.02°$.

Kelsey:

Kelseis shorter than the target, therefore she will be throwing the dart at an angle of elevation. The length of the side opposite the angle of elevation is 5.7-5.0=0.7 ft. Let y be the angle of depression.



$$\tan x = \frac{5.7-5}{8.5}$$
$$\tan x = \frac{0.7}{8.5}$$
$$x \approx 4.71°$$

Therefore, Kelsey throws at an angle of elevation of $4.71°$.

22. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate relationships between the sides and angles of triangles.

a. GEOMETRIC Draw three triangles. Make one acute, one obtuse, and one right. Label one triangle *ABC*, a second *MNP*, and the third *XYZ*. Label the side lengths and angle measures of each triangle.

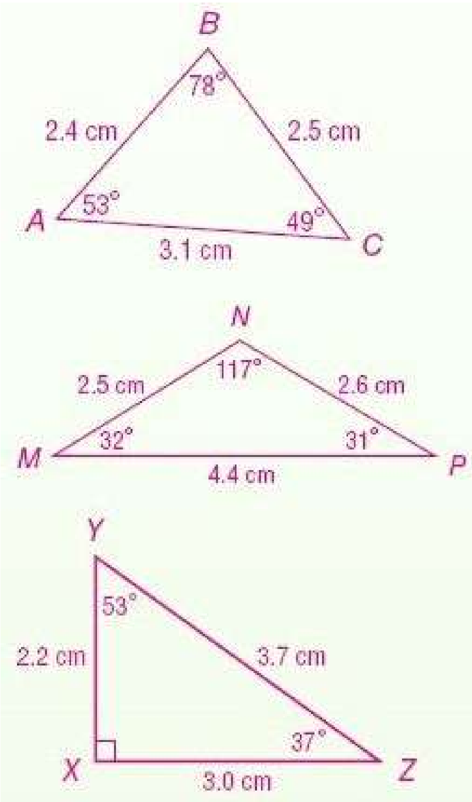
b. TABULAR Copy and complete the table below.

Triangle	Ratios		
ABC	$\frac{\sin A}{BC} =$	$\frac{\sin B}{CA} =$	$\frac{\sin C}{AB} =$
MNP	$\frac{\sin M}{NP} =$	$\frac{\sin N}{PM} =$	$\frac{\sin P}{MN} =$
XYZ	$\frac{\sin X}{YZ} =$	$\frac{\sin Y}{ZX} =$	$\frac{\sin Z}{XY} =$

c. VERBAL Make a conjecture about the ratio of the sine of an angle to the length of the leg opposite that angle for a given triangle.

SOLUTION:

a. Use a straight edge and ruler when making these three triangles. Label as directed. Carefully measure each side length, in centimeters, and angle measure.



b. Record your measurements below and find the ratios of the indicated sides.

Triangle	Ratios		
ABC	$\frac{\sin A}{BC} = 0.3$	$\frac{\sin B}{CA} = 0.3$	$\frac{\sin C}{AB} = 0.3$
MNP	$\frac{\sin M}{NP} = 0.2$	$\frac{\sin N}{PM} = 0.2$	$\frac{\sin P}{MN} = 0.2$
XYZ	$\frac{\sin X}{YZ} = 0.3$	$\frac{\sin Y}{ZX} = 0.3$	$\frac{\sin Z}{XY} = 0.3$

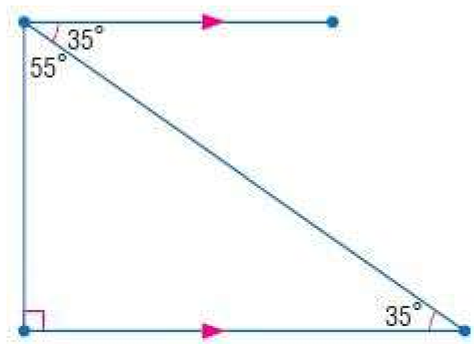
c. What pattern do you notice is evident in your calculations? Summarize what you see, using general terminology.

Sample answer: The ratio of the sine of an angle to the length of the leg opposite that angle is approximately equal for all three angles of a triangle.

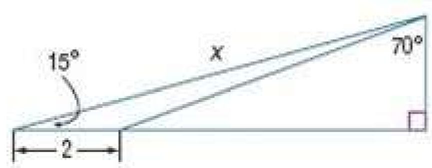
23. **ERROR ANALYSIS** Terrence and Rodrigo are trying to determine the relationship between angles of elevation and depression. Terrence says that if you are looking up at someone with an angle of elevation of $35°$, then they are looking down at you with an angle of depression of $55°$, which is the complement of $35°$. Rodrigo disagrees and says that the other person would be looking down at you with an angle of depression equal to your angle of elevation, or $35°$. Is either of them correct? Explain.

SOLUTION:

Rodrigo; sample answer: Since your horizontal line of site is parallel to the other person's horizontal line of sight, the angles of elevation and depression are congruent according to the Alternate Interior Angles Theorem.

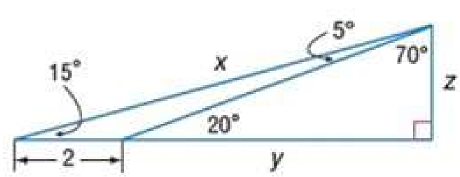


24. **CHALLENGE** Find the value of *x*. Round to the nearest tenth.



SOLUTION:

Label the three sides of the big triangle *x*, *y*+2, and *z*. Use the Triangle Sum Theorem to find the missing angle measures in the smaller right triangle and the obtuse triangle.



Consider the large right triangle, with angle measures 15, 75 and 90 degrees. We can express the value of z in terms of y, using the tangent function.

$$\tan 15° = \frac{z}{y+2}$$
$$z = y\tan 15° + 2\tan 15° \rightarrow (1)$$

Repeat this same idea for the smaller right triangle, using z and y again.

$$\tan 20° = \frac{z}{y}$$
$$z = y\tan 20° \rightarrow (2)$$

Equate (1) and (2).

$$y\tan 15° + 2\tan 15° = y\tan 20°$$
$$y\tan 20° - y\tan 15° = 2\tan 15°$$
$$y(\tan 20° - \tan 15°) = 2\tan 15°$$

$$y = \frac{2\tan 15°}{(\tan 20° - \tan 15°)}$$
$$y \approx 5.58$$

Substitute $y \approx 5.58$ in (2).

$$z \approx 5.58\tan 20°$$
$$z \approx 2.$$

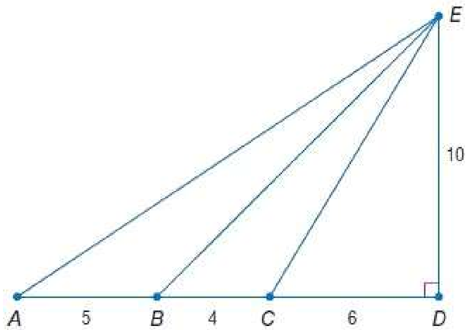
Use the Pythagorean Theorem to solve for x.

$$x^2 = (2)^2 + (5.58+2)^2$$
$$= (2)^2 + (7.58)^2$$
$$= 4 + 57.4564$$
$$= 61.4564$$
$$x \approx 7.9$$

25. **CCSS REASONING** Classify the statement below as *true* or *false*. Explain.
As a person moves closer to an object he or she is sighting, the angle of elevation increases.

SOLUTION:

True; sample answer: As a person moves closer to an object, the horizontal distance decreases, but the height of the object is constant. The tangent ratio will increase, and therefore the measure of the angle also increases.



$$\tan \angle EAB = \frac{10}{15} \qquad \tan \angle EBD = \frac{10}{10}$$
$$m\angle EAB = 33.7 \qquad m\angle EAB = 45$$
$$\tan \angle ECD = \frac{10}{6}$$
$$m\angle EAB = 59$$

26. **WRITE A QUESTION** A classmate finds the angle of elevation of an object, but she is trying to find the angle of depression. Write a question to help her solve the problem.

SOLUTION:

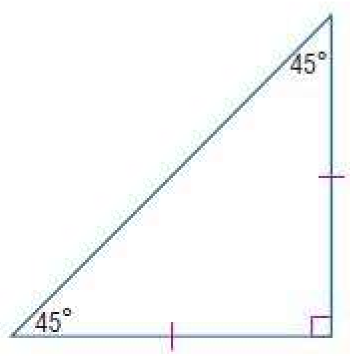
To come up with a new question regarding the relationship between the angle of depression and angle of elevation, consider how they are similar, as well as how they are different.

Sample answer: What is the relationship between the angle of elevation and angle of depression?

27. **WRITING IN MATH** Describe a way that you can estimate the height of an object without using trigonometry by choosing your angle of elevation. Explain your reasoning.

SOLUTION:

Sample answer: If you sight something with a $45°$ angle of elevation, you don't have to use trigonometry to determine the height of the object. Since the legs of a $45°$ - $45°$ - $90°$ are congruent, the height of the object will be the same as your horizontal distance from the object.

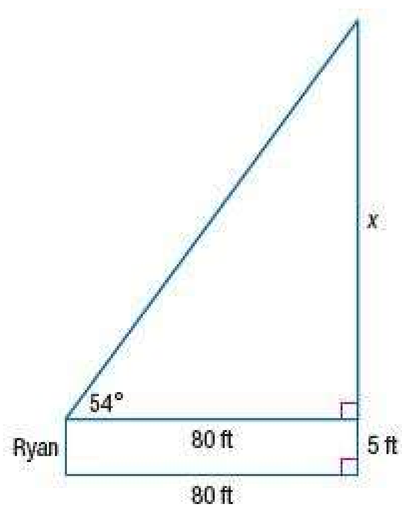


28. Ryan wanted to know the height of a cell-phone tower neighboring his property. He walked 80 feet from the base of the tower and measured the angle of elevation to the top of the tower at $54°$. If Ryan is 5 feet tall, what is the height of the cell-phone tower?

A 52 ft
B 63 ft
C 110 ft
D 115 ft

SOLUTION:

Assume that x + 5 feet is the height of the cell-phone tower because Ryan is 5 feet tall.



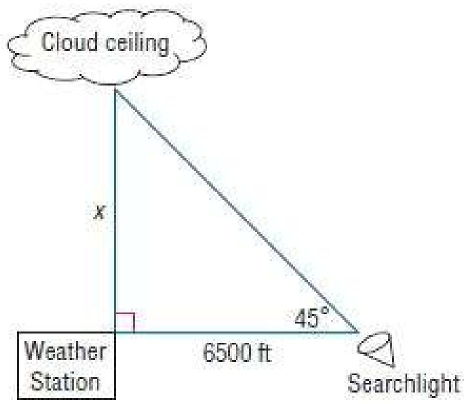
$$\tan 54° = \frac{x}{80}$$
$$x = 80(\tan 54°)$$
$$x \approx 110$$

Therefore, the height of the cell-phone tower is about 110+5=115 feet tall.
So, the correct choice is D.

29. **SHORT RESPONSE** A searchlight is 6500 feet from a weather station. If the angle of elevation to the spot of light on the clouds above the station is $45°$, how high is the cloud ceiling?

SOLUTION:

Let *x* be the height of the cloud ceiling over the weather station.



$$\tan 45 = \frac{x}{6500}$$
$$x = 6500(\tan 45)$$
$$x = 6500$$

The height of the cloud ceiling is 6500 ft.

30. **ALGEBRA** What is the solution of this system of equations?

$2x - 4y = -12$

$-x + 4y = 8$

F (4, 4)
G (–4, 1)
H (–4, –4)
J (1, –4)

SOLUTION:

$2x - 4y = -12$

$-x + 4y = 8$

Add the equations.

$x = -4$

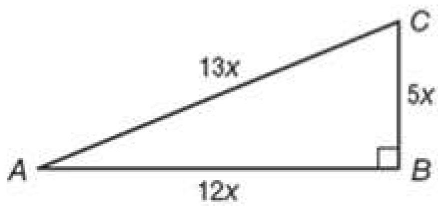
Substitute $x = -4$ in the second equation to find y.

$$-(-4) + 4y = 8$$
$$4 + 4y = 8$$
$$4y = 4$$
$$y = 1$$

The solution is (–4, 1).
So, the correct choice is G.

31. **SAT/ACT** A triangle has sides in the ratio of 5:12:13. What is the measure of the triangle's smallest angle in degrees?
A 13.34
B 22.62
C 34.14
D 42.71
E 67.83

SOLUTION:
This triangle is a right triangle, since it has sides in the ratio of 5:12:13. This ratio is equivalent to $5x$: $12x$: $13x$.

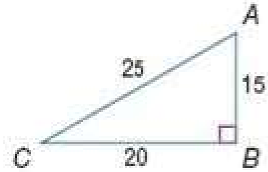


Angle A has the smallest measure, since the length of its opposite is small when comparing the other two sides. You can find either $\sin A$ or $\cos A$ or $\tan A$.

$$\sin A = \frac{5x}{13x}$$
$$\sin A = \frac{5}{13}$$
$$A \approx 22.62°$$
$$\cos A = \frac{12x}{13x}$$
$$\cos A = \frac{12}{13}$$
$$A \approx 22.62°$$
$$\tan A = \frac{5x}{12x}$$
$$\tan A = \frac{5}{12}$$
$$A \approx 22.62°$$

So, the correct choice is B.

Express each ratio as a fraction and as a decimal to the nearest hundredth.



32. $\sin C$

SOLUTION:
The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$$\sin C = \frac{AB}{AC} = \frac{15}{25} = 0.6.$$

33. $\tan A$

SOLUTION:
The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$$\tan A = \frac{BC}{AB} = \frac{20}{15} \approx 1.33.$$

34. $\cos C$

SOLUTION:
The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse. So,

$$\cos C = \frac{BC}{AC} = \frac{20}{25} = 0.8.$$

35. $\tan C$

SOLUTION:
The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$$\tan C = \frac{AB}{BC} = \frac{15}{20} = 0.75.$$

36. $\cos A$

SOLUTION:
The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse. So,

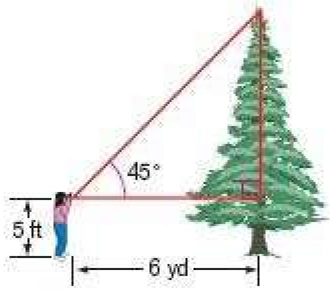
$$\cos A = \frac{AB}{AC} = \frac{15}{25} = 0.6.$$

37. $\sin A$

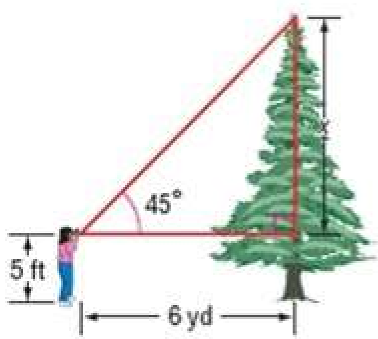
SOLUTION:
The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$$\sin A = \frac{BC}{AC} = \frac{20}{25} = 0.8.$$

38. **LANDSCAPING** Imani needs to determine the height of a tree. Holding a drafter's 45° triangle so that one leg is horizontal, she sights the top of the tree along the hypotenuse, as shown at the right. If she is 6 yards from the tree and her eyes are 5 feet from the ground, find the height of the tree.



SOLUTION:



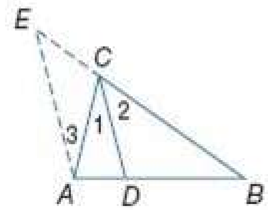
Let $x + 5$ represent the height of the tree. To solve for x, we can consider the relationships of the 45-45-90 triangle.

Since the legs are congruent, x is 6 yards, which is equivalent to 6(3)=18 feet. Therefore, the height of the tree is 18+5 or 23 feet tall.

PROOF Write a two-column proof.

39. **Given**: $\overline{CD}$ bisects $\angle ACB$. By construction, $\overline{AE} \parallel \overline{CD}$

Prove: $\frac{AD}{DB} = \frac{AC}{BC}$



SOLUTION:
Theorem 7.11 states that an angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides. Since you have a line ($\overline{CD}$) that is parallel to one side of the triangle and divides the other sides of the triangle into two parts, by the Triangle Proportionality theorem, we know that $\frac{AD}{DB} = \frac{EC}{BC}$. This looks very similar to the conclusion, except we need to replace EC with AC. Since $\overline{CD} \parallel \overline{EA}$, then $\angle E \cong \angle 2$ and $\angle 1 \cong \angle 3$. From the given information, we can also conclude that $\angle 1 \cong \angle 2$ (How?). Therefore, $\angle E \cong \angle 3$. This means that ΔECA is an isosceles triangle and $\overline{EC} \cong \overline{AC}$. Now, we can replace EC with AC in $\frac{AD}{DB} = \frac{EC}{BC}$.

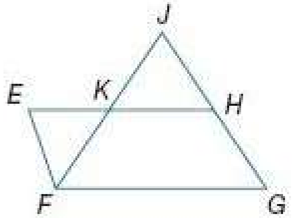
Proof:

Statements (Reasons)

1. $\overline{CD}$ bisects $\angle ACB$. By construction, $\overline{AE} \parallel \overline{CD}$. (Given)
2. $\frac{AD}{DB} = \frac{EC}{BC}$ (Triangle Proportionality Thm.)
3. $\angle 1 \cong \angle 2$ (Def. of $\angle$ Bisector)
4. $\angle 3 \cong \angle 1$ (Alt. Int. $\angle$ Thm.)
5. $\angle 2 \cong \angle E$ (Corres. $\angle$ Post.)
6. $\angle 3 \cong \angle E$ (Transitive Prop.)
7. $\overline{EC} \cong \overline{AC}$ (Isosceles Δ Thm.)
8. $EC = AC$ (Def. of $\cong$ segments)
9. $\frac{AD}{DB} = \frac{AC}{BC}$ (Substitution)

40. **Given**: $\overline{JF}$ bisects $\angle EFG$. $\overline{EH} \parallel \overline{FG}$, $\overline{EF} \parallel \overline{HG}$

Prove: $\frac{EK}{KF} = \frac{GJ}{JF}$



SOLUTION:

A good approach to this proof would be to think backwards - What do you need to prove that sides of two different triangles are proportional to each other? If you can prove $\Delta EKF \sim \Delta GJF$ by AA, then you can prove their corresponding sides are proportional.Use definition of angle bisector and angle relationships formed by parallel lines to get two pairs of corresponding angles congruent.

Proof:

Statements (Reasons)

1. $\overline{JF}$ bisects $\angle EFG$; $\overline{EH} \parallel \overline{FG}$, $\overline{EF} \parallel \overline{HG}$ (Given)
2. $\angle EFK \cong \angle KFG$ (Def. of $\angle$ bisector)
3. $\angle KFG \cong \angle JKH$ (Corr. angle Post.)
4. $\angle JKH \cong \angle EKF$ (Vertical angles are $\cong$.)
5. $\angle EFK \cong \angle EKF$ (Transitive Prop.)
6. $\angle FJH \cong \angle EKF$ (Alt. Int. angles Thm.)
7. $\angle FJH \cong \angle EFK$ (Transitive Prop.)
8. $\Delta EKF \sim \Delta GJF$ (AA Similarity)
9. $\frac{EK}{KF} = \frac{GJ}{JF}$ (Def. of $\sim \Delta$ s)

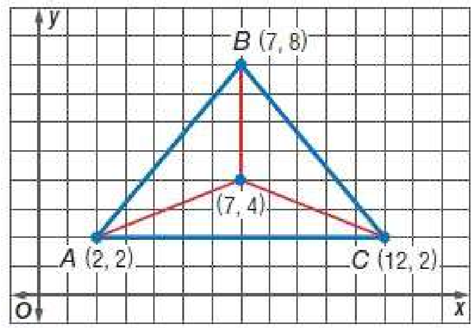
COORDINATE GEOMETRY Find the coordinates of the centroid of each triangle.

41. *A*(2, 2), *B*(7, 8), *C*(12, 2)

SOLUTION:

Use the centroid formula $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

The centroid of the triangle is $\left(\frac{2+7+12}{3}, \frac{2+8+2}{3}\right)$ or $(7, 4)$.

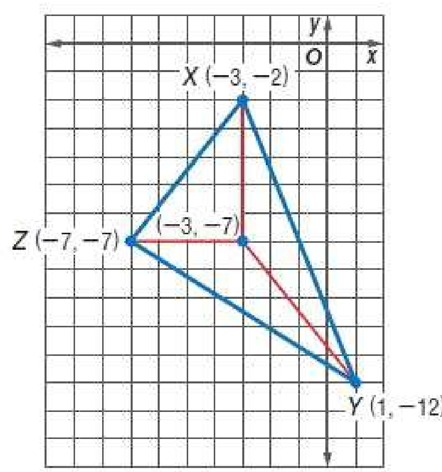


42. *X*(–3, –2), *Y*(1, –12), *Z*(–7, -7)

SOLUTION:

Use the centroid formula $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

The centroid of the triangle is $\left(\frac{-3+1-7}{3}, \frac{-2-12-7}{3}\right)$ or $(-3, -7)$.

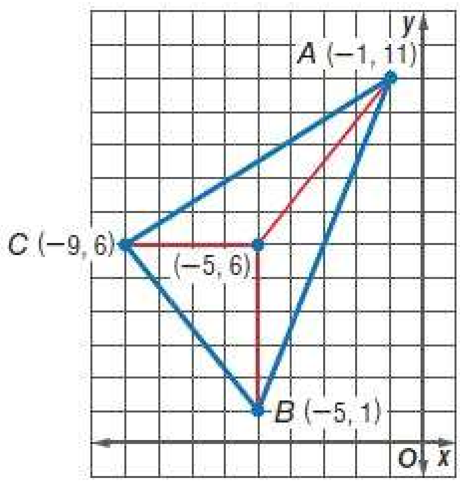


43. $A(-1, 11)$, $B(-5, 1)$, $C(-9, 6)$

SOLUTION:

Use the centroid formula $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

The centroid of the triangle is $\left(\frac{-1-5-9}{3}, \frac{11+1+6}{3}\right)$ or $(-5, 6)$.

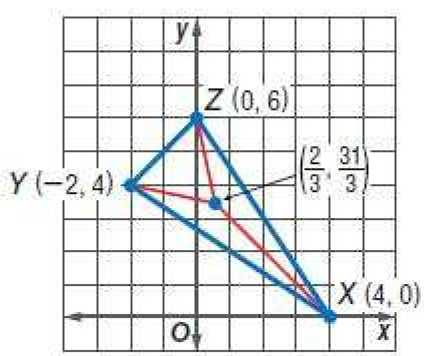


44. $X(4, 0)$, $Y(-2, 4)$, $Z(0, 6)$

SOLUTION:

Use the centroid formula $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

The centroid of the triangle is $\left(\frac{4-2+0}{3}, \frac{0+4+6}{3}\right)$ or $\left(\frac{2}{3}, 3\frac{1}{3}\right)$.



Solve each proportion.

45. $\frac{1}{5} = \frac{x}{10}$

SOLUTION:

$$\frac{1}{5} = \frac{x}{10}$$

Cross multiply.

$$5(x) = 1(10)$$

Solve for x.

$$5x = 10$$
$$x = 2$$

46. $\frac{2x}{11} = \frac{3}{8}$

SOLUTION:

$$\frac{2x}{11} = \frac{3}{8}$$

Cross multiply.

$$8(2x) = 11(3)$$

Solve for x.

$$16x = 33$$
$$x \approx 2.1$$

47. $\frac{4x}{16} = \frac{62}{118}$

SOLUTION:

$$\frac{4x}{16} = \frac{62}{118}$$

Cross multiply.

$$118(4x) = 16(62)$$

Solve for x.

$$472x = 992$$
$$x \approx 2.1$$

48. $\frac{12}{21}=\frac{45}{10x}$

SOLUTION:

$$\frac{12}{21}=\frac{45}{10x}$$

Cross multiply.

$$12(10x)=45(21)$$

Solve for x.

$$120x=945$$
$$x\approx 7.9$$