A Story of Units®

Eureka Math™

Grade 4, Module 4

Teacher Edition

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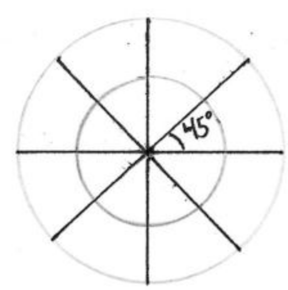
OVERVIEW

This 20-day module introduces points, lines, line segments, rays, and angles, as well as the relationships between them. Students construct, recognize, and define these geometric objects before using their new knowledge and understanding to classify figures and solve problems. With angle measure playing a key role in the work throughout the module, students learn how to create and measure angles, as well as how to create and solve equations to find unknown angle measures. In these problems, where the unknown angle is represented by a letter, students explore both measuring the unknown angle with a protractor and reasoning through the solving of an equation. This connection between the measurement tool and the numerical work lays an important foundation for success with middle-school geometry and algebra. Through decomposition and composition activities, as well as an exploration of symmetry, students recognize specific attributes present in two-dimensional figures. They further develop their understanding of these attributes as they classify two-dimensional figures.

Topic A begins with students drawing points, lines, line segments, and rays, as well as identifying these in various contexts and within familiar figures. Students recognize that two rays sharing a common endpoint form an angle (**4.MD.5**). They create right angles through a paper-folding activity, identify right angles in their environment, and see that one angle can be greater (obtuse) or less (acute) than a right angle. Next, students use their understanding of angles to explore relationships between pairs of lines as they define, draw, and recognize intersecting, perpendicular, and parallel lines (**4.G.1**).

In Topic B, students explore the definition of degree measure, beginning with a circular protractor. By dividing the circumference of a circle into 360 equal parts, they recognize one part as representing 1 degree (**4.MD.5**). Through exploration, students realize that, although the size of a circle may change, an angle spans an arc, representing a constant fraction of the circumference. By carefully distinguishing the attribute of degree measure from that of length measure, the common misconception that degrees are a measure of length is avoided. Armed with their understanding of the degree as a unit of measure, students use various types of protractors to measure angles to the nearest degree and to sketch angles of a given measure (**4.MD.6**). The idea that an angle measures the amount of *turning* in a particular direction is explored as students recognize familiar angles in varied contexts (**4.G.1**, **4.MD.5**).



Topic C begins by decomposing 360° using pattern blocks, allowing students to see that a group of angles meeting at a point with no spaces or overlaps add up to 360°. With this new understanding, students now discover that the combined measure of two adjacent angles on a line is 180° (supplementary angles), that the combined measure of two adjacent angles meeting to form a right angle is 90° (complementary angles), and that vertically opposite angles have the same measure. These properties are then used to solve unknown angle problems (**4.MD.7**).

An introduction to symmetry opens Topic D as students recognize lines of symmetry for two-dimensional figures, identify line-symmetric figures, and draw lines of symmetry (**4.G.3**). Given one half of a line-symmetric figure and the line of symmetry, students draw the other half of the figure. This leads to their work with triangles. Students are introduced to the precise definition of a triangle and then classify triangles based on angle measure and side length (**4.G.2**). For isosceles triangles, a line of symmetry is identified, and a folding activity demonstrates that base angles are equal. Folding an equilateral triangle highlights multiple lines of symmetry and establishes that all interior angles are equal. Students construct triangles given a set of classifying criteria (e.g., create a triangle that is both right and isosceles). Finally, students explore the definitions of familiar quadrilaterals and classify them based on their attributes, including angle measure and parallel and perpendicular lines (**4.G.2**). This work builds on Grade 3 reasoning about the attributes of shapes and lays a foundation for hierarchical classification of two-dimensional figures in Grade 5. The topic concludes as students compare and analyze two-dimensional figures according to their properties and use grid paper to construct two-dimensional figures given a set of criteria.

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

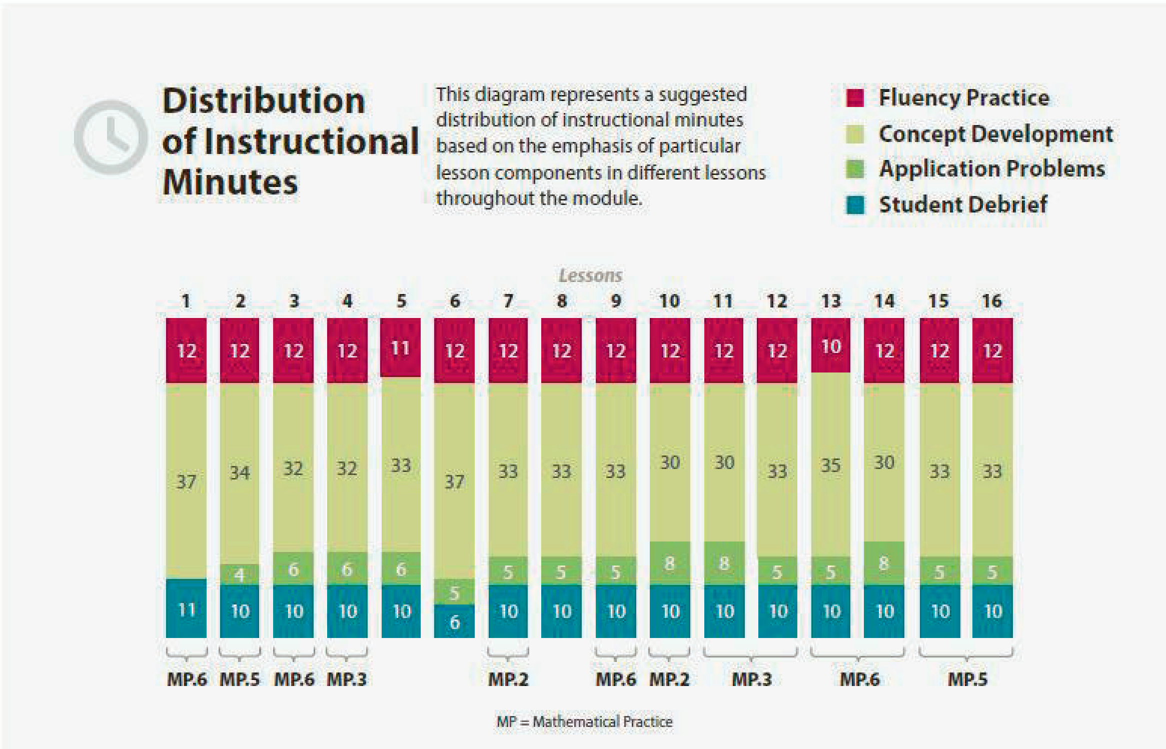
Notes on Pacing for Differentiation

The placement of Module 4 in *A Story of Units* was determined based on the New York State Education Department Pre-Post Math Standards document, which placed **4.NF.5–7** outside the testing window and **4.MD.5** inside the testing window. This is not in alignment with PARCC's Content Emphases Clusters (http://www.parcconline.org/mcf/mathematics/content-emphases-cluster-0), which reverses those priorities, labeling **4.NF.5–7** as Major Clusters and **4.MD.5** as an Additional Cluster, the status of lowest priority.

Those from outside New York State may want to teach Module 4 after Module 6 and truncate the lessons using the Preparing a Lesson protocol (see the Module Overview, just before the Assessment Overview). This would change the order of the modules to the following: Modules 1, 2, 3, 5, 6, 4, and 7.

Those from New York State might apply the following suggestions and truncate Module 4's lessons using the Preparing a Lesson protocol. Topic A could be taught simultaneously with Module 3 during an art class. Topics B and C could be taught directly following Module 3, prior to Module 5, since they offer excellent scaffolding for the fraction work of Module 5. Topic D could be taught simultaneously with Module 5, 6, or 7 during an art class when students are served well with hands-on, rigorous experiences.

Keep in mind that Topics B and C of this module are foundational to Grade 7's missing angle problems.



Focus Grade Level Standards

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Foundational Standards

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** Students represent angle measures within equations, and when determining the measure of an unknown angle, they represent the unknown angle with a letter or symbol both in the diagram and in the equation. They reason about the properties of groups of figures during classification activities.

MP.3 **Construct viable arguments and critique the reasoning of others.** Knowing and using the relationships between adjacent and vertical angles, students construct an argument for identifying the angle measures of all four angles generated by two intersecting lines when given the measure of one angle. Students explore the concepts of parallelism and perpendicularity on different types of grids with activities that require justifying whether completing specific tasks is possible on different grids.

MP.5 **Use appropriate tools strategically.** Students choose to use protractors when measuring and sketching angles, drawing perpendicular lines, and precisely constructing two-dimensional figures with specific angle measurements. They use right angle templates (set squares) and straightedges to construct parallel lines. They also choose to use straightedges for sketching lines, line segments, and rays.

MP.6 **Attend to precision.** Students use clear and precise vocabulary. They learn, for example, to cross-classify triangles by both angle size and side length (e.g., naming a shape as a right, isosceles triangle). They use right angle templates (set squares) and straightedges to construct parallel lines and become sufficiently familiar with a protractor to decide which set of numbers to use when measuring an angle whose orientation is such that it opens from either direction, or when the angle measures more than 180°.

Overview of Module Topics and Lesson Objectives

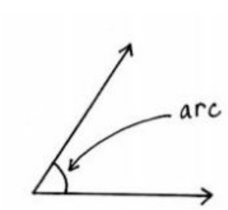
Standards		Topics and Objectives	Days
4.G.1	A	**Lines and Angles** Lesson 1: Identify and draw points, lines, line segments, rays, and angles. Recognize them in various contexts and familiar figures. Lesson 2: Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles. Lesson 3: Identify, define, and draw perpendicular lines. Lesson 4: Identify, define, and draw parallel lines.	4
4.MD.5 **4.MD.6**	B	**Angle Measurement** Lesson 5: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of a turn. Explore benchmark angles using the protractor. Lesson 6: Use varied protractors to distinguish angle measure from length measurement. Lesson 7: Measure and draw angles. Sketch given angle measures, and verify with a protractor. Lesson 8: Identify and measure angles as turns and recognize them in various contexts.	4
		Mid-Module Assessment: Topics A–B (assessment ½ day, return ½ day, remediation or further application 1 day)	2
4.MD.7	C	**Problem Solving with the Addition of Angle Measures** Lesson 9: Decompose angles using pattern blocks. Lessons 10–11: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.	3

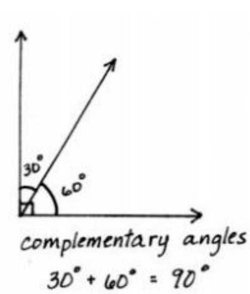
Standards		Topics and Objectives	Days
4.G.1 **4.G.2** **4.G.3**	D	**Two-Dimensional Figures and Symmetry** Lesson 12: Recognize lines of symmetry for given two-dimensional figures. Identify line-symmetric figures, and draw lines of symmetry. Lesson 13: Analyze and classify triangles based on side length, angle measure, or both. Lesson 14: Define and construct triangles from given criteria. Explore symmetry in triangles. Lesson 15: Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size. Lesson 16: Reason about attributes to construct quadrilaterals on square or triangular grid paper.	5
		End-of-Module Assessment: Topics A–D (assessment ½ day, return ½ day, remediation or further application 1 day)	2
Total Number of Instructional Days			**20**

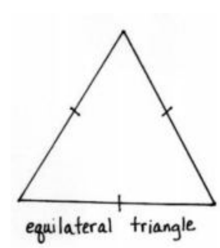
Terminology

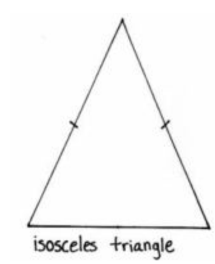
New or Recently Introduced Terms

- Acute angle (angle with a measure of less than 90°)
- Acute triangle (triangle with all interior angles measuring less than 90°)
- Adjacent angle (Two angles $\angle AOC$ and $\angle COB$, with a common side $\overrightarrow{OC}$, are *adjacent angles* if C is in the interior of $\angle AOB$.)
- Angle (union of two different rays sharing a common vertex, e.g., $\angle ABC$)
- Arc (connected portion of a circle)



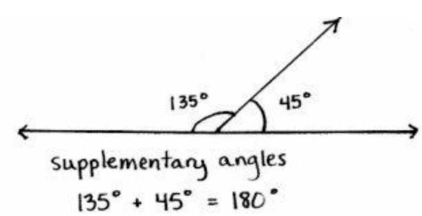






- Collinear (Three or more points are *collinear* if there is a line containing all of the points; otherwise, the points are *non-collinear*.)

- Complementary angles (two angles with a sum of 90°)
- Degree, degree measure of an angle (Subdivide the length around a circle into 360 arcs of equal length. A central angle for any of these arcs is called a *one-degree angle* and is said to have an angle measure of 1°.)
- Diagonal (straight lines joining two opposite corners of a straight-sided shape)
- Equilateral triangle (triangle with three equal sides)
- Figure (set of points in the plane)
- Interior of an angle (the convex[1] region defined by the angle)
- Intersecting lines (lines that contain at least one point in common)
- Isosceles triangle (triangle with at least two equal sides)
- Length of an arc (circular distance around the arc)
- Line (straight path with no thickness that extends in both directions without end, e.g., $\overleftrightarrow{AB}$)
- Line of symmetry (line through a figure such that when the figure is folded along the line, two halves are created that match up exactly)
- Line segment (two points, A and B, together with the set of points on $\overleftrightarrow{AB}$ between A and B, e.g., $\overline{AB}$)
- Obtuse angle (angle with a measure greater than 90°, but less than 180°)
- Obtuse triangle (triangle with an interior obtuse angle)
- Parallel (two lines in a plane that do not intersect, e.g., $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$)
- Perpendicular (Two lines are *perpendicular* if they intersect, and any of the angles formed between the lines is a 90° angle, e.g., $\overleftrightarrow{EF} \perp \overleftrightarrow{GH}$.)
- Point (precise location in the plane)
- Protractor (instrument used in measuring or sketching angles)
- Ray (The $\overrightarrow{OA}$ is the point O and the set of all points on $\overleftrightarrow{OA}$ that are on the same side of O as the point A.)
- Right angle (angle formed by perpendicular lines, measuring 90°)
- Right triangle (triangle that contains one 90° angle)
- Scalene triangle (triangle with no sides or angles equal)
- Straight angle (angle that measures 180°)
- Supplementary angles (two angles with a sum of 180°)



- Triangle (A *triangle* consists of three non-collinear points and the three line segments between them. The three segments are called the *sides* of the triangle, and the three points are called the *vertices.*)
- Vertex (a point, often used to refer to the point where two lines meet, such as in an angle or the corner of a triangle)
- Vertical angles (When two lines intersect, any two non-adjacent angles formed by those lines are called *vertical angles* or *vertically opposite angles*.)

[1]In Grade 4, a picture will suffice. A precise definition of convexity is given in high school geometry.

Familiar Terms and Symbols

- Decompose (process of separating something into smaller components)
- Parallelogram (quadrilateral with two pairs of parallel sides)
- Polygon (closed two-dimensional figure with straight sides)
- Quadrilateral (polygon with four sides)
- Rectangle (quadrilateral with four right angles)
- Rhombus (quadrilateral with all sides of equal length)
- Square (rectangle with all sides of equal length)
- Sum (result of adding two or more numbers)
- Trapezoid (quadrilateral with at least one pair of parallel sides)

Suggested Tools and Representations

- Folded paper models
- Pattern blocks
- Protractors of various diameters, including a 360° and 180° protractor
- Rectangular and triangular grid paper
- Right angle template (created in Lesson 2), set square
- Ruler (used to measure length), straightedge (used to draw straight lines)

Scaffolds[2]

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*."

[2]Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website www.p12.nysed.gov/specialed/aim for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	4.MD.5 4.MD.6 4.G.1
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	4.MD.5 4.MD.6 4.MD.7 4.G.1 4.G.2 4.G.3

Topic A

Lines and Angles

4.G.1

Focus Standard:		4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
Instructional Days:		4	
Coherence	**-Links from:**	G2–M8	Time, Shapes, and Fractions as Equal Parts of Shapes
		G3–M7	Geometry and Measurement Word Problems
	-Links to:	G5–M5	Addition and Multiplication with Volume and Area

Topic A begins with students drawing points, lines, line segments, and rays and identifying these in various contexts and familiar figures. As they continue, students recognize that two rays sharing a common endpoint form an angle. In Lesson 2, students create right angles through a paper-folding activity and identify right angles in their environment by comparison with the right angles they have made. They also draw acute, right, and obtuse angles. This represents students' first experience with angle comparison and the idea that one angle's measure can be greater (obtuse) or less (acute) than that of a right angle.

Next, students use their understanding of angles to explore relationships between pairs of lines, defining and recognizing intersecting, perpendicular, and parallel lines. In Lesson 3, students' knowledge of right angles leads them to identify, define, and construct perpendicular lines. In Lesson 4, students learn lines that never intersect are also called parallel and have a special relationship. Students use, in conjunction with a straightedge, the right-angle templates that they created in Lesson 2 to construct parallel lines (**4.G.1**). Activities using different grids provide students with the opportunity to explore the concepts of perpendicularity and parallelism.

A Teaching Sequence Toward Mastery of Lines and Angles

Objective 1: **Identify and draw points, lines, line segments, rays, and angles. Recognize them in various contexts and familiar figures.**
(Lesson 1)

Objective 2: **Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles.**
(Lesson 2)

Objective 3: **Identify, define, and draw perpendicular lines.**
(Lesson 3)

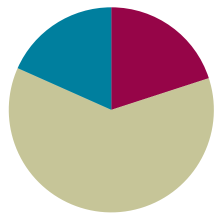
Objective 4: **Identify, define, and draw parallel lines.**
(Lesson 4)

Lesson 1

Objective: Identify and draw points, lines, line segments, rays, and angles. Recognize them in various contexts and familiar figures.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Concept Development	(37 minutes)
Student Debrief	(11 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Multiply Mentally **4.OA.4** (4 minutes)
- Add and Subtract **4.NBT.4** (4 minutes)
- Sides, Angles, and Vertices **3.G.1** (4 minutes)

Multiply Mentally (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Grade 4 Module 3 content.

T: (Write 43 × 2 =____.) Say the multiplication sentence.
S: 43 × 2 = 86.
T: (Write 43 × 2 = 86. Below it, write 43 × 20 =____.) Say the multiplication sentence.
S: 43 × 20 = 860.
T: (Write 43 × 20 = 860. Below it, write 43 × 22 =____.) On your personal white boards, solve 43 × 22.
S: (Write 43 × 22 = 946.)

Continue with the following possible sequence: 32 × 3, 32 × 20, 32 × 23, 21 × 4, 21 × 30, and 21 × 34.

Add and Subtract (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews the yearlong Grade 4 fluency standard for adding and subtracting using the standard algorithm.

T: (Write 654 thousands 289 ones.) On your personal white boards, write this number in standard form.

S: (Write 654,289.)

T: (Write 245 thousands 164 ones.) Add this number to 654,289 using the standard algorithm.

S: (Write 654,289 + 245,164 = 899,453 using the standard algorithm.)

Continue the process for 591,848 + 364,786.

T: (Write 918 thousands 670 ones.) On your board, write this number in standard form.

S: (Write 918,670.)

T: (Write 537 thousands 159 ones.) Subtract this number from 918,670 using the standard algorithm.

S: (Write 918,670 – 537,159 = 381,511 using the standard algorithm.)

Continue the process for 784,182 – 154,919 and 700,000 – 537,632.

Sides, Angles, and Vertices (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews features of various figures learned in previous grades.

T: (Project triangle.) Say the name of the shape.

S: Triangle.

T: How many sides are in a triangle?

S: Three.

T: How many angles are in a triangle?

S: Three.

T: (Point at one of the corners.) How many corners are in a triangle?

S: Three.

Continue the process for pentagon, hexagon, and rectangle.

Concept Development (37 minutes)

Materials: (T) Straightedge (S) Straightedge, blank paper

Problem 1: Draw, identify, and label points, a line segment, and a line.

T: I'd like to use my pencil to mark a specific location on my paper. How do you think I could do that?

S: You could put an X.

T: (Draw an X.) Okay, so, is this the location that I've marked? (Point to the upper right corner of the X.)

S: No! You marked the middle, where it crosses.

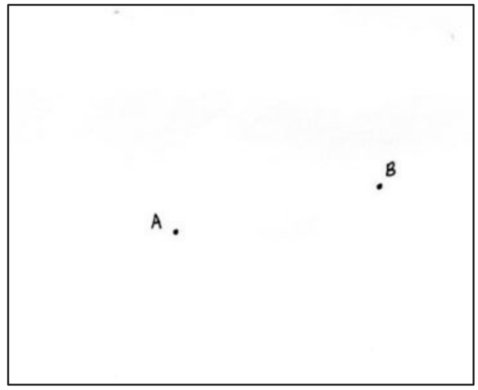
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Teachers may choose to provide square grid paper or triangle grid paper to students for today's Concept Development. If not providing grid paper, consider providing red markers for students to assist visual discrimination between the grid lines and lines they construct.

T: Oh, I see. Well, if that's all I want to mark, I don't really need all of these extra marks. Let's just mark the point with a small dot.

T: Let's try it. Mark a very specific location on your paper by drawing a small dot with your pencil tip.

T: Place your pencil tip in another location on your paper. Draw another small dot. The dots are a representation of a location.



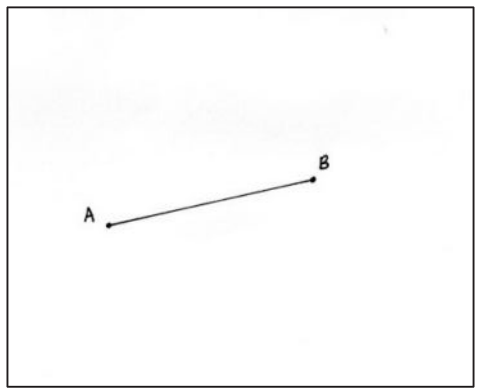
T: Notice that the dots, or **points**, that you and your neighbor drew are probably in different locations.

T: How many points could you draw on this paper?

S: A lot! → Too many to count. → I could draw points until my whole paper is filled with points.

T: When we draw our dots, they have size. However, we are trying to imagine and mark a location so precise that you couldn't even find it with the world's most powerful microscope.

T: To identify your two points, label each with a letter. (Label points A and B.)



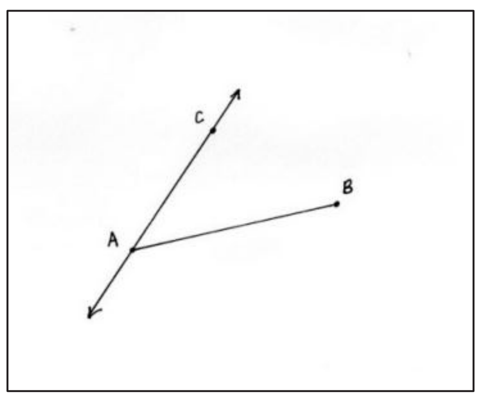
T: Use your straightedge to connect point A to point B. Compare what you drew to what your partner drew. Are your drawings the same? What is different about them?

S: One is longer than the other. → This one is horizontal, and this one looks more diagonal. → They are both straight. → They both begin at point A and end with point B.

T: Let's identify what we drew using the endpoints. We will call this **line segment** AB. (Write $\overline{AB}$.) Line segments have two endpoints.

T: We can also identify this line segment, or segment, as $\overline{BA}$.

T: Draw point C on your paper. Point C should not be located on $\overline{AB}$.

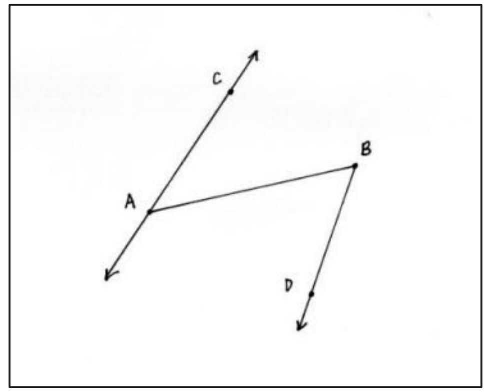


T: Using your straightedge, draw $\overline{AC}$. (Allow students time to draw.)

T: Could you extend $\overline{AC}$ to make it longer if you wanted to? If you had a really big piece of paper, could you continue to extend the segment in both directions? What if your paper extended forever? Could the segment go on forever? Let's extend $\overline{AC}$ just a bit on both ends, and draw an arrow on both ends to indicate that the line could continue going in either direction forever. We call this **line** AC. (Write $\overleftrightarrow{AC}$.)

T: What is different about $\overleftrightarrow{AC}$ and $\overline{AB}$?

S: This one is longer. → This one is shorter. → This one doesn't have points on the ends. Instead, it has arrows. → The line goes past points A and C. I guess the arrows mean that it's really longer than what we can see.

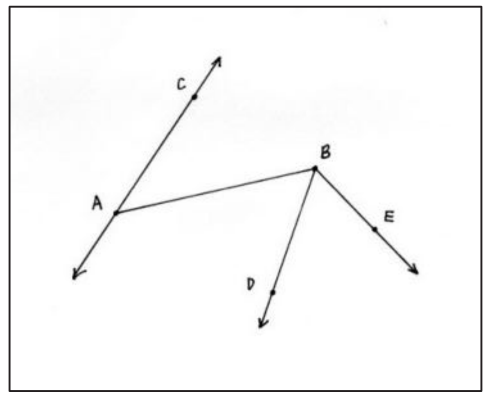


T: Yes, a line extends in both directions without an end. We show that by drawing arrows on the ends of a line. We can also represent it as line CA. (Write $\overleftrightarrow{CA}$.) We couldn't actually show a line that goes on forever. It's like trying to list every number. You just can't do it. What we actually drew is a representation of a line. A real line has no thickness, and it extends forever without end in both directions.

T: Compare the notation we used to identify line segment AB and line AC.

S: We can write them as $\overline{AB}$ or $\overline{BA}$, or $\overleftrightarrow{AC}$ or $\overleftrightarrow{CA}$. → We put a segment over the letters for a segment and a line with arrows for the line that goes on forever in both directions.

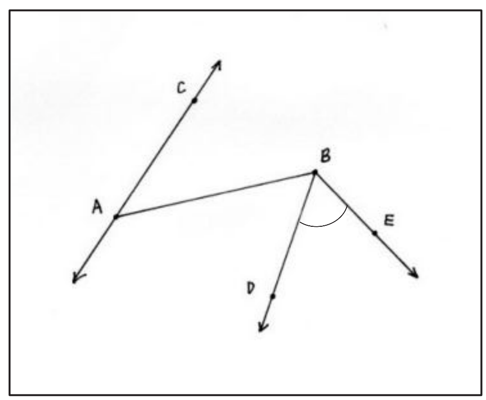
Problem 2: Draw, identify, and label rays and angles.



T: Draw point D. Point D should not be located on $\overline{AB}$ or anywhere on $\overleftrightarrow{AC}$ (including the parts where it might extend).

T: Using a straightedge, connect point B to point D. Use point B as the endpoint, and extend your line past point D. Draw an arrow at the end of this line.

Students draw. Observe their work.



T: Compare this part of the drawing, or **figure**, to the others you have drawn.

S: This one is longer. → This one is shorter. → They are all straight because I used a straightedge. → They all have two points. → This line has an endpoint and arrow.

T: Because it has an endpoint and arrow, we don't call this a line. We call it a **ray**. It has one endpoint that we think of as a starting point, and goes on forever in one direction. (Write $\overrightarrow{BD}$.) We record the letters in that order because the ray begins at point B and extends past point D. The ray symbol shows the direction of the line above the letters. Unlike before, we can't call it $\overrightarrow{DB}$ because that would imply that the ray starts at point D, which it does not.

T: Draw point E. Make sure point E does not lie in line with $\overrightarrow{BD}$, $\overline{AB}$, or $\overleftrightarrow{AC}$. Draw $\overrightarrow{BE}$.

Students draw. Observe their work.

T: Touch point B with your pencil. Trace along the line to point D. Now, touch point B. Trace along the line to point E. Discuss the connection of $\overrightarrow{BD}$ and $\overrightarrow{BE}$.

S: Both rays have the same endpoint. → Both rays are connected. → Mine looks like a corner of a rectangle.

T: Both rays originate at point B and extend out. Any two rays sharing the same endpoint create an **angle**.

T: We can call this angle DBE. (Write $\angle DBE$.)

S: Or $\angle EBD$!

T: To identify this angle in the figure, we will draw an **arc**. (Draw an arc to identify $\angle DBE$.) With your partner, identify two other angles in your figures.

Problem 3: Draw, identify, and label points, line segments, and angles in a familiar figure.

T: Quickly sketch a rectangle. Use your straightedge. Do you see any lines or line segments? Do you see any angles?

S: I see four line segments, four points where the line segments meet, and four angles!

T: Identify the line segments with your partner using the correct notation.

S: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$. There are four of them!

T: How many line segments are there in a square? A rhombus?

S: There are four in each one.

T: You mentioned angles earlier. I thought an angle was made of two rays. Where do you see rays in this picture?

S: I don't see any. → But that still looks like an angle where $\overline{AD}$ and $\overline{AB}$ meet. → I could draw an arrow on the end of $\overline{AB}$ and $\overline{AD}$ to make rays.

T: You're right. Each of the segments is part of a larger ray. However, we don't have to draw them to imagine that they're there. So, do the segments $\overline{AB}$ and $\overline{AD}$ meet to form an angle?

S: Yes!

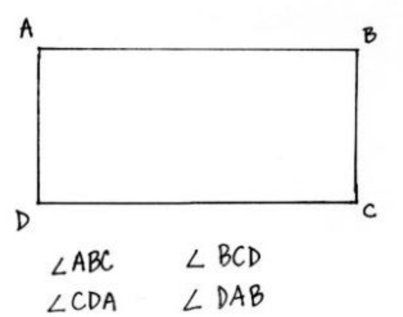
T: Name each of the angles that lie inside of the rectangle. Identify the angles using the correct notation.

S: $\angle ABC$, $\angle BCD$, $\angle CDA$, and $\angle DAB$.

NOTE TO TEACHERS ABOUT RAYS:

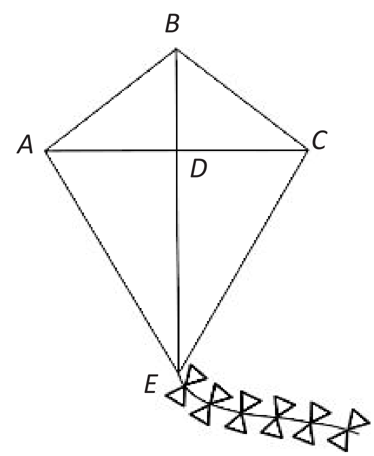
Traditionally, elementary school textbooks use a line with a single arrowhead to denote rays, for example, $\overrightarrow{BD}$. In some middle and high school texts, however, this same notation is sometimes used instead for vectors, resulting in an alternate notation for rays, a line with a half arrowhead, for example, $\overrightarrow{BD}$.

To alleviate confusion when observing the consistency and coherence of the curriculum as a whole, *A Story of Units* uses a half arrowhead as the notation for a ray, and *A Story of Ratios* uses the notation of a single arrowhead for a vector. Though the typed notation in Grade 4 always uses the half arrowhead, sample student work depicts both variations in representing rays, some with a half arrowhead, and some with the full arrowhead notation. Both representations for a *ray* can be viewed as correct. Individual classrooms may choose to adopt either convention for the *ray* notation.

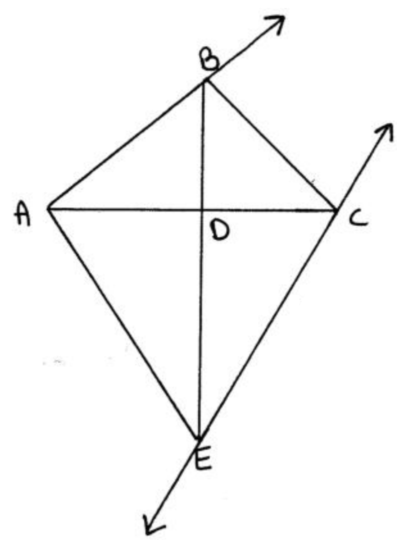


Problem 4: Analyze a familiar figure.

T: With a partner, make a list of the new terms that we learned today.
S: Point, line segment (segment), line, ray, angle, arc, and figure.
T: Let's look at the first figure that we drew. What do you see?
S: Points, line segments, lines, rays, and angles.
T: Did we create a figure that looks familiar?
S: No! It doesn't really look like anything that I've seen.
T: Look at the second figure that we drew. What do you see?
S: That looks familiar! → It has points, line segments (rays), and angles. Combined, they make a rectangle!
T: Here's another familiar figure. (Draw or project the figure of a kite.)



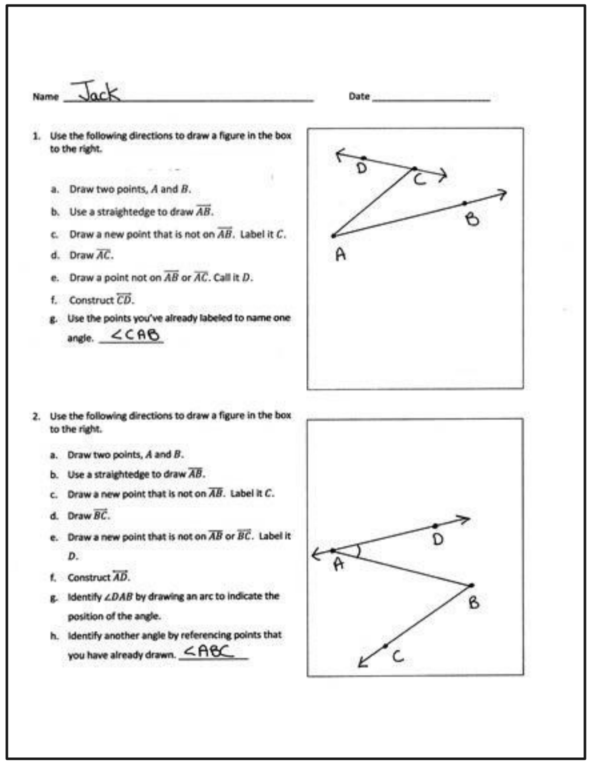
S: It's a kite!
T: Let's see if we can find points, line segments, lines, rays, and angles. Are there any points?
S: There are lots of points. → There are too many points to count.
MP.6 T: Let's identify the points that show the corners.
S: (Label points A, B, C, D, and E.)
T: What else do you see? How about segments and angles?
S: (Identify segments and angles by name, working first with a partner to identify, and then share with the whole group.)
T: Are there any rays or lines?
S: No!
T: Think again! Segments, or line segments, are just a part of a line. If we extend $\overline{AB}$ in one direction, we represent $\overrightarrow{AB}$. And if we extend $\overline{AB}$ in both directions, we represent $\overleftrightarrow{AB}$, which includes $\overline{AB}$ and $\overrightarrow{AB}$.
S: I get it! Lines, rays, and segments are all related!
T: Draw the kite and then extend the segments to represent a ray and a line. (Demonstrate how to draw the kite, starting with a *t* shape and then joining the endpoints with a straightedge.)
S: (Draw the kite, and then represent a ray and a line.)



Problem Set (10 minutes)

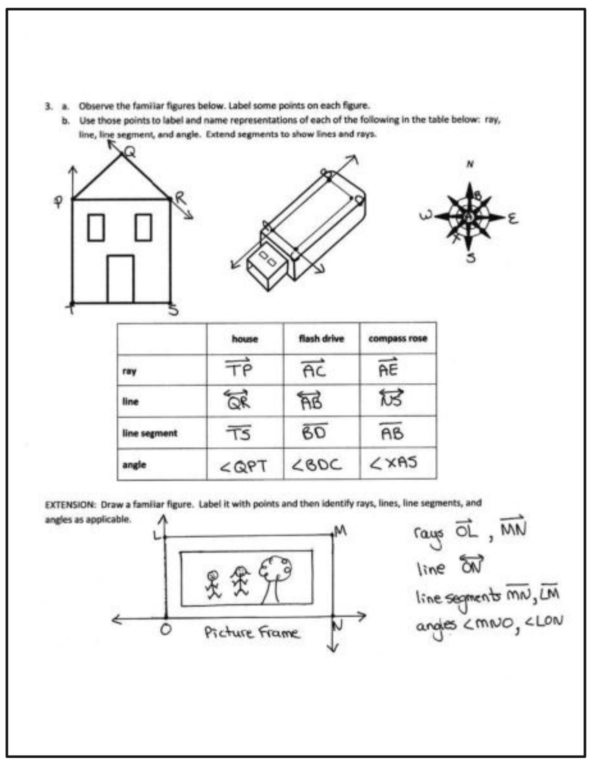
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the purposeful sequencing of the Problem Set guide your selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Consider assigning incomplete problems for homework or at another time during the day.


Name Jack Date

1. Use the following directions to draw a figure in the box to the right.
   a. Draw two points, A and B.
   b. Use a straightedge to draw $\overline{AB}$.
   c. Draw a new point that is not on $\overline{AB}$. Label it C.
   d. Draw $\overline{AC}$.
   e. Draw a point not on $\overline{AB}$ or $\overline{AC}$. Call it D.
   f. Construct $\overleftrightarrow{CD}$.
   g. Use the points you've already labeled to name one angle. <CAB

2. Use the following directions to draw a figure in the box to the right.
   a. Draw two points, A and B.
   b. Use a straightedge to draw $\overline{AB}$.
   c. Draw a new point that is not on $\overline{AB}$. Label it C.
   d. Draw $\overline{BC}$.
   e. Draw a new point that is not on $\overline{AB}$ or $\overline{BC}$. Label it D.
   f. Construct $\overrightarrow{AD}$.
   g. Identify $\angle DAB$ by drawing an arc to indicate the position of the angle.
   h. Identify another angle by referencing points that you have already drawn. <ABC


3. a. Observe the familiar figures below. Label some points on each figure.
   b. Use those points to label and name representations of each of the following in the table below: ray, line, line segment, and angle. Extend segments to show lines and rays.

	house	flash drive	compass rose
ray	$\overrightarrow{TP}$	$\overrightarrow{AC}$	$\overrightarrow{AE}$
line	$\overleftrightarrow{QR}$	$\overleftrightarrow{AB}$	$\overleftrightarrow{NS}$
line segment	$\overline{TS}$	$\overline{BD}$	$\overline{AB}$
angle	<QPT	<BDC	<XAS

EXTENSION: Draw a familiar figure. Label it with points and then identify rays, lines, line segments, and angles as applicable.

Student Debrief (11 minutes)

Lesson Objective: Identify and draw points, lines, line segments, rays, and angles. Recognize them in various contexts and familiar figures.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 3, the image of the USB drive has several lines with curved edges. We often talk about curved lines and straight lines. How are those lines different from the lines we learned about today?
- Compare your **figure** to your partner's for Problem 1. How are they alike? How are they different?

- A **point** indicates a precise location with no size, only position. Points are infinitely small. Why do we mark them with a dot? Won't our pencil marks have width? Won't our pencil marks actually cover many points since the dots we draw have width and points do not?
- Just like a point, a **line** has no thickness. Can we draw a line that has no thickness, or will we always have to imagine that particular attribute? Why do we draw it on paper with thickness?
- How is a **line segment** different from a line?
- How many corners does a triangle have? A square? A quadrilateral? How does that relate to the number of angles a polygon has?
- How are a **ray** and a line similar? How are they different?
- How are **angles** formed? Where have you seen angles before? How does an **arc** help to identify an angle?
- Why is it hard to find real life examples of lines, points, and rays?
- How does your understanding of a number line connect to this lesson on lines?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ____ Date ______________________

1. Use the following directions to draw a figure in the box to the right.

   a. Draw two points: A and B.

   b. Use a straightedge to draw $\overrightarrow{AB}$.

   c. Draw a new point that is not on $\overrightarrow{AB}$. Label it C.

   d. Draw $\overline{AC}$.

   e. Draw a point not on $\overrightarrow{AB}$ or $\overline{AC}$. Call it D.

   f. Construct $\overleftrightarrow{CD}$.

   g. Use the points you've already labeled to name one angle. ____________

2. Use the following directions to draw a figure in the box to the right.

   a. Draw two points: A and B.

   b. Use a straightedge to draw $\overline{AB}$.

   c. Draw a new point that is not on $\overline{AB}$. Label it C.

   d. Draw $\overrightarrow{BC}$.

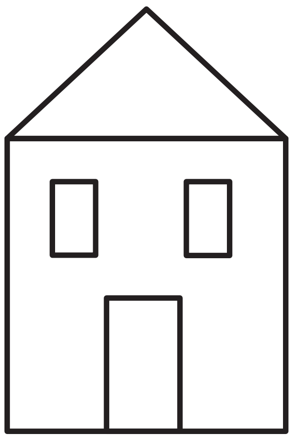
   e. Draw a new point that is not on $\overline{AB}$ or $\overrightarrow{BC}$. Label it D.

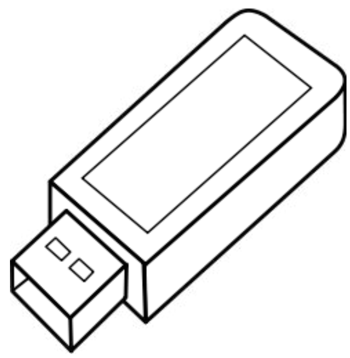
   f. Construct $\overleftrightarrow{AD}$.

   g. Identify $\angle DAB$ by drawing an arc to indicate the position of the angle.

   h. Identify another angle by referencing points that you have already drawn. _____________

3. a. Observe the familiar figures below. Label some points on each figure.
   b. Use those points to label and name representations of each of the following in the table below: ray, line, line segment, and angle. Extend segments to show lines and rays.



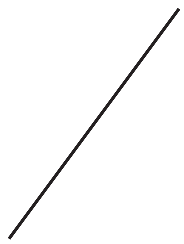


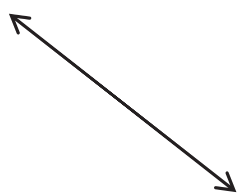
	House	Flash drive	Compass rose
Ray			
Line			
Line segment			
Angle			

Extension: Draw a familiar figure. Label it with points, and then identify rays, lines, line segments, and angles as applicable.

Name ____ Date ____________________

1. Draw a line segment to connect the word to its picture.





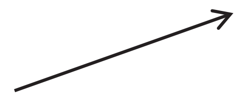
Ray

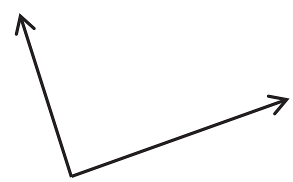
Line

Line segment

Point

Angle





2. How is a line different from a line segment?

Name ____ Date ____________________

1. Use the following directions to draw a figure in the box to the right.

   a. Draw two points: W and X.

   b. Use a straightedge to draw $\overrightarrow{WX}$.

   c. Draw a new point that is not on $\overrightarrow{WX}$. Label it Y.

   d. Draw $\overline{WY}$.

   e. Draw a point not on $\overrightarrow{WX}$ or $\overline{WY}$. Call it Z.

   f. Construct $\overleftrightarrow{YZ}$.

   g. Use the points you've already labeled to name one angle. ____________

2. Use the following directions to draw a figure in the box to the right.

   a. Draw two points: W and X.

   b. Use a straightedge to draw $\overline{WX}$.

   c. Draw a new point that is not on $\overline{WX}$. Label it Y.

   d. Draw $\overrightarrow{WY}$.

   e. Draw a new point that is not on $\overrightarrow{WY}$ or on the line containing $\overline{WX}$. Label it Z.

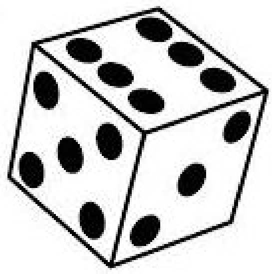
   f. Construct $\overleftrightarrow{WZ}$.

   g. Identify $\angle ZWX$ by drawing an arc to indicate the position of the angle.

   h. Identify another angle by referencing points that you have already drawn. ____________

3. a. Observe the familiar figures below. Label some points on each figure.

   b. Use those points to label and name representations of each of the following in the table below: ray, line, line segment, and angle. Extend segments to show lines and rays.



	Clock	Die	Number line
Ray			
Line			
Line segment			
Angle			

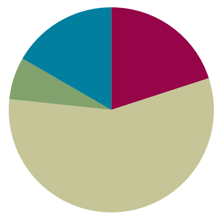
Extension: Draw a familiar figure. Label it with points, and then identify rays, lines, line segments, and angles as applicable.

Lesson 2

Objective: Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(4 minutes)
■ Concept Development	(34 minutes)
■ Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Multiply Using Partial Products **4.NBT.4** (3 minutes)
- Identify Two-Dimensional Figures **4.G.1** (4 minutes)
- Physiometry **4.G.1** (5 minutes)

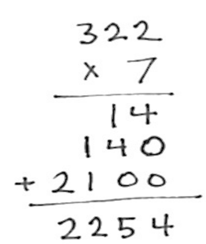
Multiply Using Partial Products (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity serves as a review of the Concept Development in Grade 4 Module 3 Lessons 7–8.

T: (Write 322 × 7.) Say 322 in unit form.
S: 3 hundreds 2 tens 2 ones.
T: Say it as a three-product addition expression in unit form.
S: 3 hundreds × 7 + 2 tens × 7 + 2 ones × 7.
T: Write 322 × 7 vertically and solve using the partial product strategy.

Continue with the following possible sequence: 5 thousands 1 hundred 3 tens 2 ones × 3 and 4 × 4,312.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Scaffold the Multiply Using Partial Products fluency activity by giving a clear example with a simpler problem, followed immediately by a similar two-digit problem.

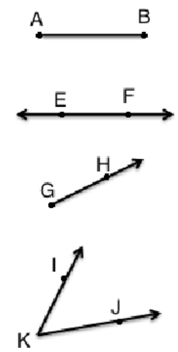
T: (Write 32 × 7.) Say 32 in unit form.
T: 3 tens × 7 + 2 ones × 7 is a two-product addition expression in unit form. What are the two products?
T: (Write 43 × 6.) Say 43 in unit form.
T: Write 43 × 6 as a two-product addition expression in unit form.

Once students are successful at the simpler level, move forward to three-digit examples.

Identify Two-Dimensional Figures (4 minutes)

Materials: (S) Personal white board, straightedge

Note: This fluency activity reviews terms learned in Lesson 1.



T: (Project $\overline{AB}$. Point to point A.) Say the term for what I'm pointing to.
S: Point A.
T: (Point to point B.) Say the term.
S: Point B.
T: (Point to $\overline{AB}$.) Say the term.
S: $\overline{AB}$.
T: Use your straightedge to draw $\overline{CD}$ on your personal white boards.
S: (Draw a segment with endpoints C and D.)

Continue with the following possible sequence: $\overleftrightarrow{EF}$, $\overrightarrow{GH}$, and $\angle IKJ$.

Physiometry (5 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews Lesson 1 terms.

T: Stand up.
S: (Stand up.)
T: (Extend arms straight so that they are parallel with the floor. Clench both hands into fists.) What kind of figure do you think I'm modeling?
S: A line segment.
T: What do you think my fists might represent?
S: Points.
T: Make a line segment with your arms.
S: (Extend arms straight so that they are parallel with the floor. Clench both hands into fists.)
T: (Keep arms extended. Open fists, and point to side walls.) What kind of figure do you think I'm modeling now?
S: A line.
T: What do you think my pointing fingers might represent?
S: Arrows.
T: Make a line.
S: (Keep arms extended, but open hands and point to the side walls.)
T: (Clench one hand in a fist, and extend arm forward to students.) Say the figure that you think I'm modeling.
S: A point.
T: Make a point.
S: (Clench one hand in a fist, and extend arm forward.)

T: (Extend arms straight so that they are parallel with the floor. Clench one hand in a fist, and leave the other hand open, pointing to a side wall.) Say the figure you think I'm modeling.

S: A ray.

T: Make a ray.

S: (Extend arms straight so that they are parallel with the floor. Clench one hand in a fist, and leave the other hand open, pointing to a side wall.)

T: (Extend arms in an acute angle.) Say the figure I'm modeling.

S: An angle.

T: Make an angle.

S: (Extend arms in an acute angle.)

Next, move between figures with the following possible sequence: ray, angle, line segment, point, angle made of two segments, and line.

Close the session by quickly cautioning students against the incorrect idea that lines and points are as thick as arms and fists when they are actually infinitely small.

Application Problem (4 minutes)

1. Figure 1 has three points. Connect points A, B, and C with as many line segments as possible.
2. Figure 2 has four points. Connect points D, E, F, and G with as many line segments as possible.

Figure 1

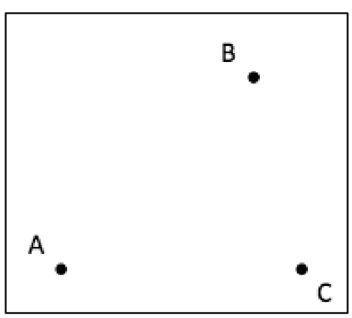


Figure 2

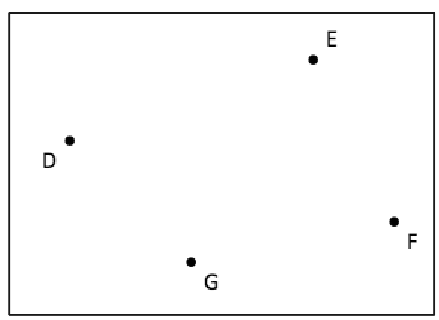


Figure 1

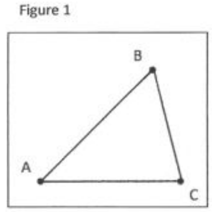
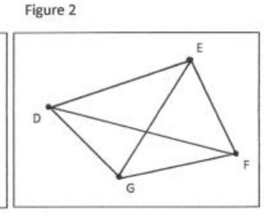


Figure 2



Note: This Application Problem builds on the previous lesson in that students use points to draw line segments. Review Lesson 1 by engaging students in a discussion about the representation of a point and how segments are related to lines and rays.

Concept Development (34 minutes)

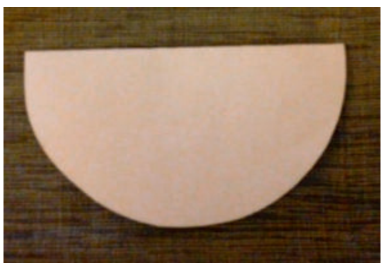
Materials: (T/S) Paper, straightedge, angles (Template)

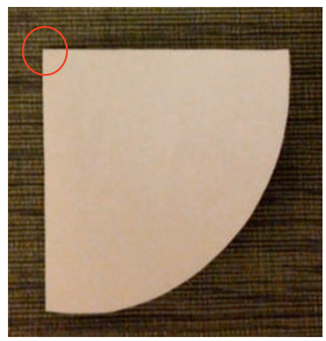
Note: The following activity and images for the paper-folding activity are modeled using a large circle. Any sized paper and any shaped paper is sufficient for this activity. Include a variety of papers for this activity. Students find that any paper folded twice results in a right angle template.

Problem 1: Create right angles through a paper folding activity.

T: Everyone, hold your circle, and fold it in half like this. (Demonstrate.)

T: Then, fold it in half again, like this. (Demonstrate.)





T: Do you notice any angles in our folded circle?

S: Yes! This corner right here!

T: Yes, that shared endpoint is where these two lines meet to form an angle.

T: Now, trace both lines with your fingers, starting at their shared endpoint.

T: Point to the angle we formed. This is called a **right angle**.

T: Using your folded circle as a reference, look around the room for right angles. With your partner, create a list of objects that have right angles.

S: Door, book, desk, floor tile, window, paper, and white board.

T: Use the words *equal to* for describing the relationship between your right angle template and the other right angles you found around the room.

S: The angles on the corners of the floor tile are equal to the right angle on my folded paper. → The corner of the door is equal to a right angle.

Problem 2: Determine whether angles are equal to, greater than, or less than a right angle.

T: Use your right angle template to find all of the right angles on the angles template. How will you know if it's a right angle?

S: The sides of the right angle template will match exactly with the sides of the angles. (Find the right angles on the angles template.)

T: Let's identify the right angles with a symbol. We put a square in the corner of the angle, or the **vertex**, to show that it is a right angle (demonstrate). It's your turn.

Students identify each right angle by putting a right angle symbol at the vertex.

T: What do you notice about the other angles on the angles template?

S: They are not right angles. → Some are less than right angles. → Some are greater than right angles.

T: But what if one looks *almost, but not quite like* a right angle?

S: It would be hard to tell. → We can use our right angle template!

T: Place your right angle template on $\angle B$ so that the corner of the template and one of the sides lines up with the corner and side of the angle. What do you notice?

S: The two rays make an opening that is smaller than the right angle. → I can only see one ray of the angle. → This angle fits inside the right angle.

T: Find the other angles that are less than a right angle. Write *less* next to them.

Students identify other angles that are less than a right angle.

MP.5

T: Are the remaining angles greater or less than a right angle?

S: Greater!

T: Place your right angle template on $\angle C$ so that the corner of the template and one of the sides lines up with the corner and side of the angle. What do you notice?

S: My right angle fits inside of it. → When I line up my right angle along this side, the other side of the angle is outside my right angle. → It's greater than a right angle.

T: Verify that each of the other remaining angles is greater than a right angle using your template. Write *greater* next to each angle.

T: We just identified three groups of angles. What are they?

S: Some are right angles. Some are less than right angles. Some are greater than right angles.

T: $\angle A$, $\angle E$, and $\angle G$ are right angles. $\angle B$, $\angle D$, and $\angle F$ are examples of another type of angle. We call them **acute angles**. Describe an acute angle.

S: An acute angle is an angle that is less than a right angle.

T: Look around the classroom for acute angles.

S: I see one by the flagpole.

T: What two objects represent the rays or sides of your acute angle?

S: The flagpole and the wall.

T: When we align the right angle template against the wall and follow the flagpole, it goes inside the interior of the right angle. (Demonstrate.)

T: $\angle C$, $\angle H$, $\angle I$, and $\angle J$ are examples of another type of angle. We call them **obtuse angles**. Describe an obtuse angle.

S: An obtuse angle is an angle that is greater than a right angle.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

To assist with building math vocabulary for English language learners and other students, point to a picture of acute, right, and obtuse angles each time they are mentioned during today's lesson. Consider building into the instruction additional checks for understanding. Additionally, learners may benefit from adding these new terms and corresponding pictures to their personal math dictionaries before or after the lesson.

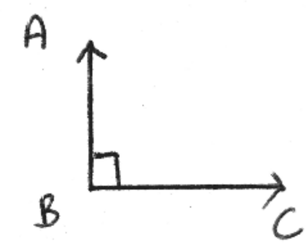
T: Look around the classroom for obtuse angles.

S: The door is creating an obtuse angle right now.

T: What two objects represent the sides composing your obtuse angle?

S: The wall and the bottom of the door.

Problem 3: Draw right, acute, and obtuse angles.



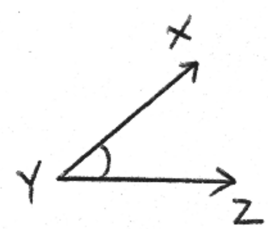
T: Using your straightedge, draw one ray. Use your right angle template as a guide. Then, draw a second ray, creating a right angle, $\angle ABC$. Will you label the two rays' shared endpoint A, B, or C?

S: The shared endpoint should be labeled B because it is $\angle ABC$. Point B is in the middle.

T: When you are finished drawing your angle, use your template to check your partner's angle. Do everyone's right angles look exactly the same?

S: Not all of them. → Our angles are facing different directions, but the angle looks exactly the same.

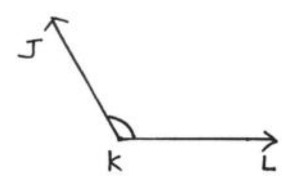
T: Right angles are represented with a little square in the angle. (Demonstrate). Add one to your angle.



T: Next, using the same process, draw an acute angle labeled $\angle XYZ$. When you are finished, check your partner's angle.

T: What did you notice?

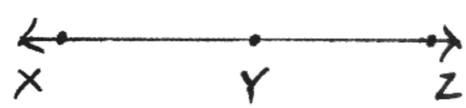
S: This time, they all look different. → I notice that our angles are facing different directions, but also, the sizes of the angles look different. → All are different sizes, but all are less than a right angle. → Right angles are exactly the same, but acute angles can be anything less than a right angle, so there are a lot of them.



T: *Acute* indicates less than a right angle, so everyone in our class may have drawn a different angle!

T: For all angles that are not equal to a right angle, we can draw an arc to show the angle. (Demonstrate.) Add one to your angle.

T: Lastly, draw an obtuse angle labeled $\angle JKL$, and draw an arc to show the angle.



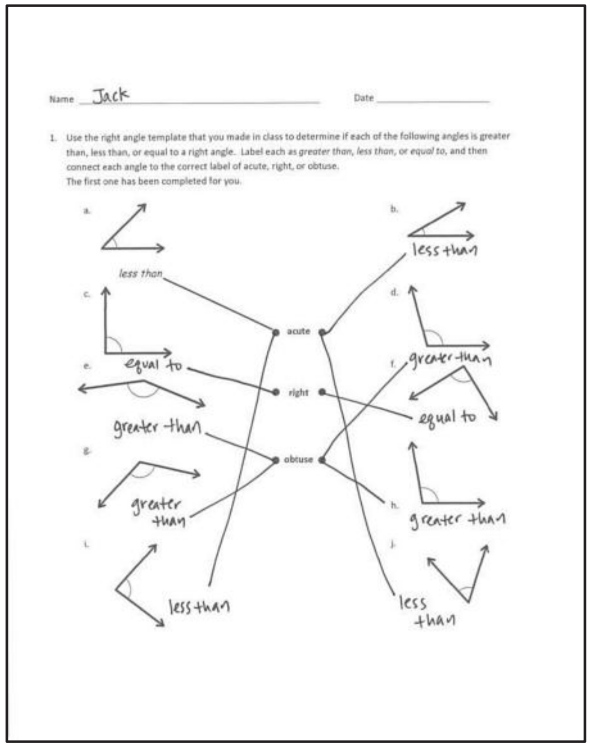
T: (Draw a straight line and label points X, Y, and Z on the line.) Identify this angle.

S: I don't see an angle. → Isn't it just a line? Line XYZ.

T: There are two rays, $\overrightarrow{YX}$ and $\overrightarrow{YZ}$. So yes, it is $\angle XYZ$. However, since all three points lie on a line, we have a special angle. We call this a **straight angle**. Obtuse angles are smaller than a straight angle, but larger than a right angle. Check your partner's work. Use your right angle template and straightedge as guides.

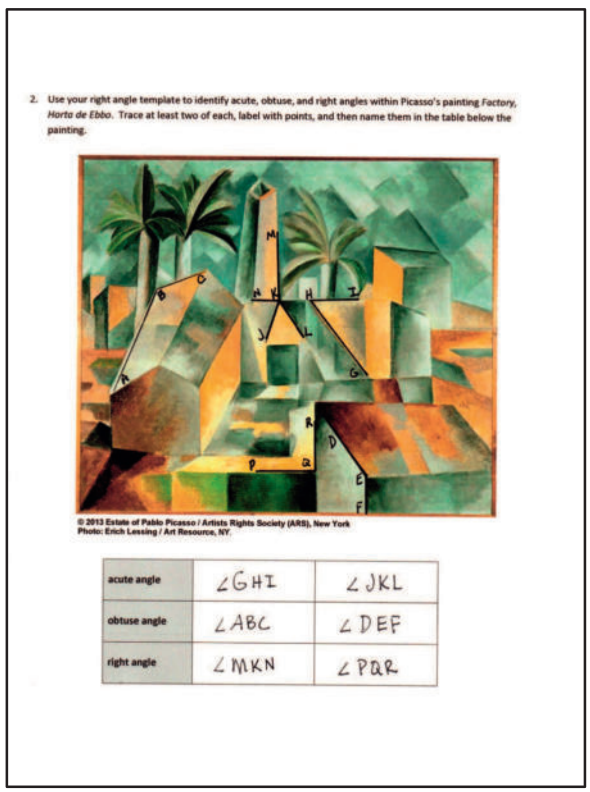
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



Name Jack Date

1. Use the right angle template that you made in class to determine if each of the following angles is greater than, less than, or equal to a right angle. Label each as *greater than, less than, or equal to*, and then connect each angle to the correct label of acute, right, or obtuse.
The first one has been completed for you.



2. Use your right angle template to identify acute, obtuse, and right angles within Picasso's painting *Factory, Horta de Ebbo*. Trace at least two of each, label with points, and then name them in the table below the painting.

© 2013 Estate of Pablo Picasso / Artists Rights Society (ARS), New York
Photo: Erich Lessing / Art Resource, NY.

acute angle	∠GHI	∠JKL
obtuse angle	∠ABC	∠DEF
right angle	∠MKN	∠PQR

Student Debrief (10 minutes)

Lesson Objective: Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles.

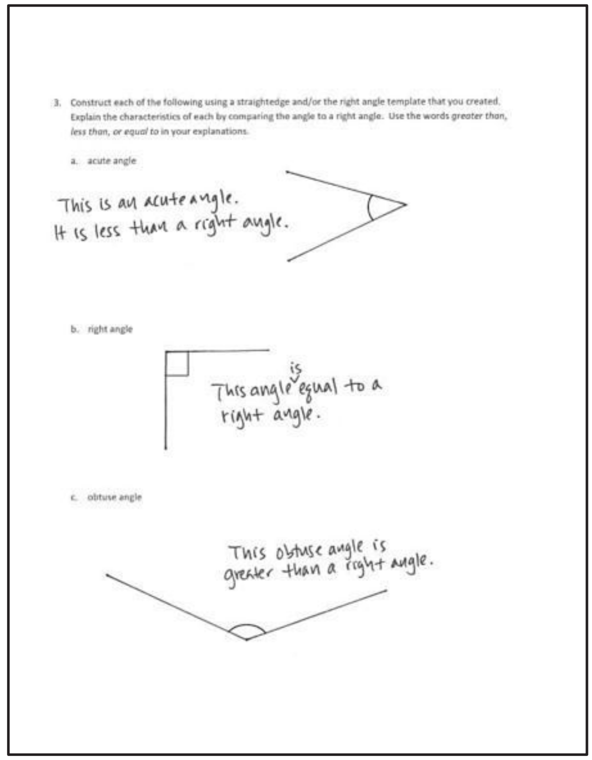
The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Problems 1(c) and 1(f) are both **right angles**. Describe their position. Does the orientation of an angle determine whether it is right, acute, or obtuse?
- In Problem 3(a), each ray shared the same endpoint. The shared endpoint is called a **vertex.** Label the points on your angles in Problem 3. Identify the vertex in Problems 3(b) and 3(c) with your partner.
- When we first found **obtuse angles**, we said that all of our examples were angles greater than a right angle, but then you learned a **straight angle** is a straight line. How did your understanding of the term *obtuse angle* grow? How did that understanding help you draw your angle for Problem 3(c)? What is the difference between a straight angle and a line?

- Where else in your environment have you seen right angles?
- How did the right angle template help you recognize and draw angles? How can a right angle template help you recognize an **acute angle**?
- How does the right angle template help you visualize the **interior of an angle**? Where would I find the interior of an angle that I've drawn? What does the exterior of an angle refer to?

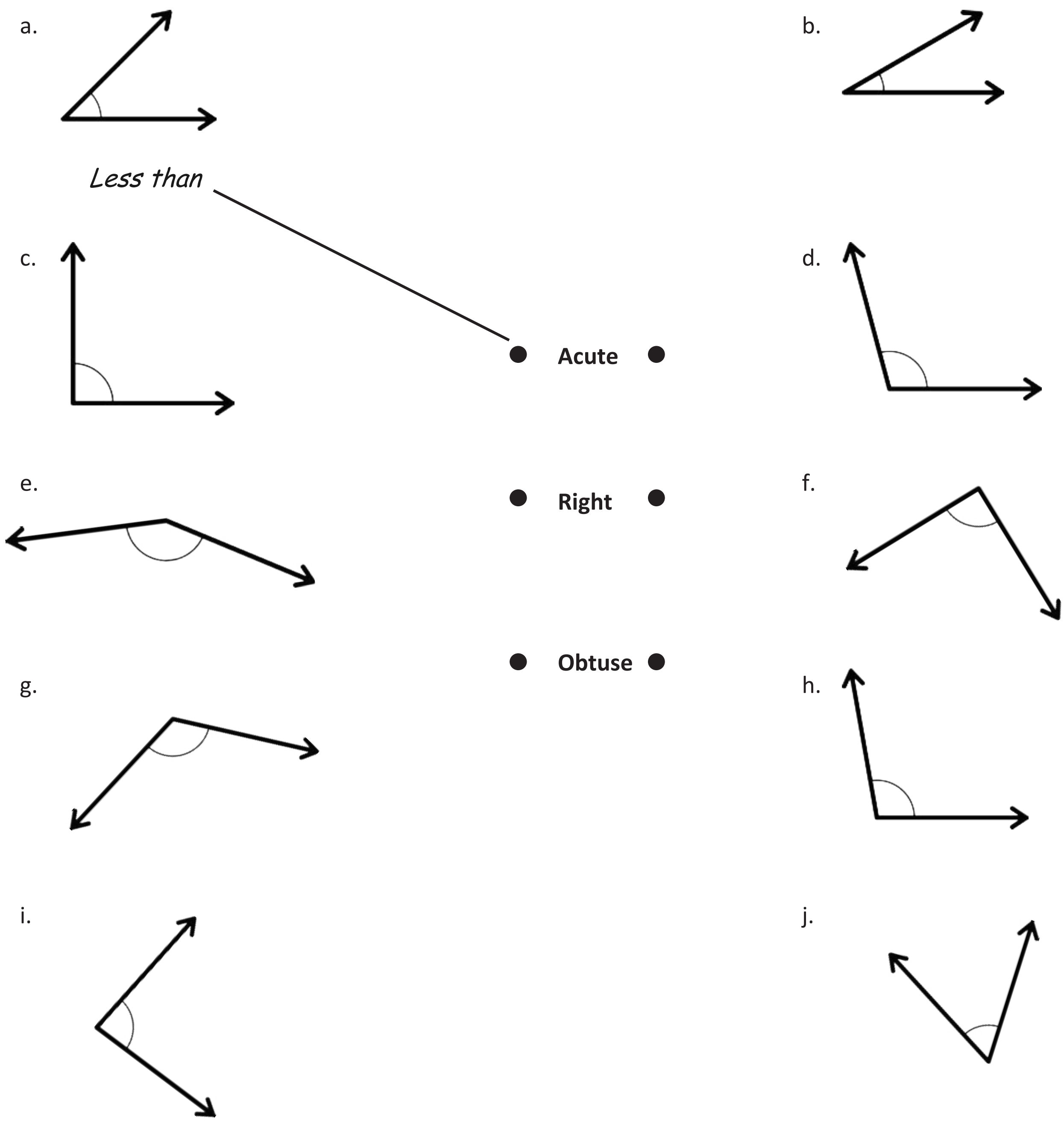


Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ____ Date ______________________

1. Use the right angle template that you made in class to determine if each of the following angles is greater than, less than, or equal to a right angle. Label each as *greater than*, *less than*, or *equal to*, and then connect each angle to the correct label of acute, right, or obtuse.
   The first one has been completed for you.



2. Use your right angle template to identify acute, obtuse, and right angles within Picasso's painting *Factory, Horta de Ebbo*. Trace at least two of each, label with points, and then name them in the table below the painting.

Acute angle		
Obtuse angle		
Right angle		

3. Construct each of the following using a straightedge and the right angle template that you created. Explain the characteristics of each by comparing the angle to a right angle. Use the words *greater than*, *less than,* or *equal to* in your explanations.

   a. Acute angle

   b. Right angle

   c. Obtuse angle

Name ____ Date ____________________

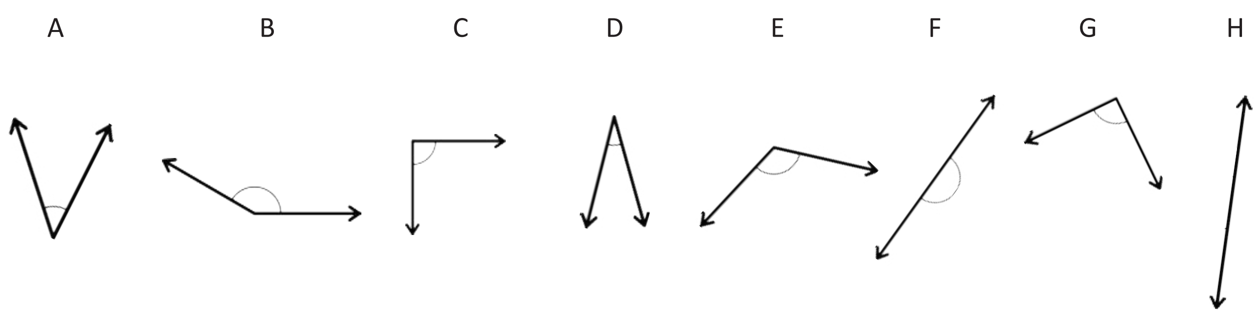
1. Fill in the blanks to make true statements using one of the following words: *acute, obtuse, right, straight.*

   a. In class, we made a _________________ angle when we folded paper twice.

   b. An _________________ angle is smaller than a right angle.

   c. An _________________ angle is larger than a right angle, but smaller than a straight angle.

2. Use a right angle template to identify the angles below.



   a. Which angles are right angles? ____

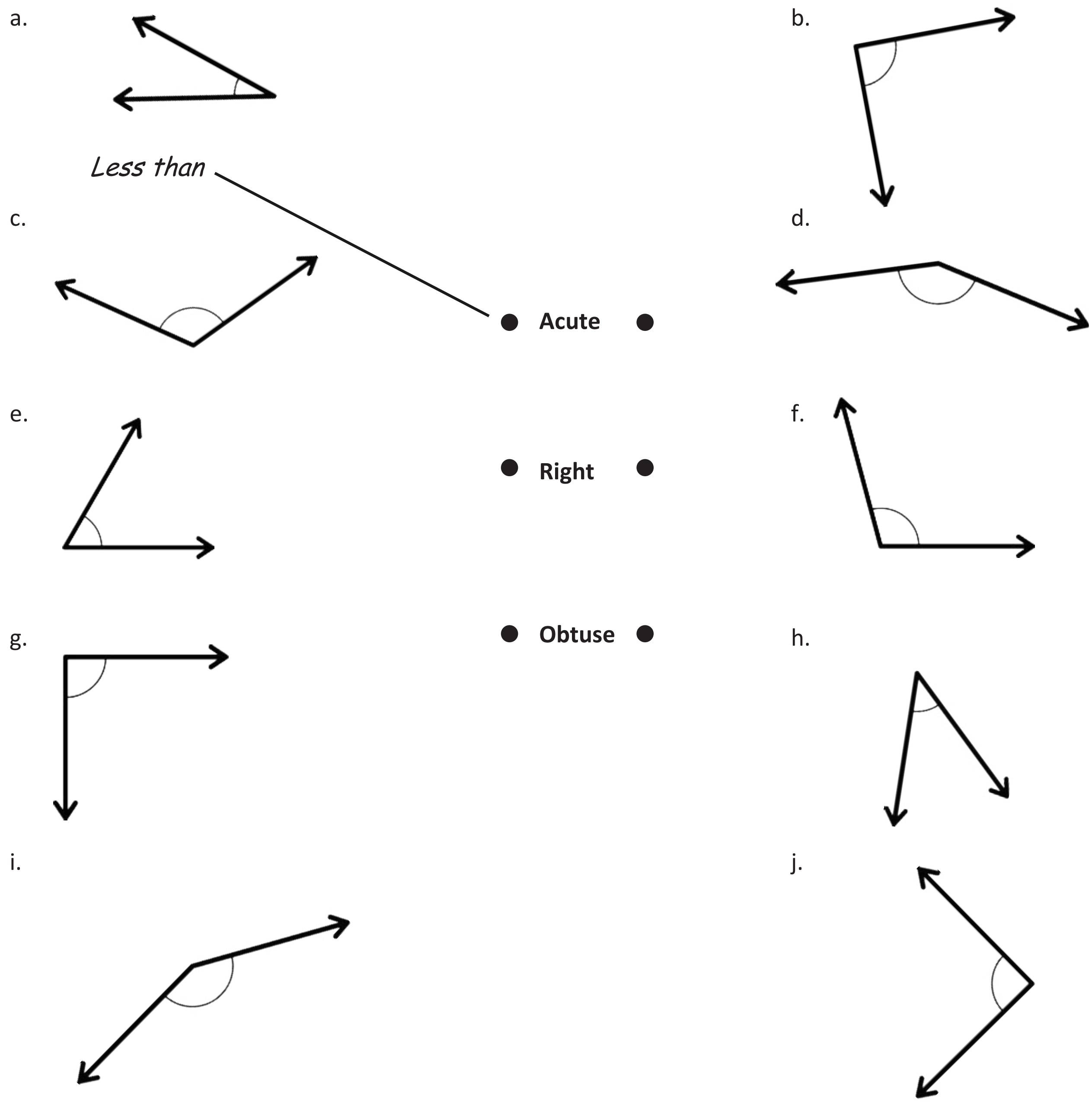
   b. Which angles are obtuse angles? ____

   c. Which angles are acute angles? ____

   d. Which angles are straight angles? ____

Name ____ Date ______________________

1. Use the right angle template that you made in class to determine if each of the following angles is greater than, less than, or equal to a right angle. Label each as *greater than*, *less than,* or *equal to*, and then connect each angle to the correct label of acute, right, or obtuse. The first one has been completed for you.



2. Use your right angle template to identify acute, obtuse, and right angles within this painting. Trace at least two of each, label with points, and then name them in the table below the painting.

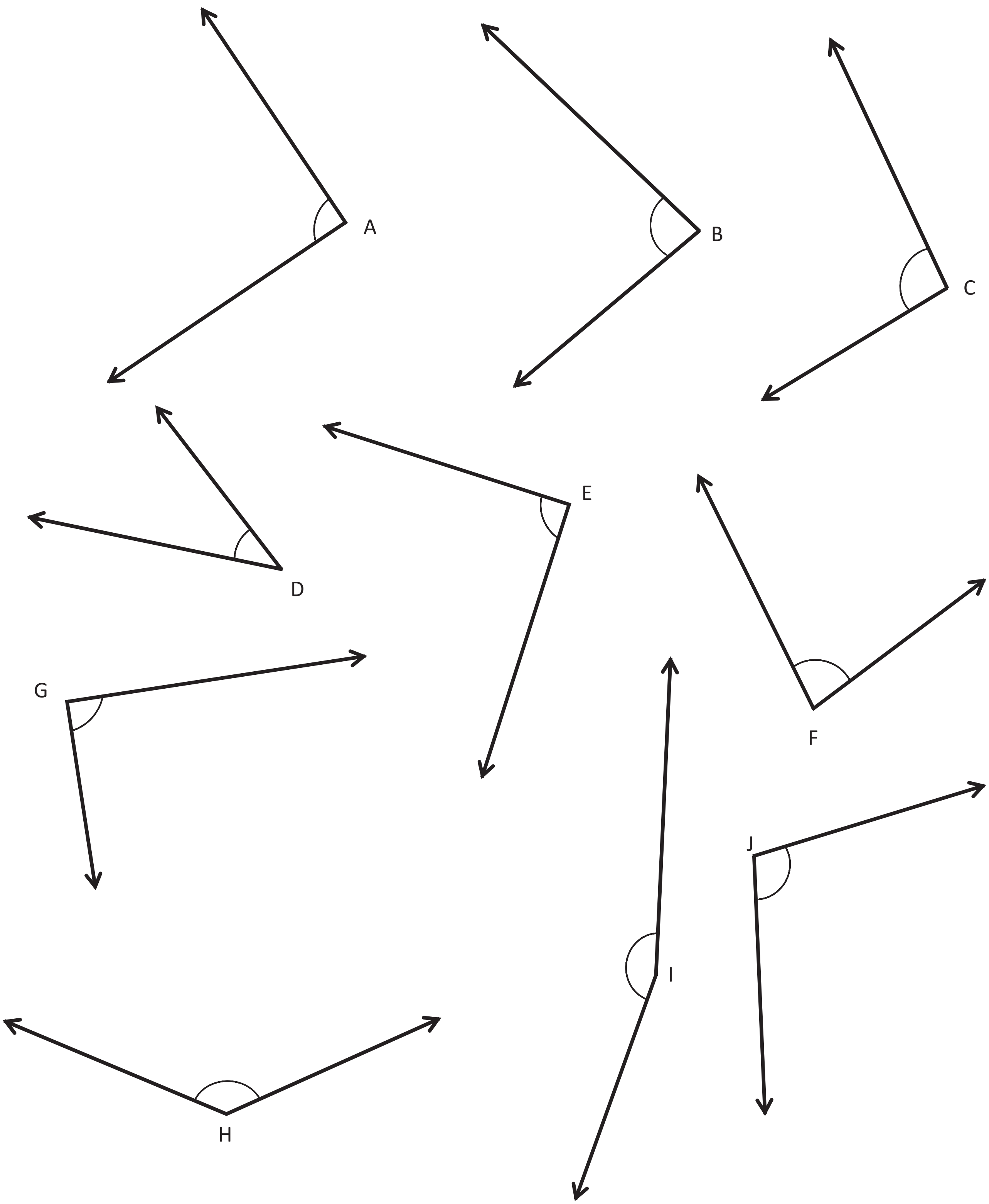
Acute angle		
Obtuse angle		
Right angle		

3. Construct each of the following using a straightedge and the right angle template that you created. Explain the characteristics of each by comparing the angle to a right angle. Use the words *greater than*, *less than,* or *equal to* in your explanations.

   a. Acute angle

   b. Right angle

   c. Obtuse angle



______________________ angles

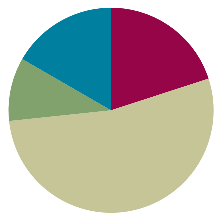
Lesson 3

Objective: Identify, define, and draw perpendicular lines.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (6 minutes)
- Concept Development (32 minutes)
- Student Debrief (10 minutes)

Total Time **(60 minutes)**



Fluency Practice (12 minutes)

- Multiply Mentally **4.NBT.4** (3 minutes)
- Identify Two-Dimensional Figures **4.G.1** (4 minutes)
- Physiometry **4.G.1** (5 minutes)

Multiply Mentally (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews the Concept Developments from Grade 4 Module 3 Lessons 34–38.

T: (Write 34 × 2.) Say the multiplication sentence.

S: 34 × 2 = 68.

T: (Write 34 × 2 = 68. Below, write 34 × 20 = ____.) Say the multiplication sentence.

S: 34 × 20 = 680.

T: (Write 34 × 20 = 680. Below, write 34 × 22 = ____.) On your personal white board, solve 34 × 22.

S: 748.

Continue with the following possible sequence: 23 × 2, 23 × 30, and 23 × 32.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

The Identify Two-Dimensional Figures fluency activity provides English language learners and other students a valuable opportunity to speak and review meanings and representations of recently introduced geometry terms. If necessary, allow extra time for students to respond.

Identify Two-Dimensional Figures (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews terms learned in Lessons 1–2.

T: (Project a line AB. Trace line AB.) Write the symbol for what I'm pointing to.

S: $\overleftrightarrow{AB}$.

T: (Point to point A.) Say the term.

S: Point A.

T: (Point to point B.) Say the term.

S: Point B.

T: On your board, draw $\overleftrightarrow{CD}$.

S: (Draw a line with points C and D on the line.)

Continue with the following possible sequence: $\overline{EF}$, $\angle GIH$, and $\overrightarrow{JK}$.

T: (Project a right angle LNM.) Name the angle.

S: $\angle LNM$.

T: What type of angle is it?

S: Right angle.

T: (Project an acute angle OQP.) Name the angle.

S: $\angle OQP$.

T: Is it greater than or less than a right angle?

S: Less than.

T: What's the term for an angle that's less than a right angle?

S: Acute angle.

T: (Project an obtuse angle RTS.) Name the angle.

S: $\angle RTS$.

T: Is it greater than or less than an acute angle?

S: Greater than.

T: Is it greater than or less than a right angle?

S: Greater than.

T: What's the term for an angle greater than a right angle but less than a straight angle?

S: Obtuse angle.

Physiometry (5 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms from Lessons 1–2.

T: Stand up.

S: (Stand up.)

T: Model a line segment.

S: (Extend arms straight so that they are parallel with the floor. Clench both hands into fists.)

T: Model a line.

S: (Extend arms straight so that they are parallel with the floor. Open both hands and point at side walls.)

T: Model a point.

S: (Clench one hand in a fist and extend arm forward.)

T: Model a ray.

S: (Extend arms straight so that they are parallel with the floor. Clench one hand in a fist, and leave the point with a finger on the other hand.)

T: Model a ray pointing in the other direction.

S: (Clench open hand, and open clenched hand. Point with a finger on the open hand.)

T: (Stretch one arm up directly at the ceiling. Stretch the other arm directly toward a wall parallel to the floor.) What type of angle do you think I'm modeling with my arms?

S: Right angle.

T: Model a right angle with your arms.

S: (Stretch one arm up directly at the ceiling. Stretch another arm directly toward a wall parallel to the floor.)

T: (Stretch the arm pointing toward a wall directly up toward the ceiling. Move the arm pointing toward the ceiling so that it points directly toward the opposite wall.) Model another right angle.

S: (Stretch the arm pointing toward a wall directly up toward the ceiling. Move the arm pointing toward the ceiling so that it points directly toward the opposite wall.)

T: Model an acute angle.

S: (Model an acute angle with arms.)

T: Model an obtuse angle.

S: (Model an obtuse angle with arms.)

Next, move between figures with the following possible sequence: right angle, ray, line segment, acute angle, line, obtuse angle, point, and right angle.

Application Problem (6 minutes)

Materials: (S) Straightedge

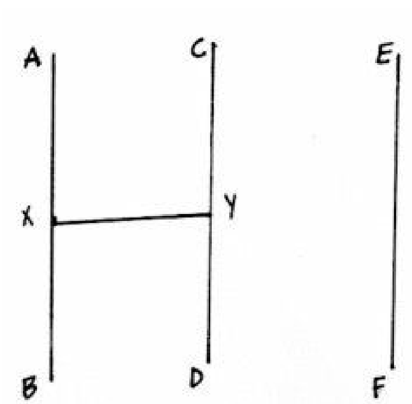
a. Use a straightedge to draw and label $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ as modeled on the board.

b. Estimate to draw point X halfway up $\overline{AB}$.

c. Estimate point Y halfway up $\overline{CD}$.

d. Draw horizontal line segment XY. What word do the segments create?

e. Erase segment XY. Draw segment CF. What word do the segments create?

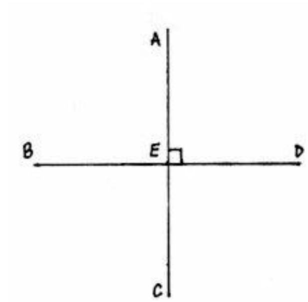


Note: This Application Problem reviews Lessons 1's introduction to and application of points and line segments. This Application Problem also transitions into today's lesson, during which students discover types of lines or line segments present in letters of the English alphabet.

Concept Development (32 minutes)

Materials: (T/S) Straightedge, right angle template (created in Lesson 2), paper, Problem Set

Problem 1: Define perpendicular lines.



T: (Draw perpendicular lines using the right angle template and a straightedge.) What do you see?

S: A right angle! → Two line segments and four right angles. → A cross. → The lowercase letter *t*. → A plus sign.

T: (Label central point E and endpoints A, B, C, and D.) $\overline{AE}$ and $\overline{ED}$ make right angles. (Mark a right angle.) With your partner, list two more segments that form a right angle.

S: $\overline{AE}$ and $\overline{BE}$. → $\overline{EB}$ and $\overline{EC}$. → $\overline{EC}$ and $\overline{ED}$. → $\overline{AC}$ and $\overline{BD}$.

T: Can you find examples of right angles in the room?

S: Yes! In my square grid paper! → In the heating grate! → I see them in the floor tiles.

MP.6

T: (Point to perpendicular lines.) These lines are **perpendicular**. They intersect to make right angles. (Draw an X.) Are these lines perpendicular? Share your thoughts with your partner.

S: Those lines cross, but they don't make right angles. They're not perpendicular.

T: No, they are not perpendicular. They are **intersecting lines**. (Point to an acute angle). What type of angle?

S: Acute.

T: (Point to an obtuse angle). What type of angle?

S: Obtuse.

T: (Draw the capital letters *T, L,* and *V*.) Discuss with your partner whether the segments in these letters are perpendicular.

S: The lines of *T* and *L* meet to make a right angle. → The segments in *T* and *L* are perpendicular. → Letter *V* doesn't have a right angle. So, those lines are not perpendicular.

Use the right angle template to verify student responses.

T: List three more capital letters of the alphabet with perpendicular lines.

S: H, F, E.

Problem 2: Identify perpendicular lines by measuring right angles with a right angle template.

T: Hold up your right angle template, and trace the right angle with your finger. (Model.) Let's use this right angle to find perpendicular lines in our room. On your desk, which objects have perpendicular lines?

S: My personal white board, rectangular eraser, straightedge, and the blank paper all have perpendicular lines. → My nametag, iPad screen, and the edges of my desk have perpendicular lines.

T: On our classroom wall, which objects have perpendicular lines?

S: Our rules poster, the calendar, the white board, the door, and the windows have perpendicular lines.

T: Take a look at Problem 4(b) on your Problem Set. Place your right angle edge on the lines of the shape. Do they match up? Does this pentagon have perpendicular lines?

S: No. The lines form obtuse angles. The lines cross, but they do not make right angles. They are not perpendicular lines.

Problem 3: Recognize and write symbols for perpendicular segments.

T: Take a look at Problem 4(a) on the Problem Set. Trace your finger across $\overline{AC}$. (Write $\overline{AC}$.) Tell your partner the name of two segments that are perpendicular to segment $\overline{AC}$.

S: $\overline{AC}$ is perpendicular to $\overline{AB}$. $\overline{CD}$ is also perpendicular to $\overline{AC}$.

T: (Write $\overline{AC} \perp \overline{AB}$ and point.) $\overline{AC}$ is perpendicular to $\overline{AB}$. Use symbols to write $\overline{CD}$ is perpendicular to $\overline{AC}$.

S: $\overline{CD} \perp \overline{AC}$.

Problem 4: Draw perpendicular line segments.

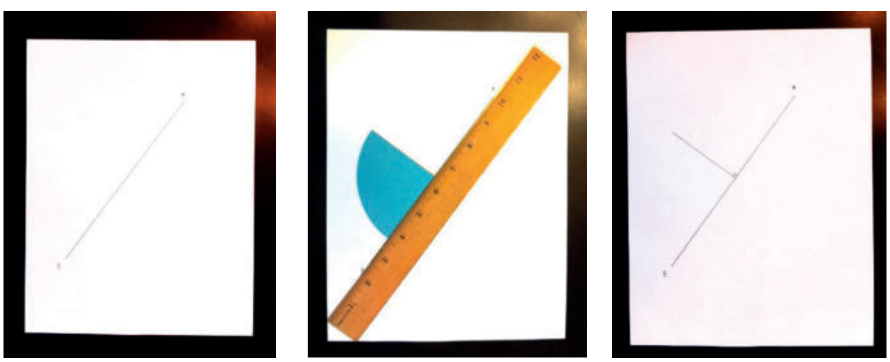
T: A line can be drawn in any direction. (Draw.) Here is a **diagonal** $\overline{AB}$. I can use my right angle template to draw a line perpendicular to $\overline{AB}$. (Model.)

NOTES ON LINES IN THE REAL WORLD:

Challenge students to search for upright and diagonal perpendicular lines in their natural and man-made environments. This activity may best be prepared beforehand with photographs of examples. Prompt observation, analysis, and discovery with the following questions:

Are perpendicular lines found in nature? Intersecting lines?

How are upright perpendicular lines used by people? Diagonal perpendicular lines? Intersecting lines?



T: What do you notice about the angles?

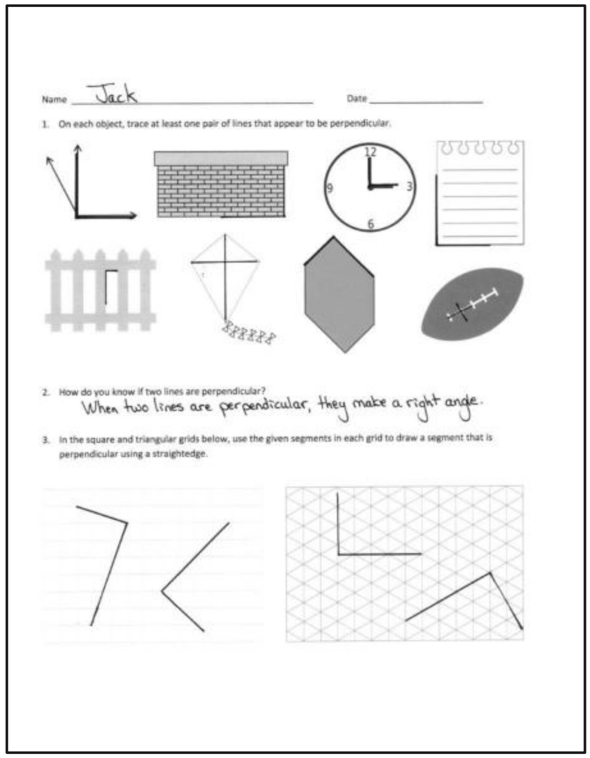
S: I notice there are two right angles. You marked one right angle with a small square.

T: On your blank paper, use your pencil and straightedge to draw $\overline{CD}$. Now, use your right angle template to draw a line perpendicular to $\overline{CD}$. Check for perpendicularity with your right angle template.

S: It's easier to draw a line perpendicular to a horizontal line. → When you drew the diagonal line, I thought it would be hard to draw a perpendicular line. So, I turned the paper to make the diagonal line horizontal to me.

T: When you're drawing or using the right angle template to identify perpendicular lines, you can turn the paper for ease, if you want. What's another helpful tip?

S: Steady the straightedge, and hold the right angle template still while you're drawing.



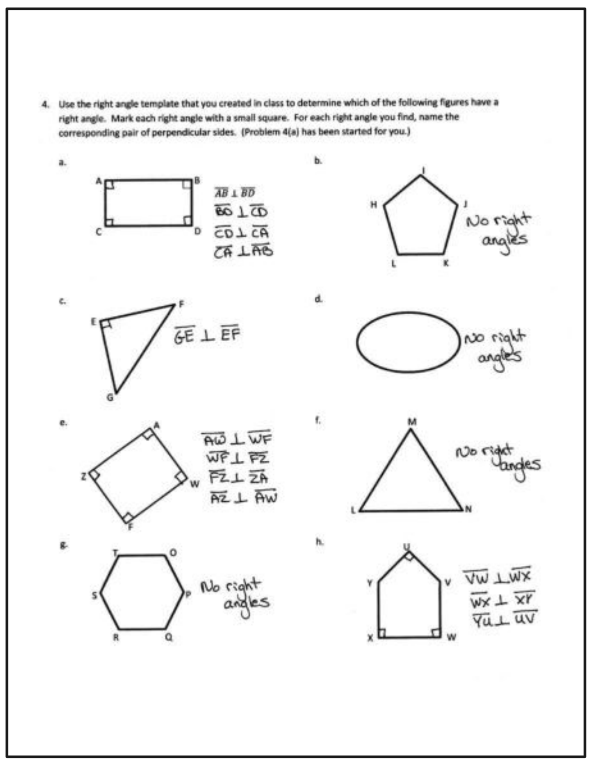
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Identify, define, and draw perpendicular lines.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

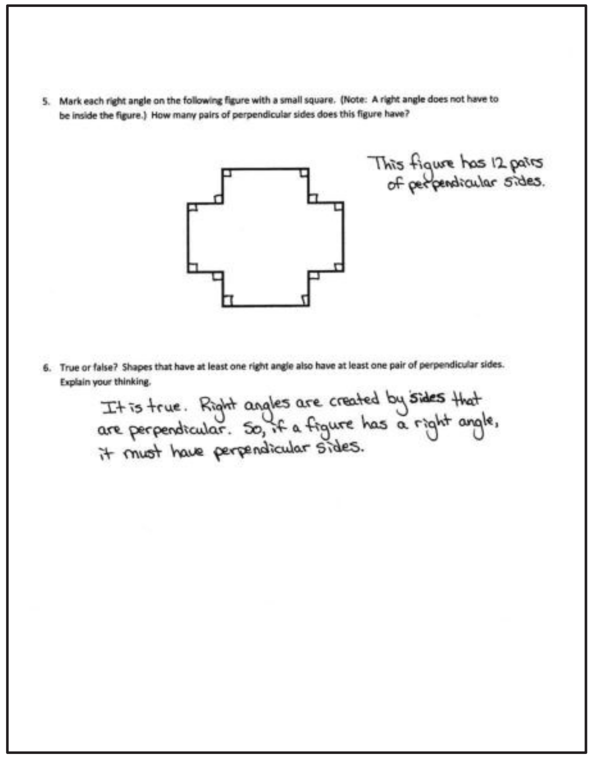


Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How did your knowledge of right angles prepare you to identify **perpendicular** lines in the figures for Problem 1?
- How can you tell if two lines are perpendicular (Problem 2)?
- In Problem 3, what was your strategy for drawing the segments perpendicular to the given segments? In what ways did the grids help you? How were the grids challenging?

- Look at the grid lines in Problem 3. Are the grid lines perpendicular or intersecting? Or both?
- In Problem 4, which figures had no perpendicular lines? Explain.
- In Problem 5, I only located eight right angles (on the interior of the figure). How many more right angles are there? What did this problem show you about locating angles on figures?
- How are perpendicular lines related to right angles? Acute angles? Obtuse angles?
- How might you use your understanding of perpendicular lines to solve a problem in real life? How might you use perpendicular lines when building something, for example?
- As you search for lines in your environment, notice if you find perpendicular or **intersecting lines** in nature. Analyze upright perpendicular lines, **diagonal** perpendicular lines, and intersecting lines as used by human beings.


5. Mark each right angle on the following figure with a small square. (Note: A right angle does not have to be inside the figure.) How many pairs of perpendicular sides does this figure have?

This figure has 12 pairs of perpendicular sides.

6. True or false? Shapes that have at least one right angle also have at least one pair of perpendicular sides. Explain your thinking.

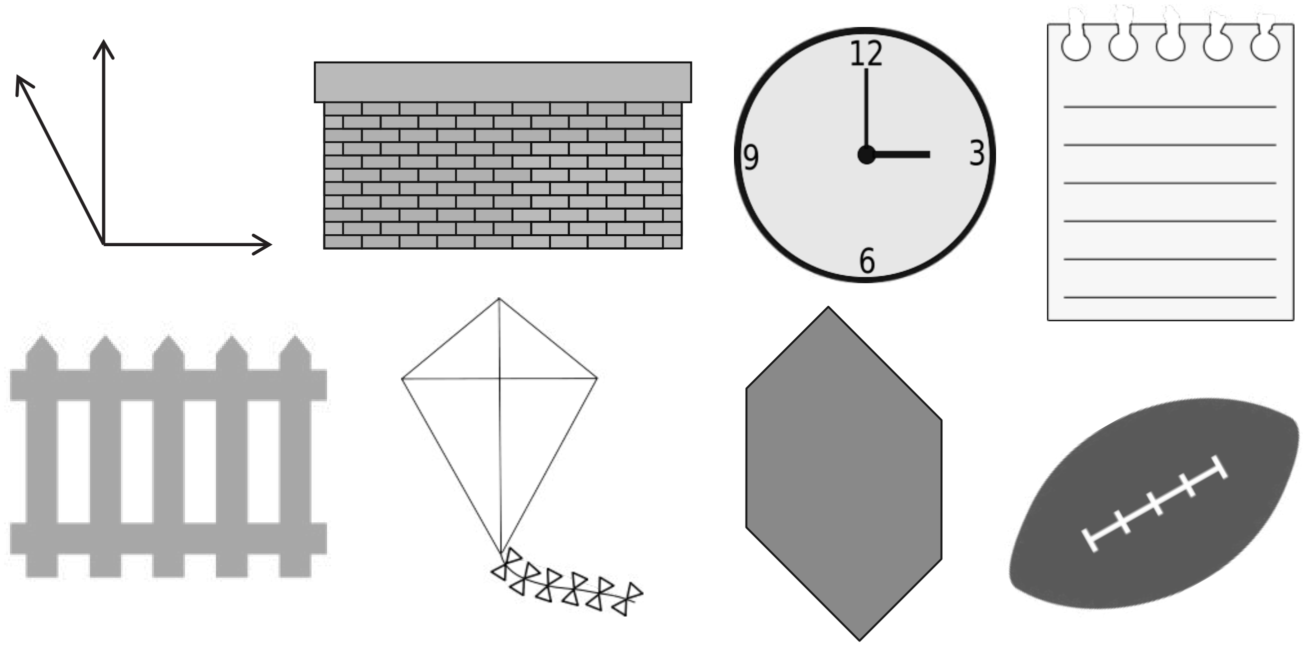
It is true. Right angles are created by sides that are perpendicular. So, if a figure has a right angle, it must have perpendicular sides.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

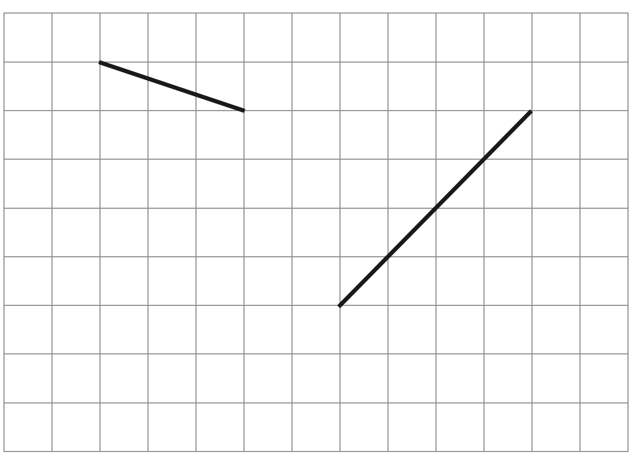
Name ______________________________ Date ________________

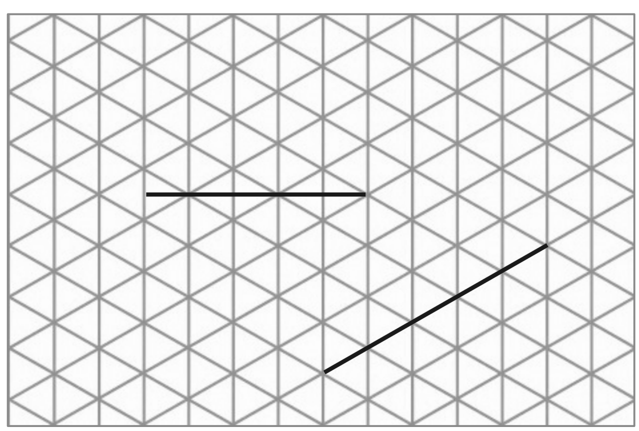
1. On each object, trace at least one pair of lines that appear to be perpendicular.



2. How do you know if two lines are perpendicular?

3. In the square and triangular grids below, use the given segments in each grid to draw a segment that is perpendicular using a straightedge.



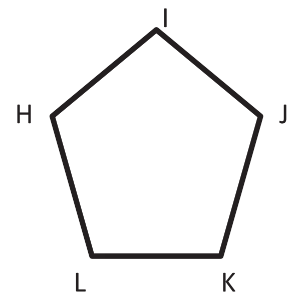


4. Use the right angle template that you created in class to determine which of the following figures have a right angle. Mark each right angle with a small square. For each right angle you find, name the corresponding pair of perpendicular sides. (Problem 4(a) has been started for you.)

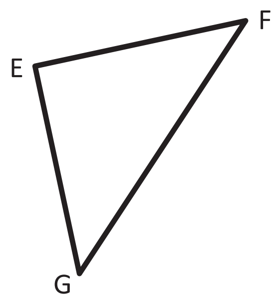
a.

$\overline{AB} \perp \overline{BD}$

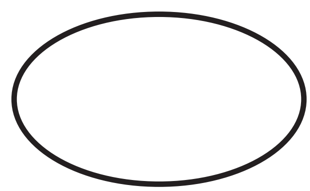
b.



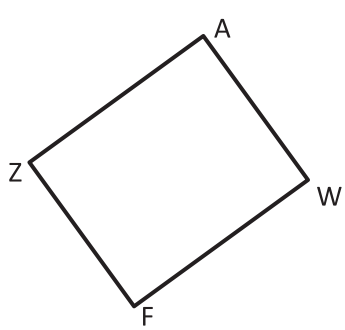
c.



d.



e.



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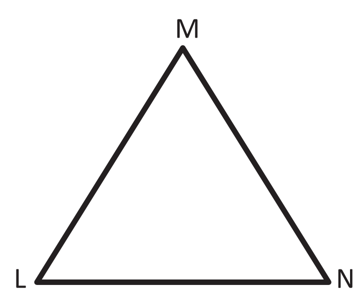

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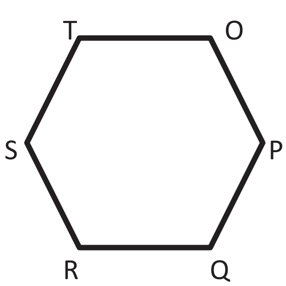

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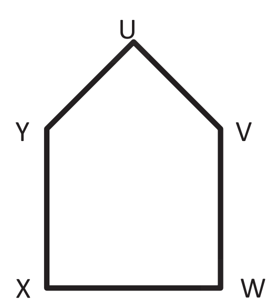
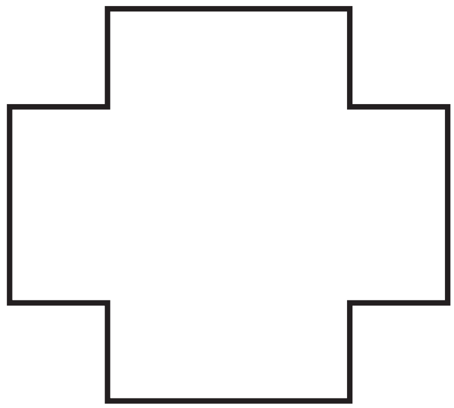


5. Mark each right angle on the following figure with a small square. (Note: A right angle does not have to be inside the figure.) How many pairs of perpendicular sides does this figure have?

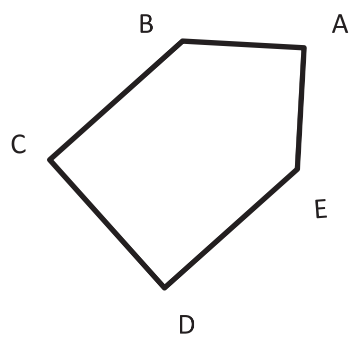


6. True or false? Shapes that have at least one right angle also have at least one pair of perpendicular sides. Explain your thinking.

Name ____ Date ____________________

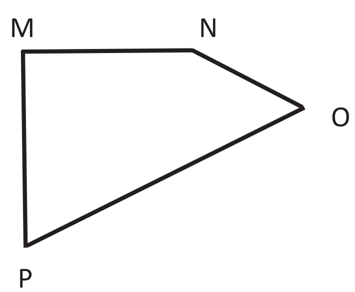
Use a right angle template to measure the angles in the following figures. Mark each right angle with a small square. Then, name all pairs of perpendicular sides.

1.



$\overline{BC}$ ⊥ _______

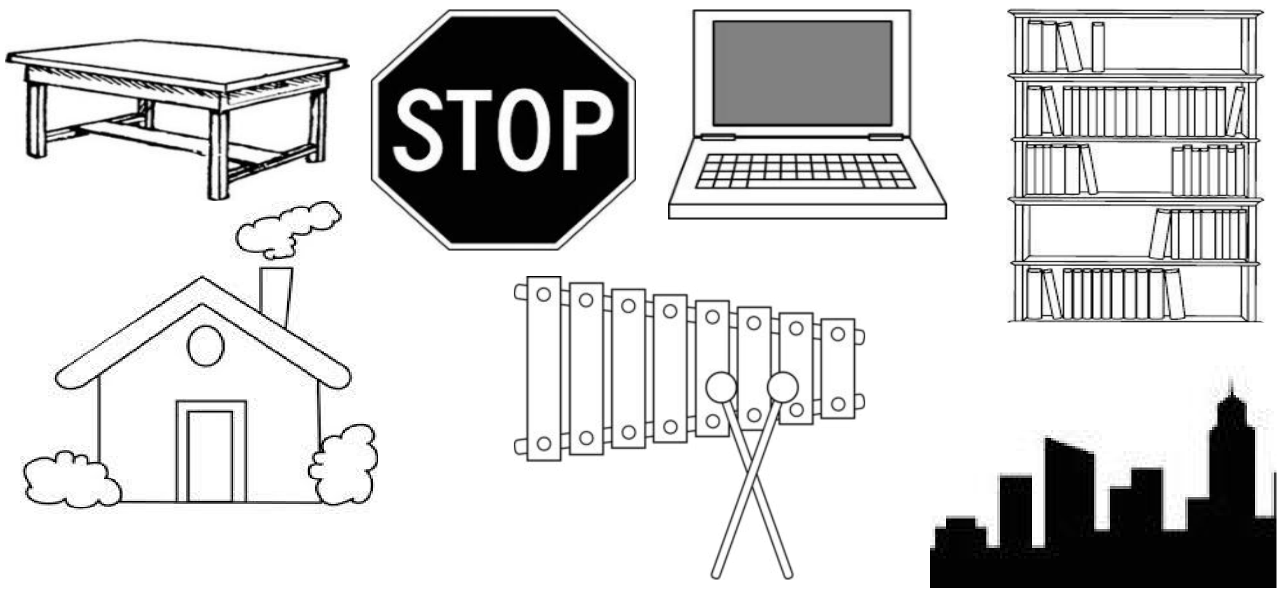
2.



$\overline{MN}$ ⊥ _______

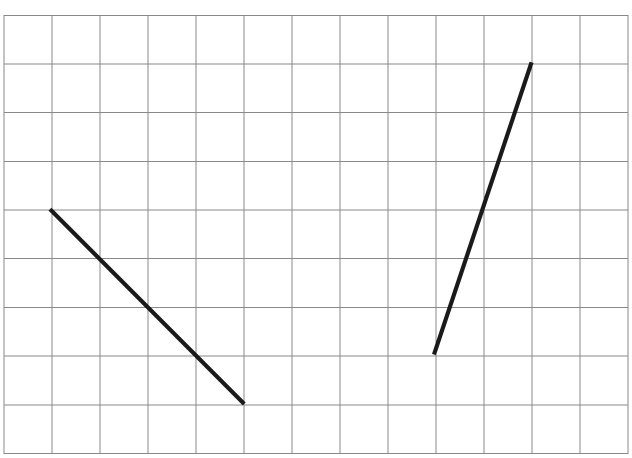
Name ____ Date ______________________

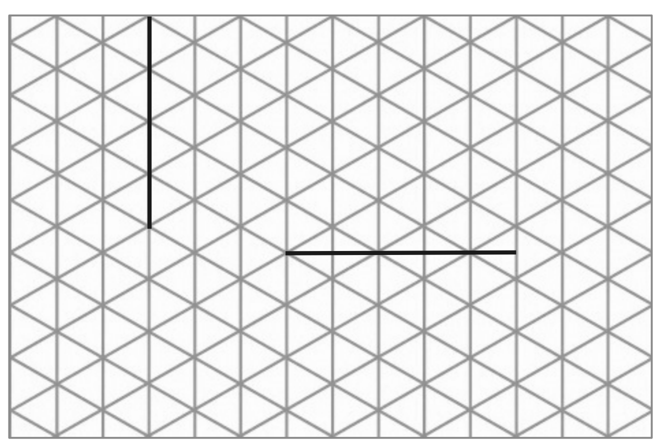
1. On each object, trace at least one pair of lines that appear to be perpendicular.



2. How do you know if two lines are perpendicular?

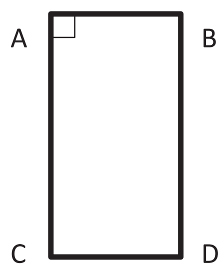
3. In the square and triangular grids below, use the given segments in each grid to draw a segment that is perpendicular. Use a straightedge.





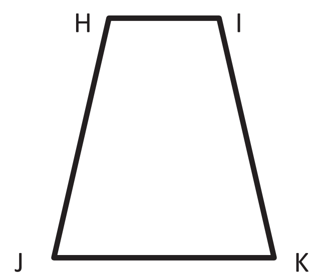
4. Use the right angle template that you created in class to determine which of the following figures have a right angle. Mark each right angle with a small square. For each right angle you find, name the corresponding pair of perpendicular sides. (Problem 4(a) has been started for you.)

a.



$\overline{CA} \perp \overline{AB}$

b.



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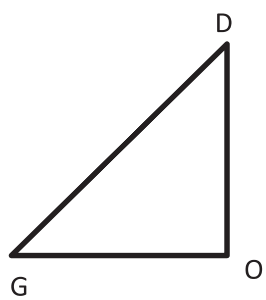

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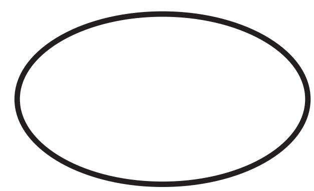

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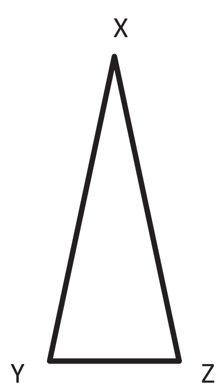

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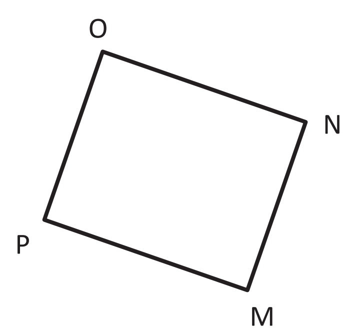

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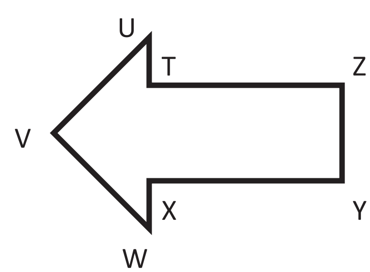
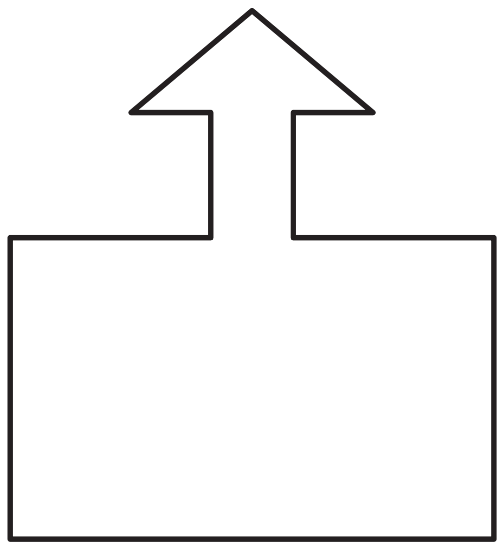


5. Use your right angle template as a guide, and mark each right angle in the following figure with a small square. (Note: A right angle does not have to be inside the figure.) How many pairs of perpendicular sides does this figure have?



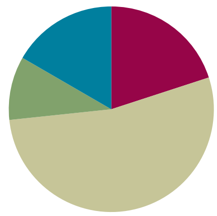
6. True or false? Shapes that have no right angles also have no perpendicular segments. Draw some figures to help explain your thinking.

Lesson 4

Objective: Identify, define, and draw parallel lines.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

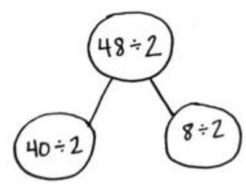


Fluency Practice (12 minutes)

- Divide Mentally **4.NBT.6** (4 minutes)
- Identify Two-Dimensional Figures **4.G.1** (4 minutes)
- Physiometry **4.G.1** (4 minutes)

Divide Mentally (4 minutes)

Note: This activity reviews Grade 4 Module 3 content.



T: (Write 40 ÷ 2.) Say the completed division sentence in unit form.
S: 4 tens ÷ 2 = 2 tens.
T: (To the right, write 8 ÷ 2.) Say the completed division sentence in unit form.
S: 8 ones ÷ 2 = 4 ones.
T: (Above both number sentences, write 48 ÷ 2. Draw a number bond to connect the 2 original problems to this problem.) Say the completed division sentence in unit form.
S: 4 tens 8 ones ÷ 2 = 2 tens 4 ones.
T: Say the division sentence in standard form.
S: 48 ÷ 2 = 24.

Continue with the following possible sequence: 48 ÷ 3, 96 ÷ 3, and 96 ÷ 4.

Identify Two-Dimensional Figures (4 minutes)

Materials: (S) Personal white board, straightedge

Note: This fluency activity reviews terms learned in Lessons 1–3.

T: (Project $\overleftrightarrow{AB}$. Trace $\overleftrightarrow{AB}$.) Name the figure.
S: $\overleftrightarrow{AB}$.

T: (Point to point A.) Say the term.

S: Point A.

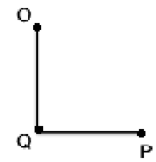
T: (Point to point B.) Say the term.

S: Point B.

T: Use your straightedge to draw $\overrightarrow{CD}$ on your personal white boards.

S: (Draw a ray with points C and D on the ray.)

Continue with the following possible sequence: $\overleftrightarrow{EF}$, $\overline{GH}$, and acute $\angle IKJ$, obtuse $\angle LMN$, and right $\angle OQP$.



T: What's the relationship between $\overline{OQ}$ and $\overline{PQ}$?

S: The line segments are perpendicular.

T: Draw $\overline{RS}$ that is perpendicular to $\overleftrightarrow{TV}$.

S: (Draw a line segment with endpoints R and S. Draw a line with points T and V that is perpendicular to $\overline{RS}$.)

Physiometry (4 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms from Lessons 1–3.

T: Stand up.

S: (Stand up.)

T: Model a ray.

S: (Extend arms straight so that they are parallel with the floor. Clench one hand in a fist, and leave the other hand open, pointing to a side wall.)

T: Model a ray pointing in the other direction.

S: (Open clenched hand, and clench open hand. Point with open hand.)

T: Model a line.

S: (Extend arms straight so that they are parallel with the floor. Open both hands, and point at the side walls.)

T: Model a point.

S: (Clench one hand in a fist, and extend arm forward.)

T: Model a line segment.

S: (Extend arms straight so that they are parallel with the floor. Clench both hands into fists.)

T: Model a right angle.

S: (Stretch one arm up, directly at the ceiling. Stretch another arm directly toward a wall, parallel to the floor.)

T: Model a different right angle.

S: (Stretch the arm pointing toward a wall directly up toward the ceiling. Move the arm pointing toward the ceiling so that it points directly toward the opposite wall.)

T: Model an acute angle.

S: (Model an acute angle with arms.)

T: Model an obtuse angle.
S: (Model an obtuse angle with arms.)

Next, move between figures with the following possible sequence: right angle, point, line, obtuse angle, line segment, acute angle, and right angle.

T: (Stretch one arm up, pointing directly at the ceiling. Stretch another arm directly pointing toward a wall, parallel to the floor.) Which type of angle do you think I'm modeling?
S: Right angle.
T: What is the relationship of the lines formed by right angles?
S: Perpendicular lines.
T: (Point at a wall to the side of the room.) Point at the walls that run perpendicular to the wall I'm pointing to.
S: (Point at the front and back walls.)
T: (Point at the back wall.)
S: (Point at the side walls.)

Continue pointing to the other side wall and front wall.

Application Problem (6 minutes)

Observe the letters *R*, *E*, *A*, and *L*.

R E A L

a. How many lines are perpendicular? Describe them.
b. How many acute angles are there? Describe them.
c. How many obtuse angles are there? Describe them.

a. There are 4 sets of perpendicular lines.
L has one where the segments meet.
E has 3 where the horizontal lines meet the vertical line.

b. There are 3 acute angles in A where the triangle is in the letter.

c. I found 2 obtuse angles in letter A where the legs meet the vertical line.

Note: This Application Problem reviews perpendicular and intersecting lines from Lesson 3. The problem can be extended in the Debrief by having students find letters with parallel lines.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Help learners keep their response to the Application Problem organized with a graphic organizer, such as the table below:

Letter	Number of Perpendicular Lines	Number of Acute Angles	Number of Obtuse Angles
R			
E			
A			
L			

Concept Development (32 minutes)

Materials: (T/S) Straightedge, personal white board, square grid paper, right angle template (created in Lesson 2)

Problem 1: Define and identify parallel lines.

T: Partners, lay your two straightedges on your desk. The straightedges cannot touch each other. Work with your partner to position your two straightedges like two roads that will never intersect.

T: Are your straightedges touching?

S: No!

T: If a car continued down your straightedge road, would it ever be on your partner's straightedge road?

S: No!

T: What do you notice?

S: Our straightedges are lined up perfectly. → Our straightedges are not perpendicular because they don't make right angles. They don't make *any* angles because they don't touch!

T: You've discovered **parallel** lines. Two lines that never touch no matter how far you extend them are parallel.

T: Look on your desk. Can you find parallel lines?

S: The opposite sides of my personal white board, desk, and book are parallel.

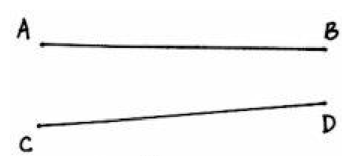
T: In our classroom, can you find parallel lines?

S: The repeating ridges in the heater are parallel. → The shelves of the bookcase are parallel.

T: (Project the letter *N*. Trace and label with arrowheads parallel segments.) Are these segments of letter *N* parallel?

S: Yes!

T: (Project $\overline{AB}$ and $\overline{CD}$ as pictured to the right.) Are these segments parallel?

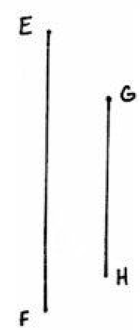


S: No!

T: Why not? I don't see an intersection?

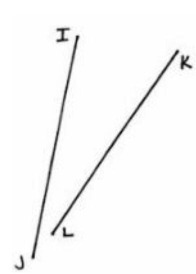
S: If you made each one longer, they'd run into each other off to the right.

T: (Project $\overline{EF}$ and $\overline{GH}$ as pictured to the right.) Are these segments parallel?



S: Yes!

T: (Project $\overline{IJ}$ and $\overline{KL}$ as pictured to the right.) Are these segments parallel?

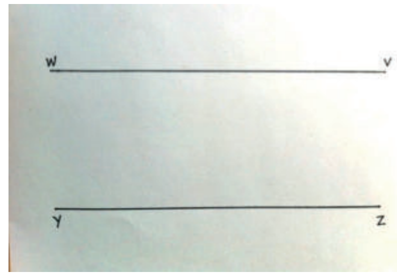


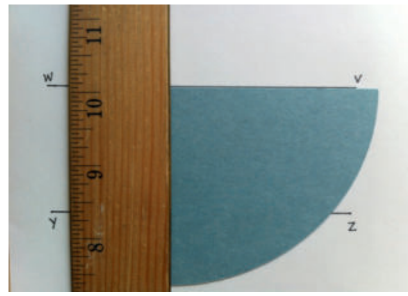
S: No!

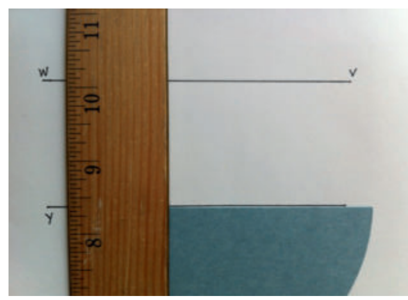
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Students have learned a significant amount of new vocabulary and math symbols in a short amount of time. Support English language learners and others by providing tools such as a word wall or dictionaries (in first and second languages in addition to pictures and symbols) that students can refer to throughout the lesson. Include bolded words, as well as familiar words, such as *horizontal.*

Problem 2: Identify parallel lines using a right angle template.







T: Partner 1, position your straightedge flat on your desk any way you like—horizontal, vertical, slanted to the right, slanted to the left. Partner 2, place your straightedge parallel to your partner's. Switch roles, and try again.

T: Use the word *parallel* in a sentence that describes your observations.

S: Parallel lines look like train tracks. → Parallel lines are side by side without touching. → Two lines that do not touch each other and are the same distance from each other at every point are parallel. → Parallel lines are not perpendicular.

T: (Project parallel segments $\overline{WV}$ and $\overline{YZ}$.) Are these segments parallel? They look parallel, but to be precise, we measure with a right angle template.

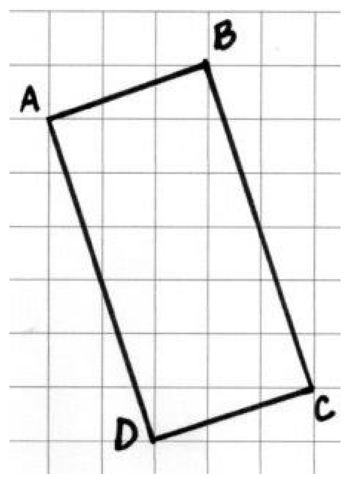
1. First, place a straightedge perpendicular across both segments.
2. Then, slide the right angle template along the straightedge to check the alignment.

T: Are these segments parallel?

S: Yes!

Repeat activity with a set of non-parallel lines following the process above.

Problem 3: Represent parallel lines with symbols.



T: On your grid paper, use your straightedge to draw rectangle $ABCD$ like mine. (Model drawing rectangle $ABCD$. Write $\overline{AB}$ on the board.)

When modeling, point out ways to confirm the lines are correctly drawn, without inferring parallelism yet, such as $\overline{AB}$ moves across three columns and up one row, as well as $\overline{DC}$. $\overline{AD}$ and $\overline{BC}$ move down six rows and across two columns. Segments can be extended and erased as needed.

T: Do you see a segment that is parallel to $\overline{AB}$? Use symbols to record your answer.

S: (Show $\overline{DC}$.) Segment DC!

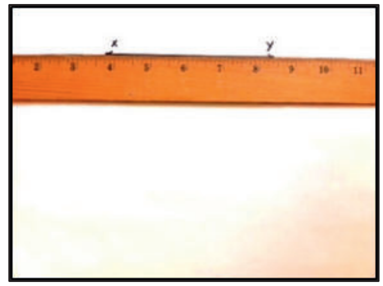
T: Let's check with our right angle template. (Model.)

S: (Check alignment using right angle template.)

T: (Assist as needed.) Are $\overline{AB}$ and $\overline{DC}$ parallel?

S: Yes.

T: (Write $\overline{AB}$ ∥ $\overline{CD}$). $\overline{AB}$ is parallel to $\overline{CD}$.) Use symbols like mine to record another parallel pair in the rectangle.



S: $\overline{AD}$ ∥ $\overline{BC}$.

T: What do you notice about sides of a rectangle and parallel lines?

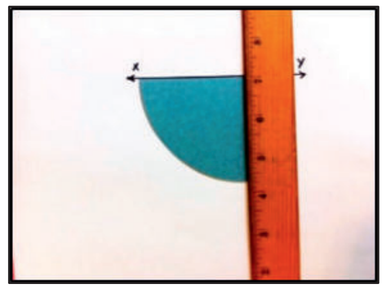
S: Opposite sides of the rectangle are parallel.

T: Is this true for all rectangles? With your partner, draw rectangles of different sizes and shapes. Use your right angle template to check for parallel segments.

MP.3

S: (Draw and verify.)

T: Does the length of the opposite sides of a rectangle change the fact that they are parallel?



S: No. Opposite sides of *all* rectangles are parallel.

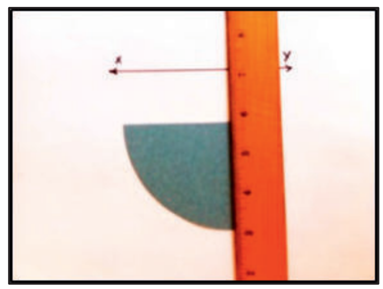
T: As you work on the Problem Set, consider if this is true for other shapes.

Problem 4: Draw parallel lines.

T: Use your straightedge to draw horizontal line XY.

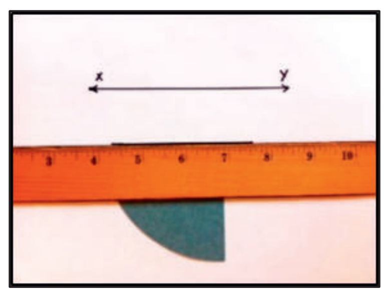
S: (Draw.)

T: We found that opposite sides of all rectangles are parallel. We also discovered in Lesson 2 that rectangles also have four right angles using our right angle template. We can use right angles to help us draw parallel lines.



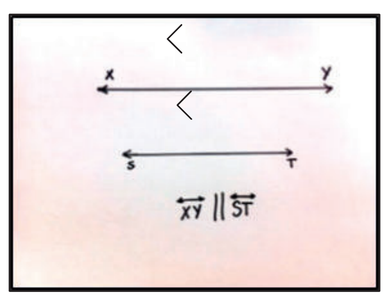
T: (Model a step at a time, checking on student progress.)

1. First, place your right angle template on $\overleftrightarrow{XY}$.
2. Second, align your straightedge along the template.
3. Next, slide your right angle template down the straightedge.
4. Align the straightedge against the other straight side of your template, and draw a line parallel to $\overleftrightarrow{XY}$.
5. Lastly, label it as $\overleftrightarrow{ST}$.



T: Use the parallel symbol to write a statement about these two lines. Draw arrowheads on each line to signify these two lines are parallel to each other.

T: Partner 1, draw a straight line—horizontal, vertical, slanted to the right, or slanted to the left. Partner 2, draw a line parallel to your partner's. Remember to draw arrowheads on the parallel lines to signal that they are, in fact, parallel. Switch roles and try again.



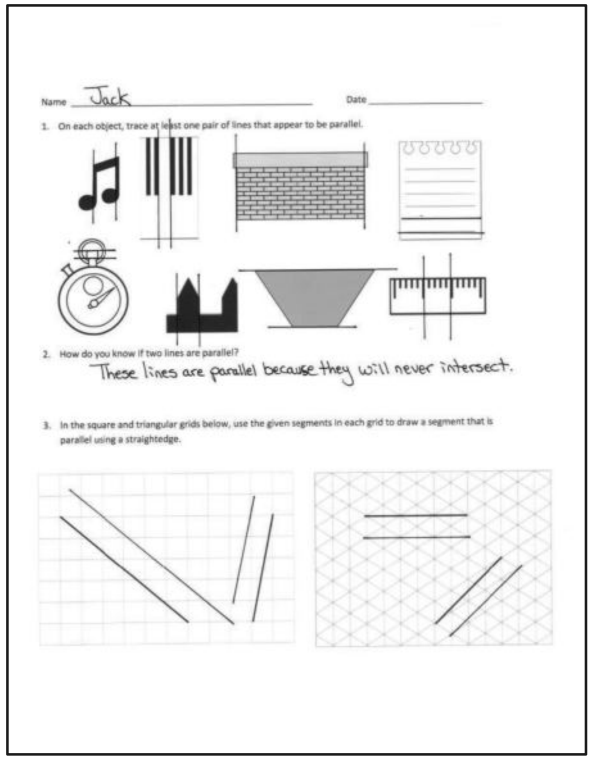
S: (Draw.)

T: What do you notice?

S: Parallel lines are the same distance from each other at every point. → It's tricky to draw a line that is parallel to a slanted line. → Turn the paper so the line is horizontal or vertical, and it's easier to draw a parallel line.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



Student Debrief (10 minutes)

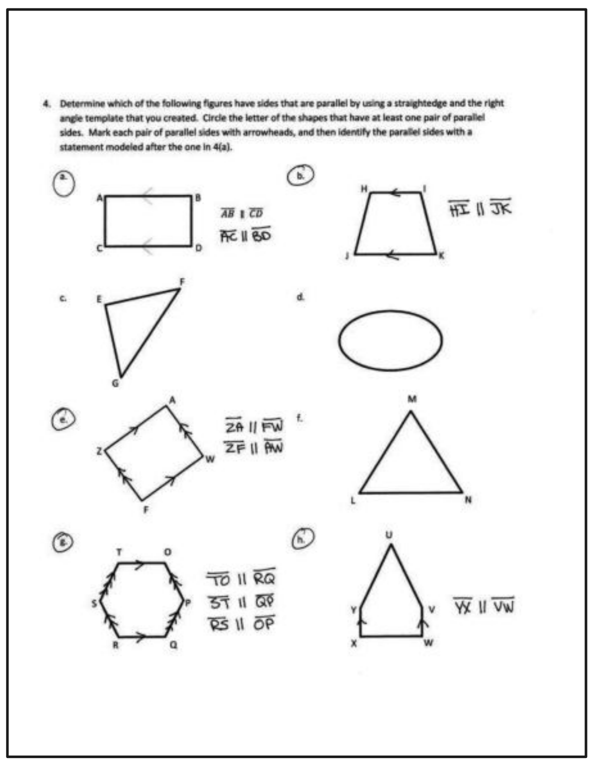
Lesson Objective: Identify, define, and draw parallel lines.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

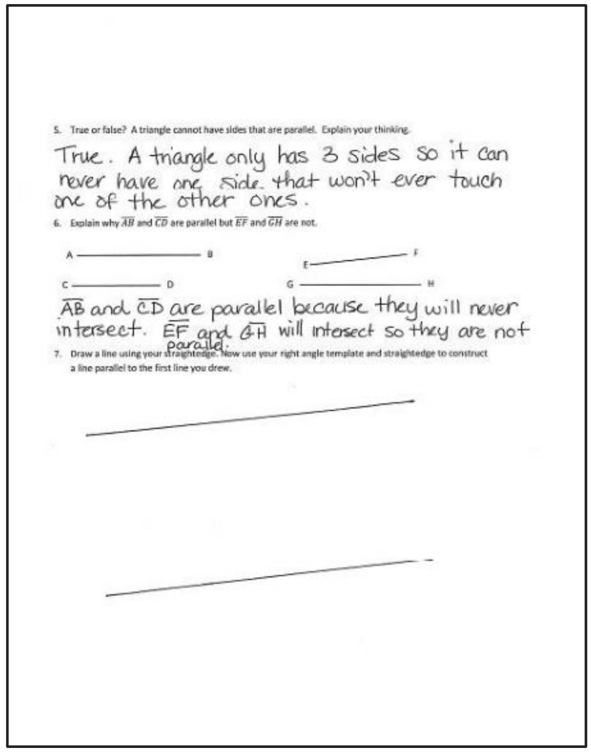
Any combination of the questions below may be used to lead the discussion.

- In Problem 1, how could your right angle template serve as a guide for identifying parallel lines?
- How do you know if two lines are **parallel** (Problem 2)?
- In Problem 3, the given line segments were not drawn on gridlines. What challenge did this pose in drawing lines parallel to the segments? What patterns did you find in the grids to help you analyze if your lines were, in fact, parallel?
- Which shapes in Problem 4 had parallel lines? Are opposite sides always parallel?
- How do parallel lines differ from perpendicular lines?
- Two segments that don't intersect must be parallel. True or false? Explain.



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



5. True or false? A triangle cannot have sides that are parallel. Explain your thinking.

True. A triangle only has 3 sides so it can never have one side that won't ever touch one of the other ones.

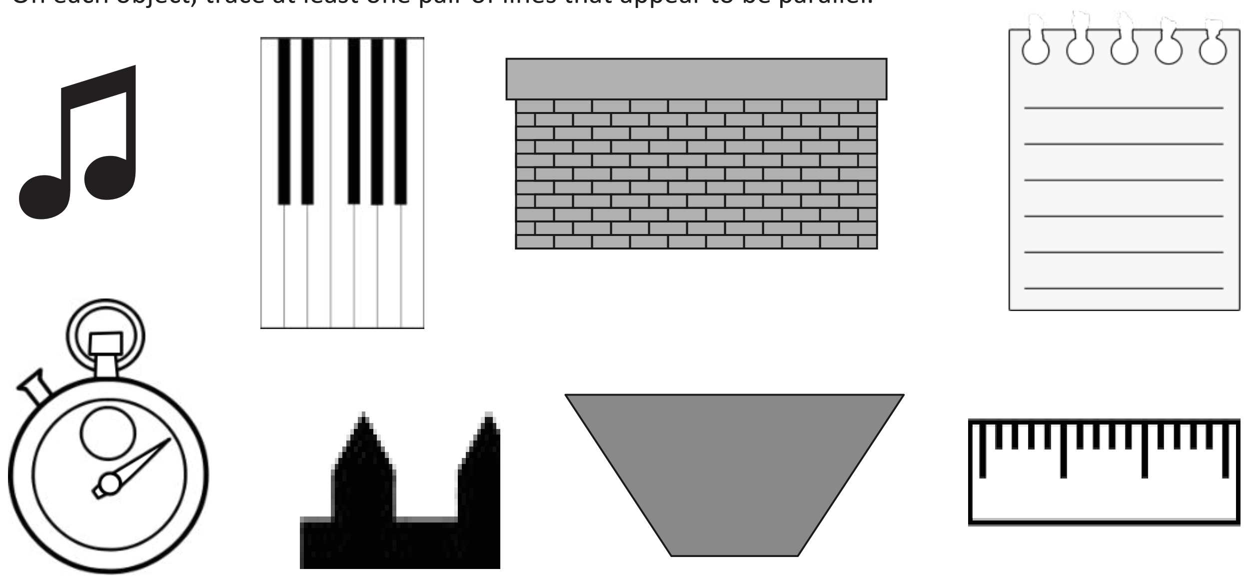
6. Explain why $\overline{AB}$ and $\overline{CD}$ are parallel but $\overline{EF}$ and $\overline{GH}$ are not.

$\overline{AB}$ and $\overline{CD}$ are parallel because they will never intersect. $\overline{EF}$ and $\overline{GH}$ will intersect so they are not parallel.

7. Draw a line using your straightedge. Now use your right angle template and straightedge to construct a line parallel to the first line you drew.

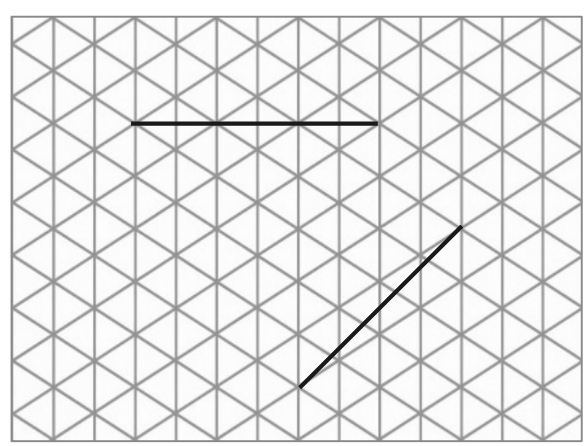
Name ______________________________ Date ________________

1. On each object, trace at least one pair of lines that appear to be parallel.



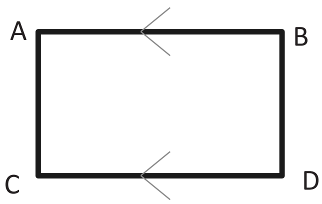
2. How do you know if two lines are parallel?

3. In the square and triangular grids below, use the given segments in each grid to draw a segment that is parallel using a straightedge.



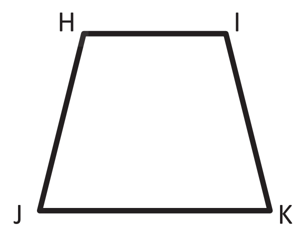
4. Determine which of the following figures have sides that are parallel by using a straightedge and the right angle template that you created. Circle the letter of the shapes that have at least one pair of parallel sides. Mark each pair of parallel sides with arrowheads, and then identify the parallel sides with a statement modeled after the one in 4(a).

a.

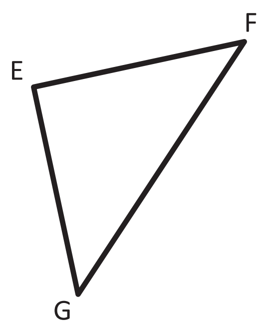


$\overline{AB} \parallel \overline{CD}$

b.



c.



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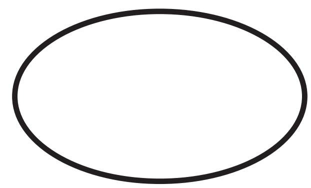
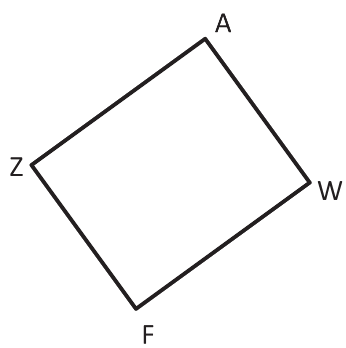
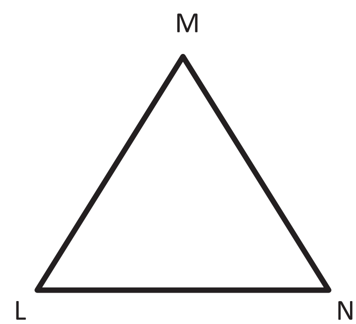


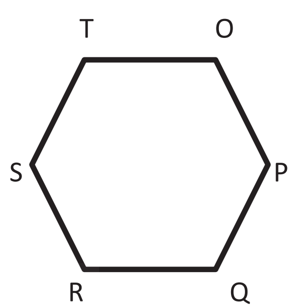
e.



f.



g.



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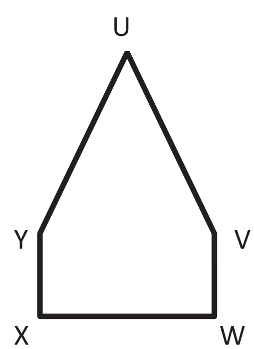


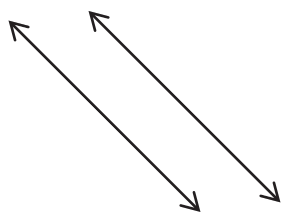
5. True or false? A triangle cannot have sides that are parallel. Explain your thinking.

6. Explain why $\overline{AB}$ and $\overline{CD}$ are parallel, but $\overline{EF}$ and $\overline{GH}$ are not.

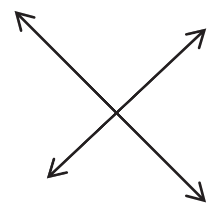
7. Draw a line using your straightedge. Now, use your right angle template and straightedge to construct a line parallel to the first line you drew.

Name ____ Date ____________________

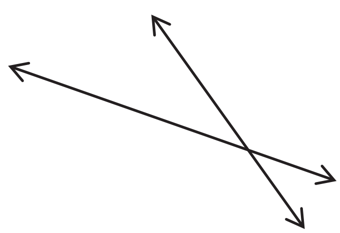
Look at the following pairs of lines. Identify if they are parallel, perpendicular, or intersecting.



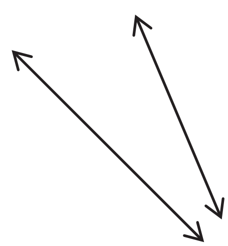
1. ____________________



2. ____________________



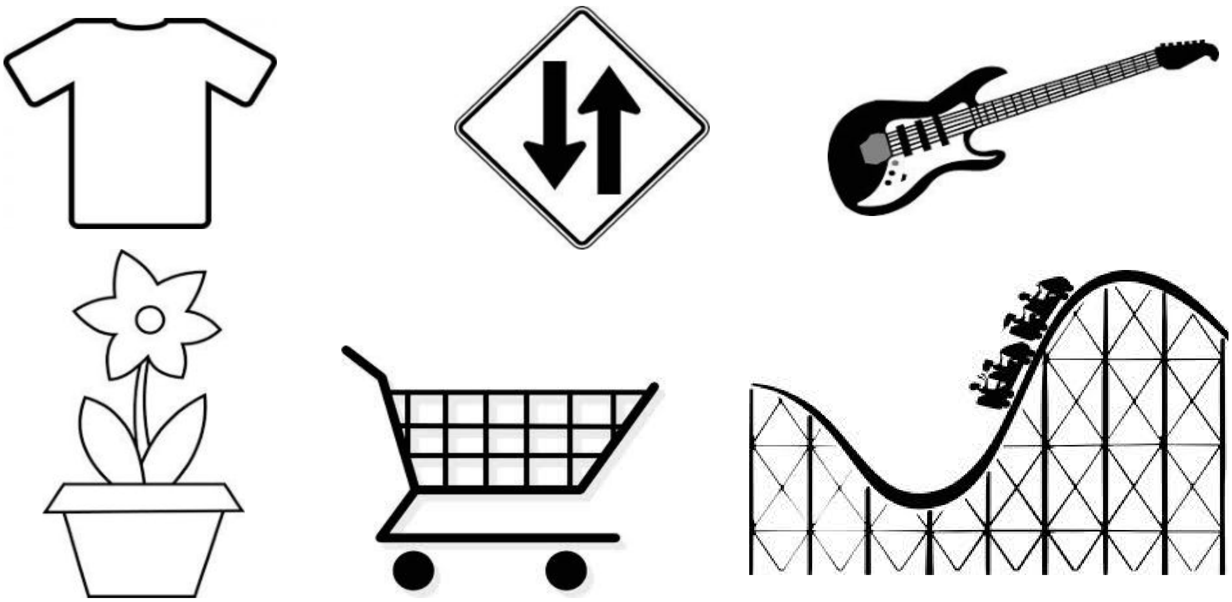
3. ____________________



4. ____________________

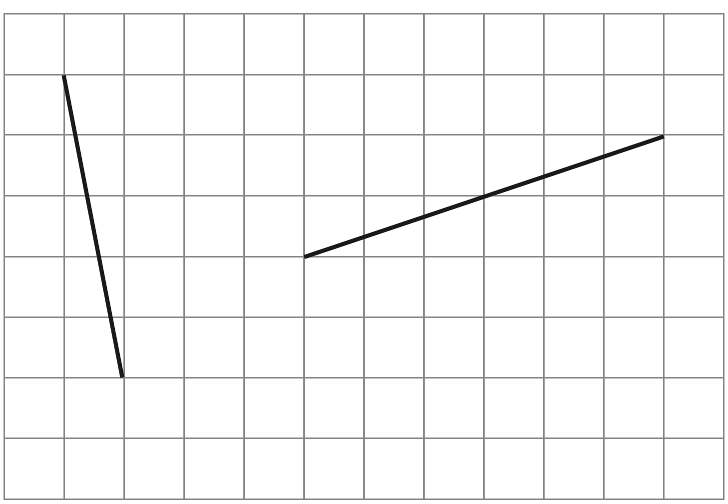
Name ______________________________ Date ________________

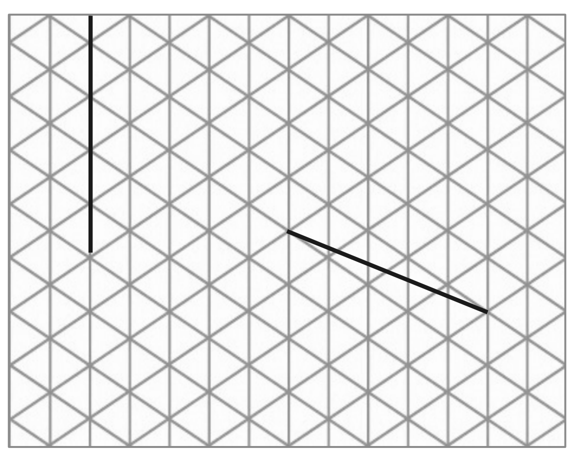
1. On each object, trace at least one pair of lines that appear to be parallel.



2. How do you know if two lines are parallel?

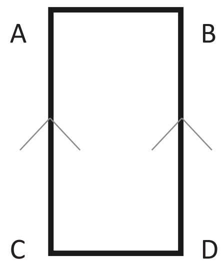
3. In the square and triangular grids below, use the given segments in each grid to draw a segment that is parallel using a straightedge.





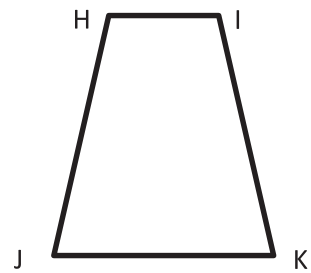
4. Determine which of the following figures have sides that are parallel by using a straightedge and the right angle template that you created. Circle the letter of the shapes that have at least one pair of parallel sides. Mark each pair of parallel sides with arrows, and then identify the parallel sides with a statement modeled after the one in 4(a).

a.



$\overline{AC} \parallel \overline{BD}$

b.



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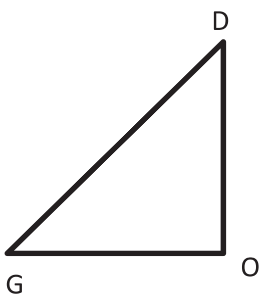

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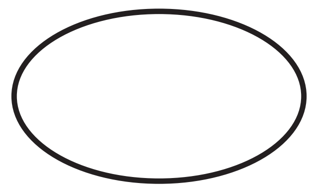

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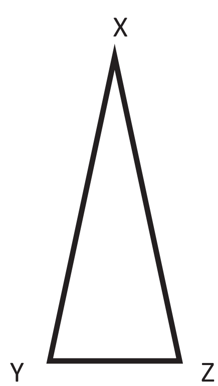

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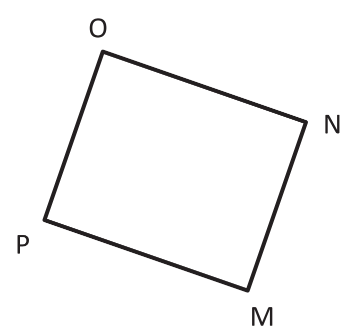

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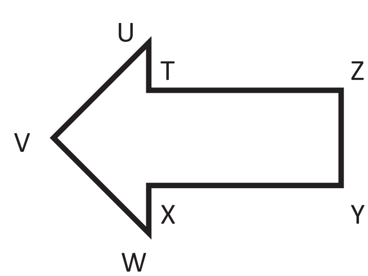
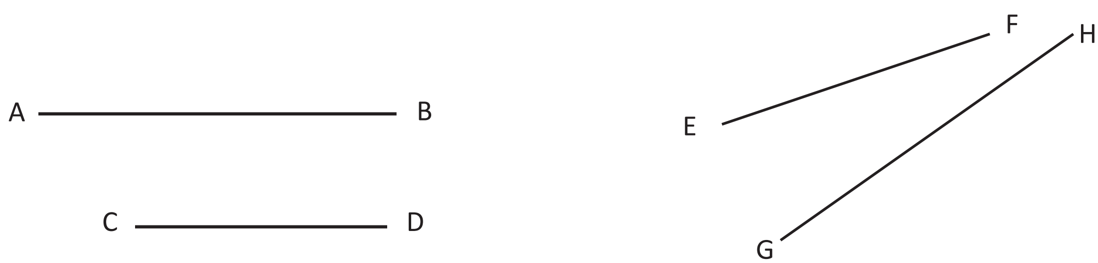


5. True or false? All shapes with a right angle have sides that are parallel. Explain your thinking.

6. Explain why $\overline{AB}$ and $\overline{CD}$ are parallel, but $\overline{EF}$ and $\overline{GH}$ are not.



7. Draw a line using your straightedge. Now, use your right angle template and straightedge to construct a line parallel to the first line you drew.

Topic B

Angle Measurement

4.MD.5, 4.MD.6

Focus Standard:	4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
	4.MD.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
Instructional Days:	4	
Coherence -Links from:	G2–M8	Time, Shapes, and Fractions as Equal Parts of Shapes

In Topic B, students explore the definition of degree measure. Beginning in Lesson 5 with a circular protractor, students divide the circumference of a circle into 360 equal parts, with each part representing 1 degree (**4.MD.5**). Students apply this understanding as they discover that a right angle measures 90° and, in turn, that the angles they know as acute measure less than 90°, and obtuse angles measure more than 90°. The idea that an angle measures the amount of *turning* in a particular direction is explored, providing students with the opportunity to recognize familiar angles in varied positions (**4.G.1**, **4.MD.5**).

Through experimentation with circles of various sizes and angles constructed to varying specifications in Lesson 6, students discover that, although the size of a circle may change, an angle spans an arc, which represents a constant fraction of the circumference. This reasoning forms the basis for the understanding that degree measure is not a measure of length. For example, as shown to the right, the 45° angle spans $\frac{1}{8}$ of the circumference of the circle, whether choosing the small or large circle.

Armed with this understanding of the degree as a unit of measure, students use various protractors in Lesson 7, including standard 180° protractors, to measure angles to the nearest degree and construct angles of a given measure (**4.MD.6**).

The topic concludes in Lesson 8 with students further exploring angle measure as an amount of turning. This provides a link to Grade 3 work with fractions as students reason that a $\frac{1}{4}$ turn is a right angle and measures 90°, a $\frac{1}{2}$ turn measures 180°, and a $\frac{3}{4}$ turn measures 270°. Students move forward to identify these angles in their environment.

A Teaching Sequence Toward Mastery of Angle Measurement

Objective 1: **Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of a turn. Explore benchmark angles using the protractor.**
(Lesson 5)

Objective 2: **Use varied protractors to distinguish angle measure from length measurement.**
(Lesson 6)

Objective 3: **Measure and draw angles. Sketch given angle measures, and verify with a protractor.**
(Lesson 7)

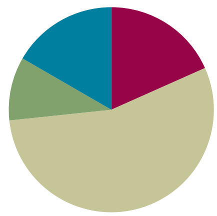
Objective 4: **Identify and measure angles as turns and recognize them in various contexts.**
(Lesson 8)

Lesson 5

Objective: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of a turn. Explore benchmark angles using the protractor.

Suggested Lesson Structure

Fluency Practice	(11 minutes)
Application Problem	(6 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (11 minutes)

- Divide Using the Standard Algorithm **4.NBT.6** (3 minutes)
- Identify Two-Dimensional Figures **4.G.1** (4 minutes)
- Physiometry **4.G.1** (4 minutes)

Divide Using the Standard Algorithm (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Grade 4 Module 3 Lesson 16 content.

T: (Write 48 ÷ 2.) On your personal white boards, solve the division problem using the vertical method.

Continue with the following possible sequence: 49 ÷ 2, 69 ÷ 3, 65 ÷ 3, 55 ÷ 5, 58 ÷ 5, 88 ÷ 4, and 86 ÷ 4.

Identify Two-Dimensional Figures (4 minutes)

Materials: (S) Personal white board, straightedge

Note: This fluency activity reviews terms learned in Lessons 1–4.

T: (Project $\overleftrightarrow{AB}$. Point to A.) Say the term for what I'm pointing to.
S: Point A.
T: (Point to B.) Say the term.
S: Point B.
T: (Point to $\overleftrightarrow{AB}$.) Say the term.
S: Line AB.

T: Use your straightedge to draw $\overleftrightarrow{CD}$ on your personal white boards.

S: (Draw a line with points C and D on the line.)

Continue with the following possible sequence: $\overrightarrow{EF}$, $\overline{GH}$, and obtuse $\angle IKJ$, acute $\angle LNM$, and right $\angle OQP$.

T: What's the relationship between $\overrightarrow{QO}$ and $\overrightarrow{QP}$?

S: The line segments are perpendicular.

T: Draw $\overline{RS}$.

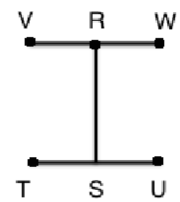
S: (Draw $\overline{RS}$.)

T: Draw $\overline{TU}$ that is perpendicular to $\overline{RS}$.

S: (Draw $\overline{TU}$.)

T: Draw $\overline{VW}$ that is perpendicular to $\overline{RS}$ and parallel to $\overline{TU}$.

S: (Draw $\overline{VW}$.)



Physiometry (4 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms from Lessons 1–4.

T: Stand up.

S: (Stand up.)

T: Show me a point.

S: (Clench one hand in a fist, and extend arm forward.)

T: Show me a line.

S: (Extend arms straight so that they are parallel with the floor. Open both hands.)

T: Show me a ray.

S: (Extend arms straight so that they are parallel with the floor. Clench one hand in a fist, and leave the other hand open.)

T: Show me a ray pointing in the other direction.

S: (Open clenched hand, and clench open hand.)

T: Show me a line segment.

S: (Extend arms straight so that they are parallel with the floor. Clench both hands into fists.)

T: Show me a right angle.

S: (Stretch one arm up directly at the ceiling. Stretch another arm directly toward a wall, parallel to the floor.)

T: Show me a different right angle.

S: (Stretch the arm pointing toward a wall directly up toward the ceiling. Move the arm pointing toward the ceiling so that it points directly toward the opposite wall.)

T: Show me an obtuse angle.

S: (Make an obtuse angle with arms.)

T: Show me an acute angle.

S: (Make an acute angle with arms.)

Continue with the following possible sequence: point, right angle, line segment, acute angle, line, right angle, and obtuse angle.

T: (Stretch one arm up directly at the ceiling. Stretch another arm directly toward a wall, parallel to the floor.) What type of angle am I making?

S: Right angle.

T: What is the relationship of the lines formed by my arms?

S: Perpendicular lines.

T: (Point to the classroom's back wall.) Point to the walls that run perpendicular to the wall I'm pointing to.

S: (Point to the side walls.)

T: (Point to the front wall.)

S: (Point to the side walls.)

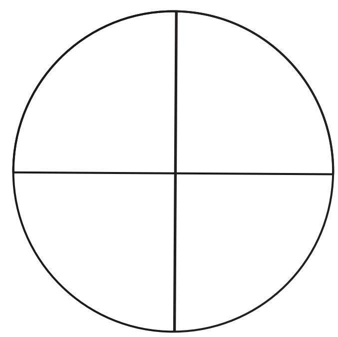
Continue pointing to one side wall, the back wall, the other side wall, and the front wall.

T: (Point to the back wall.) Point to the wall that runs parallel to the wall I'm pointing to.

S: (Point to the front wall.)

Continue pointing to one side wall, the front wall, and the other side wall.

Application Problem (6 minutes)



Materials: (S) 1 paper circle from the Concept Development

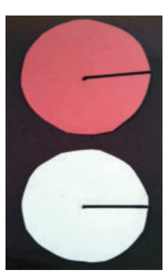
Place right angle templates on top of the circle to determine how many right angles can fit around the center point of the circle. If necessary, team up with other students to share templates. (Overlaps are not allowed.)

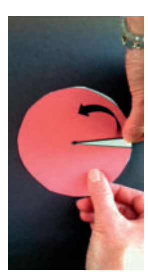
How many right angles can fit?

Note: This Application Problem bridges concepts from Lesson 2. Students use the right angle templates that they made in class to build understanding as they measure right angles around the center point of a circle.

Concept Development (33 minutes)

Materials: (T/S) 2 paper circles (5-inch diameter—one red, one white) with a radius cut into each one, delineated by a strong, straight black segment, circular protractor (Template) printed on paper or transparency





Directions for Constructing a Paper Protractor:

1. Label and cut a radius into one red and one white paper circle.
2. Stack the red circle on top of the white circle with the radii aligned and parallel to the desk's edge.
3. Pinch the corner of the white circle directly below the slit (as shown on the previous page).
4. To illustrate an angle, turn the segment given by the edge of the white region counterclockwise.

Problem 1: Reason about the number of turns necessary to make a full turn with different fractions of a full turn.

T: What do you see as you turn this segment to the left?

S: The white part is getting larger. The red part is getting smaller.

T: Do you see an angle?

S: Yes.

T: Let's agree that the white region is the interior of the angle we are focusing on.

T: (Demonstrate a quarter-turn.) Now, show a quarter-turn of the segment to the left. (Expect some confusion.)

S: (Show.)

T: Make another quarter-turn of the segment to the left. What fraction of the circular region is white now?

S: One-half. → Two-fourths.

Continue the same process until the 360° turn is complete.

T: (List the following fractions on the board.)

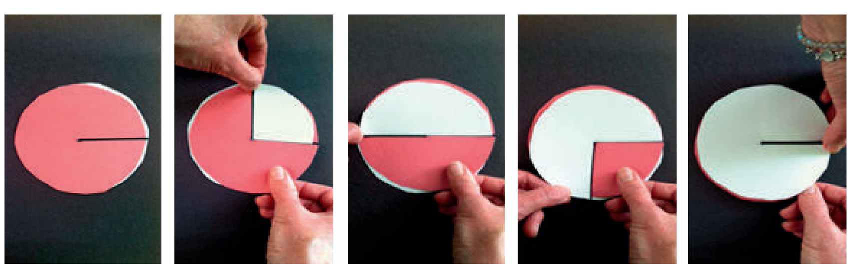
$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

T: (Point to each fraction while speaking, pausing as students manipulate the turns.) Show $\frac{1}{4}$ turn, $\frac{2}{4}$ turn, now a $\frac{3}{4}$ turn, a $\frac{4}{4}$ turn. Is the angle getting larger or smaller?

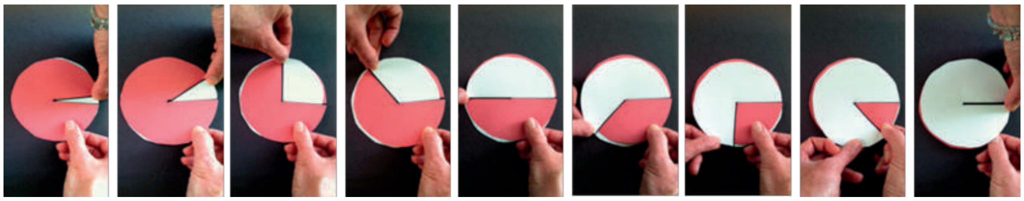
S: Larger!

T: How many fourth-turns did it take to make one full turn?

S: Four.



T: Now, I want to break up a turn into eight equal parts. Count eighths with me.



T: Will one eighth-turn be less than or greater than one fourth-turn?

S: Less than.

T: One fourth-turn is the same as two eighth-turns (point to the listed fractions). Show me what you think would be a one eighth-turn.

Repeat the same process of pointing to each eighth, in order, as the students open the angle.

T: Did it take more fourth-turns or eighth-turns to move all the way around?

S: Eighths.

T: How many eighth-turns did it take to make a whole turn?

S: Eight!

T: How many $\frac{1}{100}$ turns would it take to make a whole turn?

S: 100.

T: Would $\frac{1}{360}$ turn be smaller or larger than $\frac{1}{100}$ turn?

S: Smaller.

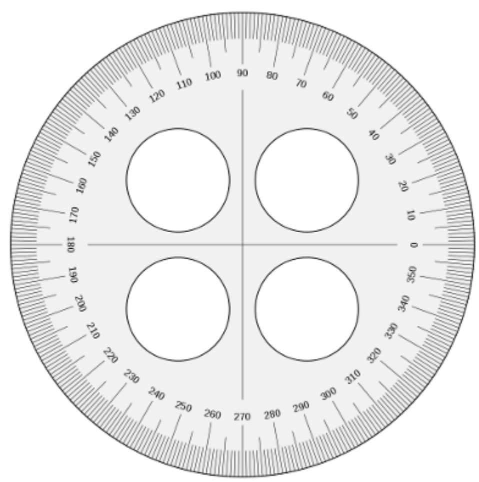
T: We have a special name for $\frac{1}{360}$ of a whole turn. It is called a **degree**! How many degrees are in one whole turn?

S: 360°.

T: Yes!

T: Here is a tool that has been partitioned and marked off to show 360°. It is called a **protractor**. The **degree measure of an angle** is measured by a protractor. Take a moment to analyze the protractor with your partner. What do you notice?

S: It is shaped like a circle. → It is counting by tens, starting at zero and going to the left. → Between each ten are tick marks showing the ones. → It has 360°. → I see four right angles! → The four right angles have the numbers 0, 90, 180, and 270.



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T: Run your finger across your protractor from zero to the center point where the bold perpendicular lines cross. Let's call that the zero line, or base line, of our protractor because it will be the starting point from where we measure angles.

Problem 2: Use a circular protractor to determine that a quarter-turn or a right angle measures 90°, a half-turn measures 180°, a three quarter-turn measures 270°, and a full rotation measures 360°.

T: Show me a quarter-turn with your circles. Keep the base segment of your angle parallel to your desk.

T: Put the zero line, or base line, on top of the bottom segment of your angle. Align the center point of the protractor with the vertex of the angle to the best of your ability.

T: Adjust the circle's angle to match your right angle template. (Pause.) Remove the template and place the protractor to measure that angle. What do you notice?

S: The quarter-turn matches the bold lines of the protractor. → It's 90°. → One fourth-turn is 90°. → A right angle measures 90°.

T: Do a half-turn and see how many degrees your angle is?

S: 180°.

T: Turn another quarter- or fourth-turn.

S: 270°.

T: And one last quarter- or fourth-turn?

S: 360°. → 0°.

T: What does your angle look like right now?

S: It's all white.

T: A zero-degree angle is when we have not turned at all. We have made one full turn of 360°. There are 360° in a full turn.

T: How many 90° angles, or right angles, are there in a full turn?

S: Four right angles.

T: How do you know?

S: Because we made four quarter-turns and each one was 90°. → It is easy to see them because of the bold perpendicular lines on the protractor.

T: Using your white circle, position your protractor with the zero or base line on top of the black segment, matching up the center point of the circle with the center point of the protractor.

T: Estimate to make a point at 90°. Draw a line segment from the center point to that point. What have you drawn?

S: A right angle. → A 90° angle. → Perpendicular lines.

T: Now, make a point at 45°. Draw a line segment from the center point to the point you just made. What have you made?

S: A 45° angle.

NOTES ON PROTRACTORS:

Circular protractors come in many different sizes and formats. Several include not one but two numbered scales. Since the lesson's objective is to understand a one-degree angle, it is recommended to use the template, which only has one set of angle measures moving counter-clockwise counting by tens. Its simplicity may help ground students' initial understanding so that when they encounter a *regular* protractor in the next lesson, they will have internalized the meaning of a degree and the measures of acute and obtuse angles.

T: Yes! What do you notice?

S: The 45° angle is half as big as the 90° angle.
→ Two 45° angles are the same as one 90° angle.
→ 2 × 45 = 90.

Problem 3: Measure and draw benchmark angles with the protractor.

T: Now, let's work to measure and draw benchmark angles using your circles and protractors.

T: We have already started Set A using your white circle. Continue turning your circle, aligning the zero or base line with each last segment drawn. Be sure to keep your protractor's center point on the center point of the circle. Draw new 45° angles until you have made a whole turn. Let me demonstrate. (Demonstrate silently.)

T: Draw Set B on your red circle just as you did for Set A, but this time, draw 30° angles. This full turn will be made of 30° angles. Draw 30° angles until you have made a whole turn.

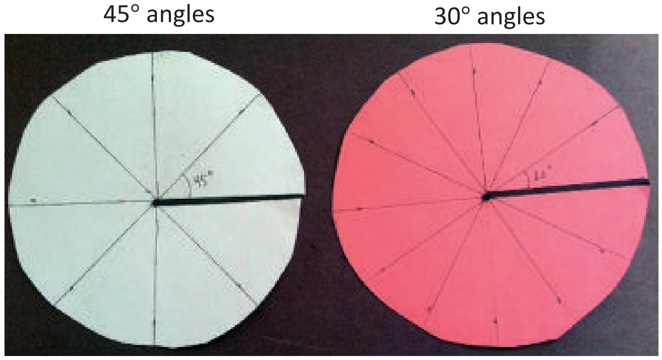
NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Ease the task of drawing benchmark angles for students with the following suggestions:

- Provide larger paper circles of thicker cardstock that may be easier to manipulate.
- Provide circles with pre-drawn markings.
- Provide the complete list of benchmark angles, liberating students to focus on reading and drawing angles with a protractor.

Set A
45° angles

Set B
30° angles



T: Place the center point of the protractor on the shared endpoints of the segments on your white circle. Align the zero line with the black segment. What are the measurements of the angles you have drawn?

S: 0°, 45°, 90°, 135°, 180°, etc.

T: Trace each angle separately with your finger, moving from the smallest to largest angles.

Repeat the process with the sequence of 30° angles.

T: All of these are benchmark angles. Let's use our Problem Set to further explore them.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set. Some students might be allowed to complete the drawing of the benchmark angles, while others start on the Problem Set. Take 10 minutes for the Problem Set, as always, with the understanding that the variation in work completed may differ considerably.

Student Debrief (10 minutes)

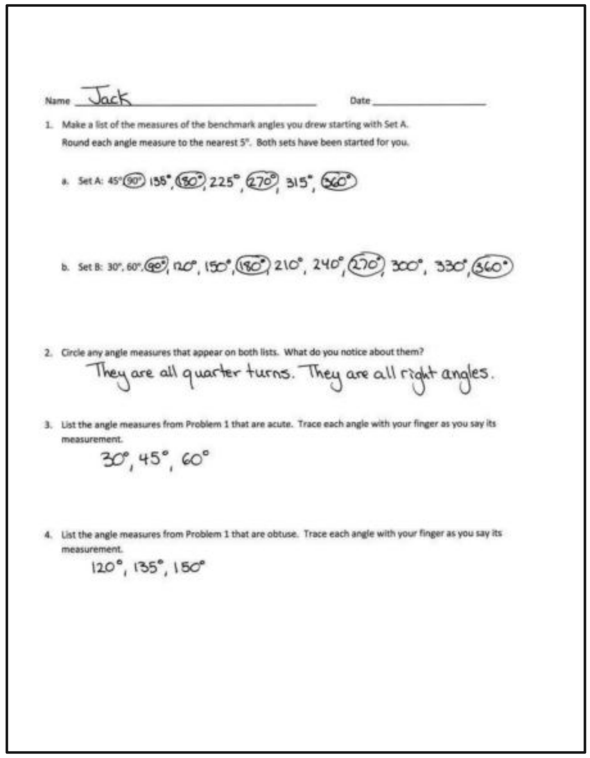
Lesson Objective: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of a turn. Explore benchmark angles using the protractor.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- When you listed the benchmark angles, did you notice any numerical patterns?
- You listed some measures of acute and obtuse angles. What would be some measurements of other acute angles? Obtuse angles?
- A full turn is 360°. What could you do to find the **degree measure of an angle** that takes 10 turns to make a whole turn?
- How did you respond to the final question?
- If you were to draw a tape diagram to represent one whole turn and the benchmark angles of Set A, what would you do? Set B?



Name Jack Date

1. Make a list of the measures of the benchmark angles you drew starting with Set A. Round each angle measure to the nearest 5°. Both sets have been started for you.

   a. Set A: 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°

   b. Set B: 30°, 60°, 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, 330°, 360°

2. Circle any angle measures that appear on both lists. What do you notice about them?

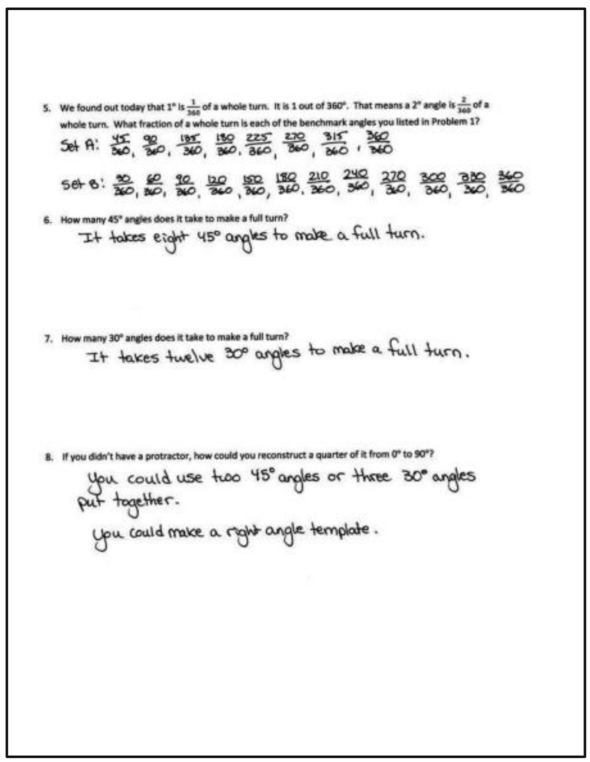
   They are all quarter turns. They are all right angles.

3. List the angle measures from Problem 1 that are acute. Trace each angle with your finger as you say its measurement.

   30°, 45°, 60°

4. List the angle measures from Problem 1 that are obtuse. Trace each angle with your finger as you say its measurement.

   120°, 135°, 150°



5. We found out today that 1° is $\frac{1}{360}$ of a whole turn. It is 1 out of 360°. That means a 2° angle is $\frac{2}{360}$ of a whole turn. What fraction of a whole turn is each of the benchmark angles you listed in Problem 1?

   Set A: $\frac{45}{360}$, $\frac{90}{360}$, $\frac{135}{360}$, $\frac{180}{360}$, $\frac{225}{360}$, $\frac{270}{360}$, $\frac{315}{360}$, $\frac{360}{360}$

   Set B: $\frac{30}{360}$, $\frac{60}{360}$, $\frac{90}{360}$, $\frac{120}{360}$, $\frac{150}{360}$, $\frac{180}{360}$, $\frac{210}{360}$, $\frac{240}{360}$, $\frac{270}{360}$, $\frac{300}{360}$, $\frac{330}{360}$, $\frac{360}{360}$

6. How many 45° angles does it take to make a full turn?

   It takes eight 45° angles to make a full turn.

7. How many 30° angles does it take to make a full turn?

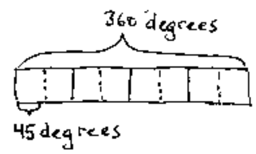
   It takes twelve 30° angles to make a full turn.

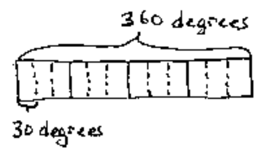
8. If you didn't have a protractor, how could you reconstruct a quarter of it from 0° to 90°?

   You could use two 45° angles or three 30° angles put together.

   You could make a right angle template.

- Shade in the region of a 45° angle on your white circle. What fraction of the whole turn is that? Do the same for your 30° angle. What if you shaded in a region defined by a 120° angle on your red circle? What fraction of the whole circle is that?
- Use your **protractor** to explain to your partner what a **degree** is.
- Use your protractor to trace some benchmark angles.
- If you didn't have a protractor, how could you construct one?





Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ____ Date ______________________

1. Make a list of the measures of the benchmark angles you drew, starting with Set A. Round each angle measure to the nearest 5°. Both sets have been started for you.

   a. Set A: 45°, 90°,

   b. Set B: 30°, 60°,

2. Circle any angle measures that appear on both lists. What do you notice about them?

3. List the angle measures from Problem 1 that are acute. Trace each angle with your finger as you say its measurement.

4. List the angle measures from Problem 1 that are obtuse. Trace each angle with your finger as you say its measurement.

5. We found out today that 1° is $\frac{1}{360}$ of a whole turn. It is 1 out of 360°. That means a 2° angle is $\frac{2}{360}$ of a whole turn. What fraction of a whole turn is each of the benchmark angles you listed in Problem 1?

6. How many 45° angles does it take to make a full turn?

7. How many 30° angles does it take to make a full turn?

8. If you didn't have a protractor, how could you reconstruct a quarter of it from 0° to 90°?

Name ____ Date ______________________

1. How many right angles make a full turn?

2. What is the measurement of a right angle?

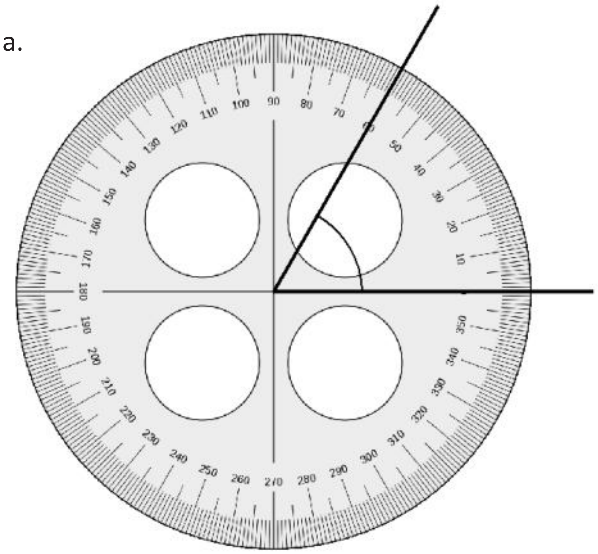
3. What fraction of a full turn is 1°?

4. Name at least four benchmark angle measurements.

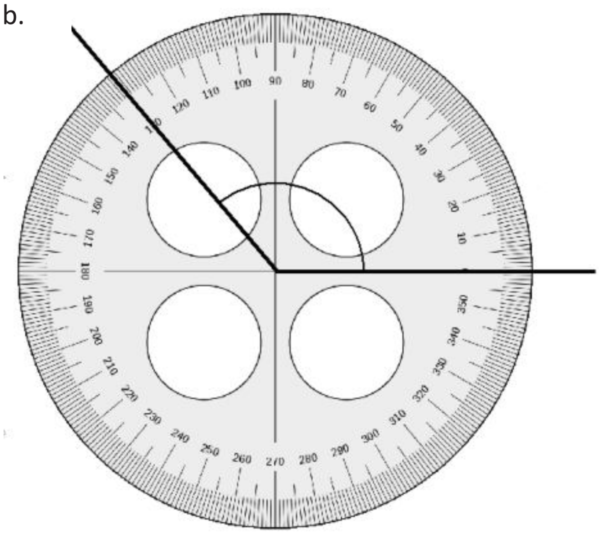
Name ____ Date ____________________

1. Identify the measures of the following angles.

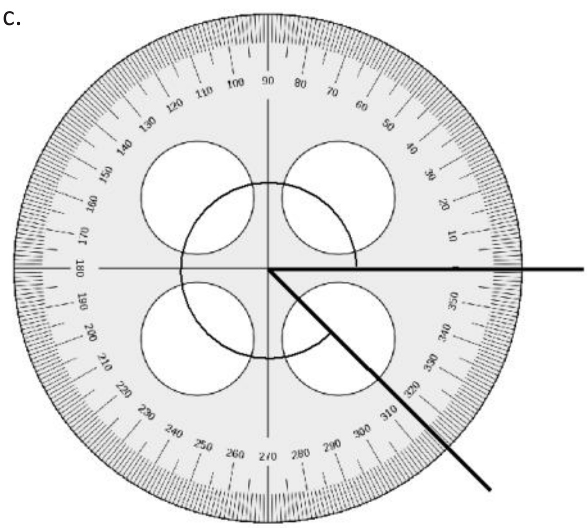
a.



b.



c.



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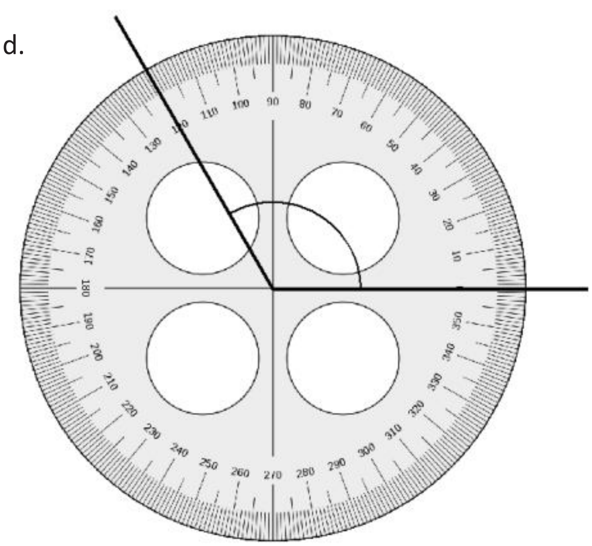
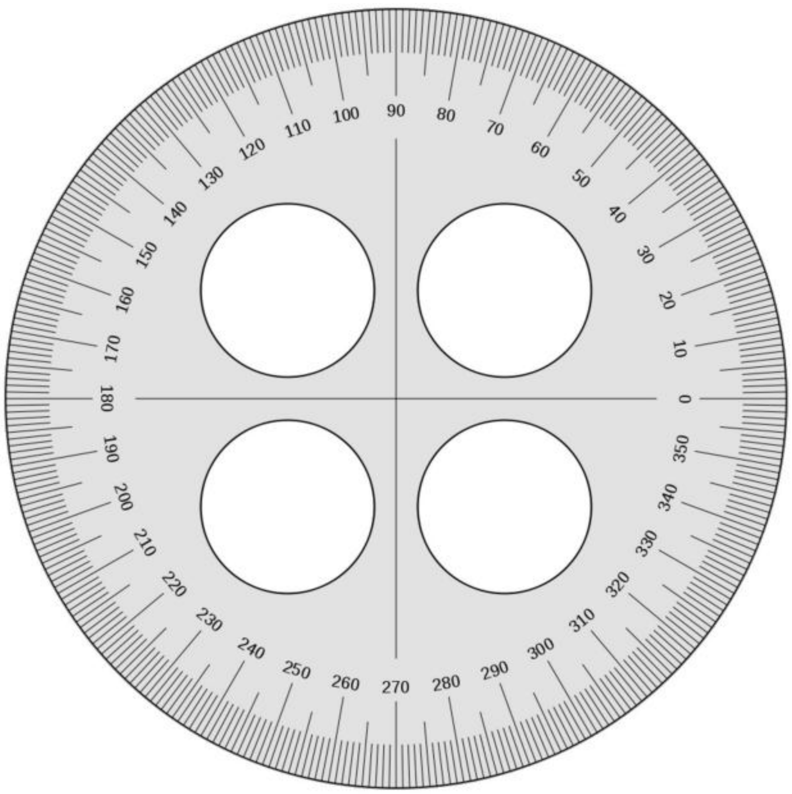


2. If you didn't have a protractor, how could you construct one? Use words, pictures, or numbers to explain in the space below.



circular protractor

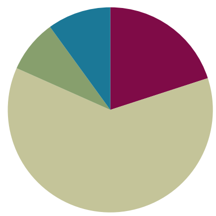
Lesson 6

Objective: Use varied protractors to distinguish angle measure from length measurement.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (5 minutes)
- Concept Development (37 minutes)
- Student Debrief (6 minutes)

Total Time (60 minutes)



Fluency Practice (12 minutes)

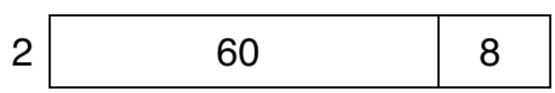
- Divide Using the Area Model **4.NBT.6** (4 minutes)
- Draw and Identify Two-Dimensional Figures **4.G.1** (4 minutes)
- Physiometry **4.G.1** (4 minutes)

Divide Using the Area Model (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Grade 4 Module 3 Lesson 20 content.

T: (Project area model that shows 68 ÷ 2.) Write a division expression for this area model.
S: (Write 68 ÷ 2.)
T: Label the length of each rectangle in the area model.
S: (Write 30 above the 60 and 4 above the 8.)
T: Solve using the standard algorithm.
S: (Solve.)



Continue with the following possible sequence: 69 ÷ 3, 78 ÷ 3, and 76 ÷ 4.

Draw and Identify Two-Dimensional Figures (4 minutes)

Materials: (S) Personal white board, straightedge

Note: This fluency activity reviews terms introduced in Lessons 1–5.

T: (Project $\overline{AB}$. Point to A.) Say the term for what I'm pointing to.
S: Point A.
T: (Point to B.) Say the term.
S: Point B.
T: (Point to $\overline{AB}$.) Say the term.
S: Line segment AB.
T: Use your straightedge to construct $\overline{CD}$ on your personal white boards.
S: (Draw $\overline{CD}$.)
T: Beneath $\overline{CD}$, draw $\overleftrightarrow{EF}$ that is parallel to $\overline{CD}$.
S: (Beneath $\overline{CD}$, draw $\overleftrightarrow{EF}$ that is parallel to $\overline{CD}$.)
T: Draw $\overrightarrow{GH}$ that begins on $\overleftrightarrow{EF}$ and runs perpendicular through $\overline{CD}$.
S: (Draw $\overrightarrow{GH}$ that begins on $\overleftrightarrow{EF}$ and runs perpendicular through $\overline{CD}$.)
T: What's the relationship between $\overrightarrow{GH}$ and $\overline{CD}$?
S: $\overrightarrow{GH}$ is perpendicular to $\overline{CD}$.
T: Draw $\overline{IJ}$ that is perpendicular to $\overline{KL}$.
S: (Draw $\overline{IJ}$. Draw $\overline{KL}$ that is perpendicular to $\overline{IJ}$.)
T: Draw $\overline{MN}$ that is perpendicular to $\overline{IJ}$ and parallel to $\overline{KL}$.
S: (Draw $\overline{MN}$ that is perpendicular to $\overline{IJ}$ and parallel to $\overline{KL}$.)
T: (Project a right $\angle ACB$.) Name the angle.
S: $\angle ACB$.
T: What type of angle is it?
S: Right angle.
T: What's the relationship of $\overrightarrow{CA}$ and $\overrightarrow{CB}$?
S: They're perpendicular.
T: How many degrees are in $\angle ACB$?
S: 90°.
T: (Project an acute $\angle DFE$.) Name the angle.
S: $\angle DFE$.
T: (Beneath $\angle DFE$, write 30° or 150°.) Estimate. Is the measure of $\angle DFE$ 30° or 150°?
S: 30°.
T: How do you know?
S: Acute angles are less than 90°.

Continue with the other given angles.

Physiometry (4 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms from Lessons 1–5.

T: Stand up.
S: (Stand up.)
T: Show me a right angle.
S: (Stretch one arm up directly at the ceiling. Stretch another arm directly toward a wall, parallel to the floor.)
T: Show me a different right angle.
S: (Stretch the arm pointing toward a wall directly up toward the ceiling. Move the arm pointing toward the ceiling so that it points directly toward the opposite wall.)
T: Show me an obtuse angle.
S: (Make an obtuse angle with arms.)
T: Show me an acute angle.
S: (Make an acute angle with arms.)
T: Show me a right angle.
S: (Make a right angle with arms.)
T: Show me an angle that measures approximately 30°.
S: (Move arms closer together, lessening the space between their arms, so that it is approximately 30°.)
T: Show me an angle that measures approximately 60°.
S: (Open arms further apart to approximately 60°.)

Continue with the following possible sequence: 90°, 120°, 150°, 50°, 170°, 70°, and 180°.

T: What is the term for a 180° angle?
S: A straight angle.
T: Show me a line segment.
S: (Close fists.)
T: (Point at the classroom's back wall.) Point to the walls that run perpendicular to the wall I'm pointing to.
S: (Point to the side walls.)
T: (Point to the front wall.)
S: (Point to the side walls.)

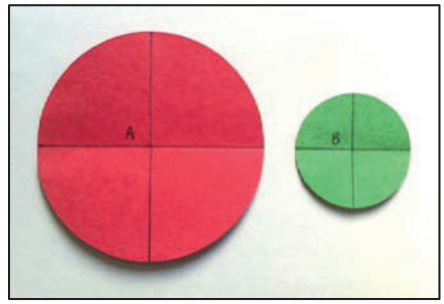
Continue pointing to one side wall, the back wall, the other side wall, and the front wall.

T: (Point to the back wall.) Point to the wall that runs parallel to the wall I'm pointing to.
S: (Point to the front wall.)

Continue pointing to one side wall, the front wall, and the other side wall.

Application Problem (5 minutes)

Materials: (S) 2 circles of different sizes (different colors, if possible)



Fold Circle A and Circle B as you would to make a right angle template. Trace the folded perpendicular lines. How many right angles do you see at the center of each circle? Did the size of the circle matter?

Note: This Application Problem connects to Lesson 5, in which students found four right angles within a circle. As an introduction to arc length measure having no effect on angle measurement, students find the number of right angles around the center point of different size circles.

Concept Development (37 minutes)

Materials: (T) 2 circle cutouts from Application Problem, 2 pieces of wire the same length as the circumference of each circle cutout, Practice Sheet, dark marker, straightedge, an assortment of protractors including at least one circular protractor and one 180° protractor (S) 2 circle cutouts from Application Problem, Practice Sheet, dark marker, straightedge, an assortment of protractors including at least one circular protractor and one 180° protractor

Note: Providing a variety of protractors allows students to distinguish angle measure from length measure. Students may share protractors during this activity. It is not necessary for every student to have two or three varied protractors of their own.

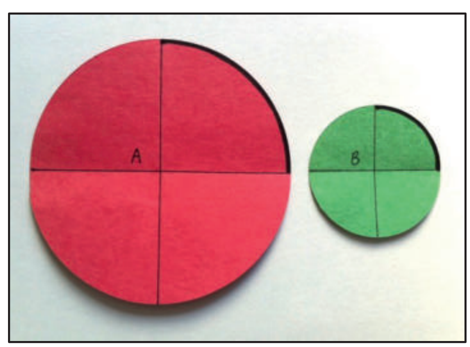
> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> Check that English language learners and others understand the meaning of the new math term *arc.* If necessary and possible, offer explanations in students' first language. Link *arc* to more familiar words or phrases such as the *Gateway Arch in St. Louis.*

Problem 1: Explore the effect of angle size on arc length. Distinguish between angle and length measurement.

T: How many degrees are in a right angle?

S: 90°.

T: Use a marker to draw an arc on Circle A and Circle B (as pictured to the right).



T: Trace your finger along each arc. Which circle has a longer arc?

S Circle A!

T: But don't both arcs measure 90°? Why are the arcs different lengths?

S: I don't know. → Circle A is bigger, so maybe it needs a bigger arc.

T: How many total degrees in this circle? (Point to Circle A.)

S: 360°.

T: How many total degrees in this circle? (Point to Circle B.)

S: 360°.

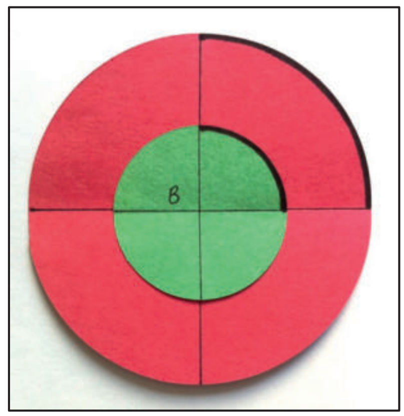
T: So, if I divide Circle A into 360°, each arc length will be a little longer than the arc lengths in Circle B. I'm still measuring a quarter turn in each circle, and each arc is one-fourth of the total distance around the circle.

T: Think of it as taking the **length of an arc** from each circle and stretching them out into a line. (Model two wires that wrap the circumference of each circle stretched out in a line.) I can chop each wire into 360 equal-size pieces. Which arc will have smaller pieces?

S: The arc from Circle B.

T: Right! 90° is one quarter of 360°. (Cut each wire into four equal parts. Show that one part from each wire is the same length as the arc of each circle.) Which arc is longer?

S: Circle A has a longer arc.



T: So, does the length of an arc determine the measure of a given angle? Discuss this with your partner.

S: No! The arcs might be longer or shorter, but they could be measuring the same size angle. → No matter where the arc is, I just have to remember that that arc is part of 360°. → Right, because I could have a super tiny circle or a really big circle, but still, the right angles measure 90°.

T: Place Circle B on top of Circle A to show that the length of an arc does not determine the degree measure.

Problem 2: Use a 180° protractor to verify angle measure.

T: (Project $\angle C$ and $\angle D$ from the Practice Sheet.) What type of angle do you see?

S: Acute!

T: Discuss what you notice about the arc length in each angle.

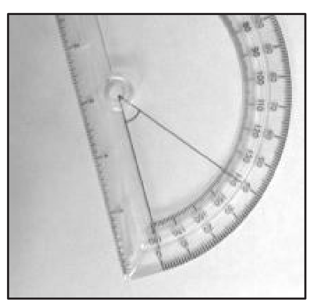
S The arc length in $\angle C$ is longer than the one in $\angle D$. → The arcs are different lengths, but the angles look like they might be the same. → It looks like $\angle C$ came from a larger circle than $\angle D$ did.

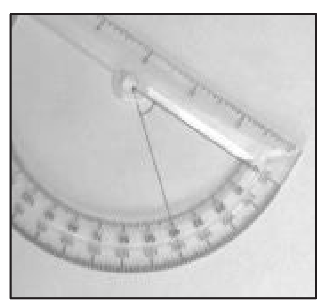
T: Let's measure to find out if the angles turn the same number of degrees.

T: (Distribute and display a 180° protractor.) What do you notice about this protractor?

S: It's half a protractor. → It's only a piece of a circular protractor. → It's got a straight edge.

T: Just like you measured angles with a circular protractor, you can measure angles with this 180° protractor. Protractors sometimes have two sets of numbers. We determine which number to read based off of the side of the angle that touches zero. (Show a 40° angle as pictured to the right, aligning both sides to zero and discussing which set of numbers to read.)





T: (Model. Place the middle notch on the vertex of the angle. Align a side with the zero or base line on the protractor. Read the number the second side length touches.)

T: With your partner, measure $\angle C$.

S: 60°. No. Wait, 120°. → It can't measure 120°. It's an acute angle. 60°. → Remember, we count up from the side of the angle at zero. So, we are using the outside numbers for this angle.

T: Measure $\angle D$.

S: 60°.

T: What did you discover? Discuss it with your partner.

S: The arc lengths are different, but the degrees are the same. → Both angles are 60°, but $\angle D$ looks different because the sides of the angle are shorter.

T: What would happen if we placed the angles on top of each other? Turn and talk. (Allow time for a brief discussion.) Let's try! (Model.)

S: They match up! → The angles are the same size!

T: Imagine a circle drawn with the vertex of $\angle D$ as its center point, the end of one segment being the length to the arc and another circle drawn in the same way around $\angle C$.

T: What could you say about the two circles?

S: The circles would be different sizes. → The lengths of the sides of $\angle C$ would make a larger circle than the sides of $\angle D$. → The arcs and sides of the angles will be different lengths, but the angle will measure the same because each angle represents a fraction of 360°.

Problem 3: Use multiple protractors to measure the same angle.

T: Look at the different protractors in front of you. What do you notice about them?

S: Some are 360° protractors, and some are 180° protractors. → Some have only one set of numbers; others have two sets. → They are all different sizes. → The base line of this one is on the bottom of the protractor, but the base line of this one is above the plastic.

T: Align your protractors using the center point, just like we did with our two circles at the beginning of the lesson. Do you see how these different protractors have different arcs?

S: Yes, some are small, and some are big.

T: Yes, but they all measure 360° of a circle.

S: But some only measure 180°.

T: That's because it is representing half a circle. Notice the tick marks on all of the different protractors.

S: Some are really close together!

T: Why is that?

S: It's on the smallest protractor, so that means the arc lengths are shorter than those of the other protractors.

T: Let's use at least three different protractors to measure $\angle E$.

Allow time for students to measure individually, with partners, or in small groups, depending on the variety of protractors available in the classroom.

S: All three protractors showed that this is a 130° angle!

T: What does that tell you about the side lengths of an angle?

S: The side lengths can be any length. → No matter where you measure on the circle, the number of degrees will always be the same. → We aren't measuring the sides of angles. The different sizes of protractors pick a different point on each segment where a circle could be and measures that.

T: Let's look at Problem 1(a) of the Problem Set together. Measure the angle that is shown.

S: I can't measure that angle. The image is too small! → I know what to do! We can make the segments of the angle longer. We just found out that the angle measure stays the same no matter what the side length is.

T: Use your straightedge to extend the sides of the angle until they are long enough for you to use the protractor to measure the angle. (Model.)

S: Now, I can measure the angle!

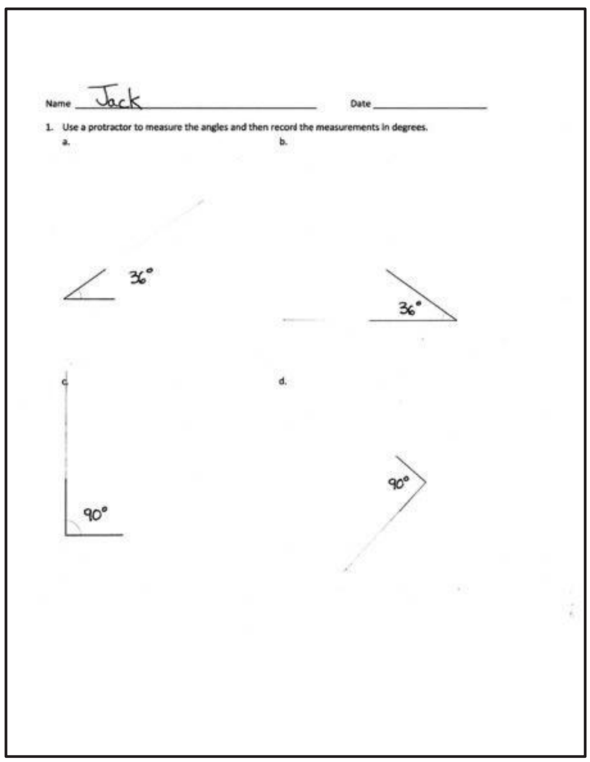
NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Students who experience frustration with manipulating and reading a protractor may find success with virtual protractors, such as those found at the following website: http://www.teacherled.com/resources/anglemeasure/angleteach.swf

Virtual protractors may be a viable option for classrooms that do not have a wide range or large number of protractors.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted time frame. For some classes, it may be appropriate to modify the assignment by specifying which problems they should work on first.



Student Debrief (6 minutes)

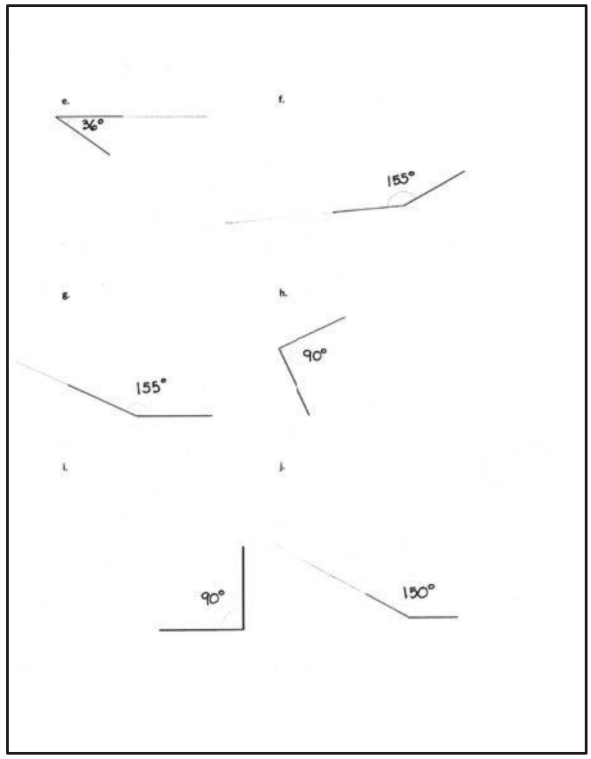
Lesson Objective: Use varied protractors to distinguish angle measure from length measurement.

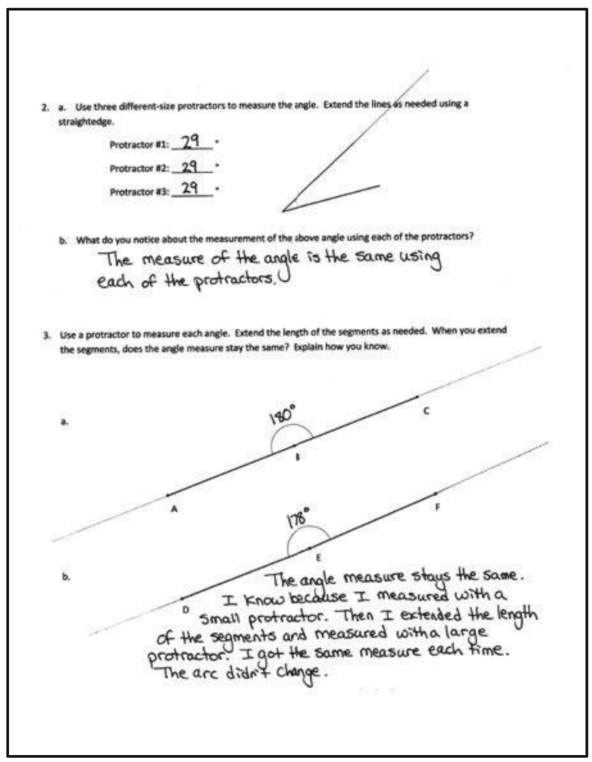
The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, which angle had the same measure as $\angle G$? $\angle I$?
- In Problem 1, which angles had the same angle measure but different side length measures?
- For Problem 2, discuss your experience of measuring with different protractors. Describe how the **length of an arc** on each protractor did or did not affect the measure of the given angle.
- How many degrees did the angles in Problem 3 measure? What type of angle is the angle in part (a)? We know a straight angle forms a straight line. Points A, B, and C create $\angle ABC$ and $\overline{ABC}$. When three or more points are found on a line, we call them **collinear** points. Are points D, E, and F collinear? Why not?
- Take a look at your 180° protractor. Find pairs of numbers that label the two scales, such as 150° and 30°. Name other pairs of numbers. What do you notice about the pairs of numbers?
- How did the Application Problem help you understand that an angle measure remains constant and is not a length measure?

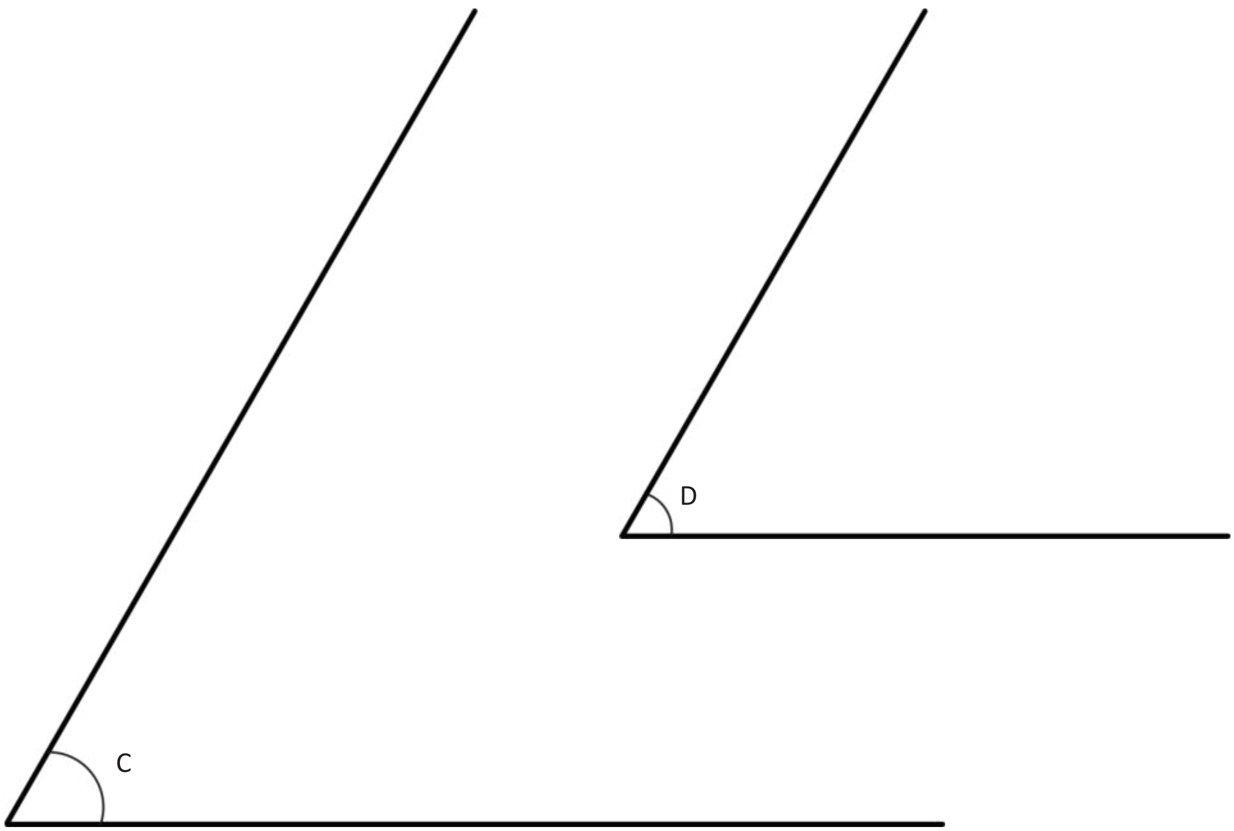


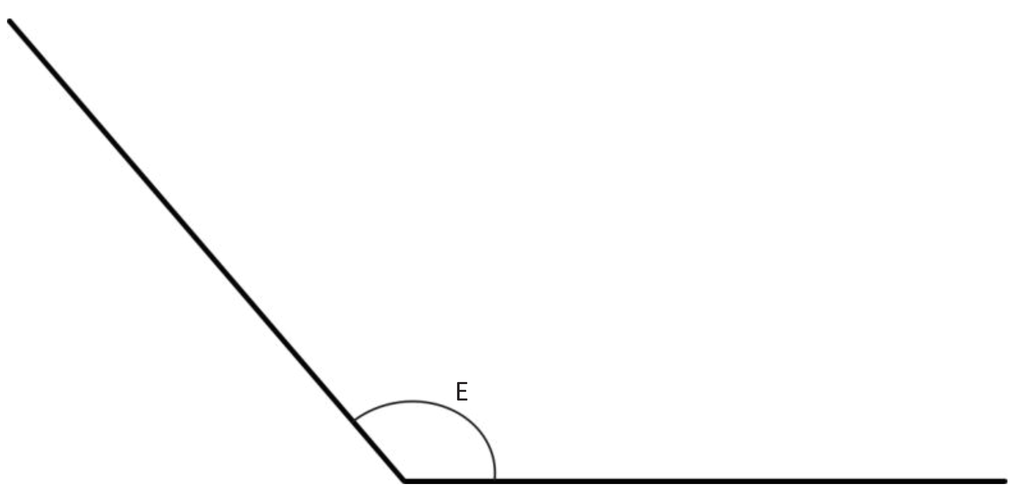


Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ______________________________ Date ____________________

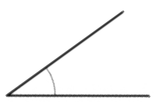




Name ________________________________ Date ____________________

1. Use a protractor to measure the angles, and then record the measurements in degrees.

a.



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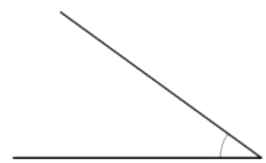

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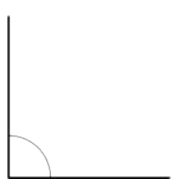

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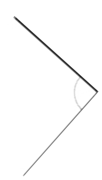
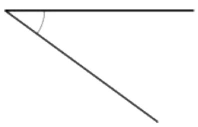


e.



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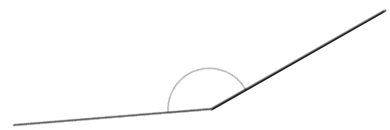

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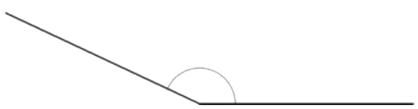

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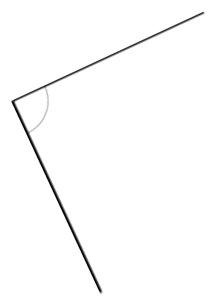

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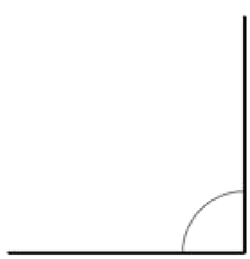


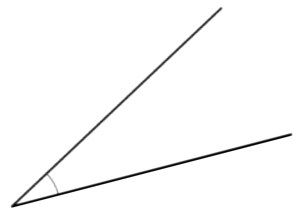
j.

2. a. Use three different-size protractors to measure the angle. Extend the lines as needed using a straightedge.

Protractor #1: ________ °

Protractor #2: ________ °

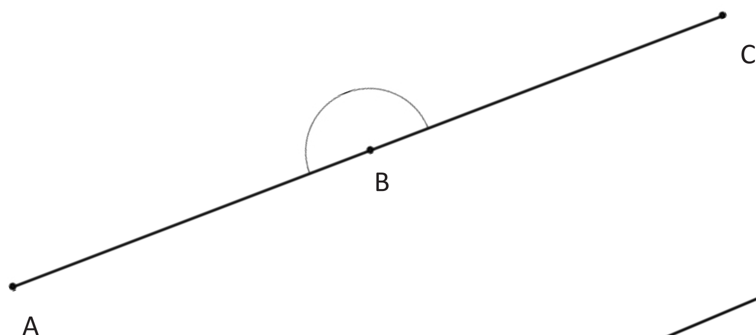
Protractor #3: ________ °



b. What do you notice about the measurement of the above angle using each of the protractors?

3. Use a protractor to measure each angle. Extend the length of the segments as needed. When you extend the segments, does the angle measure stay the same? Explain how you know.

a.



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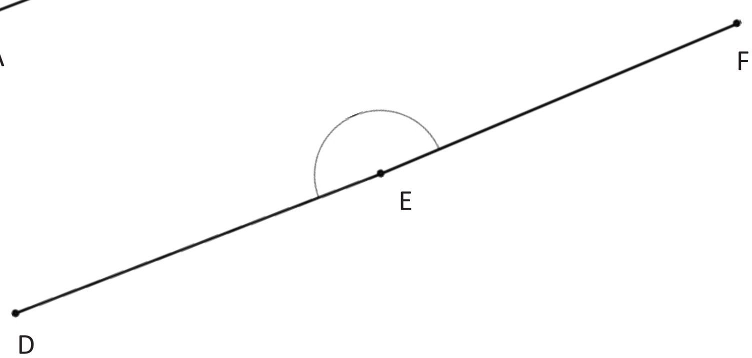
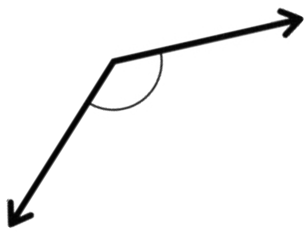


Name ____ Date ______________________

Use any protractor to measure the angles, and then record the measurements in degrees.

1.



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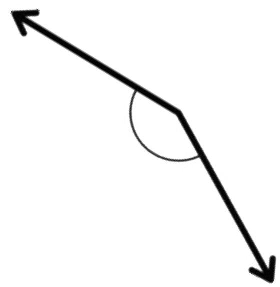

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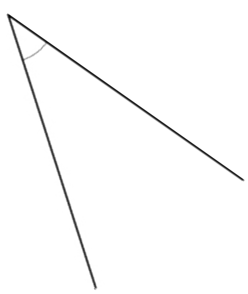

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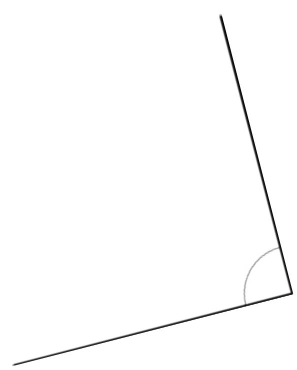
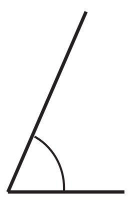


Name ____ Date ______________________

1. Use a protractor to measure the angles, and then record the measurements in degrees.

a.



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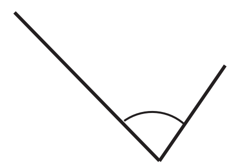

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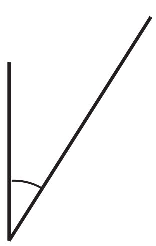

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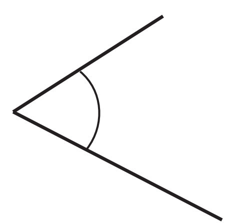

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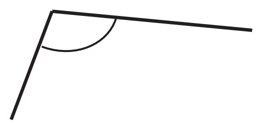

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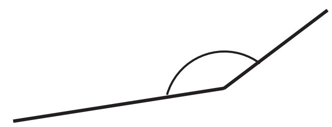

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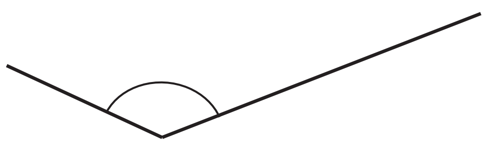

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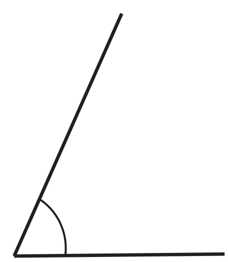

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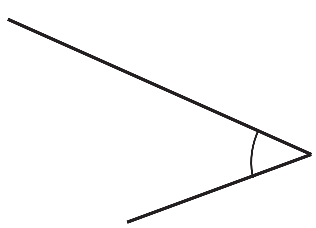

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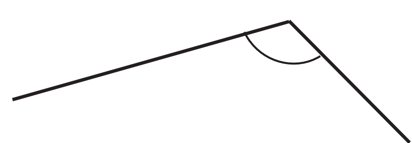
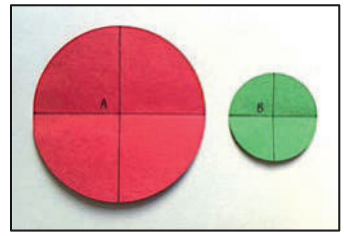
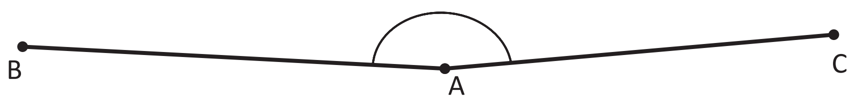


2. Using the green and red circle cutouts from today's lesson, explain to someone at home how the cutouts can be used to show that the angle measures are the same even though the circles are different sizes. Write words to explain what you told him or her.



3. Use a protractor to measure each angle. Extend the length of the segments as needed. When you extend the segments, does the angle measure stay the same? Explain how you know.

a.



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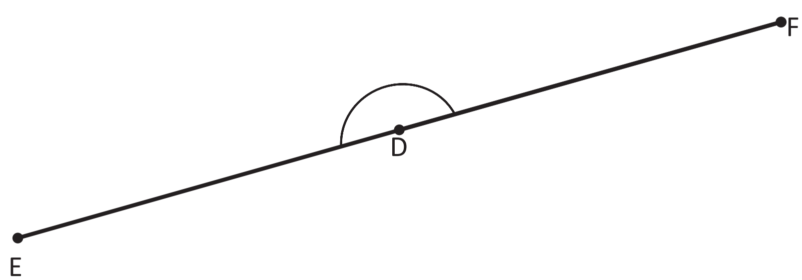
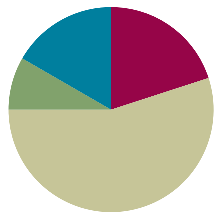


Lesson 7

Objective: Measure and draw angles. Sketch given angle measures, and verify with a protractor.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Break Apart 90, 180, and 360 **4.MD.7** (4 minutes)
- Physiometry **4.G.1** (4 minutes)
- Identify Angle Measures **4.MD.6** (4 minutes)

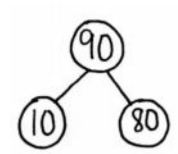
Break Apart 90, 180, and 360 (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for unknown angle problems in Lessons 10–11.

T: (Project a number bond with a whole of 90. Fill in 10 for one of the parts.) On your personal white boards, write the number bond, filling in the unknown part.

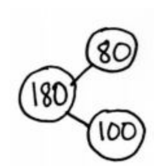
S: (Draw a number bond with a whole of 90 and with 10 and 80 as parts.)



Continue breaking apart 90 with the following possible sequence: 50, 40, and 45.

T: (Project a number bond with a whole of 180. Fill in 80 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

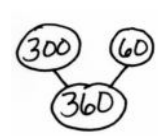
S: (Draw a number bond with a whole of 180 and with 80 and 100 as parts.)



Continue breaking apart 180 with the following possible sequence: 90, 120, 140, and 35.

T: (Project a number bond with a whole of 360. Fill in 300 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 360 and with 300 and 60 as parts.)



Continue breaking apart 360 with the following possible sequence: 100, 90, 180, 120, and 45.

Physiometry (4 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms from Lessons 1–5.

T: Stand up.
S: (Stand up.)
T: Show me an acute angle.
S: (Make an acute angle with arms.)
T: Show me an obtuse angle.
S: (Make an obtuse angle with arms.)
T: Show me a right angle.
S: (Make a right angle with arms.)
T: Show me an angle that measures approximately 80°.
S: (Move arms closer together, lessening the space between their arms, so that it's approximately 80°.)
T: Show me an angle that measures approximately 10°.
S: (Close arms more to approximately 10°.)

Continue with the following possible sequence: 90°, 100°, 170°, 150°, 60°, 140°, 70°, and 180°.

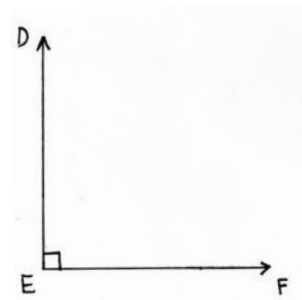
T: What is the term for a 180° angle?
S: A straight angle.
T: Show me a line segment.
S: (Close fists.)
T: Show me a ray.
S: (Open one hand while keeping the other hand clenched.)
T: Partner up with a classmate next to you. Decide who is Partner A and who is Partner B.
S: (Pair up with a partner. Decide who is Partner A and who is Partner B.)
T: Partner A, point at a side wall.
S: (Point at a side wall.)
T: Partner B, point at the walls that are perpendicular to the wall Partner A is pointing to.
S: (Point at front and back walls.)
T: Partner B, point to any wall in the room.
S: (Point at a wall.)
T: Partner A, point at the wall that is parallel to the wall Partner B is pointing to.
S: (Point at wall parallel to the wall Partner B is pointing to.)

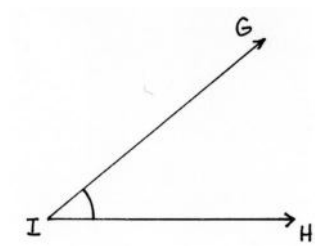
Identify Angle Measures (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 5.

T: How many degrees are in a right angle?
S: 90°.
T: (Project a right $\angle DEF$.) Name the angle.
S: $\angle DEF$.
T: What type of angle is it?
S: A right angle.
T: What's the relationship of $\overrightarrow{ED}$ and $\overrightarrow{EF}$?
S: They're perpendicular.
T: How many degrees are in $\angle DEF$?
S: 90°.
T: (Project an acute $\angle GIH$.) Name the angle.
S: $\angle GIH$.
T: (Beneath $\angle GIH$, write 40° or 140°.) Estimate. Is the measure of $\angle GIH$ 40° or 140°?
S: 40°.
T: How do you know?
S: Acute angles are less than 90°.



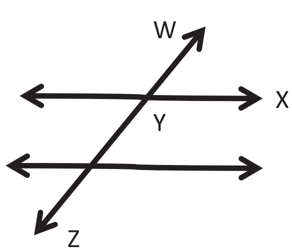


Continue with the following possible sequence: obtuse angle measuring 130° or 50°, acute angle measuring 75° or 105°, and obtuse angle measuring 92° or 88°.

Application Problem (5 minutes)

Predict the measure of $\angle XYZ$ using your right angle template. Then, find the actual measure of $\angle XYZ$ using a circular protractor and 180° protractor. Compare with your partner when you are finished.

Note: This Application Problem reviews the practice of measuring angles from Lesson 6 and transitions into the Concept Development of today's lesson, where students measure and draw angles. This figure can be found on the Practice Sheet (Figure 1).



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Provide protractor alternatives for students, if necessary. Some students may work more efficiently with large-print protractors that include a clear, moveable wand. Others may find using an angle ruler easier.

For students with poor vision and other restrictions, outline angles and shapes to be measured with pipe cleaners to make the activity tactile.

Concept Development (33 minutes)

Materials: (T/S) Circular protractor, 180° protractor, Practice Sheet

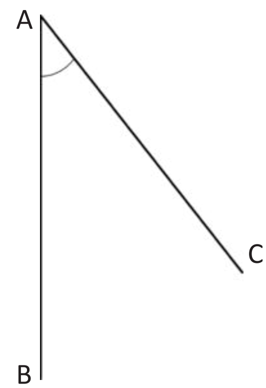
Problem 1: Measure angles less than 180° using a circular and 180° protractor.

T: In completing the Application Problem, what was your prediction for the measure of $\angle XYZ$?

S: I predicted $\angle XYZ$ to be about 100°. → I know that $\angle XYZ$ is an obtuse angle because it is greater than a right angle, so I predicted it to be about 110°.

T: How did you use the circular and 180° protractor to find the measure of $\angle XYZ$?

S: I lined up one side of the angle with the base line on the circular protractor. Then, I saw where the other side of the angle touched on the arc. → First, I put the center hole of the 180° protractor at the vertex Y. Next, I lined up $\overrightarrow{YZ}$ with the zero line on the protractor. Then, I read where $\overrightarrow{YX}$ measured on the protractor.



T: Aligning the protractor correctly is very important. Let's practice measuring $\angle CAB$ using the circular protractor. Measure $\angle CAB$ (Practice Sheet, Figure 2).

T: Now, with your partner, take the 180° protractor and measure the same angle.

T: What do you notice?

S: Both protractors say 45°. → The angle measure is the same no matter which protractor we use.

T: Look at Figure 3 on your Practice Sheet. Using either protractor, find the measure of $\angle DEF$.

S: With the circular protractor, $\angle DEF$ measures 120°. → With the 180° protractor, $\angle DEF$ measures 120°.

Problem 2: Measure an angle greater than 180° by subtracting from 360°.

T: Look at Figure 4 on your Practice Sheet. Use either protractor to measure $\angle QRS$. Which protractor will you use?

S: I am going to use the circular protractor because the 180° protractor doesn't fit right. $\angle QRS$ measures 230°. → I want to use a 180° protractor, but I am not sure how. It isn't big enough to measure the angle.

T: Let's figure out how to use the 180° protractor. The arc close to the vertex symbolizes the angle we want to measure.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

As they measure $\angle CAB$, guide students working below grade level to adjust the paper rather than the protractor.

Challenge students working above grade level to predict the measure of $\angle CAB$ before measuring. Invite students to explain their reasoning. Also, extend the task as time permits by having students measure $\angle CAB$ using each side of the angle as a base. Ask, "What do you notice?"

MP.2

T: What happens if we extend the drawing of the arc? Show me.

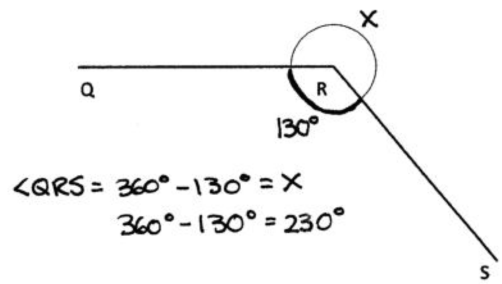
S: (Extend arc.) We have a circle with point R in the middle.

T: There are two angles represented. Talk to your partner about them.

S: One angle is shown by the arc that was already there. The other angle is shown by the arc that we just drew. → The two angles go together to represent a whole turn.

T: Which angle is easier to measure with the 180° protractor?

S: The smaller angle.



T: What is the measure of that angle? (Pause.)

S: 130°.

T: What is the total angle measure around point R?

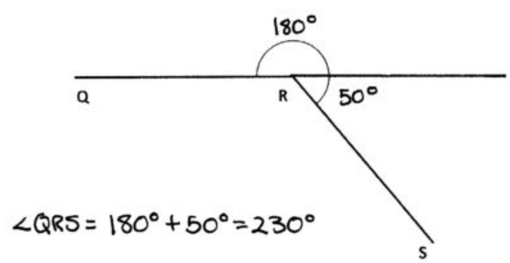
S: 360°.

T: If there are 360° in the whole, and 130° in one of the parts, figure out the measure of the other part. Talk to your partner about your strategy.

S: We could subtract. → We know that the whole minus a part equals the other part. 360° – 130° = 230°. → I counted up 2 hundreds from 130 to 330, and then added 30 more. $\angle QRS$ is 230°. → That's the same as when we measured with the circular protractor!

Problem 3: Measure an angle greater than 180° by adding on to 180°.

T: Let's explore another way to find the measure of $\angle QRS$. Erase the arc that you just drew. Now, use your straightedge to extend $\overrightarrow{QR}$ to the right.



S: (Extend $\overrightarrow{QR}$ to the right.)

T: What happened to $\angle QRS$, indicated by the arc?

S: Now, it's chopped into two smaller angles.

T: What is the angle measure of this straight line?

S: 180°.

T: Measure the new acute angle. (Pause.)

S: It's 50°.

T: Label each angle with its measure. What do you notice?

S: When I add the two angles together, I get the measure of the whole thing. 180° + 50° = 230°. Hey, it's the same!

Problem 4: Draw an angle less than 180° using a 180° protractor.

T: Now, let's practice drawing angles. Draw a ray that we can align to our 0° line.

T: Watch as I draw my ray and label my endpoint with the letter A. Now, you draw. (See Step 1 on the next page.)

T: The next ray's endpoint should also be point A so that you can form an angle.

T: Watch as I align my protractor, placing the center over the endpoint, A, and making sure my ray aligns with the 0° line. Now, it's your turn.

NOTES ON USING A PROTRACTOR:

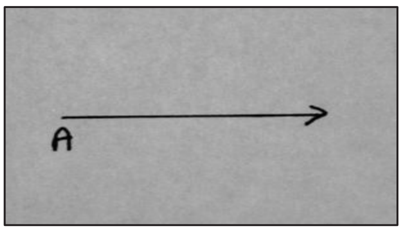
Help students measure accurately using a protractor with the following tips:

1. Place the center notch of the protractor on the vertex.
2. Put the pencil point through the notch, and move the straightedge into alignment.
3. When measuring angles, it is sometimes necessary to extend the sides of the angle so that they intersect with the protractor's scale.

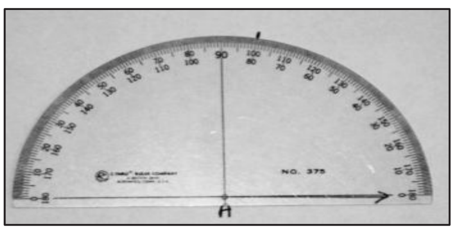
T: Next, I look to see where 80° is on the protractor. Everyone, find 80° on your protractor, and place a small point directly above 80°. (See Step 2.)

T: Use the straightedge of the protractor to draw the next ray. I create a ray beginning at point A, along my straightedge, toward the mark I made above 80°. Note that I am not going to extend my ray all the way to the point where I marked 80°. (See Steps 3 and 4.)

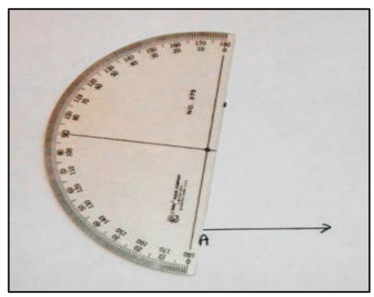
T: Now that the angle has been made, verify the measure with the protractor. Extend the ray to measure the angle. (See Step 5.)



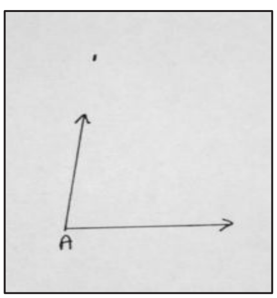
Step 1



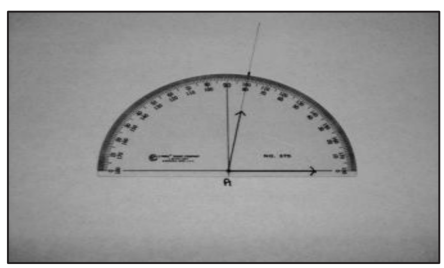
Step 2



Step 3



Step 4



Step 5

T: Let's draw another angle. Let's use our straightedge and a protractor to construct a 133° angle.

T: What's the first step?

S: We draw a ray and label the endpoint. → Let's label it with a *B* this time.

T: What do we do next?

S: We put our protractor on the ray so that the notch is directly aligned with point B and the ray is lined up with the 0° line on the protractor.

T: Next?

S: We find 133° on the protractor. → Hey! It's not there!

T: Look at the numbers that are there. Between which two numbers would you find 133?

S: Between 130 and 140.

T: Find the number 130. Let's start at 130 and count the tick marks up to 140 just like we would if we were counting on a number line.

S: 131, 132, 133, 134, 135, 136, 137, 138, 139, 140.

T: Point to the tick mark that represents 133°.

T: Make a small mark on your paper directly above the 133° mark on your protractor. Take your protractor off of your paper. What do we do next?

S: We need to draw the other ray. → We line the straightedge up with point B and the mark that we just made.

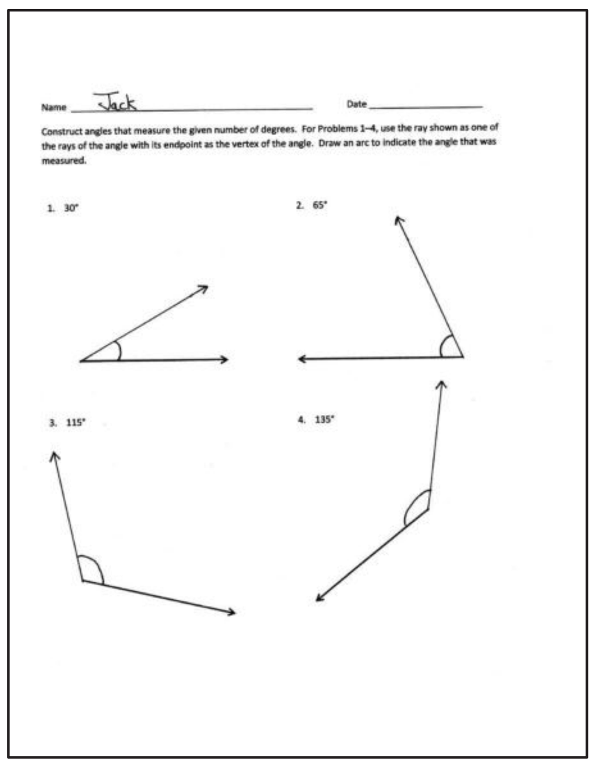
T: Place your straightedge on your paper. Be sure that point B and the tick mark are touching the edge. Draw a ray from point B beyond the tick mark.

S: We have drawn the angle! Let's verify it!

T: Remember that it is very important to place your protractor properly.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



Student Debrief (10 minutes)

Lesson Objective: Measure and draw angles. Sketch given angle measures, and verify with a protractor.

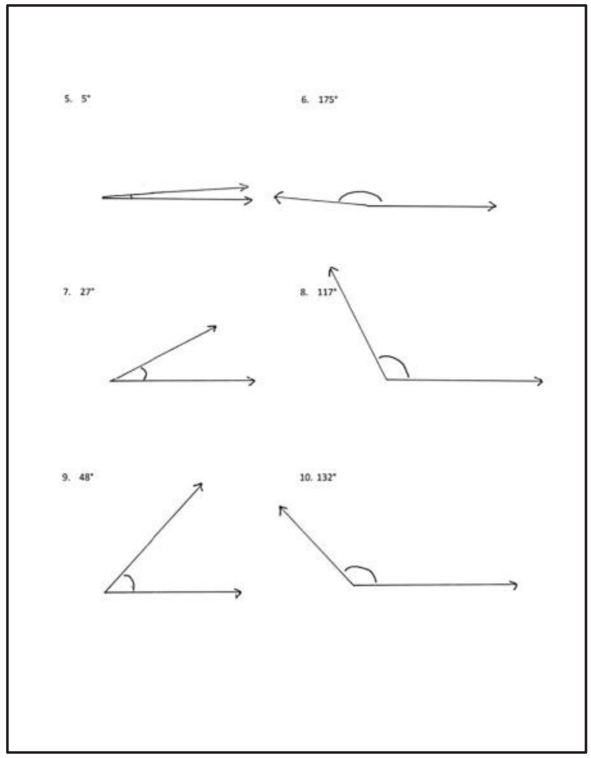
The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, how did you draw the angles with a 180° protractor?
- In Problem 1, which were the most challenging angles to draw? Explain.

- Why is it important to be precise when drawing angles? Tell your partner how you can be precise when drawing angles.
- Why do we verify our sketches with a protractor?
- It is important to learn to use the 180° protractor because it is the one you will see everywhere. Explain to your partner how to measure an angle greater than 180° with a 180° protractor.



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ____ Date ______________________

Figure 1

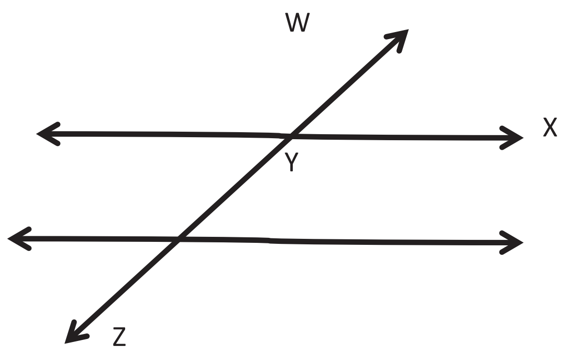


Figure 2

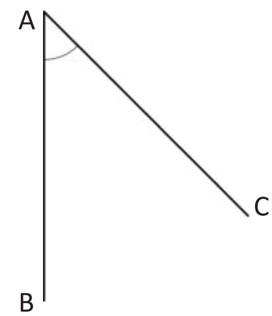


Figure 3

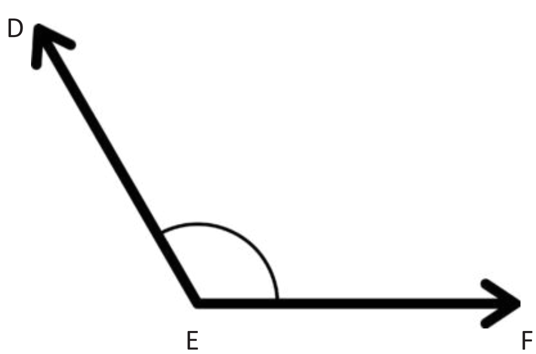
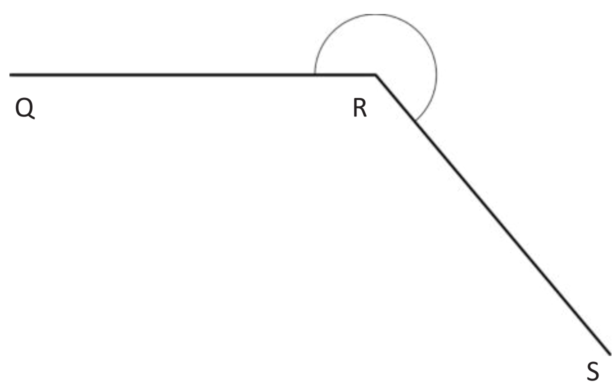


Figure 4



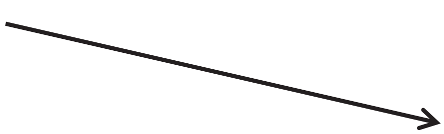
Name ____ Date ____________________

Construct angles that measure the given number of degrees. For Problems 1–4, use the ray shown as one of the rays of the angle with its endpoint as the vertex of the angle. Draw an arc to indicate the angle that was measured.

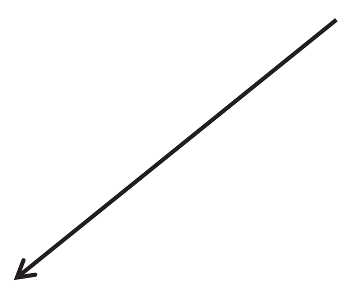
1. 30°

2. 65°

3. 115°



4. 135°



5. 5°

6. 175°

7. 27°

8. 117°

9. 48°

10. 132°

Name ____ Date ____________________

Construct angles that measure the given number of degrees. Draw an arc to indicate the angle that was measured.

1. 75°

2. 105°

3. 81°

4. 99°

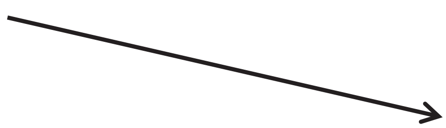
Name ____ Date ______________________

Construct angles that measure the given number of degrees. For Problems 1–4, use the ray shown as one of the rays of the angle with its endpoint as the vertex of the angle. Draw an arc to indicate the angle that was measured.

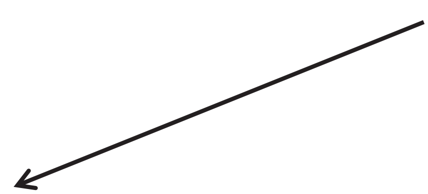
1. 25°

2. 85°

3. 140°



4. 83°



5. 108°

6. 72°

7. 25°

8. 155°

9. 45°

10. 135°

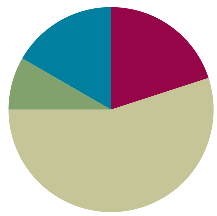
Lesson 8

Objective: Identify and measure angles as turns and recognize them in various contexts.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (5 minutes)
- Concept Development (33 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)



Fluency Practice (12 minutes)

- Count by 90° **4.MD.7** (2 minutes)
- Break Apart 90, 180, and 360 **4.MD.7** (4 minutes)
- Physiometry **4.G.1** (2 minutes)
- Sketch Angles **4.MD.6** (4 minutes)

Count by 90° (2 minutes)

Note: This fluency activity prepares students for Lesson 8. If students struggle to connect counting groups of 9, groups of 9 tens, and groups of 90, write the counting progressions on the board.

9	18	27	36
9 tens	18 tens	27 tens	36 tens
90	180	270	360
90°	180°	270°	360°

Direct students to count forward and backward:

- Nines to 36
- 9 tens to 36 tens
- 90 to 360
- 90° to 360°

Break Apart 90, 180, and 360 (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for unknown angle problems in Lessons 10–11.

T: (Project a number bond with a whole of 90. Fill in 20 for one of the parts.) On your personal white boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 90 and with 20 and 70 as parts.)

Continue breaking apart 90 with the following possible sequence: 60, 40, 50, and 45.

T: (Project a number bond with a whole of 180. Fill in 70 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 180 and with 70 and 110 as parts.)

Continue to break apart 180 with the following possible sequence: 90, 130, 40, and 135.

T: (Project a number bond with a whole of 360. Fill in 50 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 360 and with 50 and 310 as parts.)

Continue to break apart 360 with the following possible sequence: 200, 190, 180, 90, 120, and 45.

Physiometry (2 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms from Lessons 1–7.

T: Stand up.

S: (Stand up.)

T: Show me an acute angle.

S: (Make an acute angle with arms.)

T: Show me an obtuse angle.

S: (Make an obtuse angle with arms.)

T: Show me a right angle.

S: (Make a right angle with arms.)

T: Show me an angle that measures approximately 100°.

S: (Move arms further apart, increasing the space between their arms, so that it is approximately 100°.)

T: Show me an angle that measures approximately 150°.

S: (Move arms further apart to approximately 150°.)

Continue with the following possible sequence: 90°, 80°, 30°, 20°, 120°, 40°, 110°, and 180°.

T: What's another name for a 180° angle?

S: A straight angle.

T: (Point to one of the classroom's side walls.) Point to the walls that run perpendicular to the wall I'm pointing to.

S: (Point to the front and back wall.)

T: (Point to the front wall.)
S: (Point to the side walls.)

Continue pointing to the other side wall and back wall.

T: (Point to the back wall.) Point to the wall that runs parallel to the wall I'm pointing to.
S: (Point to the front wall.)

Continue pointing to one side wall, the back wall, and the other side wall.

Sketch Angles (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews terms from Lesson 7.

T: On your personal white boards, show me $\angle ABC$ that measures about 90°.
S: (Sketch $\angle ABC$ that measures approximately 90°.)
T: What do we call an angle that measures 90°?
S: Right angle.
T: On your boards, show me $\angle DEF$ that measures about 80°.
S: (Sketch $\angle DEF$ that measures approximately 80°.)
T: What type of angle did you draw?
S: Acute.

Continue with the following possible sequence: 10°, 150°, 50°, 120°, and 45°.

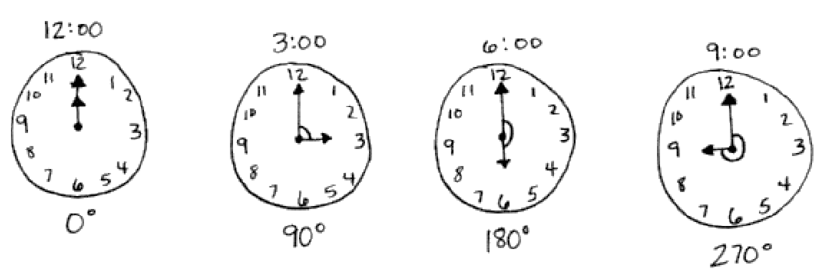
NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

If students do not recognize that the middle letter of the angle (for example, B of $\angle ABC$) denotes the vertex, then quickly review. Then, guide students to set a goal for the Sketch Angles Fluency Practice. An appropriate goal may be to consistently label the vertex as the middle letter of the angle.

Additionally, some students may benefit from sketching a 90° angle as a reference point for each angle.

Application Problem (5 minutes)

Draw a series of clocks that show 12:00, 3:00, 6:00, and 9:00. Use an arc to identify an angle and estimate the angle created by both hands on the clock.



Note: This Application Problem reviews the sketching of angles from Lesson 7 and leads up to the Concept Development of today's lesson, where students further explore angle measure on a circle. Some students may identify 3:00 as a 270° angle and 9:00 as a 90° angle. Confirm with the arcs whether the estimated measurements are accurate.

Concept Development (33 minutes)

Materials: (T) Analog clock (S) Clock (Template)

Problem 1: Explore angle measure as turning in relation to the hour hand on a clock.

T: Use your straightedge to draw a line segment that starts at the tick mark representing the hour of 12 and ends at the tick mark representing the hour of 6. Fold along the line that you just drew. What fractional units have you just created?

S: Halves!

T: Next, fold your clock in half again. Unfold and trace along the second fold. What is the new fractional unit you have created?

S: Fourths. → Quarters.

T: At 12:00, the hour hand points at the 12. Point at the 12. At 3:00, the hour hand points at the 3. Use your finger to trace along the edge of the circle from the 12 to the 3 to represent the movement of the hour hand. What fraction of the arc is that?

S: One fourth.

T: How many degrees did you just move?

S: 90°.

T: At 6:00, the hour hand points at the 6. Trace along the edge of the circle from the 3 to the 6. How many degrees did you just move?

S: 90°.

T: At 9:00, the hour hand points at the 9. Trace along the edge of the circle from the 6 to the 9. How many degrees did you just move?

S: 90°.

T: Point to the 9, and trace another quarter of the way around the clock. Where does your finger stop?

S: At the 12.

T: Talk to your partner about the total number of degrees and the number of quarter turns we just made.

S: One quarter of the way around the clock plus one quarter plus one quarter plus one quarter. That's 90° + 90° + 90° + 90°. 360°. → Four quarter-turns. → 4 × 90 = 360. → 4 × 9 tens, 36 tens, or 360°.

T: Talk to your partner about moving from the 12 to the 6 along the arc.

S: That would be halfway around the clock. → 180° because 90 + 90 = 180.

T: How about from the 12 to the 9?

S: That's three quarter-turns. → 270° because 90 + 90 + 90 = 270.

Problem 2: Explore angle measure as turning in relation to the room.

T: Everyone, stand up and face the front of the room. Let's represent turns by using our bodies. Stay where you are. If you can, show me a complete turn.

S: (Attempt to do so.)

T: How many degrees did you turn?

S: 360. A full turn is 360°. → It's just like what we showed on the clock.

T: Face the front of the room again. This time, make a half-turn. Where are you facing?

S: The back of the room.

T: How many degrees did you turn when you made a half-turn?

S: 180°. → 180° is half of 360°. → 90°+ 90° = 180°.

T: What is another turn that we can show?

S: We can show a quarter-turn. That would be 90°.

T: Everyone, face the front of the room again. Show me where you will face when you make a quarter-turn.

T: Why are people facing in different directions?

S: I turned to the left. → I turned to the right.

T: Who is correct? The students who turned to the left or right? Take a moment to discuss with your neighbor.

S: We are both correct. → We both made a quarter-turn. We just turned in different directions. → Whether you turn to the left or right, you are still turning 90°. No one said which way, just that it had to be a quarter-turn.

T: Face the front of the room. Make two quarter-turns in the same direction.

S: We are all facing the back of the room! → Two quarter-turns is the same as a half-turn. Some of us started off going to the left, and some started off going to the right, but we all ended up facing the back of the classroom.

T: We can say that we all did a 180. We were facing one direction, and then we were facing the opposite direction.

Problem 3: Recognize turning angles in various contexts.

T: When a skateboarder does a 180, what does she do?

S: She spins around to face the other way.

T: When a car loses control on an icy road and does a 360, what does the car do?

S: It spins all the way around in a circle.

T: Turn your pencil a quarter-turn.

S: (Do so.)

T: With your partner, come up with an example of something that might turn. Identify the turn using degrees or turns, and then be prepared to report back to the class.

S: My mom turned up the heat on the stove, so she moved the knob a quarter-turn. → To find the library, walk down to the end of this hall and turn 90° to the right. → The earth does a 360 every day. → When the plug didn't fit into the iPad to charge it, I flipped the charger a half-turn.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Scaffold the Problem Set with the following options:

- Put a dot in the center of the circle to assist with drawing in Problem 5.
- Guide students to count by 90° or fourths up to the desired turn.
- Clarify for English language learners that *quarters* and *fourths* are interchangeable terms.
- For Problem 7, encourage students to actually turn the Problem Set paper and count the quarter-turns to make the picture upright.

Student Debrief (10 minutes)

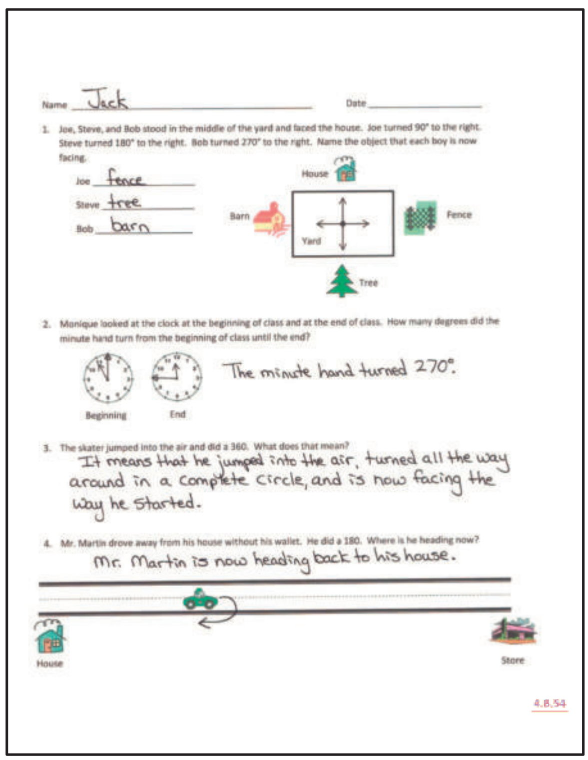
Lesson Objective: Identify and measure angles as turns and recognize them in various contexts.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Why was there confusion with turning 90°, but not with turning 180° or 360°? How can the terms clockwise and counterclockwise be used in Problem 7?
- Why is there more than one answer for Problem 7?
- Does it matter in Problem 8 if you turned 180° to the right or 180° to the left? Explain.
- What do you notice about the terms used to tell time? (All of the benchmark angles have terms, i.e., half past, quarter of, quarter past.)
- Stand face-to-face with your partner. Ask your partner to turn to the left. Why does it appear to you that she turned to the right? In each problem in this lesson, when someone turns to the right or left, it is from his or her perspective. What does this mean?


Name Jack Date

1. Joe, Steve, and Bob stood in the middle of the yard and faced the house. Joe turned 90° to the right. Steve turned 180° to the right. Bob turned 270° to the right. Name the object that each boy is now facing.

Joe fence
Steve tree
Bob barn

2. Monique looked at the clock at the beginning of class and at the end of class. How many degrees did the minute hand turn from the beginning of class until the end?

The minute hand turned 270°.

3. The skater jumped into the air and did a 360. What does that mean?

It means that he jumped into the air, turned all the way around in a complete circle, and is now facing the way he started.

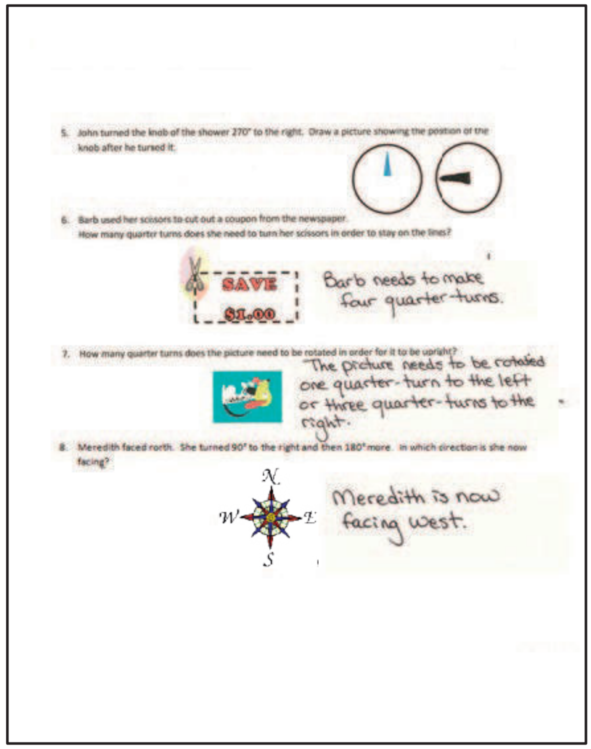
4. Mr. Martin drove away from his house without his wallet. He did a 180. Where is he heading now?

Mr. Martin is now heading back to his house.

4.B.54

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



5. John turned the knob of the shower 270° to the right. Draw a picture showing the position of the knob after he turned it.

6. Barb used her scissors to cut out a coupon from the newspaper. How many quarter turns does she need to turn her scissors in order to stay on the lines?

SAVE $1.00

Barb needs to make four quarter-turns.

7. How many quarter turns does the picture need to be rotated in order for it to be upright?

The picture needs to be rotated one quarter-turn to the left or three quarter-turns to the right.

8. Meredith faced north. She turned 90° to the right and then 180° more. In which direction is she now facing?

N
W
E
S

Meredith is now facing west.

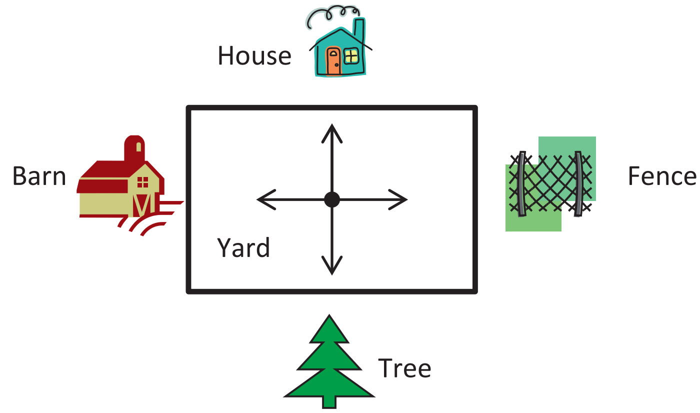
Name ____ Date ____________________

1. Joe, Steve, and Bob stood in the middle of the yard and faced the house. Joe turned 90° to the right. Steve turned 180° to the right. Bob turned 270° to the right. Name the object that each boy is now facing.

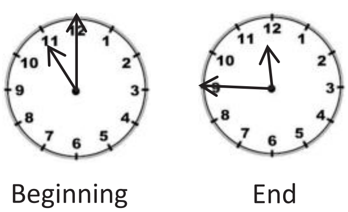
   Joe ____________________

   Steve __________________

   Bob ___________________

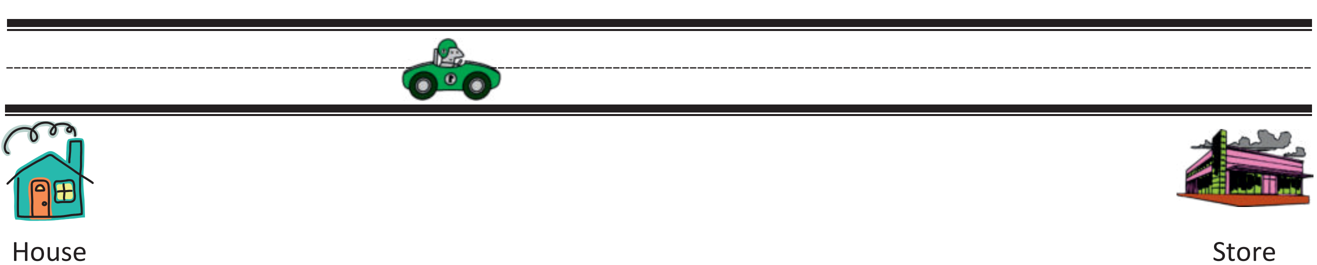


2. Monique looked at the clock at the beginning of class and at the end of class. How many degrees did the minute hand turn from the beginning of class until the end?

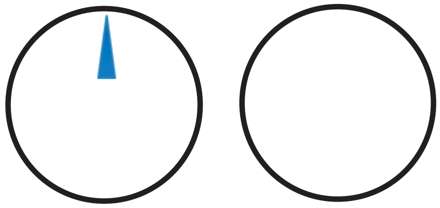


3. The skater jumped into the air and did a 360. What does that mean?

4. Mr. Martin drove away from his house without his wallet. He did a 180. Where is he heading now?



5. John turned the knob of the shower 270° to the right. Draw a picture showing the position of the knob after he turned it.

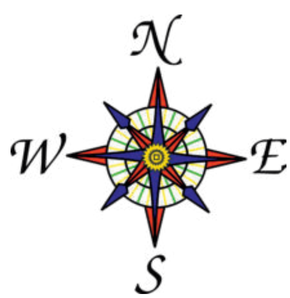


Before After

6. Barb used her scissors to cut out a coupon from the newspaper. How many quarter-turns does she need to turn the paper in order to stay on the lines?

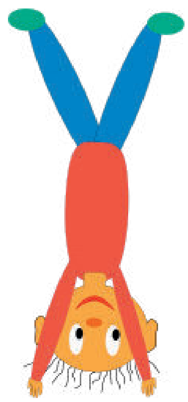
7. How many quarter-turns does the picture need to be rotated in order for it to be upright?

8. Meredith faced north. She turned 90° to the right, and then 180° more. In which direction is she now facing?

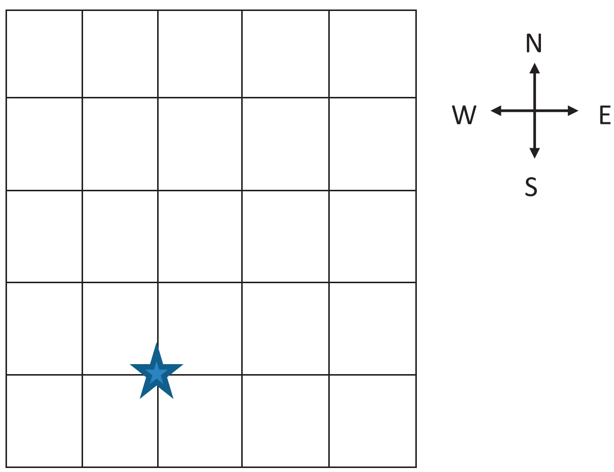


Name ____ Date ____________________

1. Marty was doing a handstand. Describe how many degrees his body will turn to be upright again.



2. Jeffrey started riding his bike at the . He travelled north for 3 blocks, then turned 90° to the right and rode for 2 blocks. In which direction was he headed? Sketch his route on the grid below. Each square unit represents 1 block.



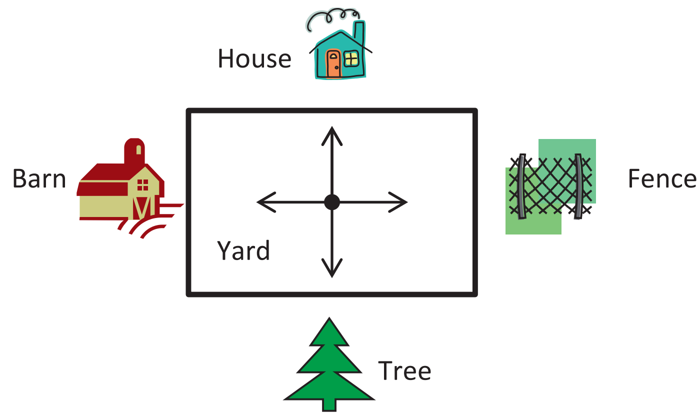
Name ____ Date ____________________

1. Jill, Shyan, and Barb stood in the middle of the yard and faced the barn. Jill turned 90° to the right. Shyan turned 180° to the left. Barb turned 270° to the left. Name the object that each girl is now facing.

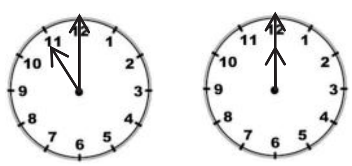
Jill ____________________

Shyan __________________

Barb ___________________



2. Allison looked at the clock at the beginning of class and at the end of class. How many degrees did the minute hand turn from the beginning of class until the end?

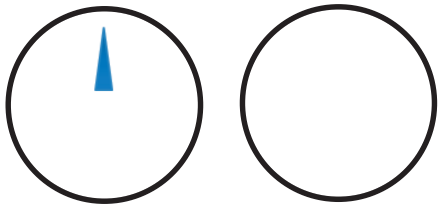


Beginning End

3. The snowboarder went off a jump and did a 180. In which direction was the snowboarder facing when he landed? How do you know?

4. As she drove down the icy road, Mrs. Campbell slammed on her brakes. Her car did a 360. Explain what happened to Mrs. Campbell's car.

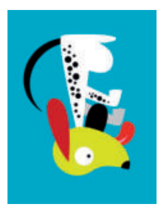
5. Jonah turned the knob of the stove two quarter-turns. Draw a picture showing the position of the knob after he turned it.



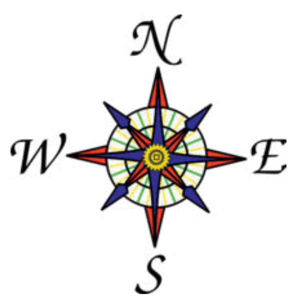
Before After

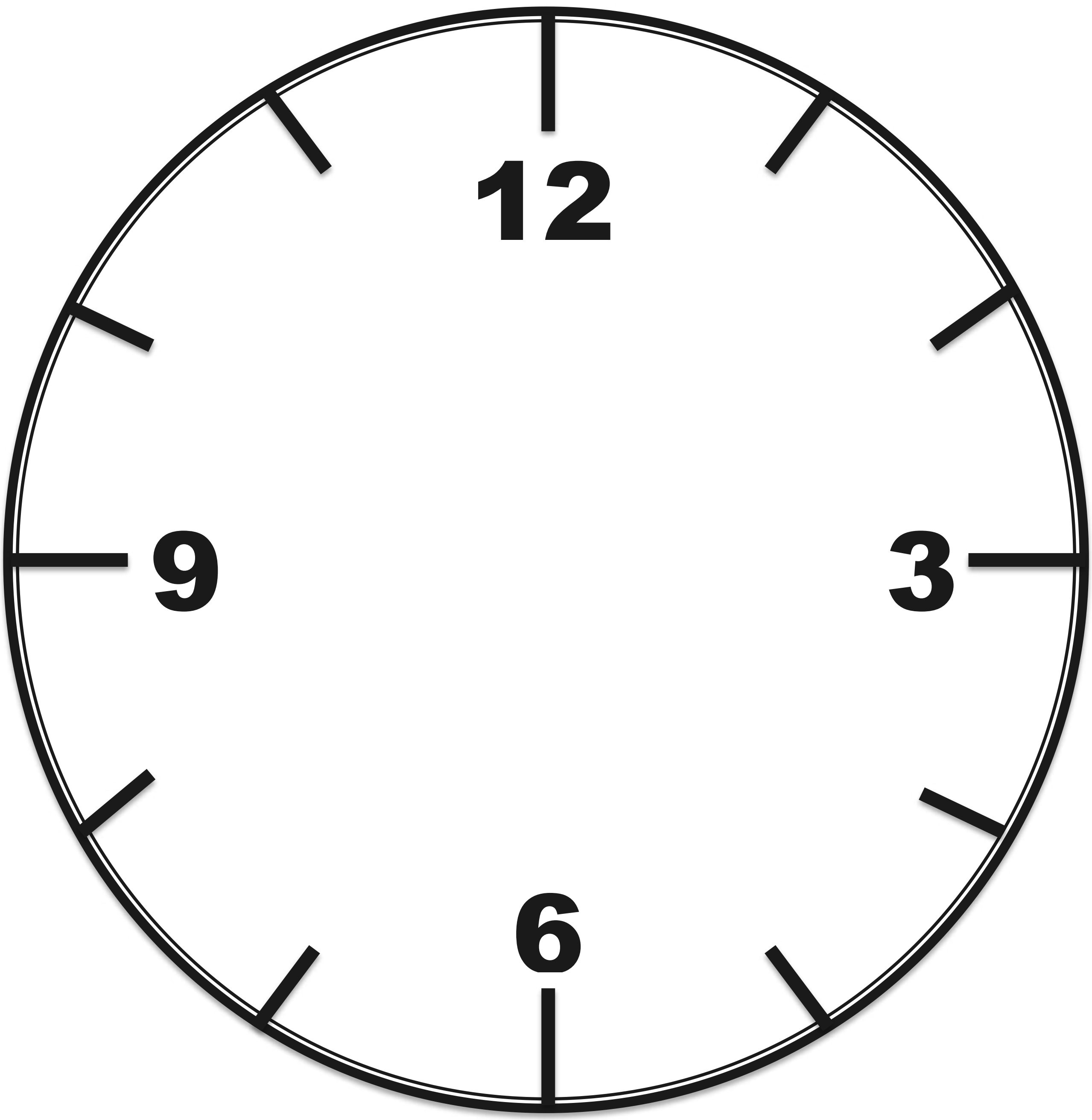
6. Betsy used her scissors to cut out a coupon from the newspaper. How many total quarter-turns will she need to rotate the paper in order to cut out the entire coupon?

7. How many quarter-turns does the picture need to be rotated in order for it to be upright?



8. David faced north. He turned 180° to the right, and then 270° to the left. In which direction is he now facing?





clock

Name ____ Date ______________________

1. Follow the directions below to draw a figure in the box below. Use a straightedge.

   a. Draw 2 points, A and B.

   b. Draw $\overleftrightarrow{AB}$.

   c. Draw point D that is not on $\overleftrightarrow{AB}$.

   d. Draw $\overrightarrow{BD}$.

   e. Draw $\overline{AD}$.

   f. Name an acute angle.

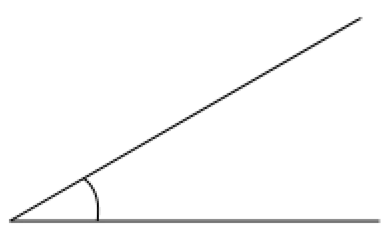
   ____

   g. Name an obtuse angle. You may have to draw and label another point.

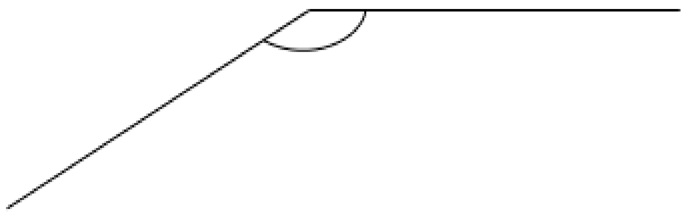
   ____

2. Use your protractor to measure the angle indicated by the arc. Classify each angle as right, acute, or obtuse. Explain how you know each angle's classification.

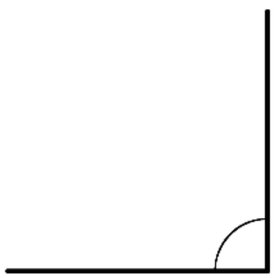
   a.



b.



c.

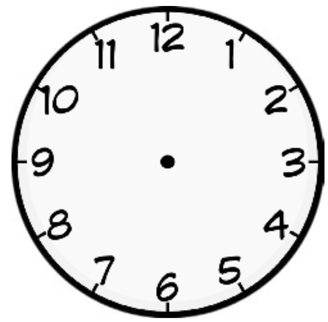


3. Use the following instructions to draw a figure in the box below.

- Using a straightedge, draw a line. Label it $\overleftrightarrow{KL}$.
- Label a point A on $\overleftrightarrow{KL}$.
- Using your protractor and ruler, draw a line perpendicular to $\overleftrightarrow{KL}$ through point A.
- Label the perpendicular line $\overleftrightarrow{PQ}$.
- Label a point B on $\overleftrightarrow{PQ}$, other than point A.
- Using your protractor and straightedge, draw a line, $\overleftrightarrow{ST}$, perpendicular to $\overleftrightarrow{PQ}$ through point B.

Which lines are parallel in your drawing? Explain why.

4. Use the clock to answer the following:



a. Use a straightedge to draw the hands as they would appear at 3:00.

b. What kind of angle is formed by the clock hands at 3:00?

c. What time will it be when the minute hand has turned 180°?

d. How many 90° turns will the minute hand make between 3:00 and 4:00?

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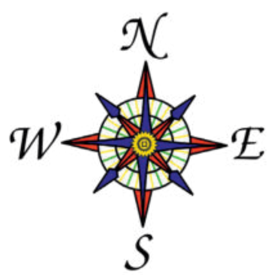
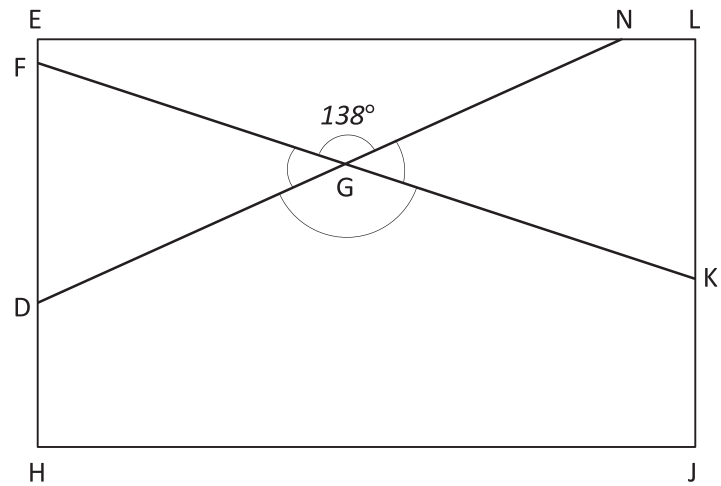
5. Use the compass rose to answer the following:



a. Maddy faced East. She turned to her right until she was facing North. How many degrees did she turn?

b. Quanisha was facing North. She turned toward her right until she faced East. Alisha was facing South. She turned toward her right until she faced West. What fraction of a full turn did each girl complete? Through how many degrees did each girl turn?

6. The town of Seaford has a large rectangular park with a biking path around its perimeter and two straight-line biking paths that cut across it as shown in the diagram below.



a. Find the measure of the following angles using a protractor.

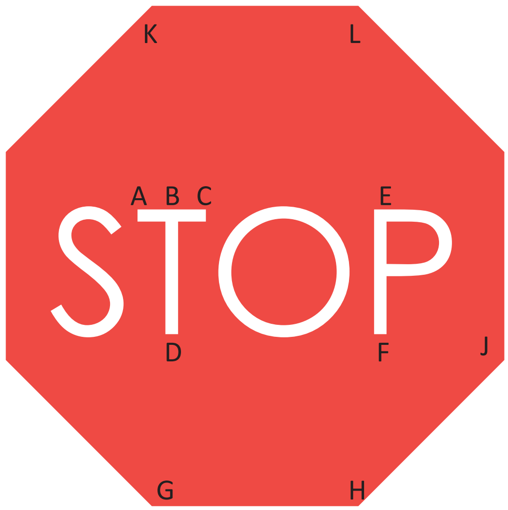
∠FGD:

∠DGK:

∠KGN:

b. In the space below, use a protractor to draw an angle with the same measure as *∠DGK*.

c. Below is a sign that bikers may encounter while riding in the park. Using the points in the figure below, identify a line segment, a right angle, an obtuse angle, a set of parallel lines, and a set of perpendicular lines. Write them in the table below.



Line Segment	
Right Angle	
Obtuse Angle	
Parallel Lines	
Perpendicular Lines	

Mid-Module Assessment Task Standards Addressed	Topics A–B

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop on their way to proficiency. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	**STEP 1 Little evidence of reasoning without a correct answer. (1 Point)**	**STEP 2 Evidence of some reasoning without a correct answer. (2 Points)**	**STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)**	**STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)**
1 **4.G.1**	The student attempts to draw some points, lines, and rays for the figure, but does so incorrectly and without correctly identifying an obtuse or acute angle.	The student correctly draws the figure but is unable to identify an obtuse or acute angle.	The student correctly draws the figure to match directions but correctly identifies only one of the two angles. OR The student follows directions to complete the figure incorrectly but correctly identifies an acute and obtuse angle.	The student correctly draws all lines, line segments, and rays as stated. The student correctly identifies an acute and obtuse angle based on the figure drawn. (Note: Drawings and angles may differ for each student.)
2 **4.MD.6** **4.G.1**	The student correctly measures and classifies fewer than two of the three angles.	The student correctly measures and classifies at least two of the three angles, providing some reasoning.	The student correctly measures at least two of the three angles and classifies them all correctly. OR The student correctly measures all three angles but does not provide solid reasoning for classifying angles.	The student correctly measures and classifies all angles and correctly explains the classifications: a. 30°; acute; the angle measures less than 90°. b. 147°; obtuse; the angle measures greater than 90°. c. 90°; right; the angle measures exactly 90°.

A Progression Toward Mastery				
3 **4.MD.6** **4.G.1**	The student attempts to draw and identify lines but does so incorrectly.	The student attempts to draw the diagram according to given directions but is only able to create one set of perpendicular lines. There are no sets of parallel lines created and little reasoning about parallel lines.	The student correctly completes the drawing according to directions, identifying the parallel lines, but is unable to provide solid reasoning about why the lines are parallel. OR The student correctly identifies parallel lines and provides solid reasoning as to why specific lines are parallel but does not draw the figure as directed.	The student correctly draws and labels all points and lines, as well as identifies $\overleftrightarrow{ST}$ as parallel to $\overleftrightarrow{KL}$. The student correctly reasons that the lines are parallel because they are perpendicular to $\overleftrightarrow{PQ}$ or because they are an equal distance apart from each other. (Drawings will vary but must contain all required elements to be considered correct.)
4 **4.MD.5**	The student is unable to complete any part or is able to complete only one part of the problem.	The student correctly completes Part (b) and one of the three remaining parts.	The student correctly completes Part (b) and two of the three remaining parts.	The student correctly completes all four parts: a. Clock hands depict 3:00. b. Possible correct responses include 90° angle, right angle, or 270° angle c. 3:30. d. Four turns.
5 **4.MD.5**	The student is unable to complete either of the two parts.	The student correctly completes one of the two parts.	The student correctly answers Part (a) but only answers one question from Part (b) correctly.	The student correctly completes both parts of the problem: a. 270°. b. Each girl turned 90°. Each turned $\frac{1}{4}$ of a full turn.

A Progression Toward Mastery				
6 **4.MD.5** **4.MD.6** **4.G.1**	The student correctly completes four or fewer of the nine components.	The student correctly completes five or six of the nine components.	The student correctly completes seven or eight of the nine components.	The student correctly completes all nine components: a. $\angle FGD$ = 42° $\angle DGK$ = 138° $\angle KGN$ = 42° (The measurements above are accurate; however, allow ± 1 degree variance for student responses.) b. Sketch of a 138° angle, labeled with an arc and points. c. The student must include one of the following choices per part: Segment: $\overline{AB}$, $\overline{AC}$, $\overline{BC}$, $\overline{BD}$, $\overline{EF}$, $\overline{GH}$, $\overline{HJ}$, $\overline{KL}$. Right angle: $\angle ABD$, $\angle CBD$. Obtuse angle: $\angle GHJ$. Parallel lines: $\overline{KL} \parallel \overline{GH}$, $\overline{BD} \parallel \overline{EF}$. Perpendicular lines: $\overline{AC} \perp \overline{BD}$, $\overline{AB} \perp \overline{BD}$, $\overline{BC} \perp \overline{BD}$.

Name Jack Date ____________

1. Follow the directions below to draw a figure in the box below. Use a straightedge.

   a. Draw 2 points, A and B.

   b. Draw $\overleftrightarrow{AB}$.

   c. Draw point D that is not on $\overleftrightarrow{AB}$.

   d. Draw $\overrightarrow{BD}$.

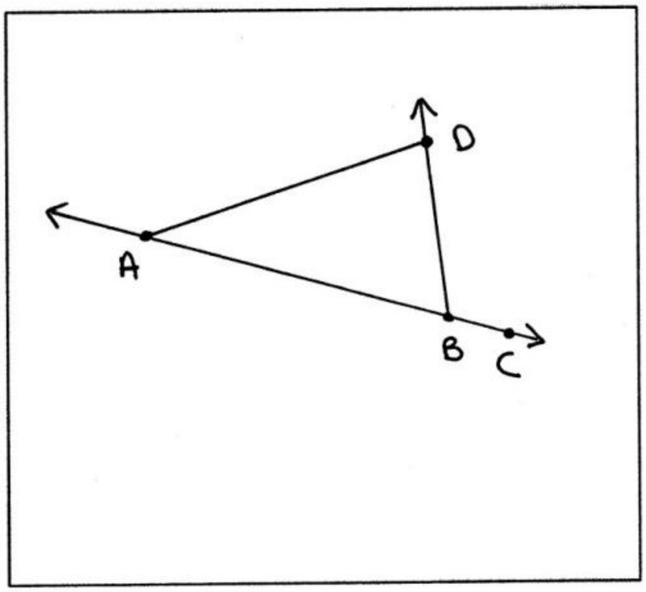
   e. Draw $\overline{AD}$.

   f. Name an acute angle.

   < BAD

   g. Name an obtuse angle. You may have to draw and label another point.

   < DBC



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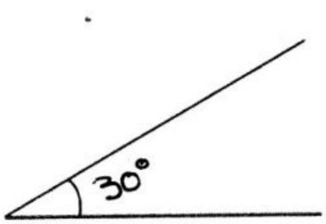
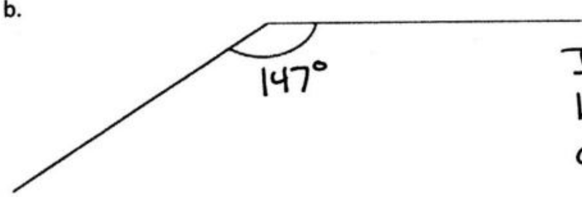
2. Use your protractor to measure the angle indicated by the arc. Classify each angle as right, acute, or obtuse. Explain how you know each angle's classification.

   a.



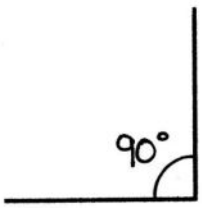
This is an acute angle. I know because it measures 30° which is less than a right angle.

b.



This is an obtuse angle. I know because it measures 147° which is greater than a right angle and less than 180°.

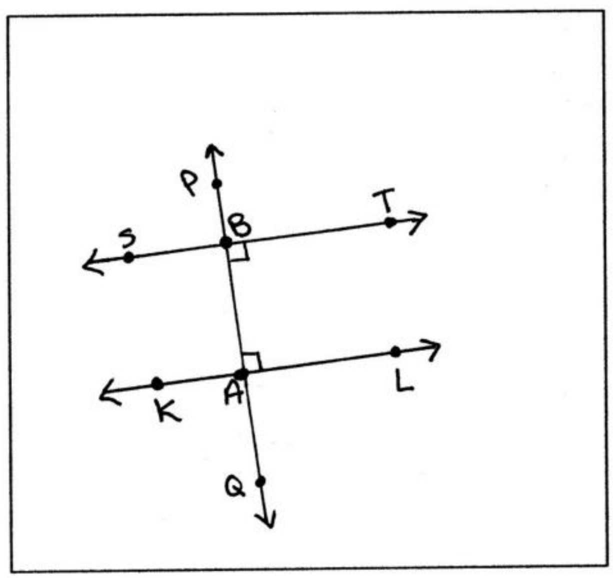
c.



This is a right angle. It measures exactly 90°.

3. Use the following instructions to draw a figure in the box below.

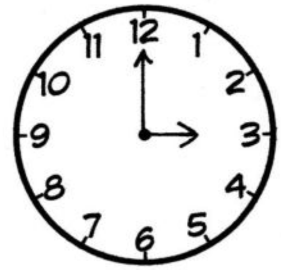
- Using a straightedge, draw a line. Label it $\overleftrightarrow{KL}$.
- Label a point A on $\overleftrightarrow{KL}$.
- Using your protractor and ruler, draw a line perpendicular to $\overleftrightarrow{KL}$ through point A.
- Label the perpendicular line $\overleftrightarrow{PQ}$.
- Label a point B on $\overleftrightarrow{PQ}$, other than point A.
- Using your protractor and straightedge, draw a line, $\overleftrightarrow{ST}$, perpendicular to $\overleftrightarrow{PQ}$ through point B.



Which lines are parallel in your drawing? Explain why.

$\overleftrightarrow{ST} \parallel \overleftrightarrow{KL}$ $\overleftrightarrow{ST}$ is parallel to $\overleftrightarrow{KL}$ because both of them are perpendicular to $\overleftrightarrow{PQ}$. It reminds me of the sides of a rectangle.

4. Use the clock to answer the following:



a. Use a straightedge to draw the hands as they would appear at 3:00.

b. What kind of angle is formed by the clock hands at 3:00?

A right angle

c. What time will it be when the minute hand has turned 180°?

It will be 3:30.

d. How many 90° turns will the minute hand make between 3:00 and 4:00?

The minute hand will make four 90° turns between 3:00 and 4:00.

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5. Use the compass rose to answer the following:

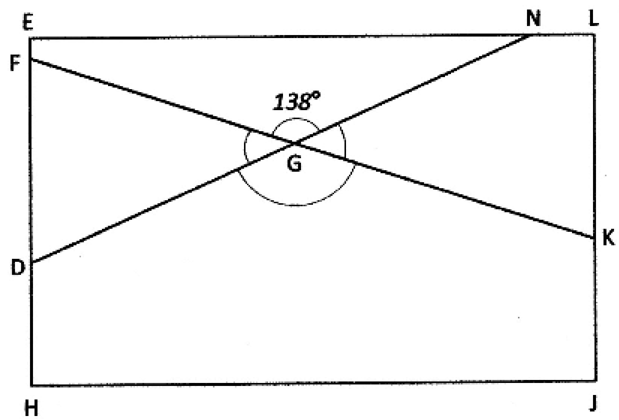
a. Maddy faced East. She turned to her right until she was facing North. How many degrees did she turn?

Maddy turned 270°.

b. Quanisha was facing North. She turned toward her right until she faced East. Alisha was facing South. She turned toward her right until she faced West. What fraction of a full turn did each girl complete? Through how many degrees did each girl turn?

Each girl completed $\frac{1}{4}$ of a full turn.
Each girl turned 90°.

6. The town of Seaford has a large rectangular park with a biking path around its perimeter and two straight-line biking paths that cut across it as shown in the diagram below.



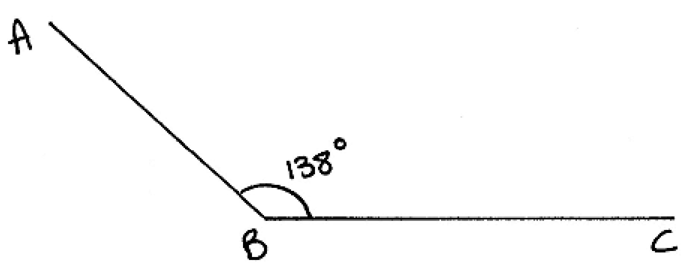
a. Find the measure of the following angles using a protractor.

∠FGD: 42°

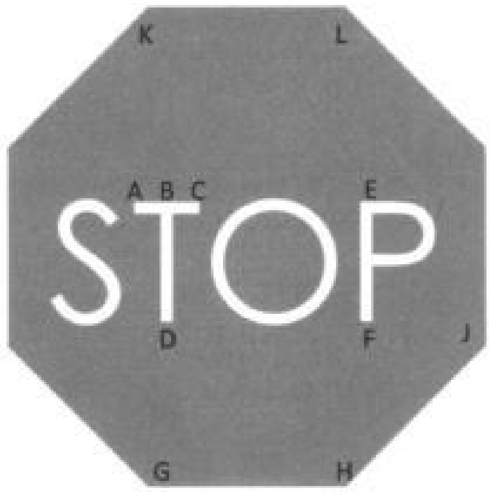
∠DGK: 138°

∠KGN: 42°

b. In the space below, use a protractor to draw an angle with the same measure as ∠DGK.



c. Below is a sign that bikers may encounter while riding in the park. Using the points in the figure below, identify a line segment, a right angle, an obtuse angle, a set of parallel lines, and a set of perpendicular lines. Write them in the table below.



Line Segment	$\overline{EF}$
Right Angle	$\angle ABD$
Obtuse Angle	$\angle GHJ$
Parallel Lines	$\overleftrightarrow{KL} \parallel \overleftrightarrow{GH}$
Perpendicular Lines	$\overleftrightarrow{AC} \perp \overline{BD}$

Topic C

Problem Solving with the Addition of Angle Measures

4.MD.7

Focus Standard:	4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
Instructional Days:	3	
Coherence -Links from:	G3–M7	Geometry and Measurement Word Problems

In Topic C, students use concrete examples to discover the additive nature of angle measurement. As they work with pattern blocks in Lesson 9, students see that the measures of all of the angles at a point, with no overlaps or gaps, add up to 360°, and they use this fact to find the measure of the pattern blocks' angles.

In Lesson 10, students use what they know about the additive nature of angle measure to reason about the relationships between pairs of adjacent angles. Students discover that the measures of two angles on a straight line add up to 180° (supplementary angles) and that the measures of two angles meeting to form a right angle add up to 90° (complementary angles).

In Lesson 11, students extend their learning by determining the measures of unknown angles for adjacent angles that add up to 360°. Additionally, through their work with angles on a line, students go on to discover that vertical angles have the same measure.

In both Lessons 10 and 11, students write addition and subtraction equations to solve unknown angle problems. Students solve these problems using a variety of pictorial and numerical strategies, combined with the use of a protractor to verify answers (**4.MD.7**).

A Teaching Sequence Toward Mastery of Problem Solving with the Addition of Angle Measures

Objective 1: **Decompose angles using pattern blocks.**
(Lesson 9)

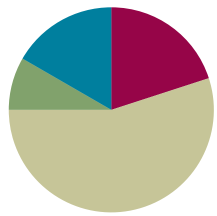
Objective 2: **Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.**
(Lessons 10–11)

Lesson 9

Objective: Decompose angles using pattern blocks.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Count by 90° **4.MD.7** (1 minute)
- Break Apart 90, 180, and 360 **4.MD.7** (4 minutes)
- Sketch Angles **4.MD.6** (3 minutes)
- Physiometry **4.G.1** (4 minutes)

Count by 90° (1 minute)

Note: This fluency activity prepares students to do problem solving that involves 90° turns.

Direct students to count forward and backward, occasionally changing the direction of the count.

- Nines to 36
- 9 tens to 36 tens
- 90 to 360
- 90° to 360° (while turning)

Break Apart 90, 180, and 360 (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise prepares students for unknown angle problems in Lessons 10 and 11.

T: (Project a number bond with a whole of 90. Fill in 30 for one of the parts.) On your personal white boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 90 and with 30 and 60 as parts.)

Continue to break apart 90 with the following possible sequence: 50, 45, 25, and 65.

T: (Project a number bond with a whole of 180. Fill in 120 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 180 and with 120 and 60 as parts.)

Continue to break apart 180 with the following possible sequence: 90, 75, 135, and 55.

T: (Project a number bond with a whole of 360. Fill in 40 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 360 and with 40 and 320 as parts.)

Continue to break apart 360 with the following possible sequence: 160, 180, 170, 270, 120, 90, and 135.

Sketch Angles (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews terms from Lesson 2.

T: Sketch $\angle ABC$ that measures 90°.

T: (Allow students time to sketch.) Is a 90° angle a right angle, an obtuse angle, or an acute angle?

S: Right angle.

T: Sketch $\angle DEF$ that measures 100°.

T: (Allow students time to sketch.) What type of angle did you draw?

S: Obtuse.

Continue with the following possible sequence: 170°, 30°, 130°, 60°, and 135°.

Physiometry (4 minutes)

Note: Kinesthetic memory is strong memory. This fluency exercise reviews terms from Lessons 1–8.

T: (Stretch one arm straight up, pointing at the ceiling. Straighten the other arm, pointing directly at a side wall.) What angle measure do you think I'm modeling with my arms?

S: 90°.

T: (Straighten both arms so that they are parallel to the floor, pointing at both side walls.) What angle measure do you think I'm modeling now?

S: 180°. → Straight angle.

T: (Keep one arm pointing directly to a side wall. Point directly down with the other arm.) Now?

S: (270°.) → 90°.

T: It could be 90°, but the angle I'm thinking of is larger than 180°, so that would be?

S: 270°.

Continue to 360°.

Quickly remind students that this is an illustration and that they should not make the mistake to think that lines and points are as thick as arms. They are actually infinitely small.

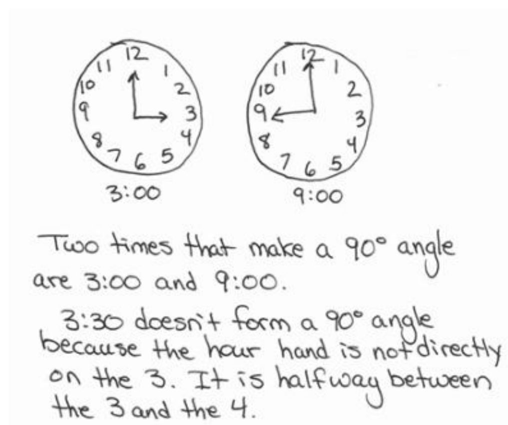
T: Stand up.
S: (Stand.)
T: Model a 90° angle.
T: Model a 180° angle.
T: Model a 270° angle.
T: Model a 360° angle.
T: Point to the walls that run perpendicular to the front of the room.
S: (Point to the side walls.)
T: (Point to the side wall.) Turn 90° to your right.
T: Turn 90° to your right.
T: Turn 90° to your right.
T: Turn 90° to your right.
T: Turn 180°.
T: Turn 90° to your left.
T: Turn 180°.

Application Problem (5 minutes)

List times on the clock in which the angle between the hour and minute hands is 90°. Use a student clock, watch, or real clock. Verify your work using a protractor.

Stay alert for this misconception: Why don't the hands at 3:30 form a 90° angle as expected?

Note: This Application Problem reviews measuring, constructing, verifying with a protractor, and recognizing in their environment 90° angles as taught in Topic B. Students use their knowledge of 90° angles to compose and decompose angles using pattern blocks in today's Concept Development.

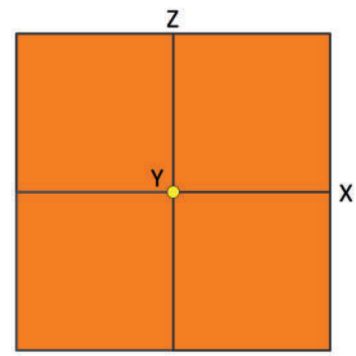


Concept Development (33 minutes)

Materials: (T) Pattern blocks for the overhead projector or an interactive white board with pattern block images, straightedge, protractor (S) Pattern blocks, Problem Set, straightedge, protractor

Note: Students record discoveries with pattern blocks on the Problem Set as indicated in this Concept Development.

Problem 1: Derive the angle measures of an equilateral triangle.



T: Place squares around a central point. (Model.) Fit them like puzzle pieces. Point to the central point. (Model.) How many right angles meet at this central Point Y?

S: 4.

T: (Trace and highlight $\angle XYZ$.) Trace $\angle XYZ$. Tell your neighbor about it.

S: It's 90°. → It's a right angle. → If $\overline{XY}$ is at 0°, $\angle XYZ$ is one quarter-turn counterclockwise. → If this were a clock, it would be 3 o'clock.

T: How many quarter-turns are there around the central point?

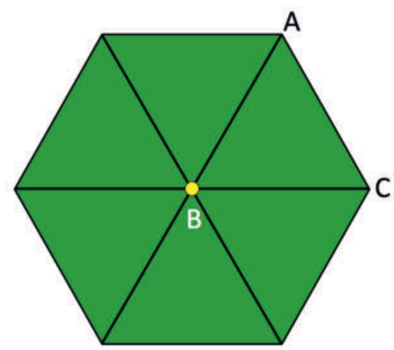
S: Four quarter-turns!

T: If we didn't know that the number of degrees in a quarter-turn is 90, how could we figure it out?

S: We could divide 360 by 4 since going all the way around in one full turn would be 360°, and there are four quarter-turns around the central point. → 360 divided by 4 is 90.

T: Tell your neighbor an addition sentence for the sum of all the right angles in degrees. Record your work on your Problem Set.

S: 90° + 90° + 90° + 90° = 360°.



T: So, the sum of the angles around a central point is…?

S: 360°.

T: Arrange a set of green triangles around a central point. (Model.) How many triangles did you fit around the central point?

S: 6.

T: Are all the central angles the same?

S: Yes!

T: How do you know?

S: I stacked all six triangles on top of each other. Each angle matched up with the others. → I turned the angles to make sure each angle aligned.

T: What is similar about the arrangement of squares and the arrangement of triangles?

S: They all fit together perfectly at their corners. → They both go all the way around a central point. → Four squares added up to 360°, so the six triangles must add up to 360°.

T: (Trace $\angle ABC$.) Work with your partner to find the angle measure of $\angle ABC$. On your Problem Set, write an equation to show your thinking.

S: $\angle ABC$ = 60°. → 60° + 60° + 60° + 60° + 60° + 60° = 360°. → 360° ÷ 6 = 60°. → 6 × 60° = 360°.

T: Let's check. Count by sixties with me. (Point to each angle as students count.) 60°, 120°, 180°°, 240°, 300°, 360°.

T: What about $\angle BCA$? $\angle BAC$? Discuss your thoughts with your partner.

S: I don't know. → I think all the angles are the same size. → 60°. → If I rotate the triangle so $\angle BAC$ is at $\angle ABC$, all the angles at the center still add to 360°.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Check that students working below grade level and others understand that the sum of the *interior*—not *exterior*—angles around a central point is 360°. Clarify the difference between exterior and interior angles.

Problem 2: Verify the equilateral triangle's angle measurements with a protractor.

T: How can we prove the angle measures in the triangle are 60°?

S: We could measure with a protractor. → But the protractor is a tool for measuring lines, not pattern blocks! → But, when I try to measure the angle of the triangle, the lines are not long enough to reach the markings on the protractor.

T: Use your straightedge and protractor to draw a 60° angle. (Demonstrate.) Now, using your protractor, verify that the angle you drew is indeed 60°. (Allow students time to measure the angle.) What angle measure do you read on the protractor?

S: 60°.

T: Align each angle of the triangle with this 60° angle. (Allow students time to perform the task.) What did you discover about the angles of this triangle?

S: All the angles measure 60°.

T: Would the angle measure change if I gave you the same triangle, just enlarged? What about a larger square pattern block?

S: No, we could still fit four squares and six triangles. → The angle measure doesn't change when the shape gets bigger or smaller. A small square or a really large square always has 90° corners. So, the angles of a smaller or larger triangle like this always measure 60°. → We learned a few days ago that degree measure isn't a length measure. So, the length of the sides on the triangle or square can grow or get smaller, but their angles always measure the same.

Problem 3: Derive the angle measure of unknown angles, and verify with a protractor.

T: Turn to Page 2 of your Problem Set. In Problem 2, find the measurement of obtuse $\angle ABC$. Discuss your thoughts with your partner.

S: I see two angles, 90° and 60°. Together, that makes 150°. → 90 + 60 is 150. This angle measures 150°.

T: The six angles of the hexagon are the same. Use your pattern blocks to find the angle measure of one angle.

S: I can place the six triangles on top of the hexagon. Two 60° angles fit in one angle of the hexagon. 60° + 60° is 120°. → 2 × 60 = 120. One of the hexagon's angles measures 120°.

T: In the margin of your Problem Set, record your observations about the relationship between the angles of the hexagon and the triangle. (Allow students time to record.) Then, write an equation to solve for the obtuse angle measure of the hexagon. Verify your answer by measuring with a protractor.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge students working above grade level and others to make predictions, find relationships, and use mental math when finding unknown angles in pattern blocks. Ask, "Is there a relationship between equal angles and equal segments in a polygon?" Ask students to make predictions for unknown angle measures and then to justify their predictions in words. Challenge them to visualize to solve mentally before using paper and pencil.

T: Look on your Problem Set. What angle do you form when you combine the triangle and the hexagon?

S: A straight angle!

T: Record the measurement of $\angle DEF$ as an addition sentence on the Problem Set.

T: Use your pattern blocks to find the angle measure for the obtuse and acute angles in the blue rhombus. Discuss and share your equations with your neighbor. Record your work in Rows (d) and (e) of Problem 1 of the Problem Set.

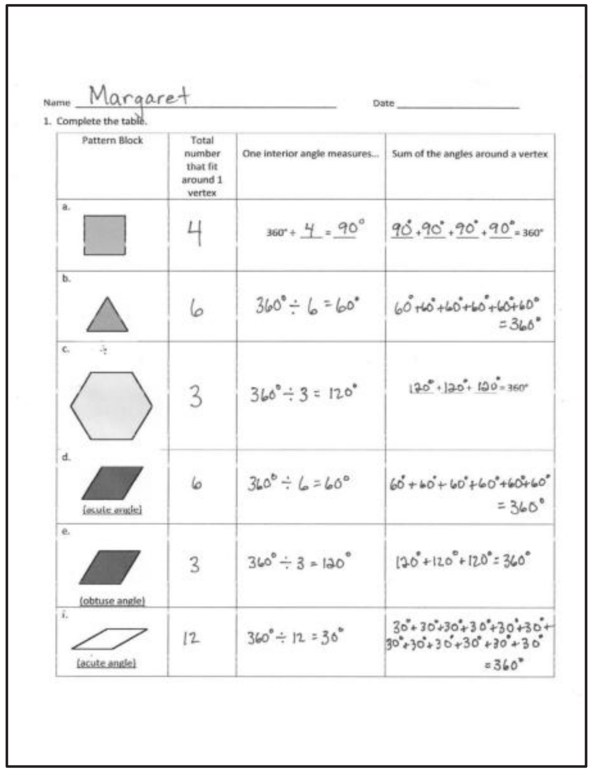
MP.6

S: I fit two triangles onto the blue rhombus. The acute angle of the rhombus is the same as the angles of the triangle. It is 60°. → The three obtuse angles can fit around the central point of a circle. We know the sum is 360°. 360 ÷ 3 = 120. The obtuse angle measures 120°. → I see two 60° angles make the obtuse angle when I align two triangles on one rhombus. 60° + 60° = 120°. → 120° + 120° + 120° = 360°.

T: How can you use what you've learned?

S: I can use what I know about the angle measurements in known shapes to find the angle measurements I don't know. → I can use the angles I know like this 60° angle to measure other angles. → I can add angle measurements to find the measurement of a larger angle.

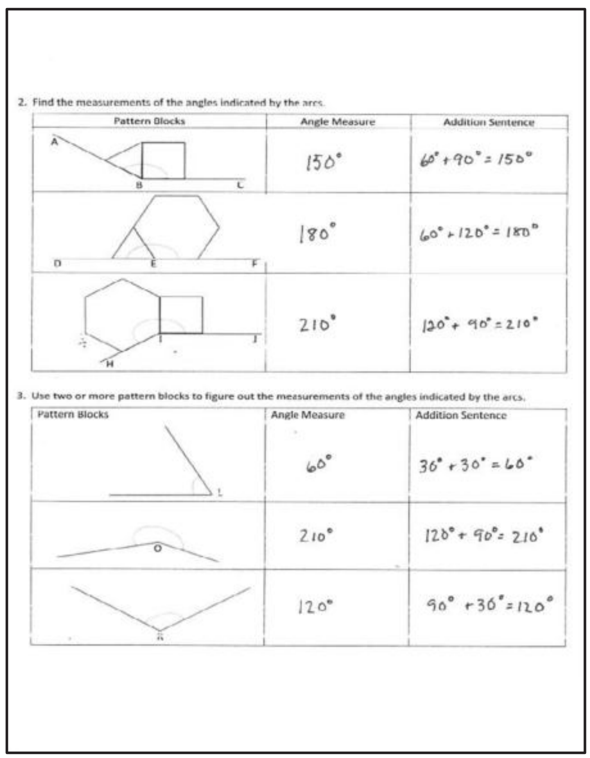
T: Work with your partner to find the measurement of the unknown angles of the tan rhombus. Then, use your pattern blocks to find the measurements of the unknown angles in Tables 2 and 3 on the Problem Set. Use words, equations, and pictures to explain your thinking.



Name Margaret Date

1. Complete the table.

Pattern Block	Total number that fit around 1 vertex	One interior angle measures...	Sum of the angles around a vertex
a.	4	360° ÷ 4 = 90°	90° + 90° + 90° + 90° = 360°
b.	6	360° ÷ 6 = 60°	60°+60°+60°+60°+60°+60° = 360°
c.	3	360° ÷ 3 = 120°	120° + 120° + 120° = 360°
d. (acute angle)	6	360° ÷ 6 = 60°	60°+60°+60°+60°+60°+60° = 360°
e. (obtuse angle)	3	360° ÷ 3 = 120°	120° + 120° + 120° = 360°
f. (acute angle)	12	360° ÷ 12 = 30°	30°+30°+30°+30°+30°+30°+30°+30°+30°+30°+30°+30° = 360°



2. Find the measurements of the angles indicated by the arcs.

Pattern Blocks	Angle Measure	Addition Sentence
A, B, C	150°	60° + 90° = 150°
D, E, F	180°	60° + 120° = 180°
H, I	210°	120° + 90° = 210°

3. Use two or more pattern blocks to figure out the measurements of the angles indicated by the arcs.

Pattern Blocks	Angle Measure	Addition Sentence
L	60°	30° + 30° = 60°
O	210°	120° + 90° = 210°
R	120°	90° + 30° = 120°

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

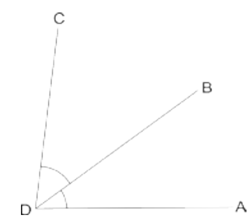
Lesson Objective: Decompose angles using pattern blocks.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

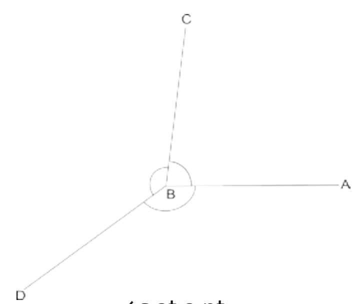
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What are the measures for the acute and obtuse angles of the cream rhombus? What did you discover when you fit the acute angles around a vertex?
- How are the different angles in the pattern blocks related?
- What was the measure of $\angle HIJ$? $\angle L$? $\angle O$? $\angle R$? How did you find the angle measures? What combination of blocks did you use? How did your method compare with your neighbor's?
- What did you learn about adding angles?
- (Write $\angle s$ add.) The angle symbol with an *s* just means *angles*. It's the plural of *angle*. "∠s add" translates as "we are adding these angles that share a side." (Write ∠ADB + ∠BDC = ∠ADC.) What are different methods for finding the sum of the pictured angles?
- (Write $\angle s$ at a pt.) In our problems today we also made use of the fact that when angles meet at a point, they add up to 360°. "∠s at a pt" simply translates as "we have angles centered around a point," which means their sum would be 360°. (Write ∠ABC + ∠CBD + ∠DBA = 360°.) Restate this in your own words to your partner.
- How can you verify an angle's measure?



$\angle s$ add:
$\angle ADB + \angle BDC = \angle ADC$



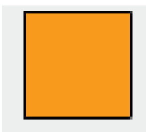
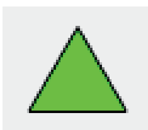
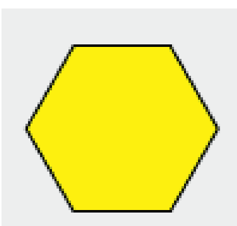
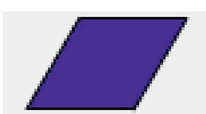
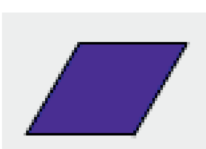
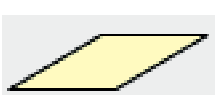
$\angle s$ at a pt:
$\angle ABC + \angle CBD + \angle DBA = 360°$

Exit Ticket (3 minutes)

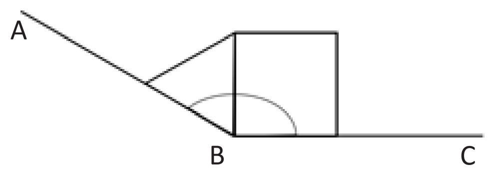
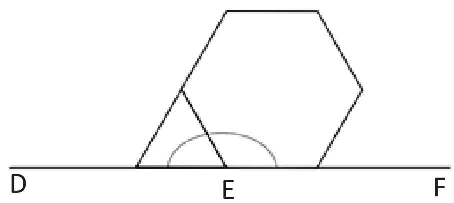
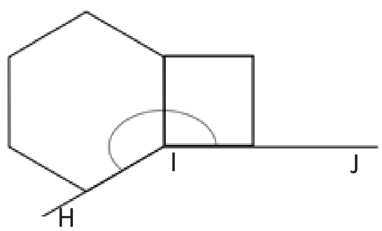
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ____ Date ______________________

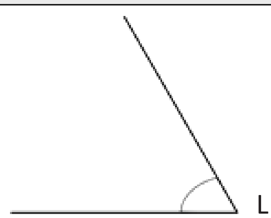
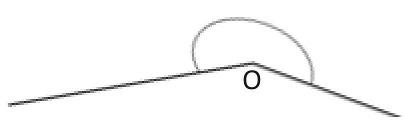
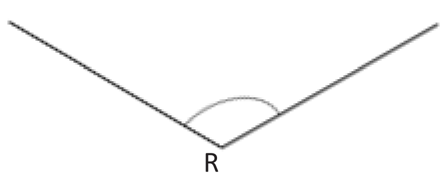
1. Complete the table.

Pattern block	Total number that fit around 1 vertex	One interior angle measures…	Sum of the angles around a vertex
a. 		360° ÷ ____ = ____	____ + ____ + ____ + ____ = 360°
b. 			
c. 			____ + ____ + ____ = 360°
d.  (Acute angle)			
e.  (Obtuse angle)			
f.  (Acute angle)			

2. Find the measurements of the angles indicated by the arcs.

Pattern blocks	Angle measure	Addition sentence
a. 		
b. 		
c. 		

3. Use two or more pattern blocks to figure out the measurements of the angles indicated by the arcs.

Pattern blocks	Angle measure	Addition sentence
a. 		
b. 		
c. 		

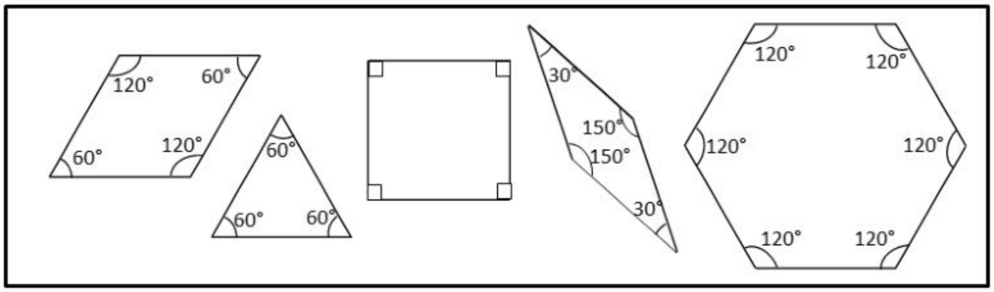
Name ____ Date ____________________

1. Describe and sketch two combinations of the blue rhombus pattern block that create a straight angle.

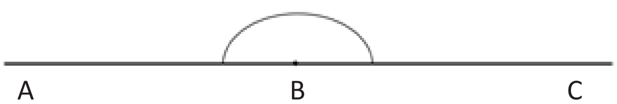
2. Describe and sketch two combinations of the green triangle and yellow hexagon pattern block that create a straight angle.

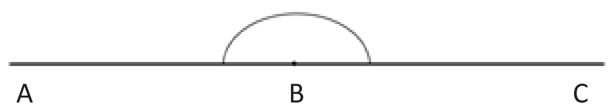
Name ____ Date _____________________

Sketch two different ways to compose the given angles using two or more pattern blocks.
Write an addition sentence to show how you composed the given angle.



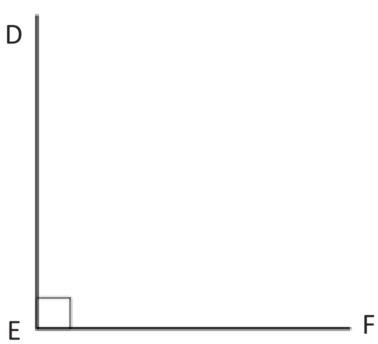
1. Points A, B, and C form a straight line.

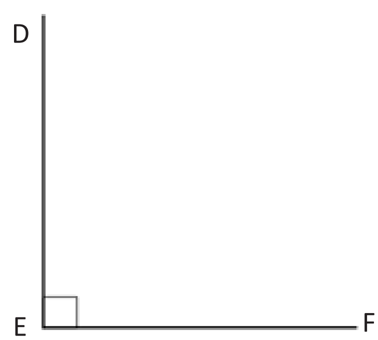




180° = ___________________________________ 180° = ___________________________________

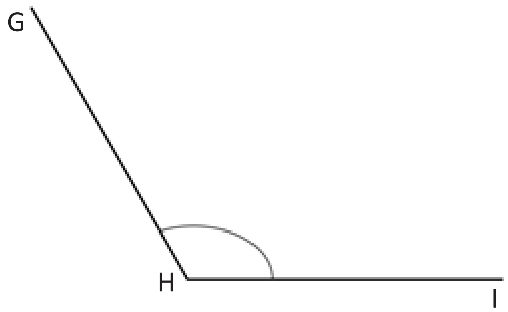
2. $\angle DEF$ = 90°

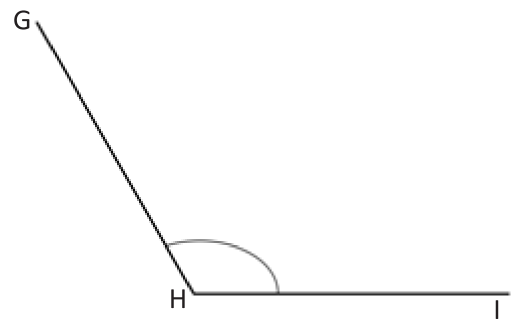




90° = ___________________________________ 90° = ___________________________________

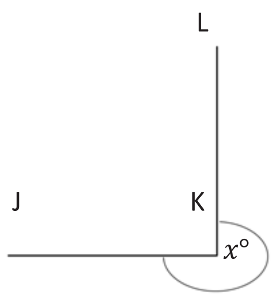
3. $\angle GHI$ = 120°

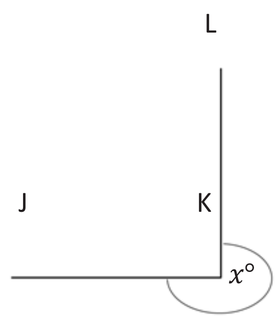




120° = ________________________________ 120° = ________________________________

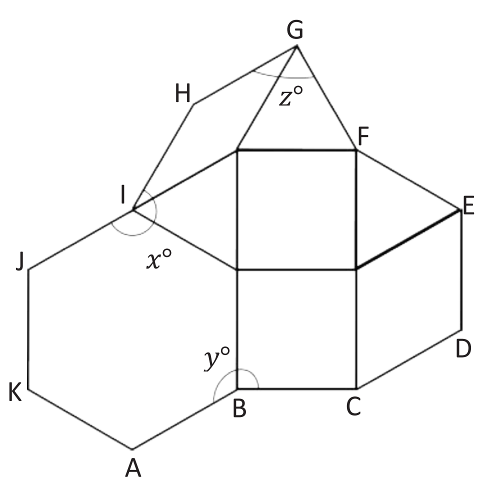
4. $x° = 270°$





270° = ________________________________ 270°= ________________________________

5. Micah built the following shape with his pattern blocks. Write an addition sentence for each angle indicated by an arc and solve. The first one is done for you.



a. $y° = 120° + 90°$

$y° = 210°$

b. $z° =$ ____________________

$z° =$ _________

c. $x° =$ ____________________

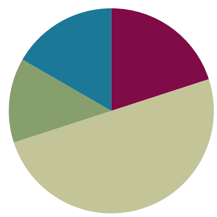
$x° =$ _________

Lesson 10

Objective: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

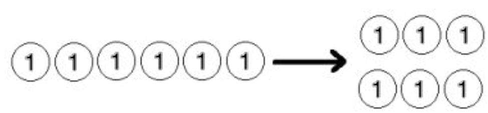
- Divide with Number Disks **4.NBT.1** (4 minutes)
- Group Count by 90° **4.MD.7** (1 minute)
- Break Apart 90, 180, and 360 **4.MD.7** (4 minutes)
- Physiometry **4.G.1** (3 minutes)

Divide with Number Disks (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 3 content.

T: (Display 6 ÷ 2.) On your personal white boards, draw number disks to represent the expression.
S: (Draw 6 ones disks, and divide them into 2 groups of 3.)
T: Say the division sentence in unit form.
S: 6 ones ÷ 2 = 3 ones.



Continue with the following possible sequence: 60 ÷ 2; 600 ÷ 2; 6,000 ÷ 2; 9 tens ÷ 3; 12 tens ÷ 4; and 12 tens ÷ 3.

Group Count by 90° (1 minute)

Note: If students struggle to connect counting groups of 9, groups of 9 tens, and groups of 90, write the counting progressions on the board.

Direct students to count forward and backward, occasionally changing the direction of the count.

- Nines to 36
- 9 tens to 36 tens
- 90 to 360
- 90° to 360° (while turning)

Break Apart 90, 180, and 360 (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise prepares students for unknown angle problems in Lessons 10 and 11.

T: (Project a number bond with a whole of 90. Fill in 45 for one of the parts.) On your personal white boards, write the number bond, filling in the unknown part.

Continue to break apart 90 with the following possible sequence: 35, 25, 65, and 15.

T: (Project a number bond with a whole of 180. Fill in 170 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

Continue to break apart 180 with the following possible sequence: 90, 85, 45, and 125.

T: (Project a number bond with a whole of 360. Fill in 180 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

Continue to break apart 360 with the following possible sequence: 90, 45, 270, 240, and 315.

Physiometry (3 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms from Lessons 1–8.

T: Stand up. (Students stand and follow the series of directions below.)
T: Model a 90° angle with your arms.
T: Model a 180° angle with your arms.
T: Model a 270° angle.
T: Model a 360° angle.
T: Point to the walls that run perpendicular to the back of the room.
T: Turn 90° to your left.
T: Turn 90° to your left.
T: Turn 90° to your left.
T: Turn 90° to your left.
T: Turn 180°.

T: Turn 90° to your left.

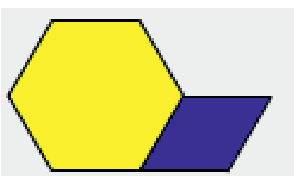
T: Turn 180°.

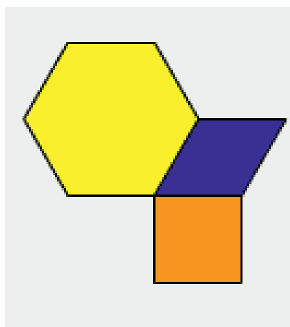
T: Turn 270° to your right.

T: Turn 180° to your left.

Application Problem (8 minutes)

Using pattern blocks of the same shape or different shapes, construct a straight angle. Which shapes did you use? Compare your representation to that of your partner. Are they the same? Which pattern block can you add to your existing shape to create a 270° angle? How can you tell?





NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Seeking to construct a straight angle, some students may place two triangles or trapezoids side by side, leaving gaps between the angle sides. Encourage them to verify 180° by adding the interior angles of the pattern blocks. Ask, "What shape could fit in this gap? How can you confirm that you've made a straight angle?"

Note: This Application Problem builds from the previous lesson, where students found the angle measures of pattern blocks and verified the measures with a protractor. In this Application Problem, students use pattern blocks to form a straight angle, and then examine the relationship of the parts to the whole and discover that there are many ways to compose and decompose a straight angle. This leads into today's Concept Development, where students deepen their understanding of the additive nature of angle measure.

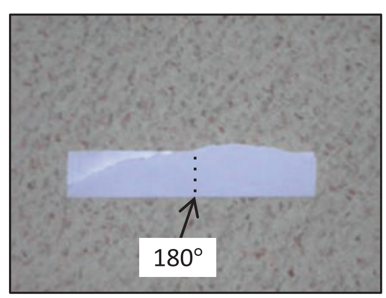
Concept Development (30 minutes)

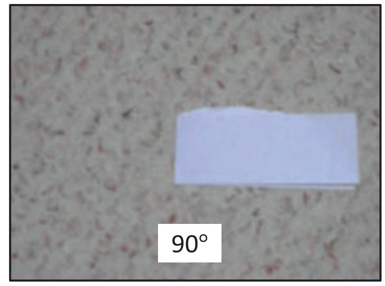
Materials: (T/S) Blank paper (full sheet of letter-size paper ripped into two pieces), personal white board, straightedge, protractor, pattern blocks

Problem 1: Use benchmark angle measures to show that angle measures are additive.

T: Grab a blank sheet of ripped paper. Fold it in half from bottom to top. Fold it from left to right. Open the paper back up one fold. (Demonstrate.) Run your finger along the line of the horizontal fold. Consider the fold. Mark the vertex with a dot. What special angle have you created?

S: A straight angle. → Its measurement is 180°.



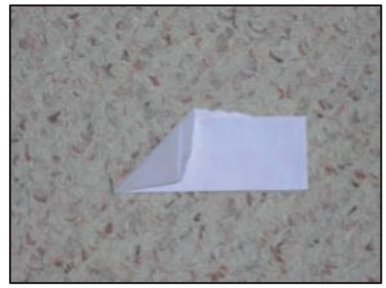


T: Fold your paper back left to right. Be sure it is folded so that the previously folded edge is directly on top of itself. Run your finger along the folded sides. What angle have you created now, if the vertex of the angle is at the corner of the folds?

S: A right angle. → A 90° angle.

T: Fold the vertical side down to match up with the horizontal side, like this. (Demonstrate.) Unfold. How many angles has the right angle been decomposed into?

S: Two!



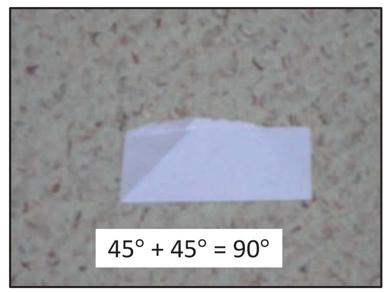
T: What do you notice about the two angles?

S: They are the same.

T: How can you tell?

S: One angle fits exactly on top of the other.

T: Discuss with your partner. How can you determine the measurement of each angle?



S: We can take 90° and divide it by two. → We can think of what number plus itself equals 90. It's 45. → We could use a protractor to measure.

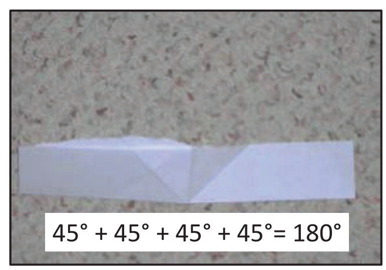
T: Unfold your paper one fold.

T: Let's look at the angles. What do we see?

S: We see the two 45° angles.

T: Say the number sentence that shows the total of the angle measurements.

S: 45° + 45° = 90°.



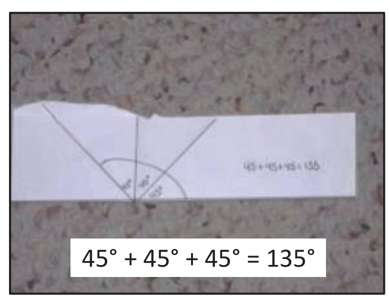
T: Unfold another fold. What do you see now?

S: Four angles.

T: What do you notice?

S: They are all the same. I can tell because if I fold the paper, they stack evenly on top of each other.

T: Say the number sentence that shows the total of the angle measurements.



S: 45° + 45° + 45° + 45° = 180°. → That makes sense because we have a straight line along this side.

T: What if we just looked at three of the angles? Draw an arc on your paper to show the angle created by looking at three of the angles together. Say the number sentence that shows the total of the angle measurements.

S: 45° + 45° + 45° = 135°. → 180° – 45° = 135°.

T: Let's verify with a protractor. Use your straightedge to trace along each crease. Measure and label each angle measure, and then measure and label the entire angle. Write the number sentence.

Students measure, label, and write the number sentence.

Problem 2: Demonstrate that the angle measure of the whole is the sum of the angle measures of the parts.

T: Fold a different ripped piece of paper to form a 90° angle as we did before.

T: Fold the upper left-hand section of your paper down. This time, the corner should not meet the bottom of your paper. (Demonstrate.)

T: Open the fold that you just created. What do you see?

S: I see two angles. They are not the same size.

T: Compare your angles to your partner's. Are they the same?

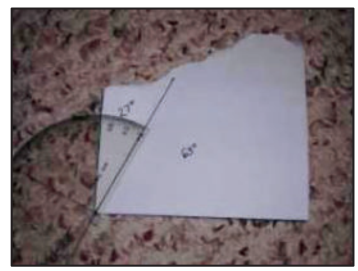
S: No, they look different.

T: Why is that?

S: We each folded our paper differently.

T: Follow these directions:

1. Use a straightedge to draw a segment on the fold.
2. Measure the two angles with your protractor.
3. Label each angle measure.
4. Write the number sentence to show the sum of the two angles.

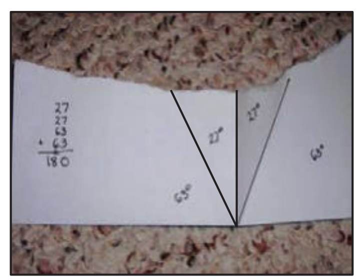


T: (Allow students time to work.) What do you notice?

S: The angles added up to 90°. 63° + 27° = 90°. That shows the whole! → Mine didn't add up to 90°. They added up to 88°. → Mine added up to 91°. That doesn't make sense because the angle we started with was 90°. If we split it into two parts, the parts should add up to the whole. It's just like when we add or subtract numbers. I must have measured the angles wrong. Let me try again!

T: Unfold your paper another time. What do you see?

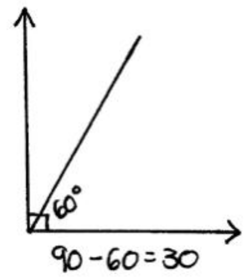
S: There are four angles instead of two. → These four angles combine to make a straight angle.



T: Repeat the same process with these four angles to find their sum. Do you need to measure all of the angles?

S: No. I know their measurements because when I folded the paper, I was making angles that are the same. One unknown angle is 27° and the other is 63°. The angles add together to make a measurement of 180°. That makes sense because the paper has a straight edge. When we fold the paper, we are splitting the original angle into parts. All of those parts have to add up to the original angle because the whole part doesn't change when we fold it. It stays the same.

Problem 3: Given the angle measure of the whole, find the unknown measure of the part. Write an equation using a symbol for the unknown angle measure.



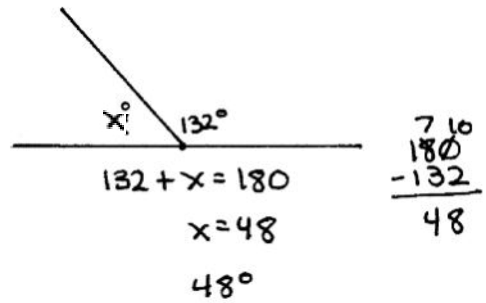
T: (Using a protractor, construct a 90° angle on the board. Within that angle, measure and label a 60° angle.)

T: Discuss with your partner how we can find the measurement of the unknown angle. Use what we just learned.

S: I know that the measurement of the large angle is 90°. If one of the parts is 60°, I can figure out the other part by subtracting 60 from 90. 90 – 60 = 30. The other angle is 30°. → I know that 60 + 30 is 90, so the other angle must be 30°.

MP.2

T: When we take a whole angle and break it into two parts, if we know the angle measurement of one part, we can find the angle measurement of the other part by subtracting.



T: (Draw a straight angle on the board. Use a protractor to draw a 132° angle. Label the angle as 132°. Indicate that we know the 180° measure and the 132° measure, but that we do not know the measure of x.)

T: Work with your partner to find the unknown angle.

S: If the straight angle measures 180° and one part is 132°, then the other angle must be 48° because 180 – 132 = 48. → I solved it because I know that 132 + 48 = 180 by counting on. → I knew that 130 plus 50 is 180, and 50 minus 2 is 48.

T: Let's write an equation and use x to represent the measure of the unknown angle. Let's start with the known part. What is the known part?

S: 132.

T: What is the total?

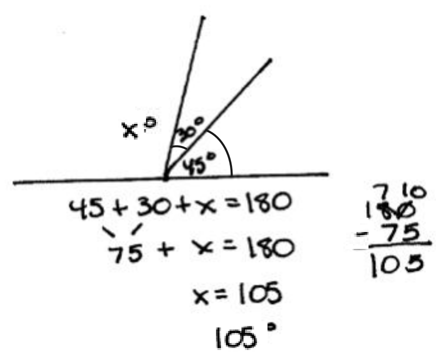
S: 180.

T: Say the equation. Start with the known part.

S: 132 + x = 180.

T: Say it in a subtraction sentence starting with the whole.

S: 180 – 132 = x.



T: (Draw a straight angle on the board. Using a protractor, measure a 75° angle. Then, using a protractor, subdivide the angle into a 45° angle and a 30° angle.)

T: What is different about this angle than the angles that we have been working with?

S: The angle is split into three parts instead of two.

T: How can we solve for the unknown angle? Write the equation.

S: 45 + 30 + x = 180. → This is just like when we have to find the unknown part when we add numbers. We find the sum of the two angles that we know, and then we subtract from the total. 45 + 30 = 75. 180 – 75 = 105.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

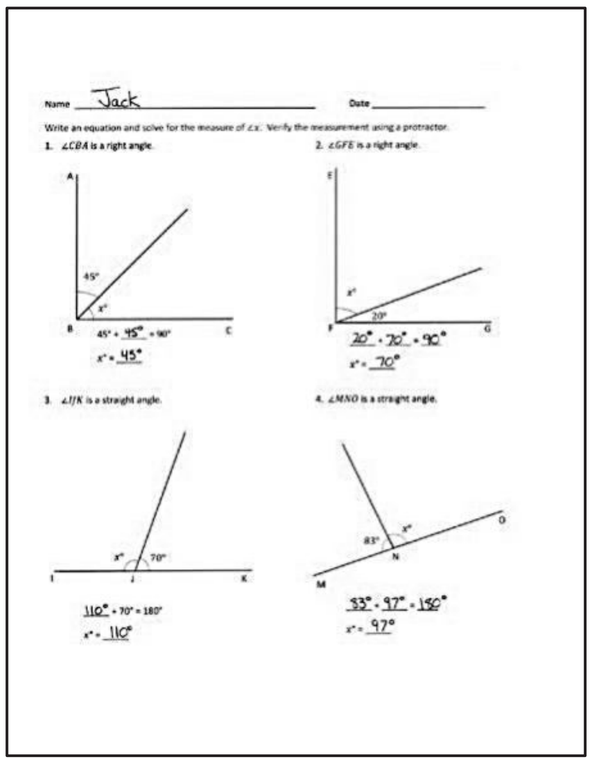
Students working below grade level and others may benefit from additional scaffolding of Page 1 of the first page of the Lesson 10 Problem Set. It may be helpful to include a subtraction sentence frame for solving for x. For example, in Problem 1, provide $90 – 45 = x$. Build student independence gradually. Have students become confident with writing their own subtraction sentence after a few examples. Then, for the final problems, encourage students to subtract mentally.

Student Debrief (10 minutes)

Lesson Objective: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

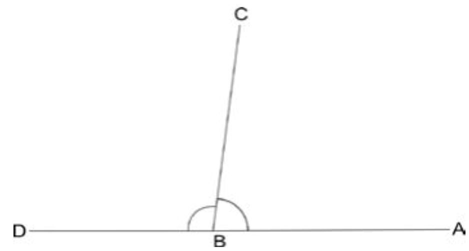
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.



Any combination of the questions below may be used to lead the discussion.

- For Problems 1–6, why is it important to know that we are starting with a right angle or a straight angle?
- For Problem 7, why is it important to know that ACDE is a rectangle?
- Why is it important to be precise when measuring angles?
- When two angles add to 90°, we say that they are **complementary angles**. When two angles add to 180°, we say that they are **supplementary angles**. What examples did we have of complementary angles? Of supplementary angles?

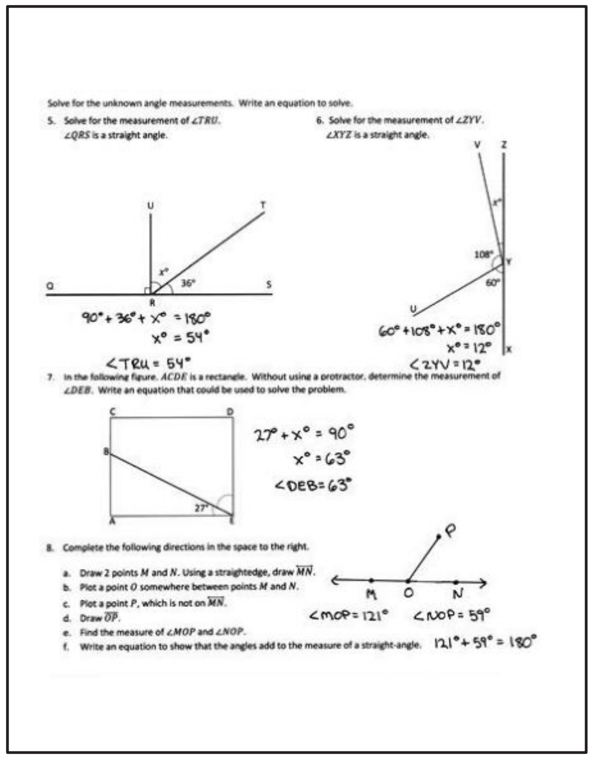
- (Write ∠ABC + ∠CBD = 180°.) When two or more angles meet to form a straight line, we saw that the angle measures add up to 180° (as shown to the right). As we saw yesterday, the angle symbol with an *s* just means *angles*. It's the plural of *angle*. (Write ∠*s* on a line.) ∠*s* on a line translates as "we have angles that together add up to make a line." How can we use the sum of angles on a line being 180° to solve problems?
- What new (or significant) math vocabulary did we use today to communicate precisely?
- How did the Application Problem connect to today's lesson?



∠s on a line:
$\angle ABC + \angle CBD = 180°$

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



Solve for the unknown angle measurements. Write an equation to solve.

5. Solve for the measurement of ∠*TRU*. ∠*QRS* is a straight angle.

$90° + 36° + x° = 180°$
$x° = 54°$
∠TRU = 54°

6. Solve for the measurement of ∠*ZYV*. ∠*XYZ* is a straight angle.

$60° + 108° + x° = 180°$
$x° = 12°$
∠ZYV = 12°

7. In the following figure, *ACDE* is a rectangle. Without using a protractor, determine the measurement of ∠*DEB*. Write an equation that could be used to solve the problem.

$27° + x° = 90°$
$x° = 63°$
∠DEB = 63°

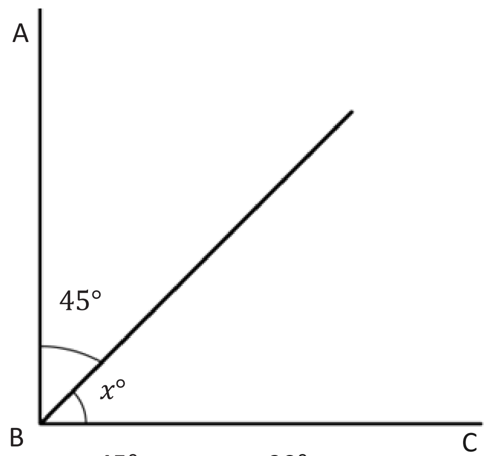
8. Complete the following directions in the space to the right.
   a. Draw 2 points *M* and *N*. Using a straightedge, draw $\overleftrightarrow{MN}$.
   b. Plot a point *O* somewhere between points *M* and *N*.
   c. Plot a point *P*, which is not on $\overleftrightarrow{MN}$.
   d. Draw $\overrightarrow{OP}$.
   e. Find the measure of ∠*MOP* and ∠*NOP*.
   f. Write an equation to show that the angles add to the measure of a straight-angle.

∠MOP = 121° ∠NOP = 59°
$121° + 59° = 180°$

Name ____ Date ______________________

Write an equation, and solve for the measure of $\angle x$. Verify the measurement using a protractor.

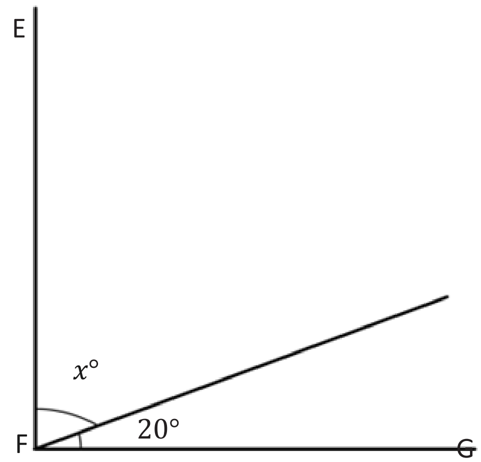
1. $\angle CBA$ is a right angle.



45° + ______ = 90°

x° = ______

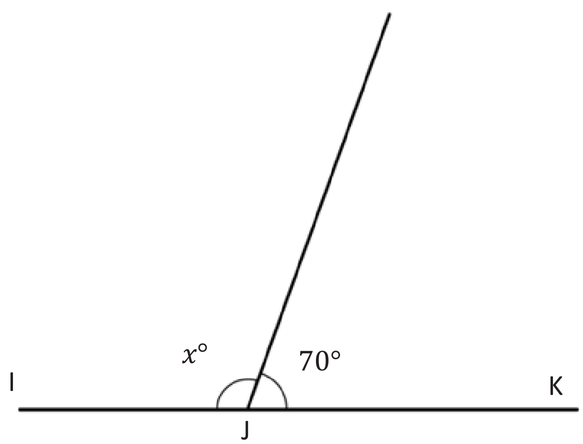
2. $\angle GFE$ is a right angle.



______ + ______ = _____

x° = ______

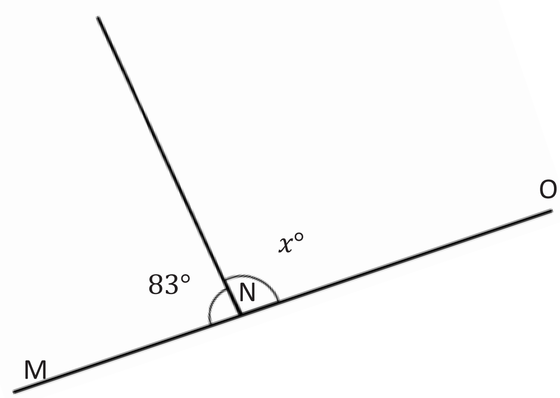
3. $\angle IJK$ is a straight angle.



______ + 70° = 180°

x° = ______

4. $\angle MNO$ is a straight angle.

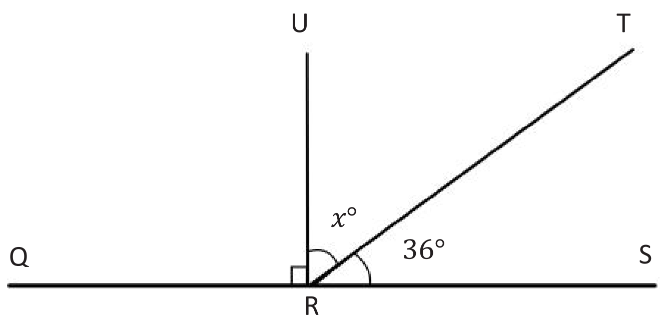


______ + ______ = _____

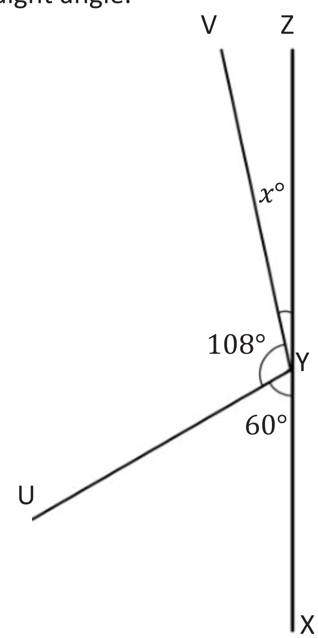
x° = ______

Solve for the unknown angle measurements. Write an equation to solve.

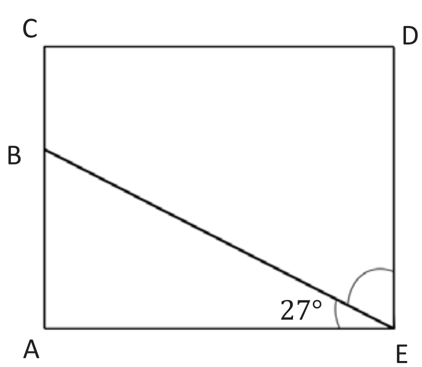
5. Solve for the measurement of $\angle TRU$.
$\angle QRS$ is a straight angle.



6. Solve for the measurement of $\angle ZYV$.
$\angle XYZ$ is a straight angle.



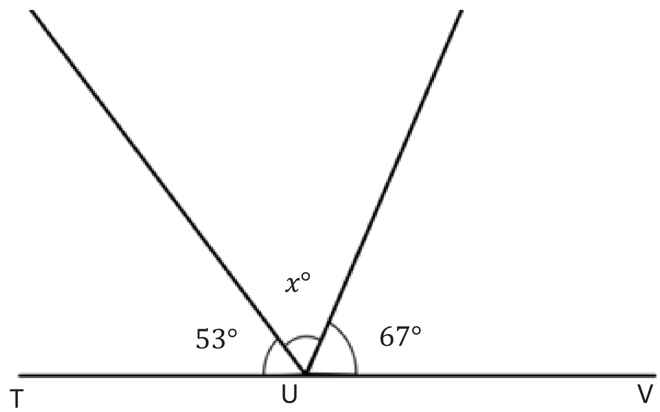
7. In the following figure, $ACDE$ is a rectangle. Without using a protractor, determine the measurement of $\angle DEB$. Write an equation that could be used to solve the problem.



8. Complete the following directions in the space to the right.
   a. Draw 2 points: M and N. Using a straightedge, draw $\overleftrightarrow{MN}$.
   b. Plot a point O somewhere between points M and N.
   c. Plot a point P, which is not on $\overleftrightarrow{MN}$.
   d. Draw $\overline{OP}$.
   e. Find the measure of $\angle MOP$ and $\angle NOP$.
   f. Write an equation to show that the angles add to the measure of a straight angle.

Name ____ Date ______________________

Write an equation, and solve for x. $\angle TUV$ is a straight angle.



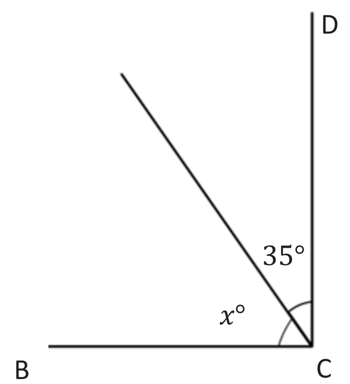
Equation: ____

$x° =$ ____________

Name ____ Date ______________________

Write an equation, and solve for the measurement of $\angle x$. Verify the measurement using a protractor.

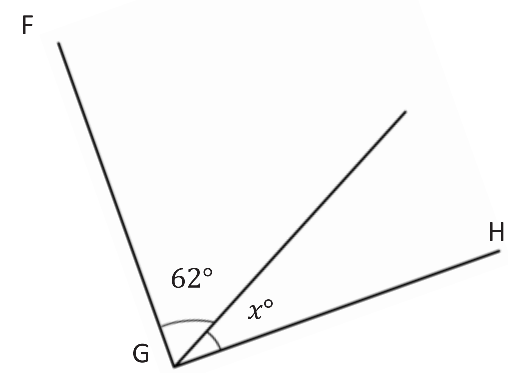
1. $\angle DCB$ is a right angle.



______ + 35° = 90°

$x°$ = ______

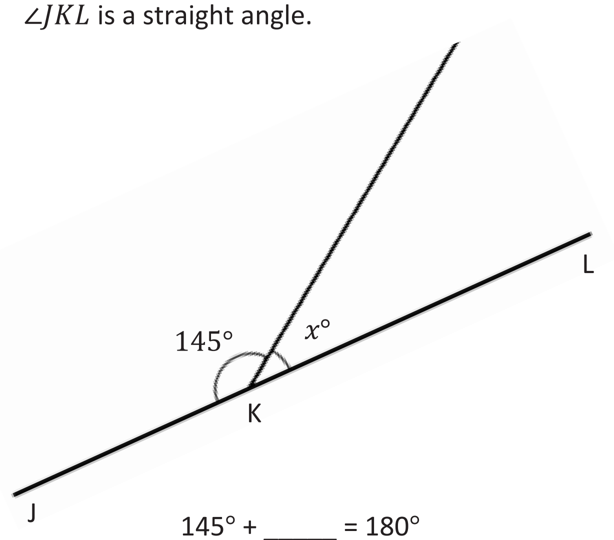
2. $\angle HGF$ is a right angle.



______ + ______ = _____

$x°$ = ______

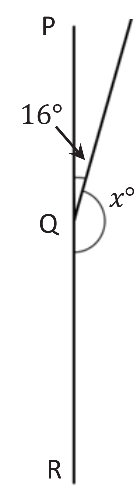
3. $\angle JKL$ is a straight angle.



145° + ______ = 180°

$x°$ = ______

4. $\angle PQR$ is a straight angle.

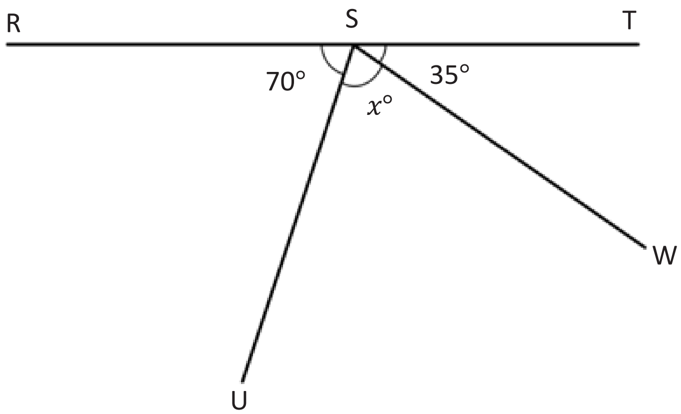


______ + ______ = _____

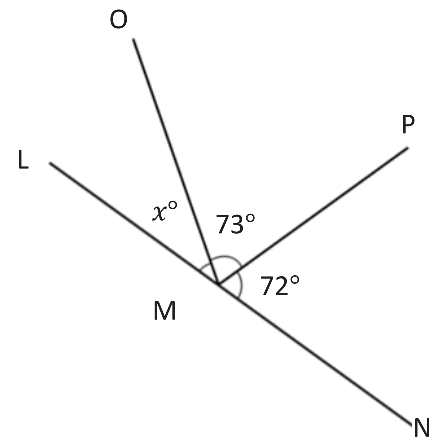
$x°$ = ______

Write an equation, and solve for the unknown angle measurements.

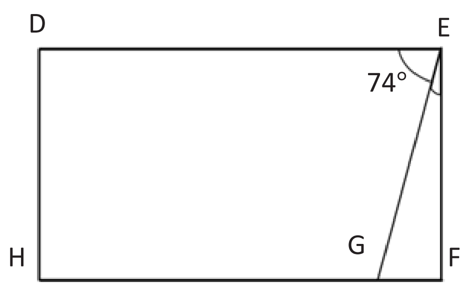
5. Solve for the measurement of $\angle USW$.
$\angle RST$ is a straight angle.



6. Solve for the measurement of $\angle OML$.
$\angle LMN$ is a straight angle.



7. In the following figure, $DEFH$ is a rectangle. Without using a protractor, determine the measurement of $\angle GEF$. Write an equation that could be used to solve the problem.



8. Complete the following directions in the space to the right.

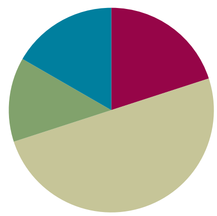
a. Draw 2 points: Q and R. Using a straightedge, draw $\overleftrightarrow{QR}$.
b. Plot a point S somewhere between points Q and R.
c. Plot a point T, which is not on $\overleftrightarrow{QR}$.
d. Draw $\overline{TS}$.
e. Find the measure of $\angle QST$ and $\angle RST$.
f. Write an equation to show that the angles add to the measure of a straight angle.

Lesson 11

Objective: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Divide Different Units **4.NBT.1** (4 minutes)
- Break Apart 90, 180, and 360 **4.MD.7** (4 minutes)
- Find the Unknown Angle **4.MD.7** (4 minutes)

Divide Different Units (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 3 content.

T: (Write 6 ÷ 2 = ___.) Say the division sentence in unit form.

S: 6 ones ÷ 2 = 3 ones.

T: (Write 6 ÷ 2 = 3. To the right, write 60 ÷ 2 = ___.) Say the division sentence in unit form.

S: 6 tens ÷ 2 = 3 tens.

T: (Write 60 ÷ 2 = 30. To the right, write 600 ÷ 2 = ___.) Say the division sentence in unit form.

S: 6 hundreds ÷ 2 = 3 hundreds.

T: (Write 600 ÷ 2 = 300. To the right, write 6,000 ÷ 2 = ___.) Say the division sentence in unit form.

S: 6 thousands ÷ 2 = 3 thousands.

T: (Write 6,000 ÷ 2 = 3,000.)

6 ÷ 2 = 3

60 ÷ 2 = 30

600 ÷ 2 = 300

6,000 ÷ 2 = 3,000

T: (Write 8 tens ÷ 2 = ____.) On your personal white boards, write the division sentence in standard form.

S: (Write 80 ÷ 2 = 40.)

Continue with the following possible sequence: 8 tens ÷ 2, 25 tens ÷ 5, 12 hundreds ÷ 4, 24 hundreds ÷ 4, 27 tens ÷ 3, 32 tens ÷ 4, 30 tens ÷ 5, and 40 hundreds ÷ 5.

Break Apart 90, 180, and 360 (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise prepares students for unknown angle problems in Lesson 11.

T: (Project a number bond with a whole of 90. Fill in 9 for one of the parts.) On your personal white boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 90 and with 9 and 81 as parts.)

Continue to break apart 90 with the following possible sequence: 55, 35, and 75.

T: (Project a number bond with a whole of 180. Fill in 142 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 180 and with 142 and 38 as parts.)

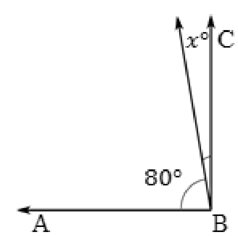
Continue to break apart 180 with the following possible sequence: 47, 133, and 116.

T: (Project a number bond with a whole of 360. Fill in 58 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 360 and with 58 and 302 as parts.)

Continue to break apart 360 with the following possible sequence: 93, 261, and 48.

Find the Unknown Angle (4 minutes)



Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 10.

T: (Project $\angle ABC$.) Angle ABC is a right angle. Say the given angle.

S: 80°.

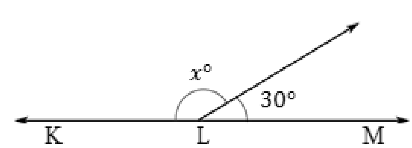
T: On your personal white boards, write the measure of $\angle x$. If you need to, write a subtraction sentence to find the answer.

S: (Write x° = 10°)

Continue with right angles using the following possible sequence: x° = 30°, and x° = 45°.

T: (Project $\angle KLM$.) KLM is a straight angle. What's the measurement of a straight angle?

S: 180°.

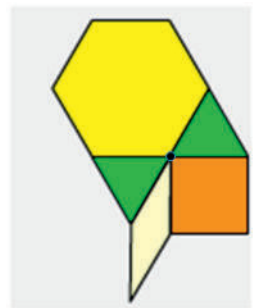


T: On your boards, write the measure of $\angle x$. If you need to, write a subtraction sentence to find the answer.

S: (Write 150°.)

Continue with straight angles using the following possible sequence: $x°$ = 60°, $x°$ = 90°, and $x°$ = 135°.

Application Problem (8 minutes)



Use patterns blocks of various types to create a design in which you can see a decomposition of 360°. Which shapes did you use? Compare your representation to that of your partner. Are they the same? Write an equation to show how you composed 360°. Refer to the pattern block chart to help with the angle measures of the pattern blocks, as needed.

120° + 60° + 90° + 30° + 60° = 360°

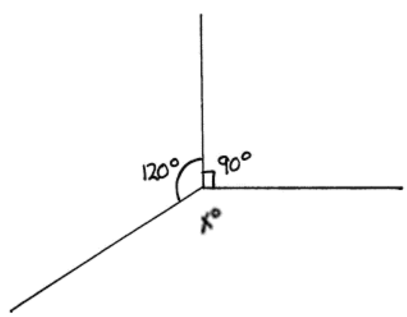
I used the hexagon, triangle, square, and rhombus. My shapes are different from my partner's. But when we put our shapes around a central point, the angles add up to 360°.

Note: This Application Problem builds from the previous lesson where students examined the relationship of the degree measure of parts of an angle to the whole and discovered that there are different ways to compose and decompose angles. This leads into today's Concept Development where students further their discovery of the additive nature of angle measure by exploring angles that add to 360°.

Concept Development (30 minutes)

Materials: (T) Blank paper, personal white board, protractor, pattern blocks, straightedge, red marker, blue marker, chart of pattern block angle measures (S) Blank paper, personal white board, protractor, pattern blocks, straightedge, red and blue pencils, markers, or crayons

Problem 1: Decompose a 360° angle into smaller angles. Recognize that the smaller angles add up to 360°.



T: Take one of your pattern blocks away from the shape that you made in the Application Problem. Now, there is a missing piece. Write an equation to show the total using x to represent the measurement of the angle of the missing piece.

S: 120° + 120° + $x°$ = 360°. → 120° + 60° + 30° + 30° + $x°$ = 360°.

T: Challenge your partner to determine the unknown angle. How can we solve?

S: Add together all of the known parts, and then subtract the total from the whole, which is 360°.

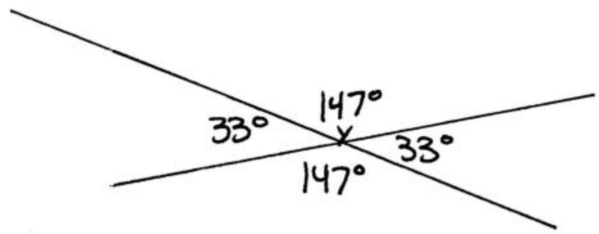
T: Does it matter how many parts there are?

S: The parts will always add to make the whole. → There could be as few as three parts or as many as twelve if we are using pattern blocks. → The parts will always add to 360°.

T: (Project image as shown below to the right.) How can we solve for the unknown angle?

S: It's like what we just did with the pattern blocks. We know that 90 + 120 = 210. 360 – 210 = 150. The angle must be 150°.

T: Let's use a protractor to verify.



T: Now, use your straightedge to draw two intersecting lines. Locate where they intersect, and label that point Y. Measure each angle that composes the angle around point Y. What do you notice?

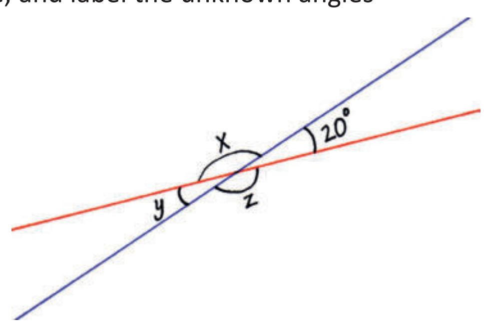
S: The angles that are across from each other are the same.

T: Write a number sentence to show the total. What will the total be?

S: The total will be 360° because all of the angles surround one point. 33° + 147° + 33° + 147° = 360°.

Problem 2: Given two intersecting lines and the measurement of one angle, determine the measurement of the other three angles.

Draw a line on the board using a red marker. Draw an intersecting line in blue, decomposing the straight angle into two smaller angles, one of which is 20°. Label the 20° angle, and label the unknown angles pictured to the right with variables.



T: What do you see?

S: Two intersecting lines. → Two straight angles in two parts. One angle is 20°. The other angles are unknown.

T: (Point to the red line.) Determine the unknown angle, $\angle x$.

S: 180° – 20° = x. → x = 160° → 160°.

T: Now, look at the blue line. Notice the measure of $\angle y$ is unknown. How can we solve for it?

S: We know that $\angle x$ is 160°. 180° – 160° = y or 160° + ___ = 180°. y = 20°.

T: (Point to the red line.) Let's look at the red line again. How can we determine $\angle z$?

S: 180° – 20° = z. z + 20° = 180°. z = 160°. → Those angles are the same as the angles that we started with!

T: Let's try another one. (Draw two intersecting lines, one red and one blue. Measure with a protractor to make one angle 110°, and label the angle.) Show this on your personal white boards, and then work with a partner to determine the unknown angles.

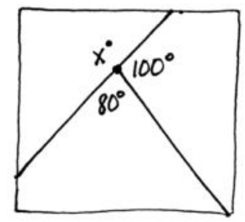
S: The unknown angles are 70°, 110°, and 70°. Hey, the angles that are across from each other are the same!

Problem 3: Solve a practical application word problem involving unknown angles.

T: Cyndi is making a quilt square. The blue, pink, and green pieces meet at a point. At the point, the blue piece has an angle measurement of 100°, and the pink has an angle measurement of 80°. What is the angle measurement determined by the green piece?

T: Draw a picture to show a representation of the quilt square. Tell your partner what your picture shows. What do we want to know?

S: There are three pieces of fabric sewn together. The angles are 100° and 80°. We need to know the measurement of the third angle.



MP.3

T: How did you know what a 100° angle looks like without a protractor?

S: I know that 100° is slightly larger than a 90° angle. I know what a 90° angle looks like, so I can draw my angle so that it's pretty close.

T: How about the 80°?

S: 80° is less than 90° so I can draw that pretty close too.

T: Write the equation that you will need to solve to find the measure of the last piece.

S: 100° + 80° + x° = 360°.

T: Solve.

S: 100° + 80° = 180°. 360° – 180° = 180°. x° = 180°

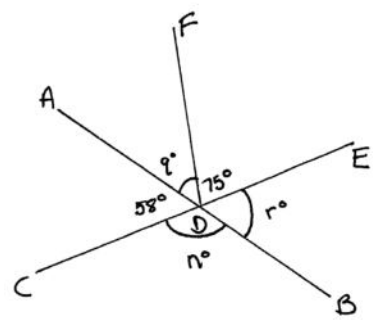
Problem 4: Determine the unknown angle measures surrounding a point.

T: (Project image as shown below to the right.) $\overline{AB}$ and $\overline{CE}$ are intersecting segments. $\overline{FD}$ meets $\overline{AB}$ and $\overline{CE}$ at point D, which is the intersection of $\overline{AB}$ and $\overline{CE}$. What angles do we know?

S: $\angle ADC$ is 58°, and $\angle FDE$ is 75°. We can solve for q°. 58° + 75° = 133°. → 180 – 133 = 47. q° is 47°.

T: We now know three angle measures. How can we figure out the measure of r?

S: 75 + 47 = 122, and 180 – 122 = 58. r° = 58°. → 122 + 8 is 130. 130 + 50 is 180, so r° is 58°. → It's 58°, since the angle directly across from it is 58°.



T: Can we solve for the last angle?

S: 58° + n° = 180°. n° = 122°.

T: What will the sum of the angles be?

S: 58° + 47° + 75° + 58° + 122° = 360°.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

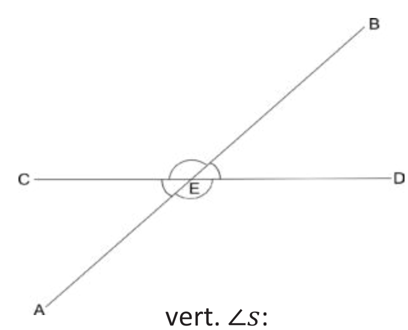
Lesson Objective: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What prior knowledge did you need in order to determine the two unknown angles for Problem 3?
- For Problem 4, how did knowing the angle measure of a neighboring or touching angle assist you in solving for the unknown angles? Try using the term **adjacent angle** to describe the neighboring or touching angle.
- How does your knowledge of a line assist you in solving Problem 5?
- Describe how you used the lines to solve Problem 6. Did your method for solving involve adding up angles to 180° or 360° or a combination?
- In our lesson today, we used what we know to see that when two lines intersect, the vertically opposite angles are equal in measure. (Point to the angles within the figure below.) (Write vert. $\angle s$ $\angle BEC = \angle AED$.) Why do you think they are called **vertical angles**?
- For the last two days, we have seen the new symbol for the plural of angles. (Allow students time to write each symbol.) On your personal white boards, show me how to write the symbols for angles add, angles at a point, angles on a line, and finally, our new one, vertical angles vert. $\angle s$. Check your work with your partner, and explain, in your own words, the meaning of each symbol. You may draw to explain.
- How did the Application Problem connect to today's lesson?



vert. $\angle s$:
$\angle BEC = \angle AED$

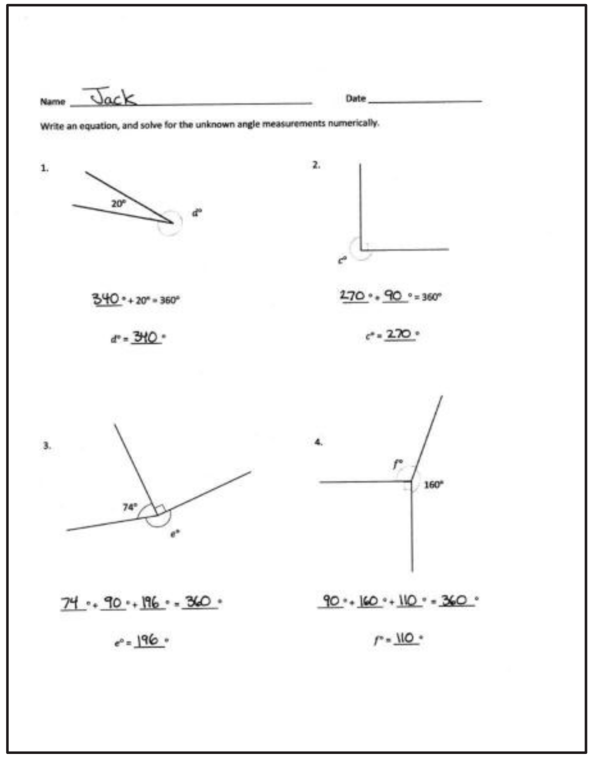
NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Allow students working above grade level and others more choice and autonomy for the Problem Set. Extend or offer as an alternative the following opportunities:

- Invite students to precisely construct and accurately label the angles made by a pair of intersecting lines or perpendicular lines, such as in Problems 6 and 7.
- Have students locate similar intersecting segments and angles within street maps, concrete or virtual, using online mapping tools pointed at familiar landmarks (for example, Times Square).
- Invite students to complete the following: Draw a pizza that is sliced for five friends to share equally. Label the angles of each slice. Use words and numbers to explain your thinking.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.


Name Jack Date

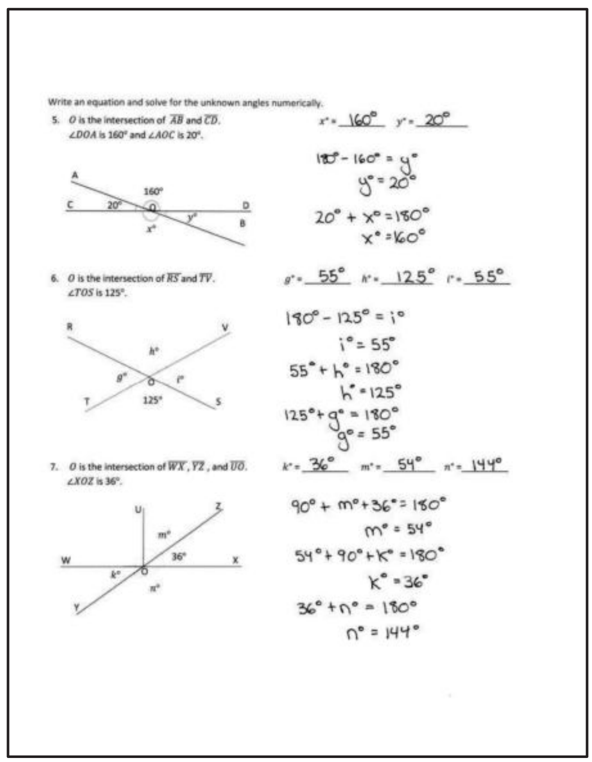
Write an equation, and solve for the unknown angle measurements numerically.

1. 340° + 20° = 360°
d° = 340°

2. 270° + 90° = 360°
c° = 270°

3. 74° + 90° + 196° = 360°
e° = 196°

4. 90° + 160° + 110° = 360°
f° = 110°


Write an equation and solve for the unknown angles numerically.

5. O is the intersection of $\overline{AB}$ and $\overline{CD}$. ∠DOA is 160° and ∠AOC is 20°.
x° = 160° y° = 20°
180° − 160° = y°
y° = 20°
20° + x° = 180°
x° = 160°

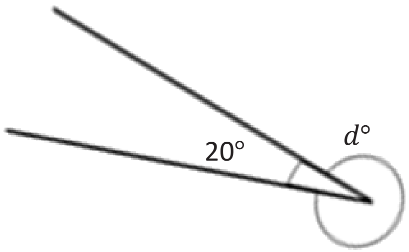
6. O is the intersection of $\overline{RS}$ and $\overline{TV}$. ∠TOS is 125°.
g° = 55° h° = 125° i° = 55°
180° − 125° = i°
i° = 55°
55° + h° = 180°
h° = 125°
125° + g° = 180°
g° = 55°

7. O is the intersection of $\overline{WX}$, $\overline{YZ}$, and $\overline{UO}$. ∠XOZ is 36°.
k° = 36° m° = 54° n° = 144°
90° + m° + 36° = 180°
m° = 54°
54° + 90° + k° = 180°
k° = 36°
36° + n° = 180°
n° = 144°

Name ____ Date ______________________

Write an equation, and solve for the unknown angle measurements numerically.

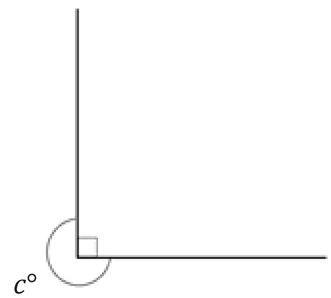
1.



_____° + 20° = 360°

d° = ______°

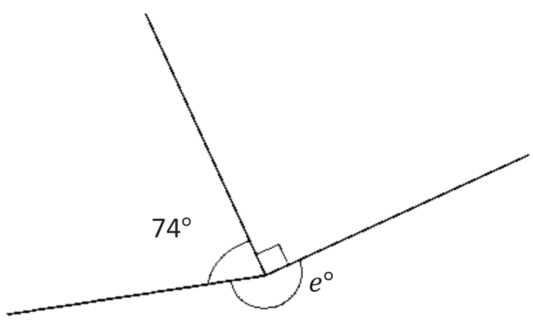
2.



_____° + _____° = 360°

c° = ______°

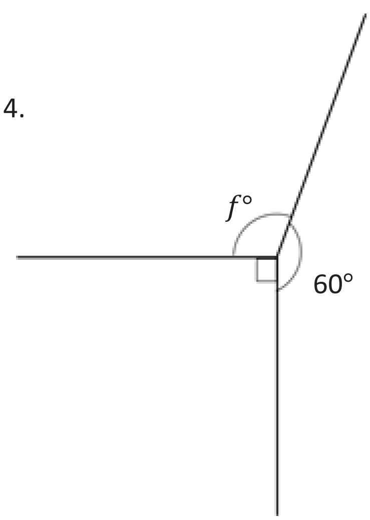
3.



_____° + _____° + _____° = _______°

e° = ______°

4.

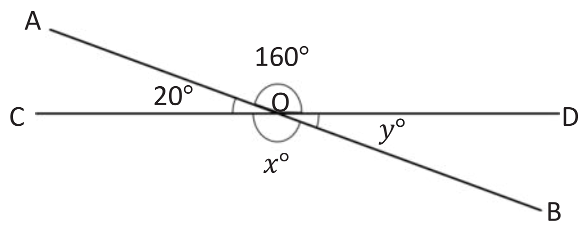


_____° + _____° + _____° = _______°

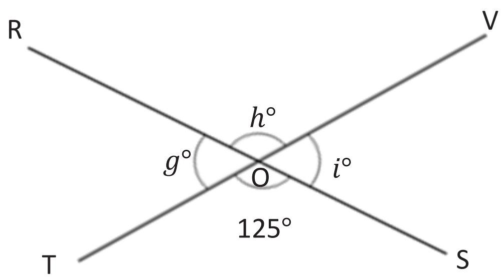
f° = _____°

Write an equation, and solve for the unknown angles numerically.

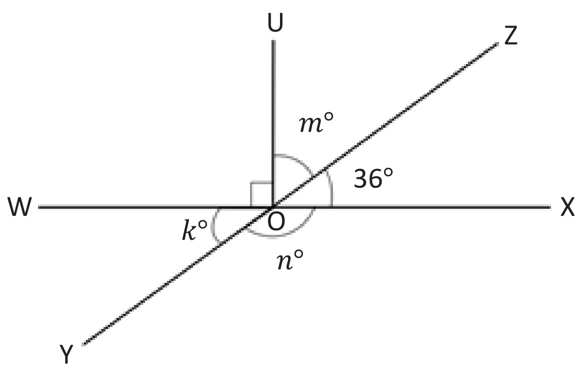
5. O is the intersection of $\overline{AB}$ and $\overline{CD}$. $x°$ = __________ $y°$ = ___________
   $\angle DOA$ is 160°, and $\angle AOC$ is 20°.



6. O is the intersection of $\overline{RS}$ and $\overline{TV}$. $g°$ = __________ $h°$ = ___________ $i°$ = __________
   $\angle TOS$ is 125°.

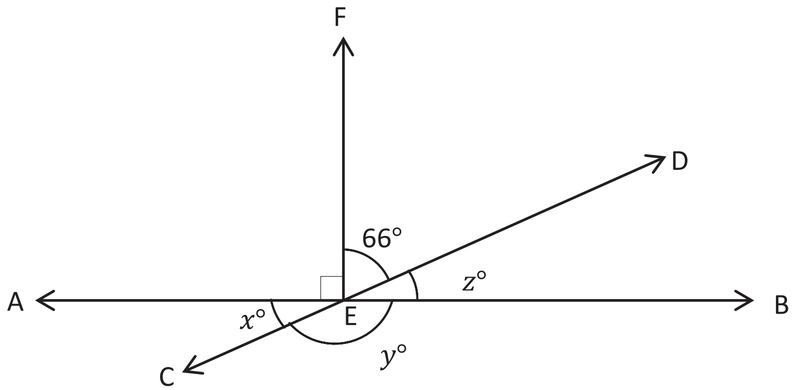


7. O is the intersection of $\overline{WX}$, $\overline{YZ}$, and $\overline{UO}$. $k°$ = __________ $m°$ = ___________ $n°$ =__________
   $\angle XOZ$ is 36°.



Name ____ Date ______________________

Write equations using variables to represent the unknown angle measurements. Find the unknown angle measurements numerically.



1. $x°$ =

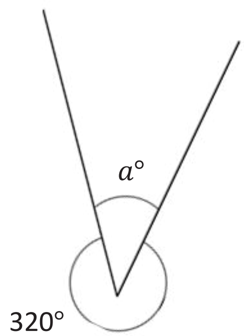
2. $y°$ =

3. $z°$ =

Name ____ Date ____________________

Write an equation, and solve for the unknown angle measurements numerically.

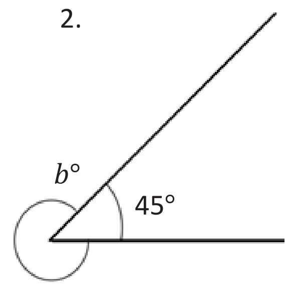
1.



_____° + 320° = 360°

$a°$ = ______°

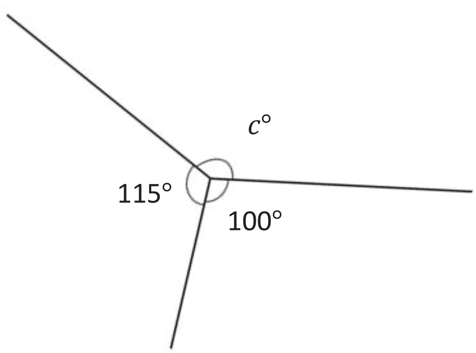
2.



_____° + _____° = 360°

$b°$ = ______°

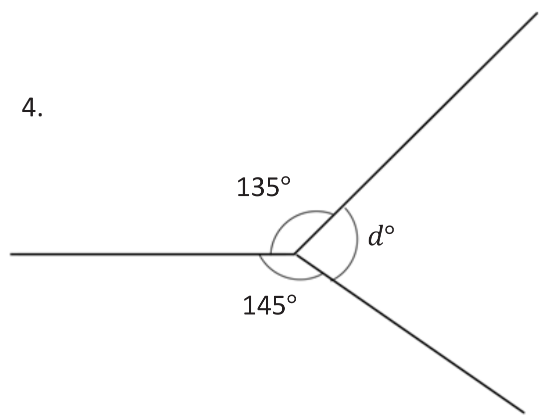
3.



_____° + _____° + _____° = _______°

$c°$ = ______°

4.

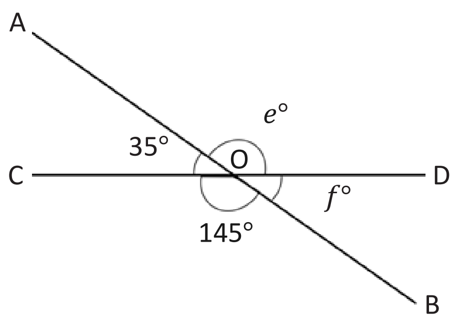


_____° + _____° + _____° = _______°

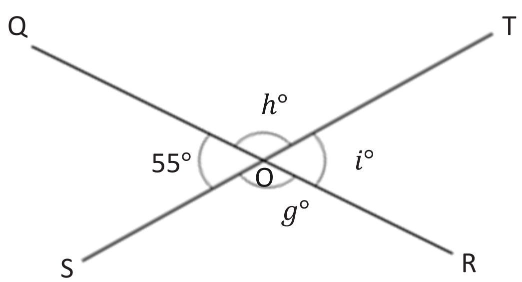
$d°$ = ______°

Write an equation, and solve for the unknown angles numerically.

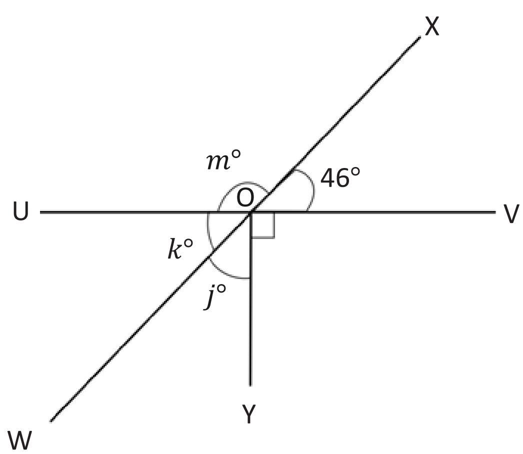
5. O is the intersection of $\overline{AB}$ and $\overline{CD}$. $e°$ = __________ $f°$ = ___________
$\angle COB$ is 145°, and $\angle AOC$ is 35°.



6. O is the intersection of $\overline{QR}$ and $\overline{ST}$. $g°$ = __________ $h°$ = ___________ $i°$ = __________
$\angle QOS$ is 55°.



7. O is the intersection of $\overline{UV}$, $\overline{WX}$, and $\overline{YO}$. $j°$ = __________ $k°$ = ___________ $m°$ = _________
$\angle VOX$ is 46°.



Topic D

Two-Dimensional Figures and Symmetry

4.G.1, 4.G.2, 4.G.3

Focus Standard:		4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
		4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
		4.G.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Instructional Days:		5	
Coherence	**-Links from:**	G3–M7	Geometry and Measurement Word Problems
	-Links to:	G5–M5	Addition and Multiplication with Volume and Area

An introduction to symmetry opens Topic D. In Lesson 12, students recognize lines of symmetry for two-dimensional figures, identify line-symmetric figures, and draw lines of symmetry. Given half of a figure and a line of symmetry, they draw the missing half. The topic then builds on students' prior knowledge of two-dimensional figures and allows students time to explore each figure's properties. Throughout this culminating topic, students use all of their prior knowledge of line and angle measure to classify and construct two-dimensional figures (**4.G.2**, **4.G.3**).

In Lesson 13, students are introduced to the precise definition of a triangle and further their understanding of right, acute, and obtuse angles by identifying them in triangles. They then classify triangles as right, acute, or obtuse based on angle measurements. Through a paper-folding activity with a right triangle, students see that the non-right angles of a right triangle are complementary. They also learn that triangles can be classified as equilateral, isosceles, or scalene based on side lengths. For isosceles triangles, lines of symmetry are identified, and a folding activity demonstrates that base angles are equal. Folding an equilateral triangle highlights multiple lines of symmetry and proves that not only are all sides equal in length, but also that all interior angles have the same measure. Students apply their understanding of triangle classification in Lesson 14 as they construct triangles given a set of classifying criteria (e.g., create a triangle that is both right and isosceles).

As the topic progresses into Lesson 15, students explore the definitions of familiar quadrilaterals and reason about their attributes, including angle measure and parallel and perpendicular lines. This work builds on Grade 3 reasoning about the attributes of shapes and lays a foundation for hierarchical classification of two-dimensional figures in Grade 5. In Lesson 16, students compare and analyze two-dimensional figures according to their properties and use grid paper to construct two-dimensional figures given a set of criteria.

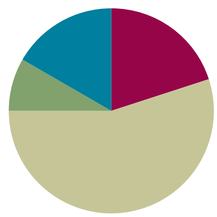
A Teaching Sequence Toward Mastery of Two-Dimensional Figures and Symmetry	
Objective 1:	**Recognize lines of symmetry for given two-dimensional figures. Identify line-symmetric figures, and draw lines of symmetry.** **(Lesson 12)**
Objective 2:	**Analyze and classify triangles based on side length, angle measure, or both.** **(Lesson 13)**
Objective 3:	**Define and construct triangles from given criteria. Explore symmetry in triangles.** **(Lesson 14)**
Objective 4:	**Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size.** **(Lesson 15)**
Objective 5:	**Reason about attributes to construct quadrilaterals on square or triangular grid paper.** **(Lesson 16)**

Lesson 12

Objective: Recognize lines of symmetry for given two-dimensional figures. Identify line-symmetric figures, and draw lines of symmetry.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Add and Subtract **4.NBT.4** (4 minutes)
- Find the Quotient and Remainder **4.NBT.6** (4 minutes)
- Find the Unknown Angle **4.MD.7** (4 minutes)

Add and Subtract (4 minutes)

Materials: (S) Personal white board

Notes: This concept reviews adding and subtracting using the standard algorithm.

T: (Write 756 thousands 498 ones.) On your personal white boards, write this number in standard form.

S: (Write 756,498.)

T: (Write 175 thousands 645 ones.) Add this number to 756,498 using the standard algorithm.

S: (Write 756,498 + 175,645 = 932,143 using the standard algorithm.)

Continue with the following possible sequence: 482,949 + 375,678.

T: (Write 800 thousands.) On your boards, write this number in standard form.

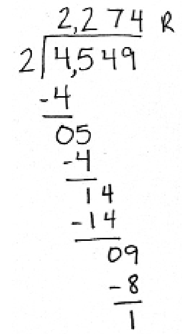
S: (Write 800,000.)

T: (Write 648 thousands 745 ones.) Subtract this number from 800,000 using the standard algorithm.

S: (Write 800,000 – 648,745 = 151,255 using the standard algorithm.)

Continue with the following possible sequence: 754,912 – 154,189.

Find the Quotient and Remainder (4 minutes)



Materials: (S) Personal white board

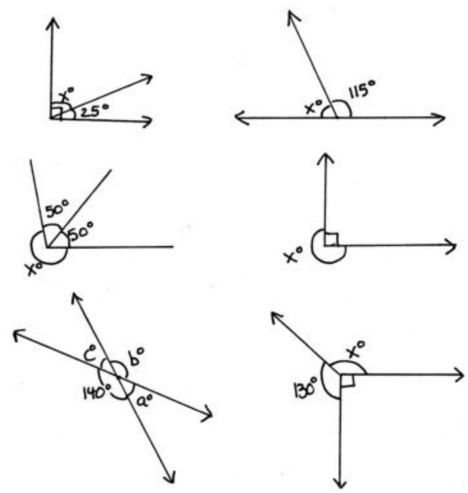
Note: This fluency activity reviews Module 3 Lesson 28's Concept Development.

T: (Write 4,549 ÷ 2.) On your personal white boards, find the quotient and remainder.

S: (Write the quotient and remainder.)

Continue with the following possible sequence: 6,761 ÷ 5; 1,665 ÷ 4; and 1,335 ÷ 4.

Find the Unknown Angle (4 minutes)



Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 10.

T: (Project the first unknown angle problem. Run finger over the larger angle.) This is a right angle. On your personal white boards, write a number sentence to find the measure of $\angle x$.

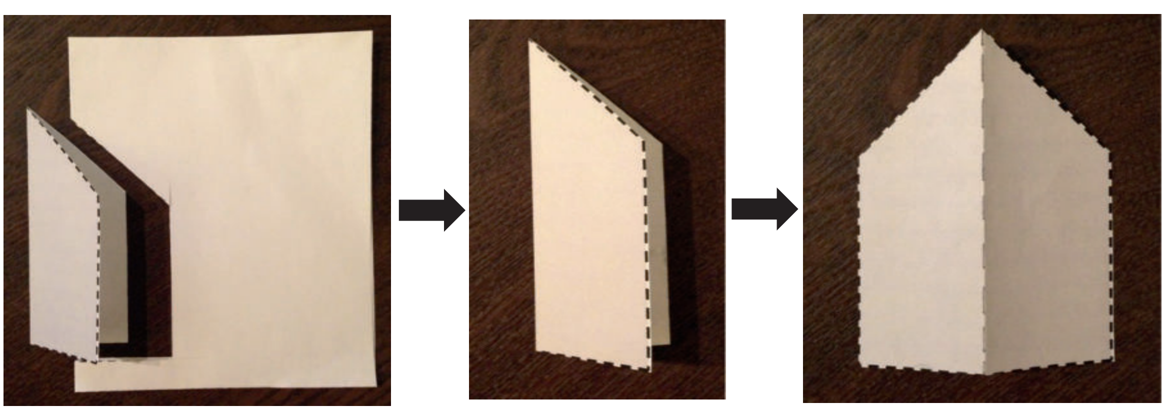
S: (Write 90° – 25° = x°. Below it, write x° = 65°.)

Continue with the remaining unknown angle problems.

Application Problem (5 minutes)

Materials: (S) Pentagon (Template 1), pre-folded as shown below, scissors.

Cut along the dotted line, and unfold the figure. Notice how each side of the folded line matches. Fold another way, and see if the sides match. Discuss the attributes of the figure and your observations with your partner.



Note: This Application Problem leads into today's Concept Development on lines of symmetry.

Concept Development (33 minutes)

Materials: (T) Pentagon (Template 1), 1 paper cutout of each of the following shapes: rectangle, square, parallelogram, rhombus, trapezoid, and circle, lines of symmetry (Template 2) (S) Pentagon (Template 1), paper cutout of 1 rectangle and 1 square (per pair), straightedge, lines of symmetry (Template 2)

Problem 1: Recognize folded symmetry.

T: What did you notice about the pentagon you cut out in the Application Problem?

S: It was a pentagon. → It had two right angles, two obtuse angles, and one acute angle. → When I cut it out, it was folded in half. → Both sides matched perfectly when folded in half. → When I folded it other ways, the sides did not match perfectly.

T: We can show the fold that cut the pentagon in half by using our straightedge and tracing that line.

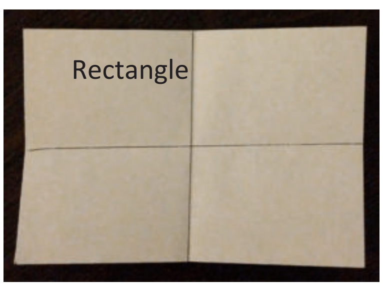
Model for students, and allow time for students to trace the line created by the fold.

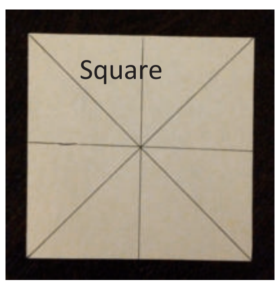
Distribute one rectangle and one square to each pair of students.

T: In your pair, one person folds the rectangle and the other the square in as many ways as you can so that when it is folded, the shapes on either side of the fold match. If you find a fold that creates two shapes that match, use a straightedge to record the line created by the fold.

Allow time for students to fold.

T: What did you notice when you folded these?





S: The square had more folds than the rectangle. → We folded the square four different ways, and the sides matched perfectly each time. The rectangle only matched when folded two ways. → The rectangle folded into smaller rectangles, but the square folded into smaller rectangles and triangles!

T: Why do you think the square had more folds with sides that matched than the rectangle?

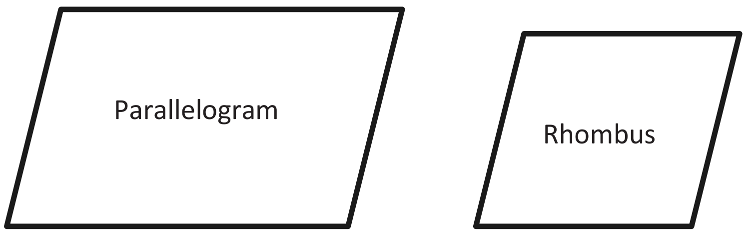
S: Because all the sides of the square are the same but not in the rectangle. → You could fold a square diagonally because all four sides are the same, but you can't do that with the rectangle because two of its sides are longer.

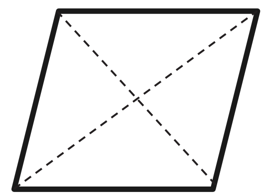
NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

For students who may find folding paper challenging at first, offer the following:

- Model folding along lines of symmetry in a triangle.
- Provide step-by-step directions.
- Provide a pre-folded rectangle and square model that students can study and practice with before attempting their own.
- As a last resort, offer shapes that have fold lines to guide student folding. Then, offer students a second opportunity to fold shapes independently.

Display the parallelogram and rhombus on the board, and show cutouts.



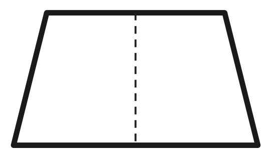


T: Here I have a parallelogram and a rhombus. I want to know how many times I can fold these so that the shapes created by the folds match.

Invite students to come fold the cutouts.

S: There are two folds in the rhombus that match, but none in the parallelogram. → But a rectangle is a lot like a parallelogram, and we found four lines of symmetry in a rectangle. → I guess it was the four right angles of the rectangle that made it work.

Display a trapezoid on the board and show cutout.



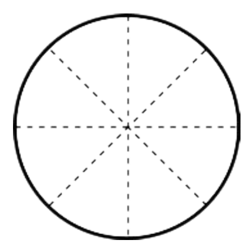
T: Here is a trapezoid. Watch as I fold it. Let me know when you see a fold that matches.

S: There is only one fold that matches in this trapezoid.

T: Just like the pentagon we cut out in the Application Problem, this trapezoid also has just one fold that matches. Will that be true for all trapezoids? Sketch some trapezoids on your boards, and try to imagine their folds.

S: No, it is not true. A rectangle is a trapezoid, and it has two folds. A square is a trapezoid too, and it has four folds.

Display a circle on the board and show cutout.



MP.3

T: Here, I have a circle. With your partner, discuss how many folds you think will match in a circle.

S: I think there will be four folds, just like a square. → No, there are eight, like in a pizza. → There are too many folds to count!

T: Watch as I start making folds in my circle. Any way I make my fold, the sides match! Why is that?

S: Well, a circle doesn't have sides, so I guess the round edges just let it fold a lot of different ways. → But I don't think we could fold an oval many different ways, and an oval doesn't have straight sides, just like circles. → The circle must be special since it has so many different folds.

T: A circle is a set of points that are the same distance from a center point. Measuring from the center point to the edge at any point on the circle always measures the same length. On a square, when you measure from the center to a point on the side, you can get different lengths. (Demonstrate with a ruler.) This special attribute of a circle allows it to have an infinite number of folds that go through its center point.

Problem 2: Identify lines of symmetry in familiar figures.

Display and distribute lines of symmetry (Template 2).

T: With your partner, look at each image and determine whether there is a fold or folds that let the figure fold perfectly in half. If you find a fold that creates two shapes that match, use a straightedge to draw it. (Allow partners time to work.)

T: Which images had one fold that matched?

S: The letter *A*, smiley face, heart, lobster, and butterfly.

T: Which images had more than one fold that matched?

S: The letter *H* and the star.

T: Watch as I use my straightedge to show the folds that create two shapes that match. Check to make sure you have the same lines drawn.

T: Does everyone see that? When we fold each of these images along the line, both halves match exactly. This line is called a **line of symmetry**.

T: Which images had no such folds?

S: The car, the hand, and the curved arrow.

T: We can say that these figures had no lines of symmetry. Discuss with your partner why these images don't have lines of symmetry.

S: If we fold the car in half, the front and the back of the car are different. One side has headlights, the other taillights. And the door doesn't match on both sides. If I folded it top to bottom, the top of the car doesn't have tires! → The hand can't fold left to right because all five fingers are different. The thumb and pinkie don't match. Top to bottom, the fingertips meet the wrist. Those aren't the same. → The arrow doesn't even work if we folded it diagonally because one end is flat, and the other has the arrow.

Problem 3: Draw lines of symmetry.

Display Figure 1.

T: This figure is incomplete. The dashed line is the line of symmetry. To complete the figure, we need to make a mirror image of the figure that is already drawn. Use the grid to complete the figure. Discuss with your partner how to complete the figure so that it is symmetrical.

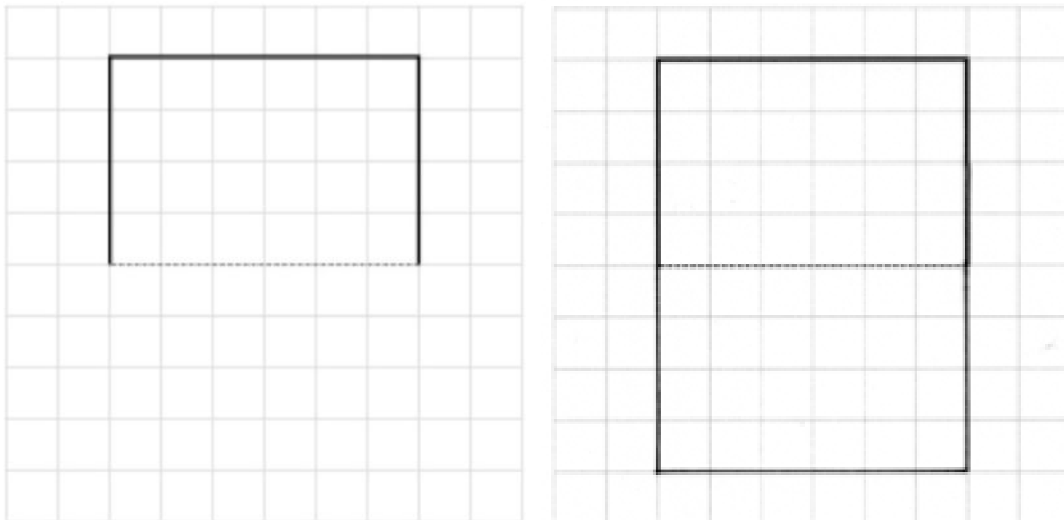
S: I can count the number of squares to help me draw. → We can use a straightedge to make sure the lines are straight. → It's 4 units wide, so we need to double that to make it 8 units wide. → It's 6 units long, but we can just connect the vertical lines.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge students working above grade level to draw an incomplete image on grid paper. After drawing the incomplete image, students can exchange papers and challenge a partner to complete the image using the line of symmetry as a point of reference.

T: Complete Figure 1.

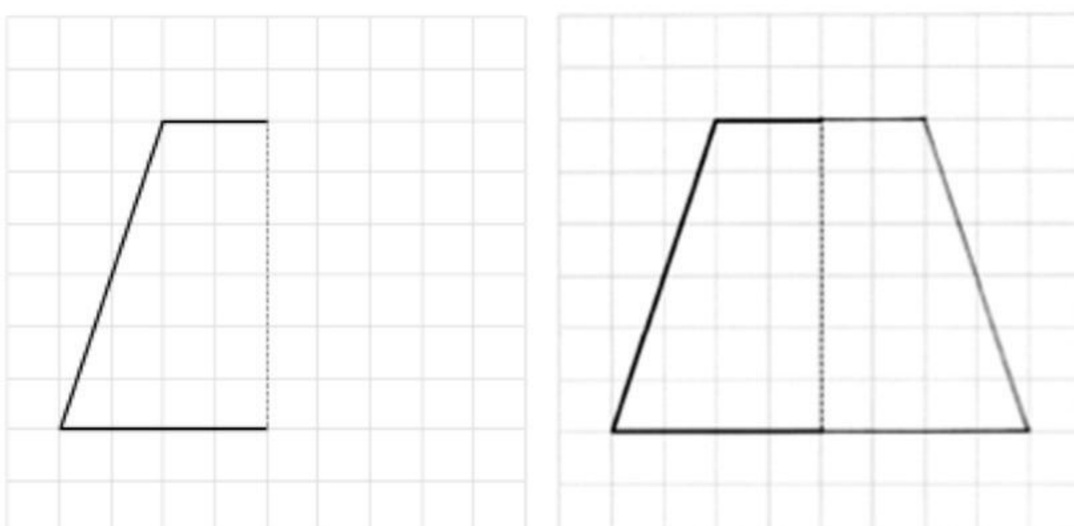
Figure 1



T: With your partner, complete Figure 2.

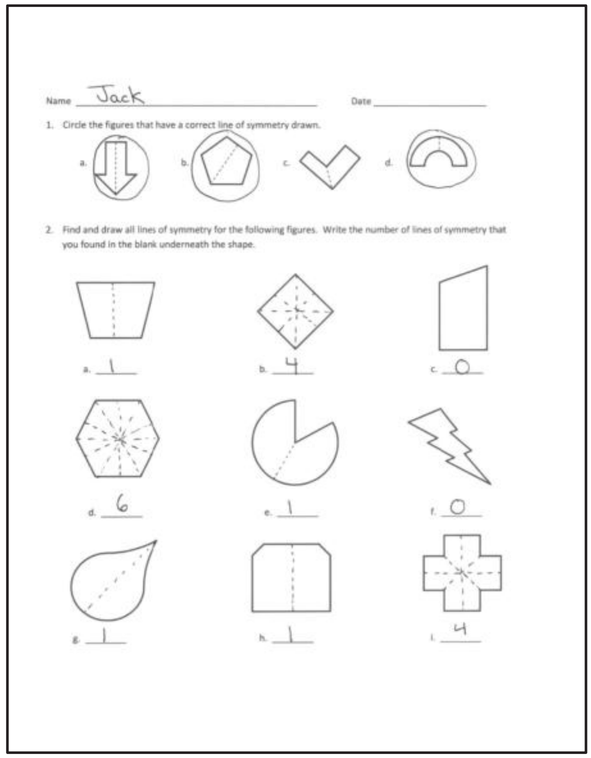
S: I draw the horizontal lines first because they are connected to the figure. I don't know where to start drawing the slanted line. → I counted two squares for the top segment and four squares for the bottom segment. Then, I just connected them with a slanted segment, but I counted it to make sure it went up six and over two so that it matched the left side.

Figure 2



Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


Name Jack Date

1. Circle the figures that have a correct line of symmetry drawn.

a. b. c. d.

2. Find and draw all lines of symmetry for the following figures. Write the number of lines of symmetry that you found in the blank underneath the shape.

a. 1 b. 4 c. 0

d. 6 e. 1 f. 0

g. 1 h. 1 i. 4

Student Debrief (10 minutes)

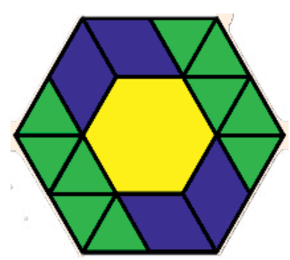
Lesson Objective: Recognize lines of symmetry for given two-dimensional figures. Identify line-symmetric figures, and draw lines of symmetry.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

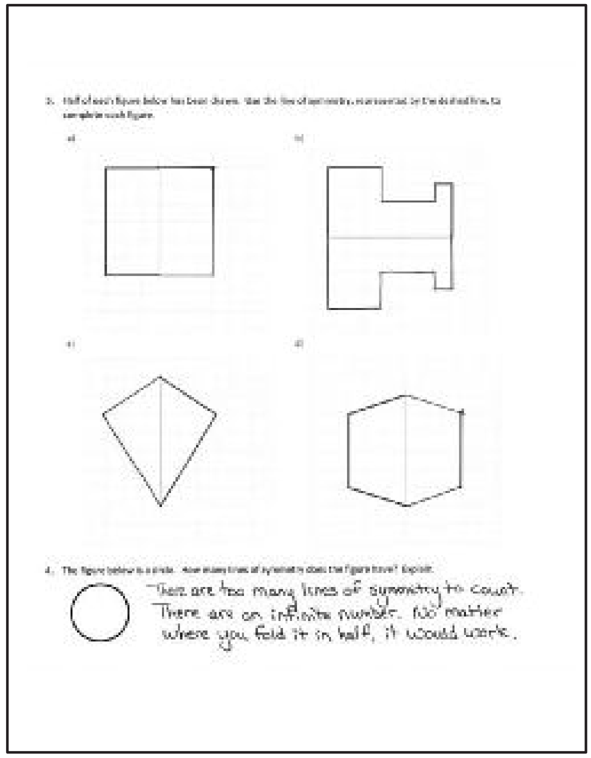
Any combination of the questions below may be used to lead the discussion.

- In Problem 2, which figures had lines of symmetry that were most difficult to see? Why were some easier and others more difficult?
- In Problem 3, what method did you use to complete each figure? How would you complete the figure if there were no graph paper?
- In Problem 4, why does a circle have an infinite number of **lines of symmetry**?
- Identify objects around the classroom or in nature that have lines of symmetry.
- In what ways are our bodies symmetrical, and in what ways are they not symmetrical?
- How can you be sure objects have lines of symmetry?
- How can lines of symmetry help to solve problems quicker? Consider this shape to the right. How would finding a line of symmetry allow you to more quickly count the number of green triangles in the figure?



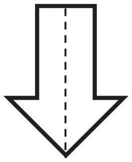
Exit Ticket (3 minutes)

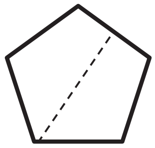
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

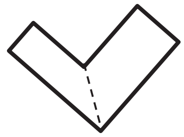


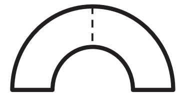
Name ____ Date ______________________

1. Circle the figures that have a correct line of symmetry drawn.

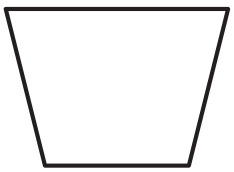
a. 

b. 

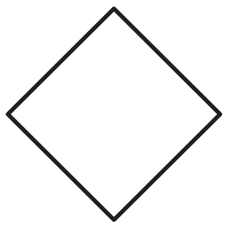
c. 

d. 

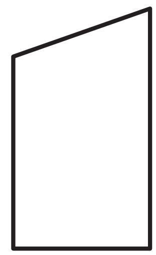
2. Find and draw all lines of symmetry for the following figures. Write the number of lines of symmetry that you found in the blank underneath the shape.



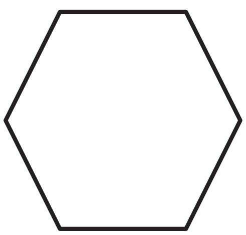
a. ________



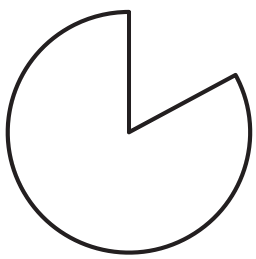
b. ________



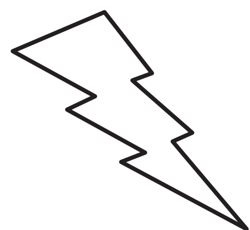
c. ________



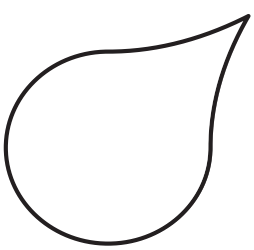
d. ________



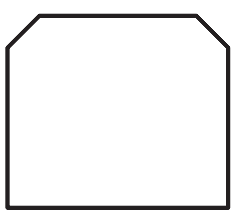
e. ________



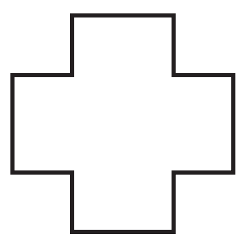
f. ________



g. ________



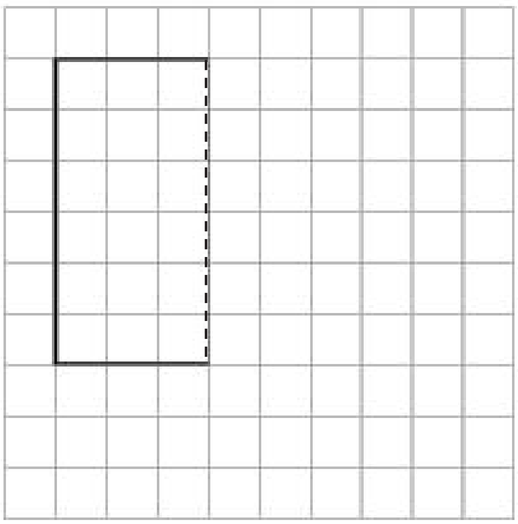
h. ________



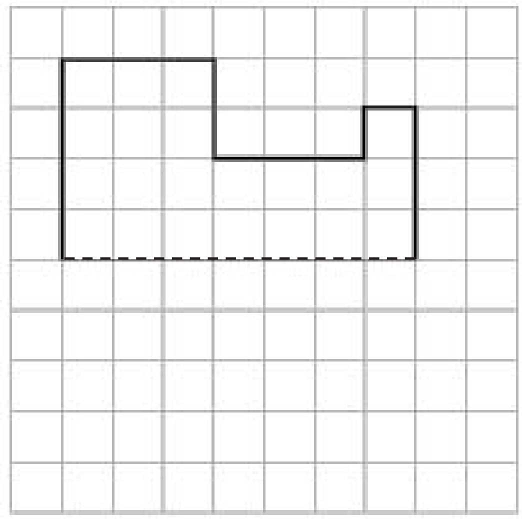
i. ________

3. Half of each figure below has been drawn. Use the line of symmetry, represented by the dashed line, to complete each figure.

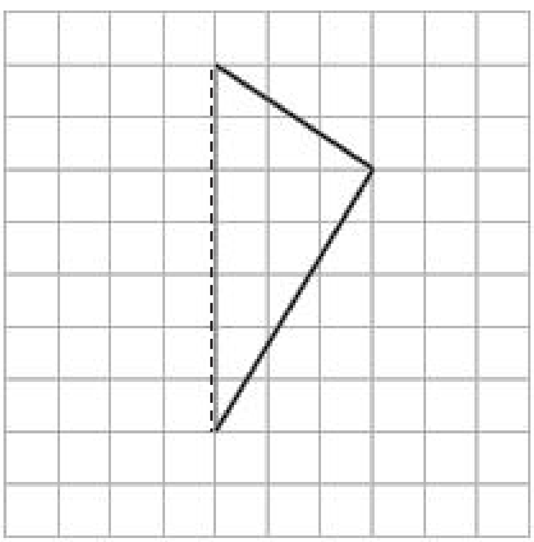
a.



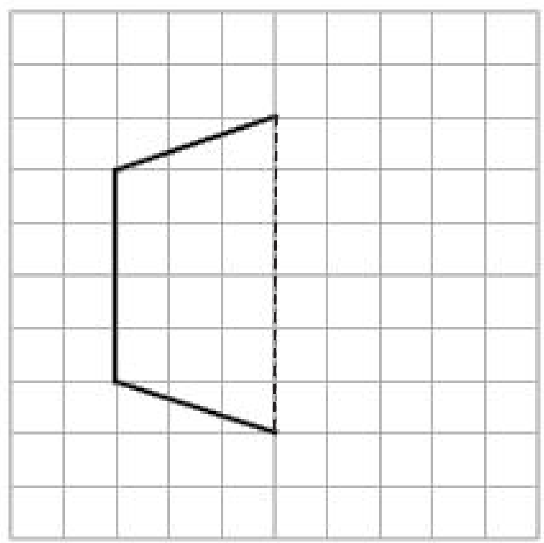
b.



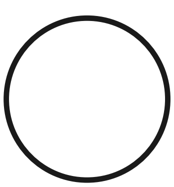
c.



d.

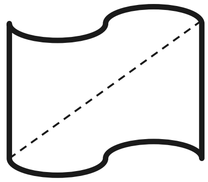


4. The figure below is a circle. How many lines of symmetry does the figure have? Explain.

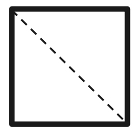


Name ____ Date ____________________

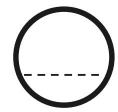
1. Is the line drawn a line of symmetry? Circle your choice.



Yes No

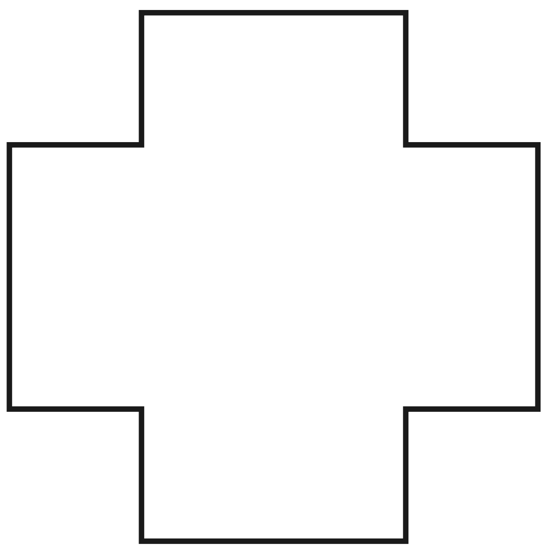


Yes No



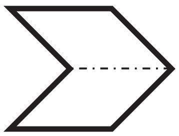
Yes No

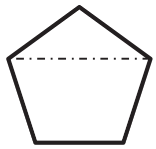
2. Draw as many lines of symmetry as you can find in the figure below.

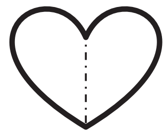


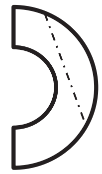
Name ____ Date ______________________

1. Circle the figures that have a correct line of symmetry drawn.

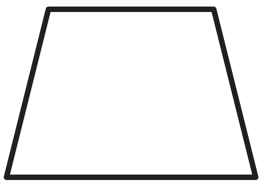
a. 

b. 

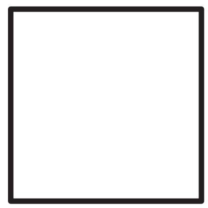
c. 

d. 

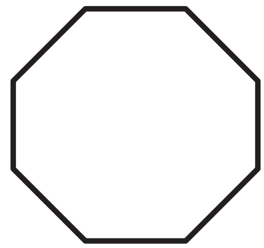
2. Find and draw all lines of symmetry for the following figures. Write the number of lines of symmetry that you found in the blank underneath the shape.



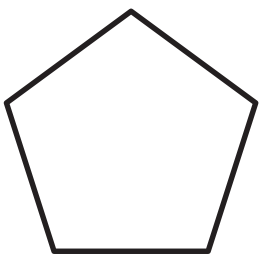
a. ________



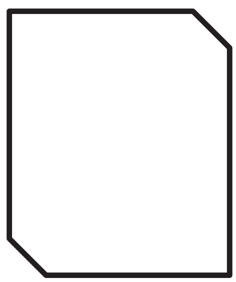
b. ________



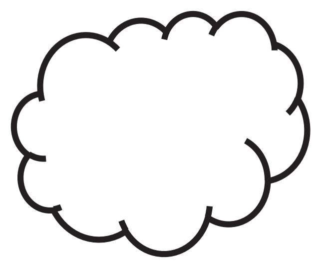
c. ________



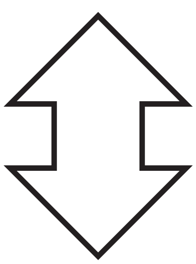
d. ________



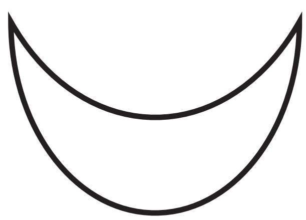
e. ________



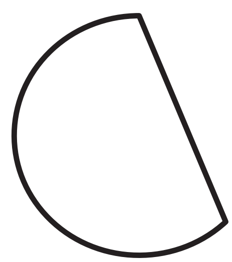
f. ________



g. ________



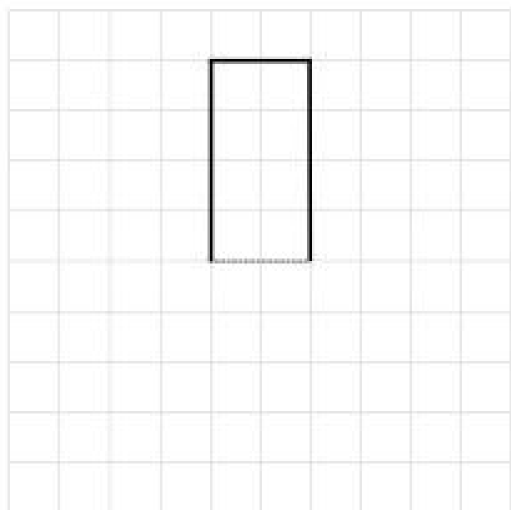
h. ________



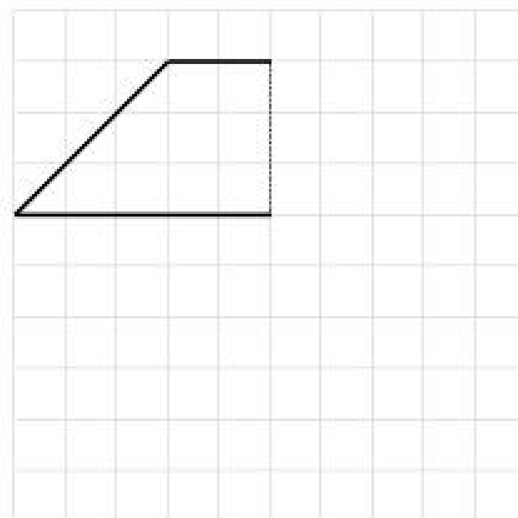
i. ________

3. Half of each figure below has been drawn. Use the line of symmetry, represented by the dashed line, to complete each figure.

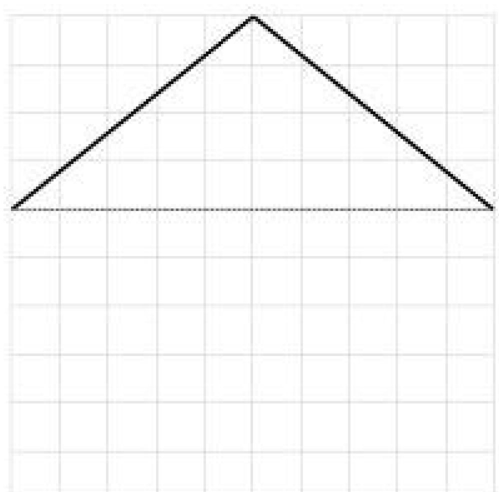
a.



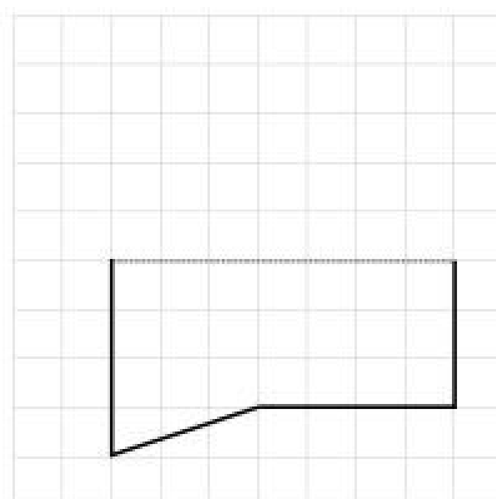
b.



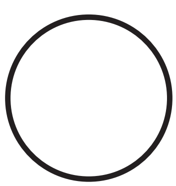
c.

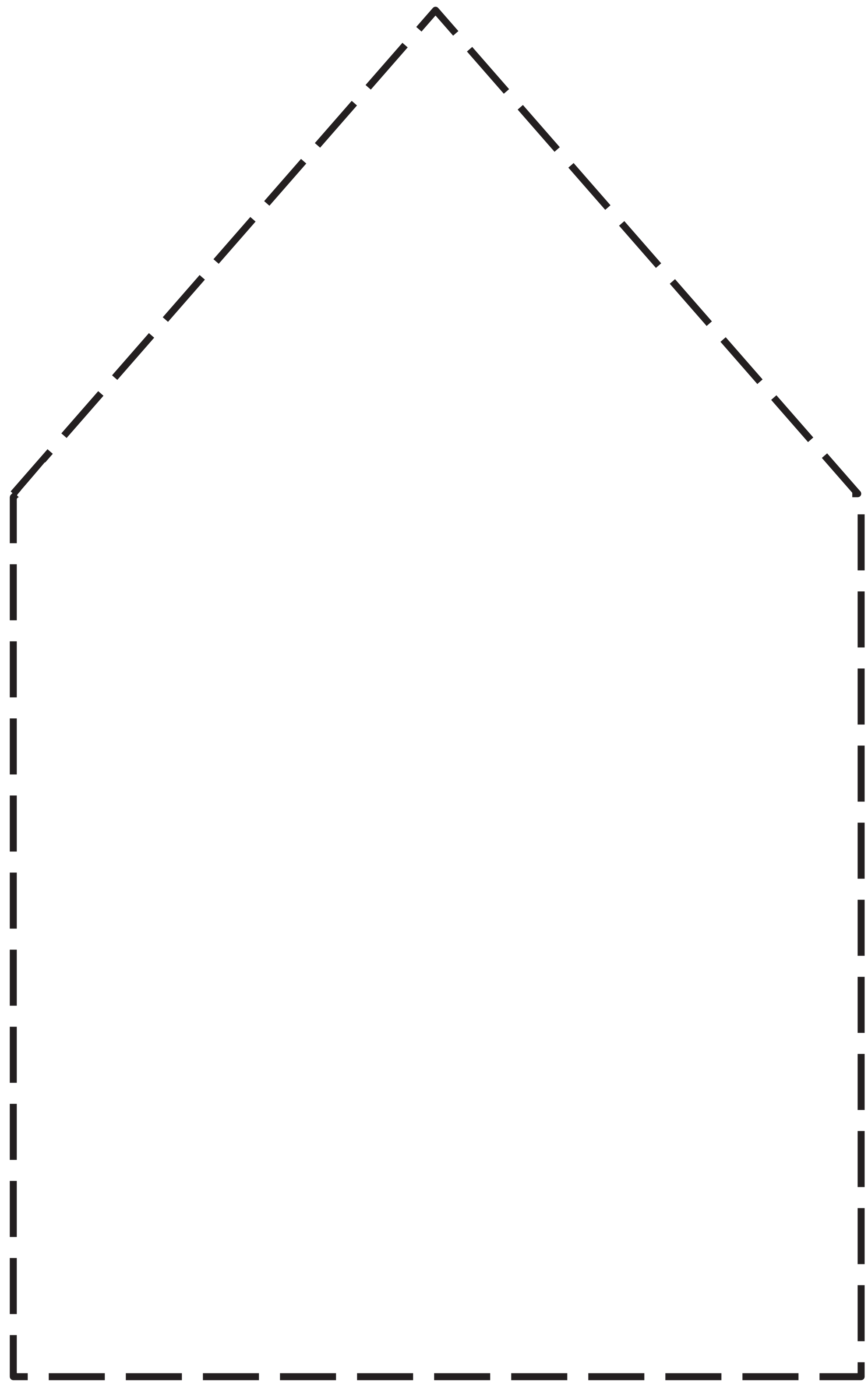


d.



4. Is there another shape that has the same number of lines of symmetry as a circle? Explain.





pentagon

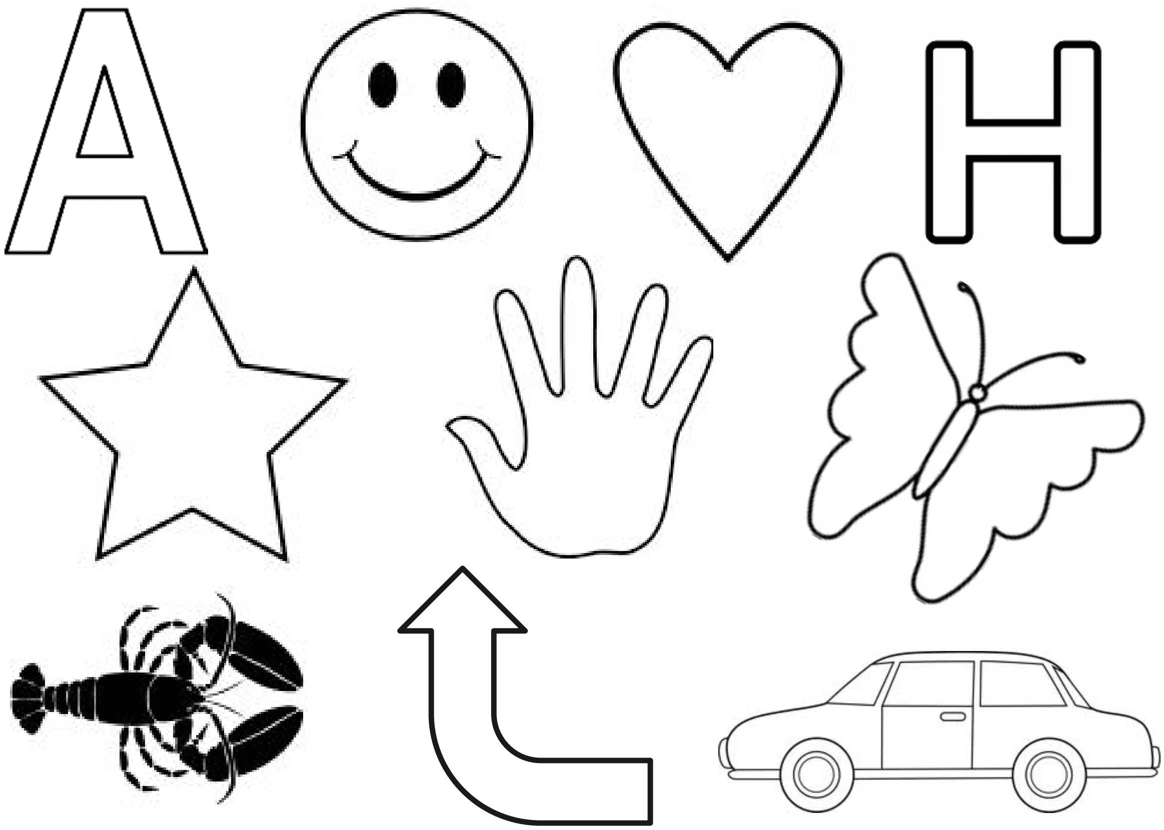


Figure 1

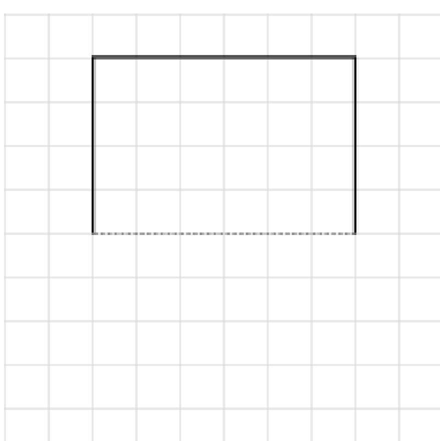
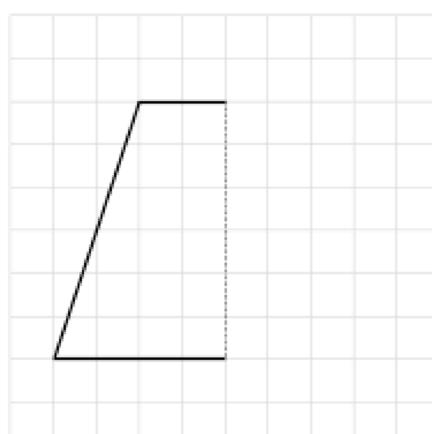


Figure 2



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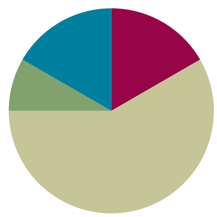
lines of symmetry

Lesson 13

Objective: Analyze and classify triangles based on side length, angle measure, or both.

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Application Problem	(5 minutes)
Concept Development	(35 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (10 minutes)

- Divide Three Different Ways **4.NBT.6** (5 minutes)
- Physiometry **4.G.3** (3 minutes)
- Lines of Symmetry **4.G.3** (2 minutes)

Divide Three Different Ways (5 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews concepts covered in Module 3. Alternately, have students choose to solve the division problem using just one of the three methods.

T: (Write 532 ÷ 4.) Solve this problem by drawing place value disks.
S: (Solve by drawing place value disks.)
T: Solve 532 ÷ 4 using the area model.
S: (Solve using the area model.)
T: Solve 532 ÷ 4 using the standard algorithm.
S: (Solve using the standard algorithm.)

Continue with this possible sequence: 854 ÷ 3.

Physiometry (3 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms learned in Lesson 12.

T: Stand up.
T: Am I trying to make my body position look symmetrical?

T: (Raise left arm so fingers point directly to the wall. Leave the other arm hanging down.) Is my position symmetrical now?

S: No.

Continue with other symmetrical and non-symmetrical positions.

T: With your arms, model a line that runs parallel to the floor. Are you modeling a symmetrical position?

S: Yes.

T: Model a right angle. Are you modeling a symmetrical position?

S: No.

T: Model a line segment that runs parallel to the floor. Are you modeling a symmetrical position?

S: Yes.

Lines of Symmetry (2 minutes)

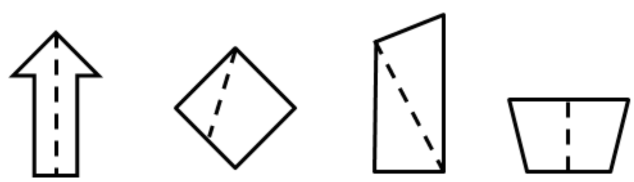
Note: This fluency exercise reviews Lesson 12.

T: (Project arrow with a line of symmetry. Point to the line of symmetry.) Is this a line of symmetry?

S: Yes.

T: (Project the diamond. Point to the non-symmetrical line.) Is this a line of symmetry?

S: No.



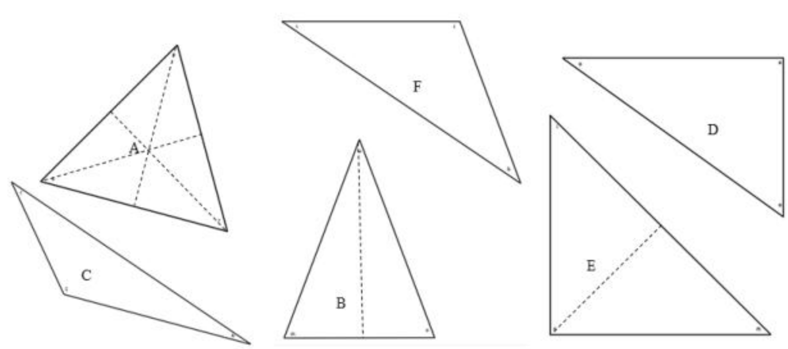
Continue process for the remaining graphics.

Application Problem (5 minutes)

Materials: (T/S) Triangles (Template)

Fold Triangles A, B, and C to show their lines of symmetry. Use a straightedge to trace each fold. Discuss with your partner the relationships of symmetric shapes to angles and side lengths.

Note: This Application Problem connects lines of symmetry in Lesson 12 to discovering the attributes of triangles in today's lesson. Prepare the triangles ahead of time by cutting them out from the triangles Template. Each student or partner group should have his or her own copy.



Concept Development (35 minutes)

Materials: (T) Triangles (Template), Practice Sheet, graph paper, ruler (S) Triangles (Template) one set per group, Practice Sheet, ruler, protractor, graph paper

Problem 1: Discover the attributes of various triangles.

T: What types of attributes can triangles have?

S: Well, they must have three sides, so they also have three angles. → But their sides can be different. Some are short, and some are long, or sometimes they are the same length. → Yeah, triangles can also have the same or different types of angles, like acute, obtuse, or right. → And some have lines of symmetry, and others don't.

T: Think about the types of angles and the lengths of the sides of triangles as we complete this activity.

Separate students into small groups of three students each. Provide each group with one of each triangle from Templates 1, 2, and 3. Instruct students to investigate the given triangle cutouts using rulers and protractors. Students should record their findings in the Attributes column of the Practice Sheet, including measures of sides and angles, as well as other general observations. It may be helpful for students to record the angle and side length measurements on the cutouts as well. Students should quickly sketch each triangle in the first column. Allow students six to eight minutes for this activity.

T: Now, take a moment with your group to compare your findings. Discuss ways in which some triangles might be classified into different groups.

Students discuss.

Problem 2: Classify triangles by side length and angle measure.

MP.6

T: Tell me how you sorted your triangles by side length.

S: Triangles B, E, and F each had two sides that were the same length. → Triangles C and D had sides that all measured different lengths. → Triangle A is the only triangle that has three sides that are all the same length!

T: Let's record your findings. You just classified some triangles by the length of their sides. Let's label the first of the classification columns as *Side Length*.

T: There are three kinds of triangles you discovered. **Equilateral triangles**, such as Triangle A, have all sides that are equal in length.

S: That's easy to remember because *equilateral* starts with the same sound as the word *equal*.

T: **Isosceles triangles** are like Triangles B, E, and F. They have at least two sides with the same length.

T: Triangles C and D are classified as **scalene triangles**. None of their side lengths are the same.

T: To show that certain sides are the same length, we draw a tick mark on each same length segment.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Remembering the names that classify triangles may present a challenge for English language learners and others. Present helpful mnemonic devices. The word *isosceles*, for example, starts with the sound *eyes*. We have *two* eyes; similarly, an isosceles triangle has at least *two* equal sides. Encourage students to come up with their own way to remember, and then to share with others.

MP.6

T: (Draw a tick mark on each side of the equilateral triangle.) It's your turn. Which other triangles need tick marks?

S: Triangles B, E, and F need just two tick marks.

T: Why don't Triangles C and D need tick marks?

S: All their sides have different lengths.

T: Tell me about how you classified the triangles based on the angles you measured.

S: Triangles D and E had one right angle.

S: Triangles C and F had one obtuse angle.

S: All of the triangles had acute angles. → Triangles A and B had only acute angles.

T: Label the second of the classification columns as *Angle Measure*. Record your findings. If a triangle has an obtuse angle, we classify it as an **obtuse triangle**. If a triangle has one right angle, we call it a **right triangle**. What are triangles called that have only acute angles?

S: **Acute triangles**!

T: What angle symbol do we know to show the classification of right triangles?

S: The small square!

Problem 3: Determine the presence of angles of specific measure in triangles.

T: Fold Triangle B on its line of symmetry. What do you notice about the two sides that line up?

S: They are the same length! That means we measured correctly. It is an isosceles triangle.

T: What about the two base angles that folded on top of each other?

S: The two angles are the same size! I wonder if that has something to do with the two sides being the same.

T: Let's check. Fold another isosceles triangle, Triangle E or F.

S: Those sides that fold together are the same, and the angles are too!

T: Use those findings to draw some conclusions about equilateral triangles. Fold Triangle A on each of its lines of symmetry.

S: No matter which symmetry line we folded, the sides were the same length, and the angles matched up. → So, if all of the angles are lining up, doesn't that mean all of the angles have the same measure? → Yeah! And that means all the sides are the same length. And we knew that when we measured with our rulers and protractors. → Equilateral triangles are a lot like isosceles triangles.

T: An isosceles triangle has at least two sides that measure the same length. Do equilateral triangles have two sides that are the same length?

S: Yes. → Yes, but actually three.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

English language learners and others may feel overwhelmed with the many new terms introduced in this lesson. Encourage students to record *isosceles, equilateral,* and *scalene* in their personal math dictionaries. Students may, for example, draw an example of each type of triangle and then define the triangles in their first language, if helpful. Create a classroom chart with examples for each type of triangle so that students may reference it during the Problem Set and further triangle work.

T: An isosceles triangle has two angles with the same measure. Do equilateral triangles have two angles with the same measure?

S: Yes. → Yes, but actually three.

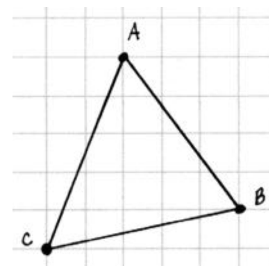
T: We can say that an equilateral triangle is a special isosceles triangle. It has everything an isosceles triangle has, but it also has a little more, such as three sides and three angles with the same measure, not just two.

T: Triangle D has a right angle. Fold the other two angles into the right angle. (Demonstrate.) It's your turn.

S: Neat, the two other angles fit perfectly into the right angle.

T: What does that tell you about the measure of both of the other angles in a right triangle?

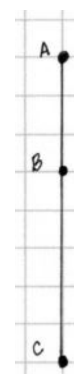
S: The other two angles add together to make 90°.



Problem 4: Define triangle.

T: What do we know about triangles that will help us to draw one?

S: Triangles have three sides and three angles. → We could draw three segments that meet together. → Those three segments will make the three angles. → When we learned about angles, we drew them by drawing two rays from one point.



T: On graph paper, plot three points, and label them *A, B*, and *C*. Connect those points with segments. What have you created?

S: A triangle!

T: (Plot three collinear points labeled *A, B*, and *C*.) What is the problem here?

S: If you connect the points with segments, *A, B*, and *C* will all be on one line. *A, B*, and *C* don't make a triangle.

T: Use your triangle to help you define the word **triangle** to your partner.

S: My triangle has three segments and three angles. → My triangle was formed from three points connected by three segments. → My triangle was formed from three points that were not in a line and were connected by segments. → Two of my points can be in a line, but not all three.

T: Identify your triangle as $\triangle ABC$. (Write $\triangle ABC$.) Classify your triangle by side length and angle measure.

S: (Identify and classify.)

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

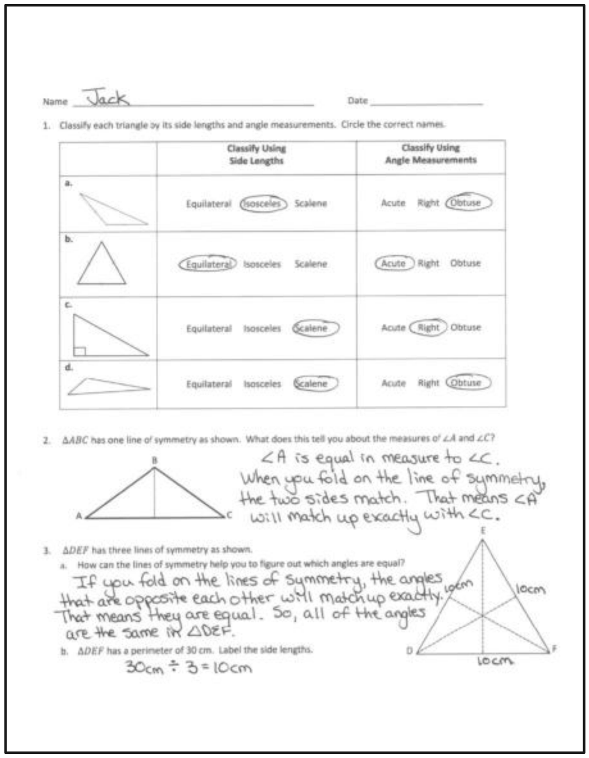
Lesson Objective: Analyze and classify triangles based on side length, angle measure, or both.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How do the tick marks and angle symbols allow classification of triangles without using tools in Problem 1?
- What strategy did you use to solve Problem 3(b)?
- Explain your answer to Problem 5(b). Recall from Lesson 6 that the word *collinear* describes three points that are on a line.
- A **triangle** can be defined as three points that are not collinear and that have line segments between them. Discuss this definition with your partner. Make sure you understand it completely.
- How many lines of symmetry can be found in **scalene triangles**? **Equilateral triangles**? **Isosceles triangles**?
- Can you determine if a triangle will have a line of symmetry just by knowing whether it is an **acute triangle** or an **obtuse triangle**? How about scalene or isosceles? Sketch an example of a scalene and isosceles triangle to verify your answer.
- Sketch some examples to prove your answer to Problem 6. How many acute angles do **right triangles** have?
- How did the Application Problem connect to today's lesson?


Name Jack Date

1. Classify each triangle by its side lengths and angle measurements. Circle the correct names.

	Classify Using Side Lengths	Classify Using Angle Measurements
a.	Equilateral (Isosceles) Scalene	Acute Right (Obtuse)
b.	(Equilateral) Isosceles Scalene	(Acute) Right Obtuse
c.	Equilateral Isosceles (Scalene)	Acute (Right) Obtuse
d.	Equilateral Isosceles (Scalene)	Acute Right (Obtuse)

2. ΔABC has one line of symmetry as shown. What does this tell you about the measures of ∠A and ∠C?

∠A is equal in measure to ∠C. When you fold on the line of symmetry, the two sides match. That means ∠A will match up exactly with ∠C.

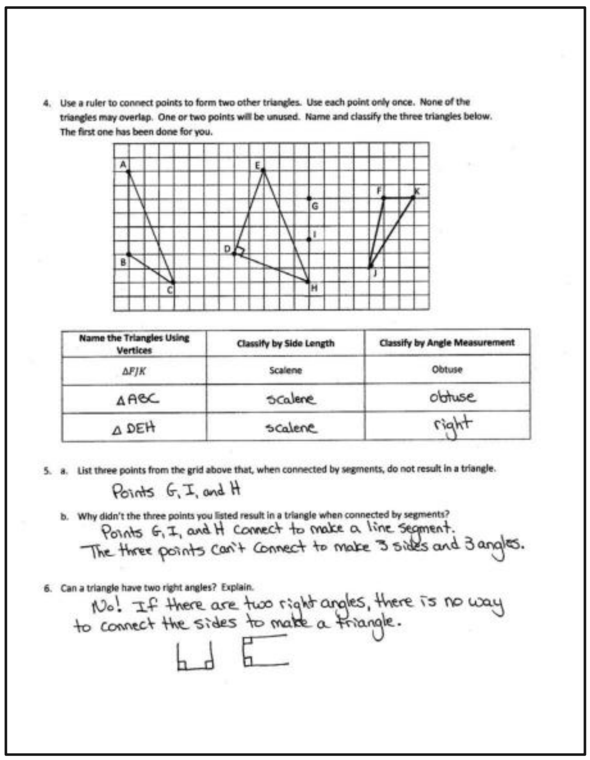
3. ΔDEF has three lines of symmetry as shown.

a. How can the lines of symmetry help you to figure out which angles are equal?

If you fold on the lines of symmetry, the angles that are opposite each other will match up exactly. That means they are equal. So, all of the angles are the same in ΔDEF.

b. ΔDEF has a perimeter of 30 cm. Label the side lengths.

30cm ÷ 3 = 10cm


4. Use a ruler to connect points to form two other triangles. Use each point only once. None of the triangles may overlap. One or two points will be unused. Name and classify the three triangles below. The first one has been done for you.

Name the Triangles Using Vertices	Classify by Side Length	Classify by Angle Measurement
ΔFJK	Scalene	Obtuse
ΔABC	scalene	obtuse
ΔDEH	scalene	right

5. a. List three points from the grid above that, when connected by segments, do not result in a triangle.

Points G, I, and H

b. Why didn't the three points you listed result in a triangle when connected by segments?

Points G, I, and H connect to make a line segment. The three points can't connect to make 3 sides and 3 angles.

6. Can a triangle have two right angles? Explain.

No! If there are two right angles, there is no way to connect the sides to make a triangle.

Exit Ticket (3 minutes)

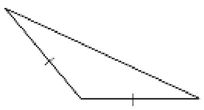
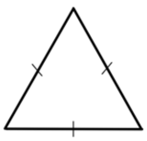
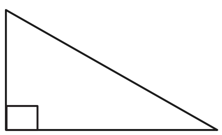
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ______________________________ Date ____________________

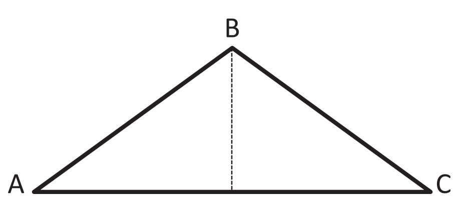
Sketch of Triangle	Attributes (Include side lengths and angle measures.)	Classification	
A			
B			
C			
D			
E			
F			

Name ____ Date ______________________

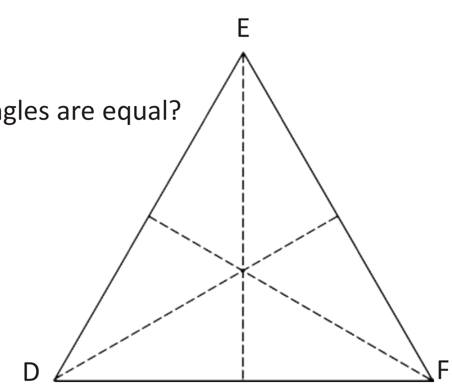
1. Classify each triangle by its side lengths and angle measurements. Circle the correct names.

	Classify Using Side Lengths	Classify Using Angle Measurements
a. 	Equilateral Isosceles Scalene	Acute Right Obtuse
b. 	Equilateral Isosceles Scalene	Acute Right Obtuse
c. 	Equilateral Isosceles Scalene	Acute Right Obtuse
d. 	Equilateral Isosceles Scalene	Acute Right Obtuse

2. $\triangle ABC$ has one line of symmetry as shown. What does this tell you about the measures of $\angle A$ and $\angle C$?



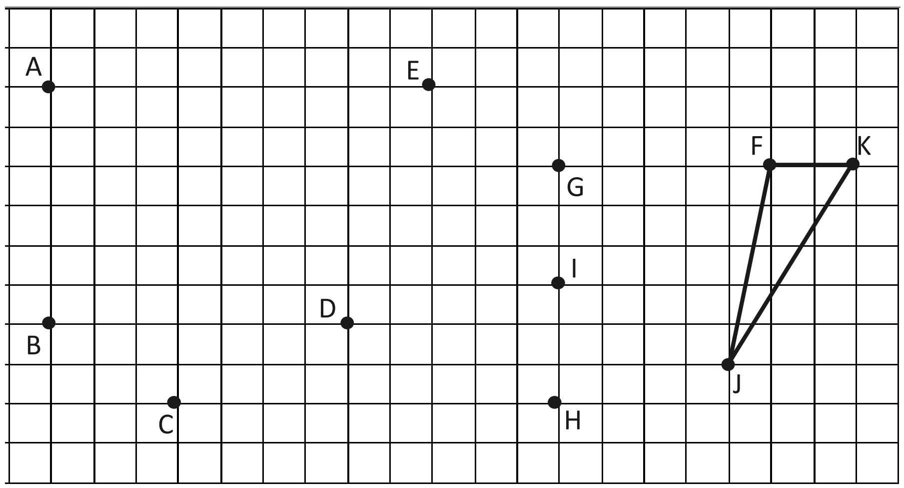
3. $\triangle DEF$ has three lines of symmetry as shown.



a. How can the lines of symmetry help you to figure out which angles are equal?

b. $\triangle DEF$ has a perimeter of 30 cm. Label the side lengths.

4. Use a ruler to connect points to form two other triangles. Use each point only once. None of the triangles may overlap. One or two points will be unused. Name and classify the three triangles below. The first one has been done for you.



Name the Triangles Using Vertices	Classify by Side Length	Classify by Angle Measurement
$\triangle FJK$	Scalene	Obtuse

5. a. List three points from the grid above that, when connected by segments, do not result in a triangle.

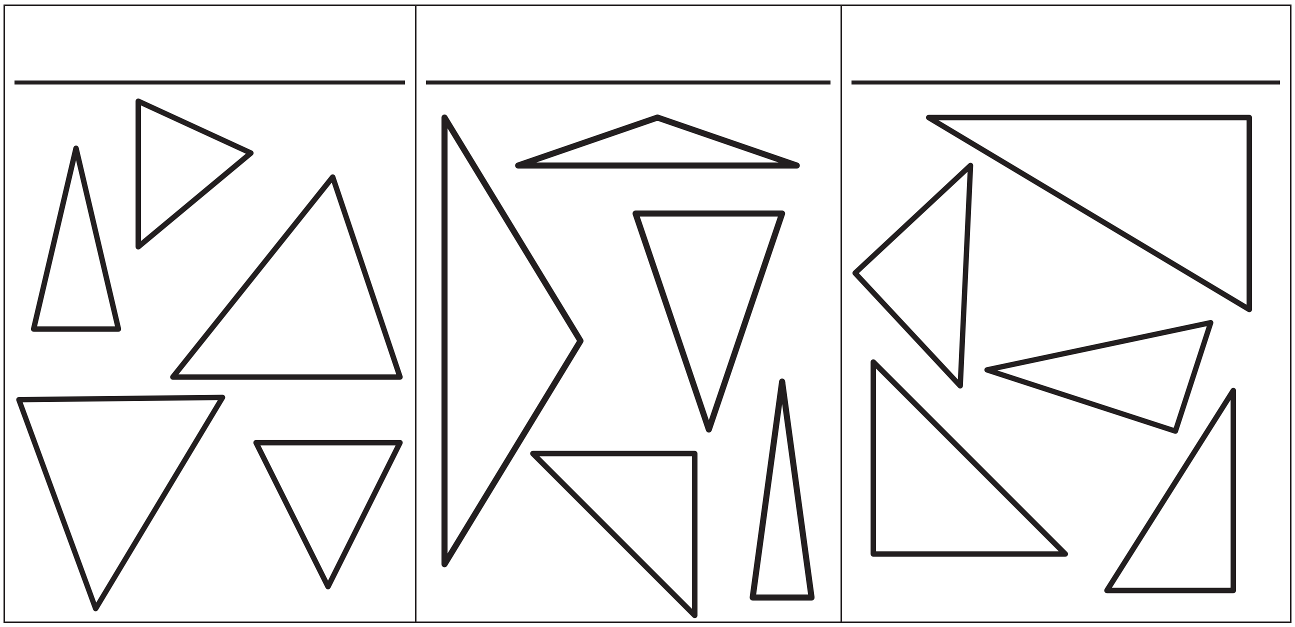
b. Why didn't the three points you listed result in a triangle when connected by segments?

6. Can a triangle have two right angles? Explain.

Name ____ Date ______________________

Use appropriate tools to solve the following problems.

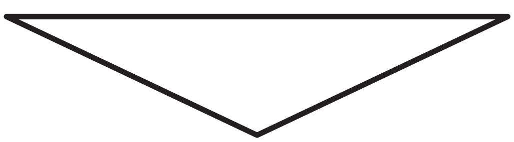
1. The triangles below have been classified by shared attributes (side length *or* angle type). Use the words *acute, right, obtuse, scalene, isosceles,* or *equilateral* to label the headings to identify the way the triangles have been sorted.



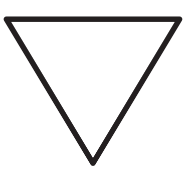
2. Draw lines to identify each triangle according to angle type *and* side length.

a.

b.



c.



Acute

Obtuse

Right

Isosceles

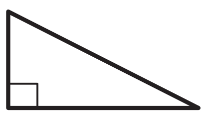
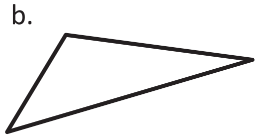
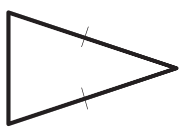
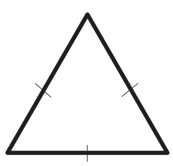
Equilateral

Scalene

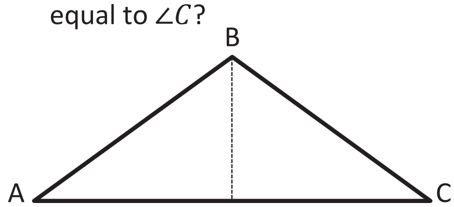
3. Identify and draw any lines of symmetry in the triangles in Problem 2.

Name ______________________________ Date ________________

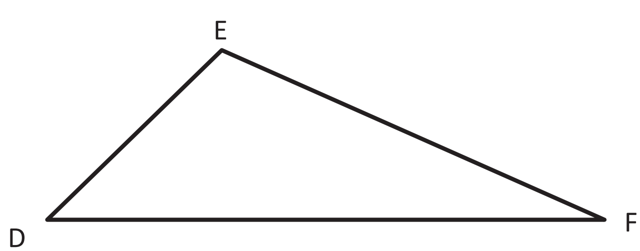
1. Classify each triangle by its side lengths and angle measurements. Circle the correct names.

	Classify Using Side Lengths	**Classify Using Angle Measurements**
a. 	Equilateral Isosceles Scalene	Acute Right Obtuse
b. 	Equilateral Isosceles Scalene	Acute Right Obtuse
c. 	Equilateral Isosceles Scalene	Acute Right Obtuse
d. 	Equilateral Isosceles Scalene	Acute Right Obtuse

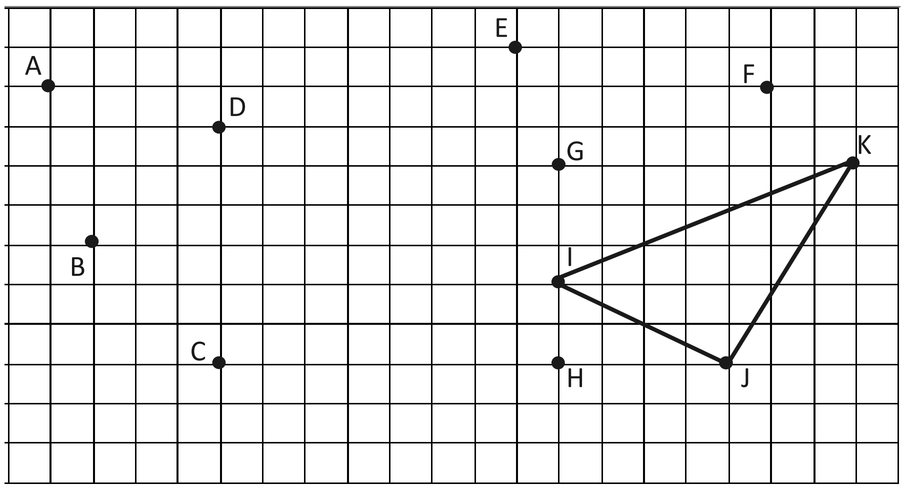
2. a. $\triangle ABC$ has one line of symmetry as shown. Is the measure of $\angle A$ greater than, less than, or equal to $\angle C$?



b. $\triangle DEF$ is scalene. What do you observe about its angles? Explain.



3. Use a ruler to connect points to form two other triangles. Use each point only once. None of the triangles may overlap. Two points will be unused. Name and classify the three triangles below.

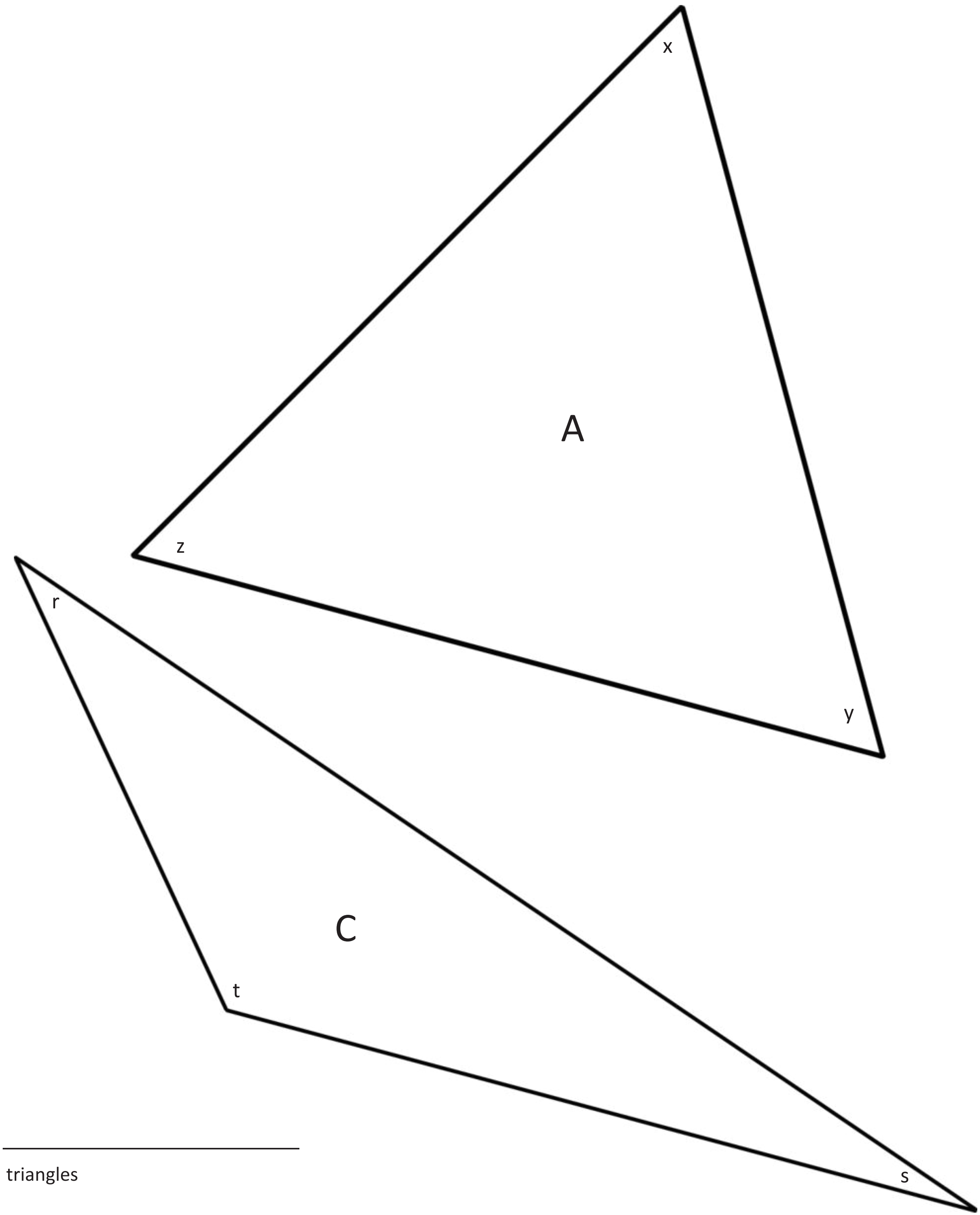


Name the Triangles Using Vertices	Classify by Side Length	Classify by Angle Measurement
$\triangle IJK$		

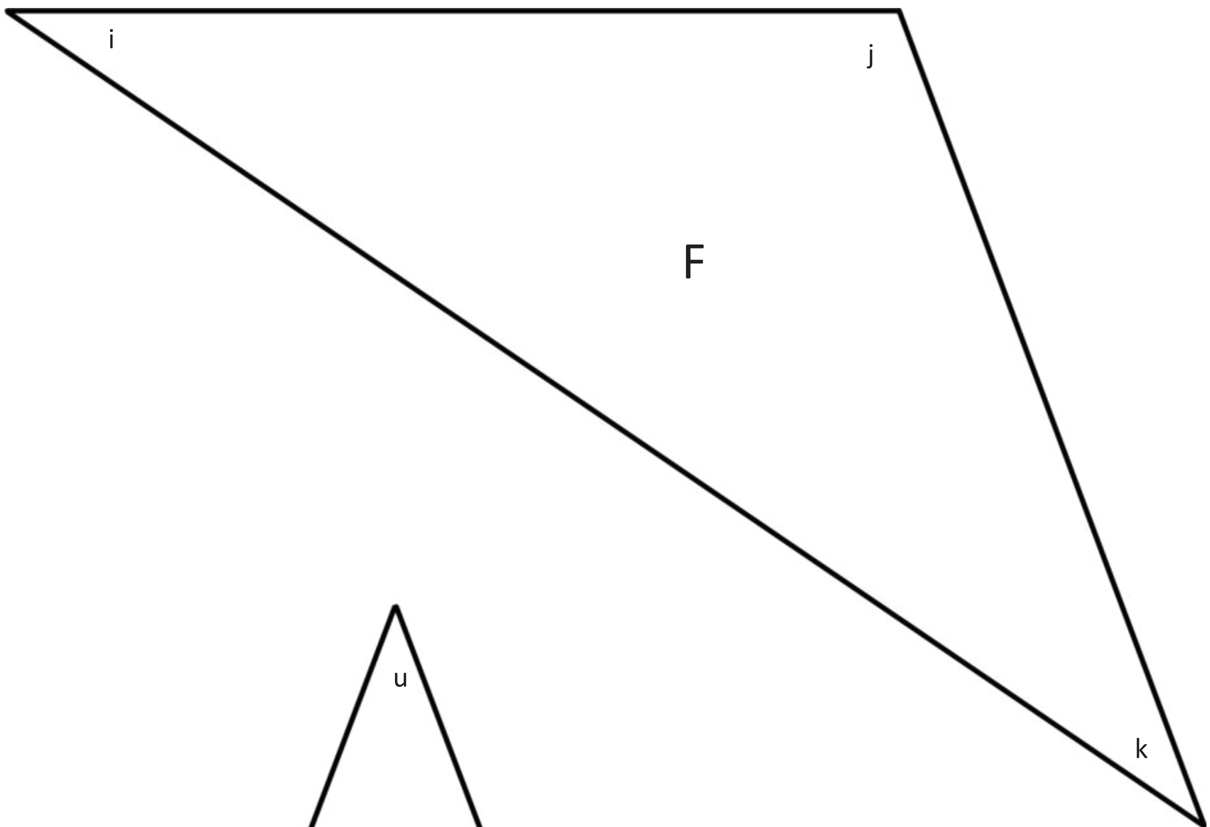
4. If the perimeter of an equilateral triangle is 15 cm, what is the length of each side?

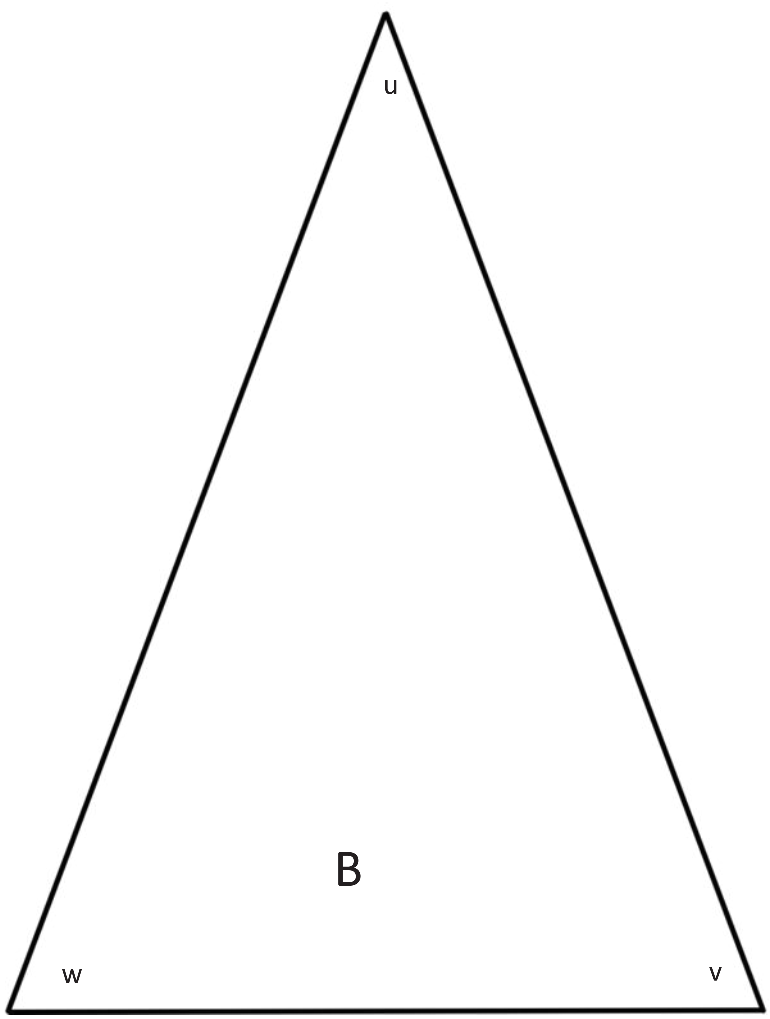
5. Can a triangle have more than one obtuse angle? Explain.

6. Can a triangle have one obtuse angle and one right angle? Explain.

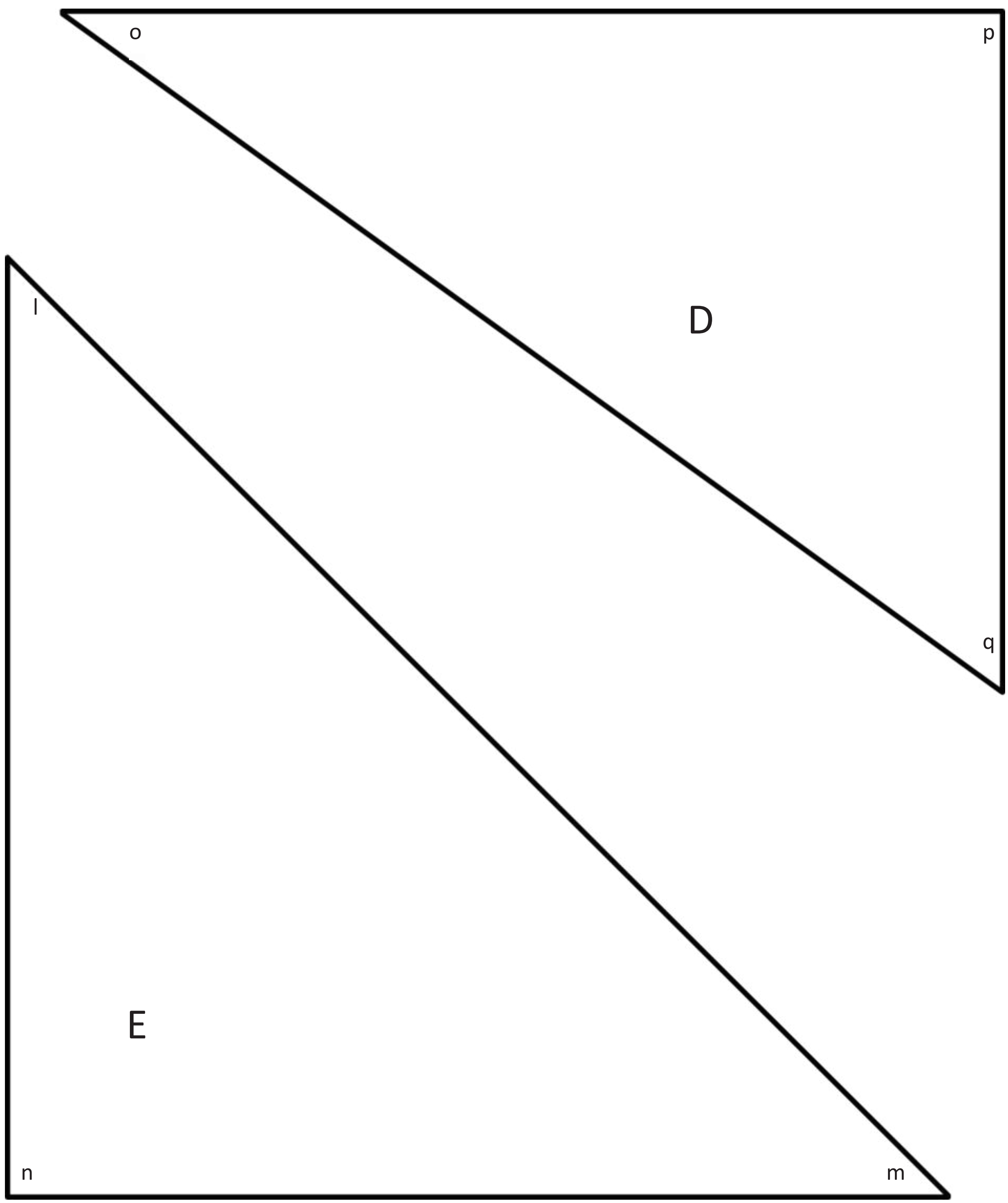


________________ triangles





triangles



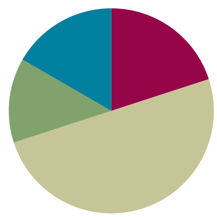
triangles

Lesson 14

Objective: Define and construct triangles from given criteria. Explore symmetry in triangles.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Divide Three Different Ways **4.NBT.6** (4 minutes)
- Physiometry **4.G.3** (4 minutes)
- Classify the Triangle **4.G.2** (4 minutes)

Divide Three Different Ways (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews the content of Module 3 Lessons 28–30. Alternatively, have students select a solution strategy.

T: (Write 148 ÷ 3.) Find the quotient by drawing place value disks.
S: (Solve by drawing place value disks.)
T: Find the quotient using the area model.
S: (Solve using the area model.)
T: Find the quotient using the standard algorithm.
S: (Solve using the standard algorithm.)

Continue with 1,008 ÷ 4.

Physiometry (4 minutes)

Note: Kinesthetic memory is strong memory. This fluency exercise reviews terms learned in Lesson 12.

T: Stand up.
T: I'm trying to make my body position look symmetrical.

T: (Raise left arm so fingers point directly to the wall. Leave the other arm hanging down.) Is my position symmetrical now?

S: No.

Continue with other symmetrical and non-symmetrical positions.

T: With your arms, model a line that runs parallel to the floor. Are you modeling a position that has symmetry?

S: Yes.

T: Model a ray. Are you modeling a position of symmetry?

S: No.

T: Model a line segment. Are you modeling a position of symmetry?

S: Yes.

Classify the Triangle (4 minutes)

Note: This fluency activity reviews Lesson 13.

T: (Project triangle.) What's the measure of the largest given angle in this triangle?

S: 110°.

T: Is the triangle equilateral, scalene, or isosceles?

S: Scalene.

T: Why?

S: Because all the sides are different lengths.

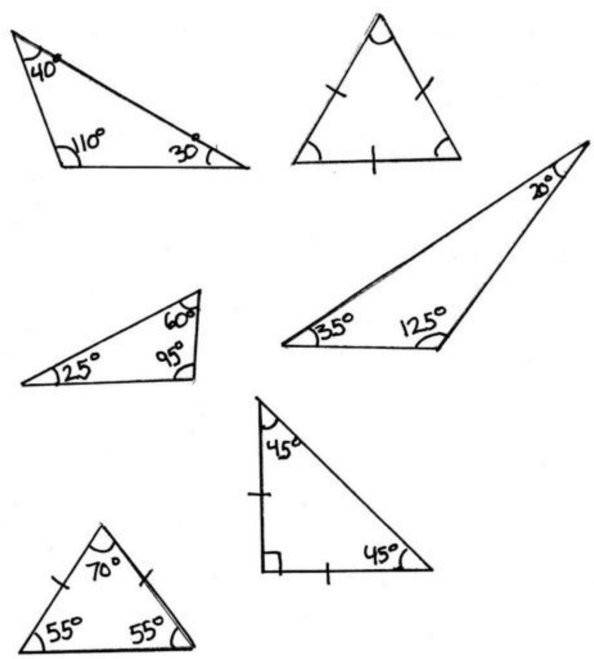
T: Is the same triangle acute, right, or obtuse?

S: Obtuse.

T: Why?

S: Because there's an angle greater than 90°.

Continue the process for the other triangles.

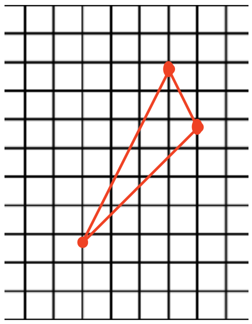


Application Problem (8 minutes)

Materials: (S) Square grid paper, ruler, protractor

Draw three points on your grid paper so that, when connected, they form a triangle. Use your straightedge to connect the three points to form a triangle. Switch papers with your partner. Determine how the triangle your partner constructed can be classified: right, acute, obtuse, equilateral, isosceles, or scalene.

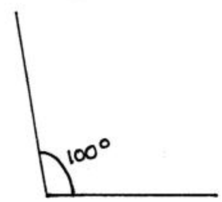
- How can you classify your partner's triangle?
- What attributes did you look at to classify the triangle?
- What tools did you use to help draw your triangle and classify your partner's triangle?



My partner's triangle is an obtuse scalene triangle. I looked at the angle measures and the side lengths in order to classify the triangle. I used my protractor, ruler, and grid paper to help me draw my triangle and to classify my partner's triangle.

Note: This Application Problem reviews Lesson 13. Students classify the triangle according to both side length and angle measure. Through discussion, students are reminded that each triangle can be classified in at least two ways. Some discover that, if they have drawn an equilateral triangle, it can be classified in three different ways. (Note that, because students are drawing triangles by connecting three random points, there may not be examples of equilateral or isosceles triangles.) The Application Problem bridges to today's Concept Development, where students construct triangles from given criteria.

Concept Development (30 minutes)



Materials: (T/S) Square grid paper, ruler, protractor

Problem 1: Construct an obtuse isosceles triangle.

T: Let's construct an obtuse triangle that is also isosceles. What tools should we use?

S: We can use a protractor to measure an angle larger than 90°. Let's make it 100°.

T: (Model.) Now, it's your turn.

S: (Draw a 100° angle.)

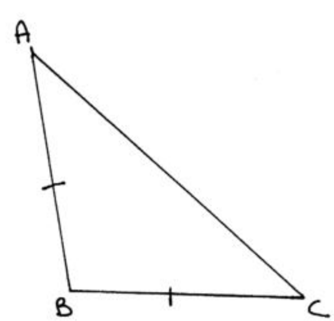
T: Now, what? What do we know about the sides of an isosceles triangle?

MP.6

S: At least two of the sides have to be the same length.

T: Use your ruler to measure each of the sides that are next to the angle. Let's make them each 2 inches.

S: (Measure and draw each side to be 2 inches.)



S: Now, we just have to connect the endpoints of the first two sides to form the triangle.

S: (Finish drawing the triangle.)

T: Do we have an obtuse triangle that is also an isosceles triangle? It looks like it, but let's measure to be sure. First, let's see if it's an obtuse triangle. What does an obtuse triangle need to have?

S: An obtuse angle. → We have one angle that measures 100°. That makes it obtuse.

MP.6

T: Now, let's see if it's an isosceles triangle. What did we do to make sure that this triangle is isosceles?

S: We made at least two of the sides the same length. → Two of the sides measure 2 inches. That makes it isosceles.

S: It's both isosceles and obtuse!

T: Let's call it $\triangle ABC$. Mark the triangle to show the relationship of the sides.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge students working above grade level to construct and classify triangles of a given criteria. For example, say, "Construct triangles having a 45° angle and side lengths of 2 cm and 3 cm. How many types of triangles can you make?" Students may work independently or in pairs.

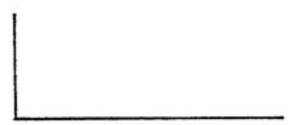
Problem 2: Construct a right scalene triangle.

T: Let's try another. Let's construct a right scalene triangle. Talk to your partner about what to draw first.

S: Let's draw two sides of the triangle. We know that they have to be different lengths. → No, that doesn't work because maybe we won't have a right angle. We have to draw the right angle first.

T: Construct a right angle.

S: (Construct a right angle.)

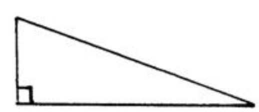


T: Now what?

S: Well, if it's scalene, we need three different side lengths. We already drew two of the sides, but we need to make sure that they are different lengths.

T: Measure to be sure that they are different lengths.

S: (Measure.)



S: Oops! Two of my sides are the same length. That would make it isosceles. I need to try again.

T: What next?

S: Now, we can connect the two sides that we just drew so that we have a triangle. (Draw the triangle's third segment.)

T: Ok. Talk over the final step with your partner.

S: We need to make sure it's both right and scalene. → We can use the protractor to make sure there is a 90° angle. → Yes, it's 90°. Now, we can measure the sides to make sure that they are all different lengths. → I have a right scalene triangle.

T: Let's remember to label and mark the triangle with symbols to show angles and side lengths if necessary. Will this triangle have tick marks?

S: No! Only isosceles and equilateral triangles will.

Problem 3: Explore classifications of triangles.

T: Look back at the triangle that you drew for today's Application Problem. Raise your hand if you drew a scalene triangle. Raise your hand if you drew an equilateral triangle. Raise your hand if you drew a scalene equilateral triangle.

S: That's silly. You can't have a scalene equilateral triangle!

T: Discuss with a partner: True or false? A triangle can be both scalene and equilateral. Explain.

S: That's false. All of the sides have to be the same length if it's equilateral, but a scalene triangle has to have sides that are all different lengths. The sides can't be the same length and different lengths at the same time!

T: True or false? An equilateral triangle is also obtuse?

S: False. You can't do that either. The sides won't be equal. One of them will be longer. → We know that equilateral triangles have three acute angles that measure the same.

T: I'm imagining an equilateral right triangle. Can it exist?

S: No. Equilateral triangles have three acute angles that measure the same.

T: I'm imagining a scalene acute triangle. Can it exist?

S: Yes! The triangle that I drew is classified that way!

T: I'm imagining a triangle that is isosceles and equilateral. Can it exist?

S: Yes! An equilateral triangle is an isosceles triangle, too, because it has at least two equal sides. That means it can have three equal sides.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Scaffold naming triangles using two criteria for English language learners and others. Refer to definitions and accompanying diagrams of *equilateral, isosceles, scalene, acute, obtuse,* and *right triangles* on a word wall, or have students refer to their personal math dictionaries. Before constructing triangles, it may be beneficial to show examples of triangles that students classify and discuss in the language of their choice.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Define and construct triangles from given criteria. Explore symmetry in triangles.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

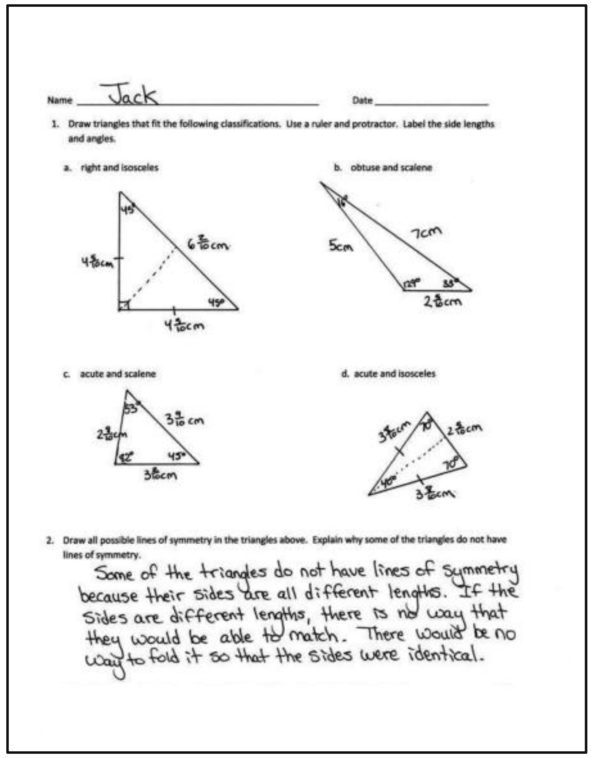
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 4, explain why you answered true or false.
- Discuss your answer to Problem 6. How are these two triangles closely related?
- In Problem 1, which of the triangles was most challenging to draw? Why?
- When you were drawing a triangle that had two attributes, how did you determine what to draw first—the side length or the angle measure?
- From Problem 2, can you determine which types of triangles never have lines of symmetry?
- If a triangle has one line of symmetry, what kind of triangle does it have to be? If a triangle has three lines of symmetry, what kind of triangle does it have to be?
- Why is it important to verify our triangles' attributes after we have constructed them?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.


Name Jack Date

1. Draw triangles that fit the following classifications. Use a ruler and protractor. Label the side lengths and angles.

a. right and isosceles

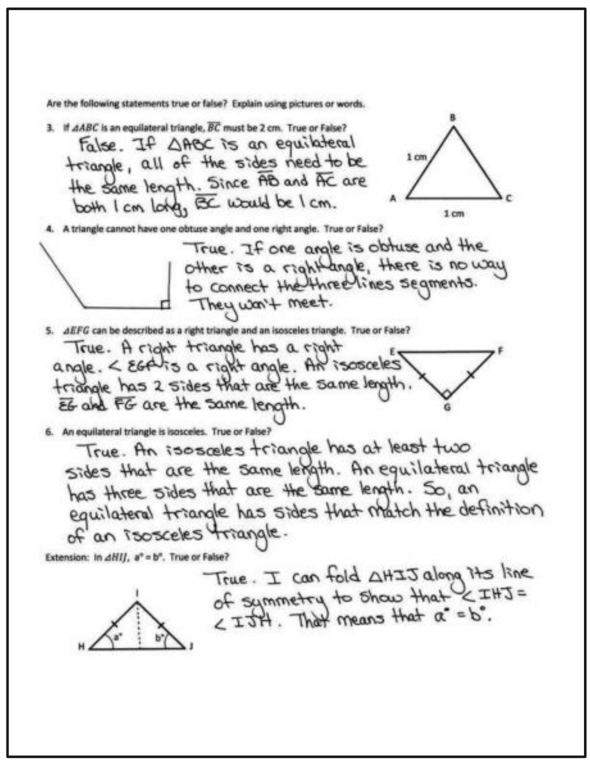
b. obtuse and scalene

c. acute and scalene

d. acute and isosceles

2. Draw all possible lines of symmetry in the triangles above. Explain why some of the triangles do not have lines of symmetry.

Some of the triangles do not have lines of symmetry because their sides are all different lengths. If the sides are different lengths, there is no way that they would be able to match. There would be no way to fold it so that the sides were identical.


Are the following statements true or false? Explain using pictures or words.

3. If $\triangle ABC$ is an equilateral triangle, $\overline{BC}$ must be 2 cm. True or False?

False. If $\triangle ABC$ is an equilateral triangle, all of the sides need to be the same length. Since $\overline{AB}$ and $\overline{AC}$ are both 1 cm long, $\overline{BC}$ would be 1 cm.

4. A triangle cannot have one obtuse angle and one right angle. True or False?

True. If one angle is obtuse and the other is a right angle, there is no way to connect the three lines segments. They won't meet.

5. $\triangle EFG$ can be described as a right triangle and an isosceles triangle. True or False?

True. A right triangle has a right angle. $\angle EGF$ is a right angle. An isosceles triangle has 2 sides that are the same length. $\overline{EG}$ and $\overline{FG}$ are the same length.

6. An equilateral triangle is isosceles. True or False?

True. An isosceles triangle has at least two sides that are the same length. An equilateral triangle has three sides that are the same length. So, an equilateral triangle has sides that match the definition of an isosceles triangle.

Extension: In $\triangle HIJ$, $a° = b°$. True or False?

True. I can fold $\triangle HIJ$ along its line of symmetry to show that $\angle IHJ = \angle IJH$. That means that $a° = b°$.

Name ____ Date ______________________

1. Draw triangles that fit the following classifications. Use a ruler and protractor. Label the side lengths and angles.

   a. Right and isosceles

   b. Obtuse and scalene

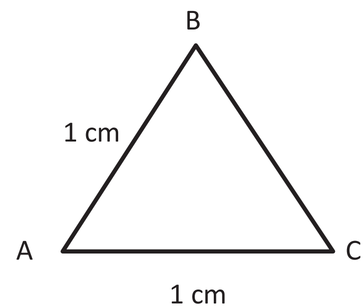
   c. Acute and scalene

   d. Acute and isosceles

2. Draw all possible lines of symmetry in the triangles above. Explain why some of the triangles do not have lines of symmetry.

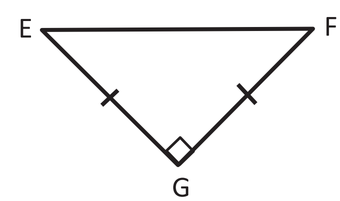
Are the following statements true or false? Explain using pictures or words.

3. If $\triangle ABC$ is an equilateral triangle, $\overline{BC}$ must be 2 cm. True or False?



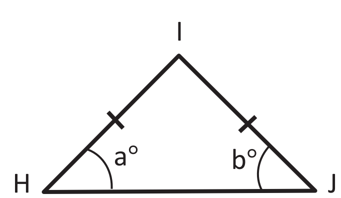
4. A triangle cannot have one obtuse angle and one right angle. True or False?

5. $\triangle EFG$ can be described as a right triangle and an isosceles triangle. True or False?



6. An equilateral triangle is isosceles. True or False?

Extension: In $\triangle HIJ$, a = b. True or False?



Name ____ Date ______________________

1. Draw an obtuse isosceles triangle, and then draw any lines of symmetry if they exist.

2. Draw a right scalene triangle, and then draw any lines of symmetry if they exist.

3. Every triangle has at least ____ acute angles.

Name ____ Date ______________________

1. Draw triangles that fit the following classifications. Use a ruler and protractor. Label the side lengths and angles.

   a. Right and isosceles

   b. Right and scalene

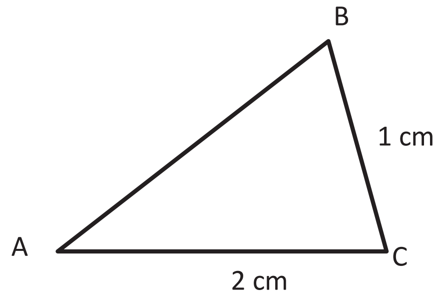
   c. Obtuse and isosceles

   d. Acute and scalene

2. Draw all possible lines of symmetry in the triangles above. Explain why some of the triangles do not have lines of symmetry.

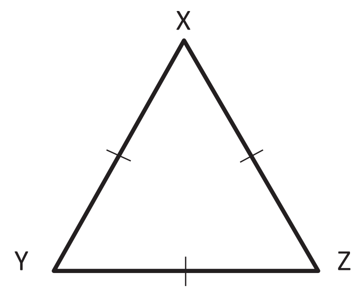
Are the following statements true or false? Explain.

3. $\triangle ABC$ is an isosceles triangle. $\overline{AB}$ must be 2 cm. True or False?



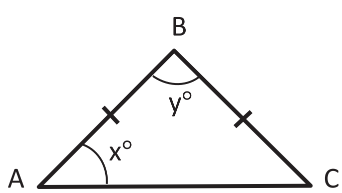
4. A triangle cannot have both an acute angle and a right angle. True or False?

5. $\triangle XYZ$ can be described as both equilateral and acute. True or False?



6. A right triangle is always scalene. True or False?

Extension: In $\triangle ABC$, x = y. True or False?

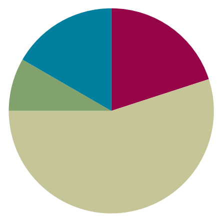


Lesson 15

Objective: Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Add and Subtract **4.NBT.4** (4 minutes)
- Classify the Triangle **4.G.2** (3 minutes)
- Find the Unknown Angle **4.MD.5** (5 minutes)

Add and Subtract (4 minutes)

Materials: (S) Personal white board

Note: This concept reviews the yearlong Grade 4 fluency standard for adding and subtracting using the standard algorithm.

T: (Write 543 thousands 178 ones.) On your personal white boards, write this number in standard form.

S: (Write 543,178.)

T: (Write 134 thousands 153 ones.) Add this number to 543,178 using the standard algorithm.

S: (Write 543,178 + 134,153 = 677,331 using the standard algorithm.)

Continue with the following possible sequence: 481,737 + 253,675.

T: (Write 817 thousands 560 ones.) On your boards, write this number in standard form.

S: (Write 817,560.)

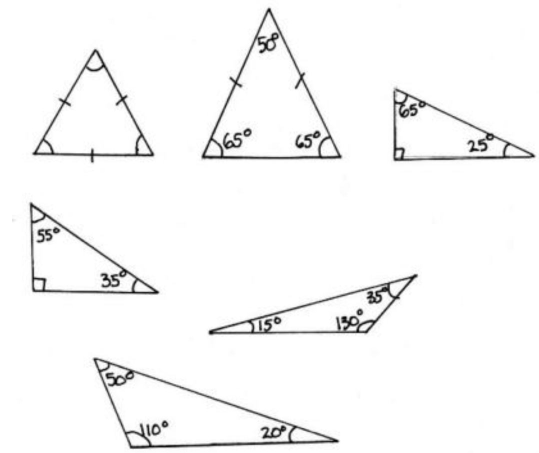
T: (Write 426 thousands 145 ones.) Subtract this number from 817,560 using the standard algorithm.

S: (Write 817,560 – 426,145 = 391,415 using the standard algorithm.)

Continue with the following possible sequence: 673,172 – 143,818 and 600,000 – 426,521.

Classify the Triangle (3 minutes)

Note: This fluency activity reviews Lesson 13.



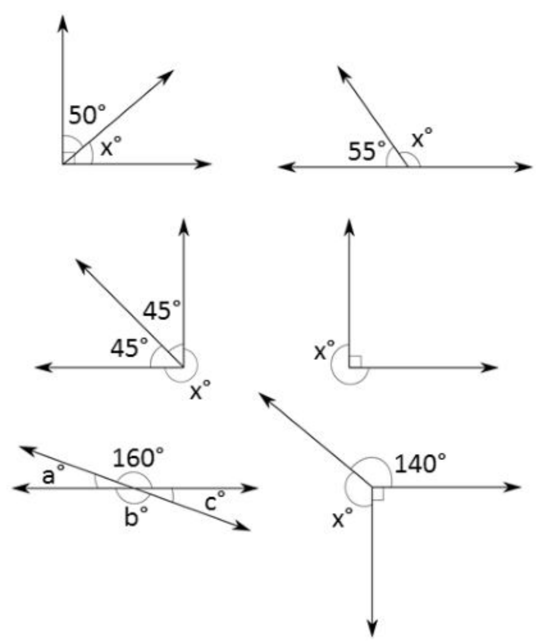
T: (Project triangle.) Is the triangle equilateral, scalene, or isosceles?
S: Equilateral.
T: Why?
S: Because all the sides are the same length.
T: Is it acute, right, or obtuse?
S: Acute.
T: Why?
S: Because all the angles are less than 90°.
T: (Project triangle.) Say the measure of the largest angle.
S: 130°.
T: Is the triangle equilateral, scalene, or isosceles?
S: Scalene.
T: Why?
S: Because all the sides are different.
T: Is the triangle acute, right, or obtuse?
S: Obtuse.
T: Why?
S: Because it has an angle greater than 90°.

Continue the process for the other triangles.

Find the Unknown Angle (5 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews Lesson 10.



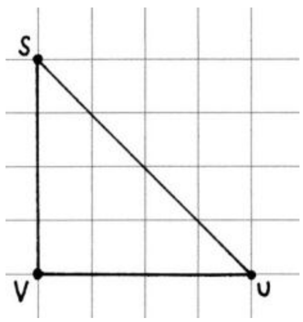
T: (Project the first unknown angle problem. Run a finger over the largest angle.) This is a right angle. On your personal white boards, write a number sentence to find the measure of $\angle x$.
S: (Write 90 – 50 = x. Below it, write x° = 40°.)

Continue with the remaining unknown angle problems.

Application Problem (5 minutes)

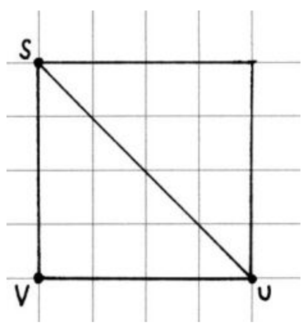
Materials: (S) Square grid paper

a. On grid paper, draw two perpendicular line segments, each measuring 4 units, which extend from a point V. Identify the segments as $\overline{SV}$ and $\overline{UV}$. Draw $\overline{SU}$. What shape did you construct? Classify it.



I constructed a triangle – ΔSUV. It is a right and isosceles triangle.

b. Imagine $\overline{SU}$ is a line of symmetry. Construct the other half of the figure. What figure did you construct? How can you tell?



This is a square. I know because each side is 4 units long and it has 4 right angles.

Note: This Application Problem reviews segments and points from Lesson 1, perpendicular lines from Lesson 3, lines of symmetry from Lesson 12, classifying triangles from Lesson 13, and constructing triangles from Lesson 14. It also links knowledge of the attributes of a square from previous grades, bridging to this lesson's objective of classifying quadrilaterals.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Provide alternatives to constructing polygons with pencil and paper to students working below grade level and others. For example, tactile learners may use geoboards, while others may benefit from using a virtual geoboard such as that found at the following link (which can be enlarged and made tactile using an interactive white board): http://www.mathplayground.com/geoboard.html

Alternatively, provide grid paper to ease the task of drawing.

Concept Development (33 minutes)

Materials: (T/S) Problem Set, ruler, right angle template (created in Lesson 2)

Problem 1: Construct and define trapezoids.

T: What do you know about quadrilaterals?

S: They have four straight sides. → They can be shapes, like a square or a rectangle.

MP.5

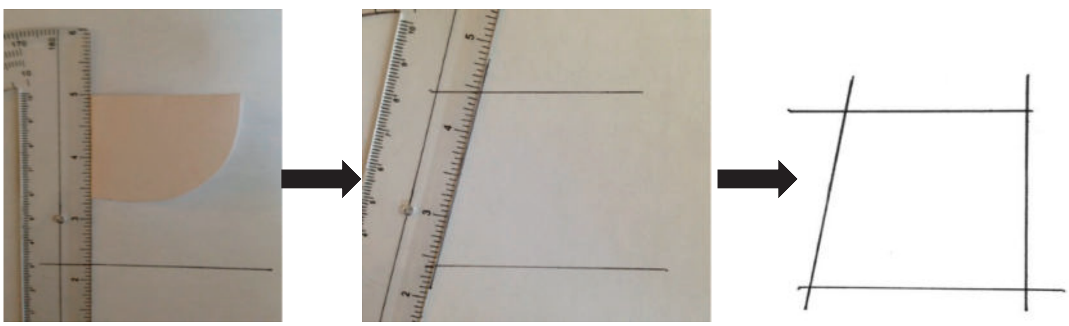
T: Use Problem 1 on your Problem Set to construct a quadrilateral with at least one set of parallel sides. (Guide students through the process of drawing the quadrilateral.)

Step 1. Draw a straight, horizontal segment.

Step 2. Use your right angle template and ruler to draw a segment parallel to that segment.

Step 3. Draw a third segment that crosses both.

Step 4. Draw a fourth different segment that crosses both, but does not cross the third segment.

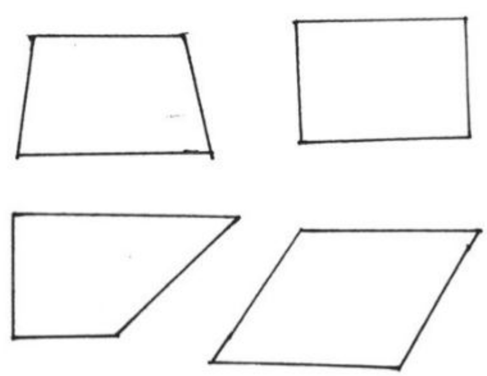


Steps 1 and 2 Step 3 Step 4

T: Compare your quadrilateral with those of your group, looking at angle size and side length.

S: The sides of mine are all different lengths. → Mine has two obtuse angles and two acute angles. → Mine looks more like a rectangle. → Mine has two right angles, an acute angle, and an obtuse angle. → Mine has angles of different sizes. One set of opposite sides look equal. → Yes, but we all have shapes with one set of parallel sides.

T: All of our quadrilaterals have at least one set of parallel sides, which means all of our quadrilaterals are trapezoids. However, some of your trapezoids might have other familiar names, like *rectangle*.



Other possible trapezoids

Be sure that students identify the pair of parallel sides in a square, rectangle, non-rectilinear parallelogram, and rhombus.

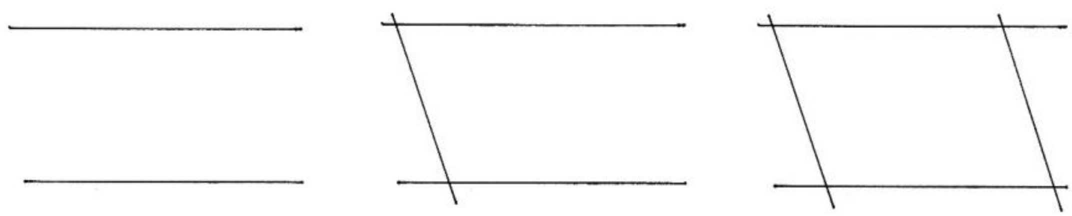
T: Construct two more trapezoids for Problem 1. Ask your partners for suggestions on how they constructed their trapezoids as you construct a new one.

Allow time for students to construct two more trapezoids.

Problem 2: Construct and define parallelograms.

T: Under Problem 2, let's construct a quadrilateral with two sets of parallel sides. Start by drawing one set of parallel segments, the same way you did in Problem 1. (Guide students through the process of drawing the quadrilateral.)

1. Draw a straight, horizontal segment.
2. Use your right angle template and ruler to draw a segment parallel to that segment.
3. Draw a third segment that crosses both.
4. Using your ruler and right angle template, draw a fourth different segment that crosses the first two segments and that is parallel to the third segment.



Steps 1 and 2 Step 3 Step 4

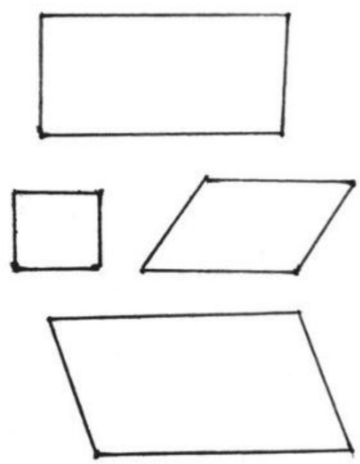
T: Verify both sets of lines are parallel. Compare your quadrilateral with those of your group.

Students discuss similar and contrasting features of their figures.

T: Are all of these shapes drawn in Problem 2 trapezoids?

S: They don't look like the trapezoid I drew. → Yeah, this one looks like mine from Problem 1. → A trapezoid has to have at least one set of parallel sides. Mine has two! → So these must be trapezoids since they all have at least one pair of parallel sides.

T: All of the trapezoids we constructed for Problem 2 have two sets of parallel sides. We call quadrilaterals with two pairs of parallel sides parallelograms. Again, I see some figures that I might give another name to, but all of the shapes we've constructed are parallelograms. Record the word *parallelogram* for Problem 2. Construct two more parallelograms for Problem 2. Ask your partners for suggestions on how they constructed their parallelograms, or construct a new one.



Other possible parallelograms

T: Did anyone draw the same quadrilateral in Problems 1 and 2?

S: Yes, I drew a parallelogram in Problem 1. So, a parallelogram has two names?

T: A trapezoid must have at least one set of parallel sides. A parallelogram is a special trapezoid. It has two sets of parallel sides. To be specific, we call the quadrilaterals in Problem 2 parallelograms.

Problem 3: Construct and define rectangles.

T: For Problem 3, we need to make a parallelogram with four right angles. What do we call two lines that intersect at a right angle?

S: Perpendicular lines.

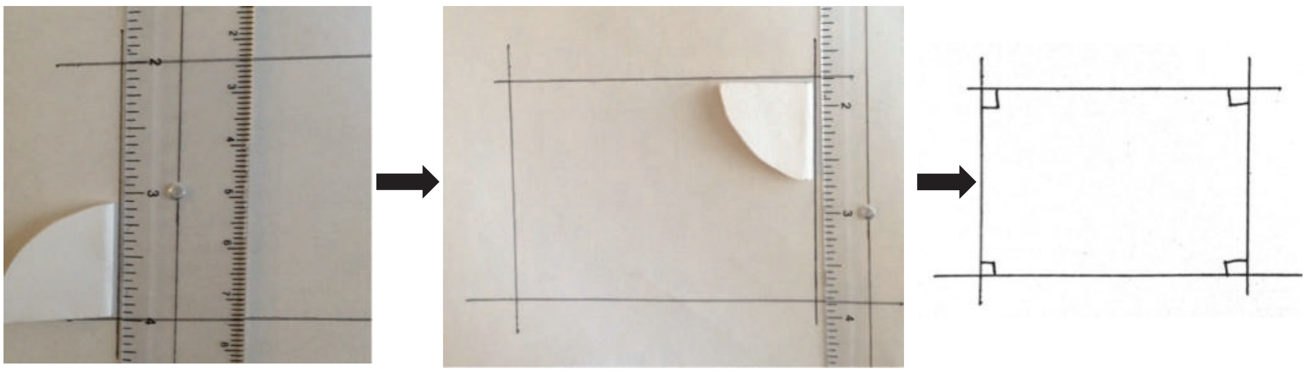
T: (Guide students through the process of drawing the parallelogram.)

Step 1. Draw a straight, horizontal segment.

Step 2. Use your right angle template and ruler to draw a segment parallel to that segment.

Step 3. Draw a third segment with a right angle, perpendicular to the base line.

Step 4. Draw a fourth segment that is also perpendicular to the first segment.



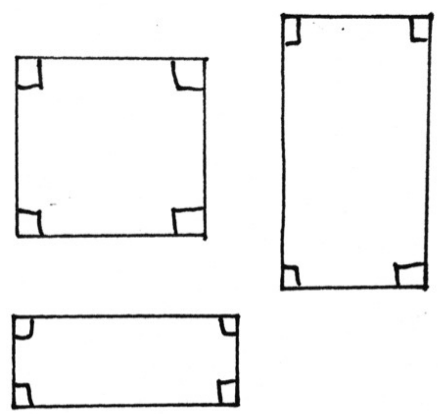
Steps 1, 2, and 3 Step 4

T: Compare your quadrilateral with those of your group, looking at angle measure and side length.

S: The fourth segment is parallel to the third one. → It has two sets of parallel sides. That means it is a parallelogram. → Mine has four right angles. → The opposite sides are the same length. → It looks like a rectangle. → Mine looks like a square.

T: These quadrilaterals all have two sets of parallel sides, so they are parallelograms and trapezoids. However, our figures have another special attribute—four right angles, so they are also rectangles.

T: Construct two more rectangles for Problem 3. (A square is a special rectangle, so at least one should be evidenced in the examples.)



Problem 4: Construct and define squares.

T: Problem 4 requires us to draw a rectangle with sides that are all the same length. Discuss with your group how you might do that.

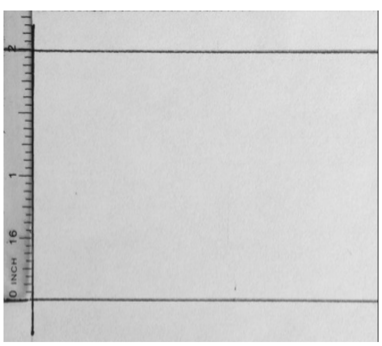
S: Draw each side the same length. → We can draw the parallel sides, then one of the perpendicular sides. Then, we will have to measure some sides.

T: (Guide students through the process of drawing the rectangle.)

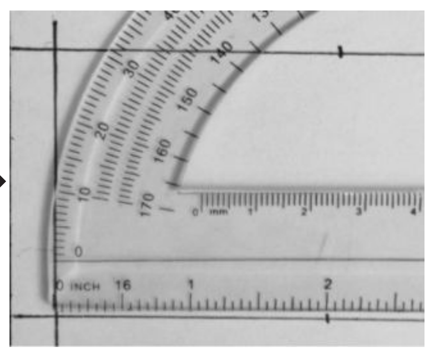
1. Draw a straight, horizontal segment.
2. Use your right angle template and ruler to draw a segment parallel to that segment.
3. Draw a third segment with a right angle, perpendicular to the base line.
4. Measure the length of the third side, and mark the same length on both of the first segments. Start the measurement at the third side.
5. Draw a fourth segment perpendicular to the first segment through those marks.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

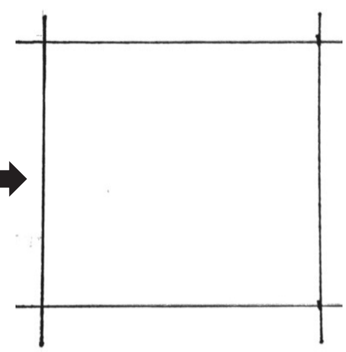
Support math language acquisition for English language learners and others. Post on the word wall, and have students add to their personal math dictionaries *quadrilateral, parallelogram,* and *trapezoid* and corresponding pictures. Guide student connections amongst the quadrilaterals using graphic organizers, such as a Venn diagram. Teach the etymology or meaningful word parts, if helpful. Offer or facilitate student-made mnemonic devices. Challenge students working above grade level to research connections between similar words, such as *trapeze* and *trapezoid* and *quarter* and *quadrilateral.*



Steps 1, 2, and 3



Step 4



Step 5

S: We made a square!

T: Yes, a square is a special rectangle and has all sides the same length. Construct two more squares.

S: If a square is a rectangle, then a square can also be a parallelogram. → And a trapezoid!

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

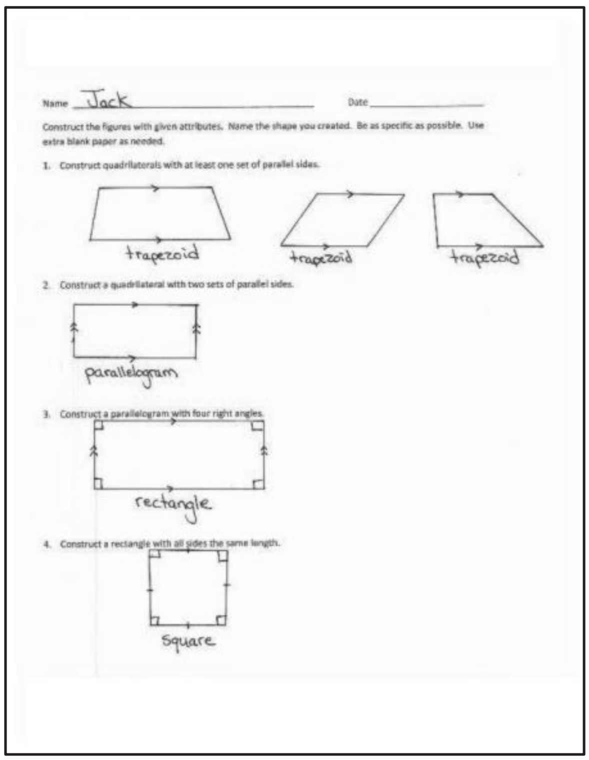
Lesson Objective: Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

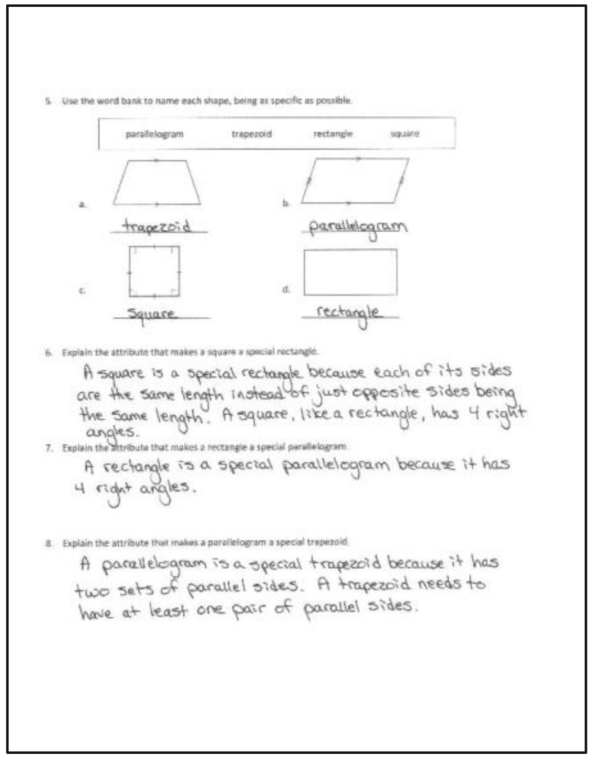
Any combination of the questions below may be used to lead the discussion.

- For Problem 6, what makes a square different from a rectangle? Why is it important to define a square as a rectangle with four equal length sides and not as a quadrilateral with four equal length sides?
- What are some attributes that every square has in common? How is a square a special case of a rectangle, a parallelogram, and a trapezoid?
- If your teacher asked you to draw a trapezoid, and you drew a parallelogram, explain to your teacher why a parallelogram is also a trapezoid.
- Can a trapezoid be defined as a square? What attributes of a square are not present in a trapezoid? Why does it only work in the reverse: a square is also a trapezoid? What attributes of a trapezoid are present in a square?
- We have seen today that a figure can belong to different categories. That is often true in life. For example, consider the following words: woman, mother, sister, and aunt. A woman can be a mother, but only is a mother if she has children. A woman isn't a sister unless she has a sister or a brother. Each classification has a defining attribute. A mother, sister, and aunt are all women just as a parallelogram, rectangle, and square are all trapezoids and, ultimately, all quadrilaterals. Talk to your partner about the following set of words: clothes, pants, and jeans.



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



5. Use the word bank to name each shape, being as specific as possible.

parallelogram	trapezoid	rectangle	square

a. trapezoid

b. parallelogram

c. Square

d. rectangle

6. Explain the attribute that makes a square a special rectangle.

A square is a special rectangle because each of its sides are the same length instead of just opposite sides being the same length. A square, like a rectangle, has 4 right angles.

7. Explain the attribute that makes a rectangle a special parallelogram.

A rectangle is a special parallelogram because it has 4 right angles.

8. Explain the attribute that makes a parallelogram a special trapezoid.

A parallelogram is a special trapezoid because it has two sets of parallel sides. A trapezoid needs to have at least one pair of parallel sides.

Name ____ Date ____________________

Construct the figures with the given attributes. Name the shape you created. Be as specific as possible. Use extra blank paper as needed.

1. Construct quadrilaterals with at least one set of parallel sides.

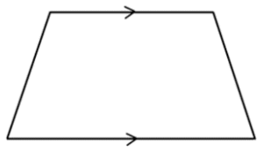
2. Construct a quadrilateral with two sets of parallel sides.

3. Construct a parallelogram with four right angles.

4. Construct a rectangle with all sides the same length.

5. Use the word bank to name each shape, being as specific as possible.

Parallelogram	Trapezoid	Rectangle	Square

a. 

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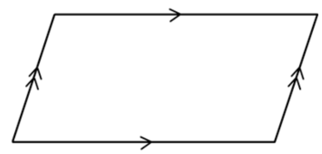
b. 

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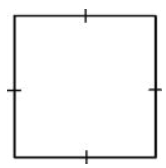
c. 

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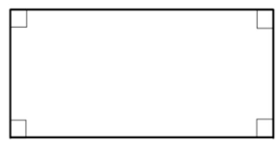
d. 

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6. Explain the attribute that makes a square a special rectangle.

7. Explain the attribute that makes a rectangle a special parallelogram.

8. Explain the attribute that makes a parallelogram a special trapezoid.

Name ____ Date _____________________

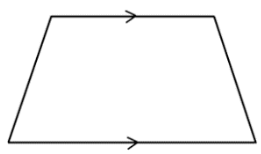
1. In the space below, draw a parallelogram.

2. Explain why a rectangle is a special parallelogram.

Name ______________________________ Date ________________

1. Use the word bank to name each shape, being as specific as possible.

Parallelogram	Trapezoid	Rectangle	Square

a. 

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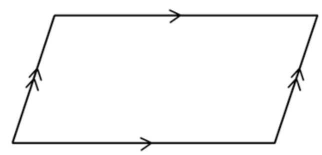
b. 

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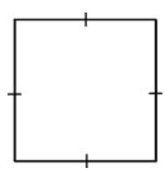
c. 

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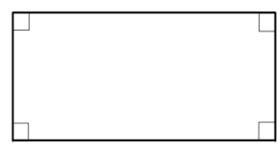
d. 

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2. Explain the attribute that makes a square a special rectangle.

3. Explain the attribute that makes a rectangle a special parallelogram.

4. Explain the attribute that makes a parallelogram a special trapezoid.

5. Construct the following figures based on the given attributes. Give a name to each figure you construct. Be as specific as possible.

a. A quadrilateral with four sides the same length and four right angles.

b. A quadrilateral with two sets of parallel sides.

c. A quadrilateral with only one set of parallel sides.

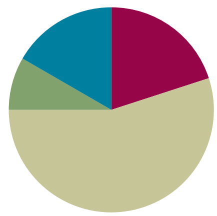
d. A parallelogram with four right angles.

Lesson 16

Objective: Reason about attributes to construct quadrilaterals on square or triangular grid paper.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Add and Subtract **4.NBT.4** (4 minutes)
- Find the Unknown Angle **4.MD.5** (5 minutes)
- Classify the Quadrilateral **4.G.2** (3 minutes)

Add and Subtract (4 minutes)

Materials: (S) Personal white board

Notes: This concept reviews the yearlong Grade 4 fluency standard for adding and subtracting using the standard algorithm.

T: (Write 765 thousands 198 ones.) On your personal white boards, write this number in standard form.

S: (Write 765,198.)

T: (Write 156 thousands 185 ones.) Add this number to 765,198 using the standard algorithm.

S: (Write 765,198 + 156,185 = 921,383 using the standard algorithm.)

Continue with the following possible sequence: 681,959 + 175,845.

T: (Write 716 thousands 450 ones.) On your boards, write this number in standard form.

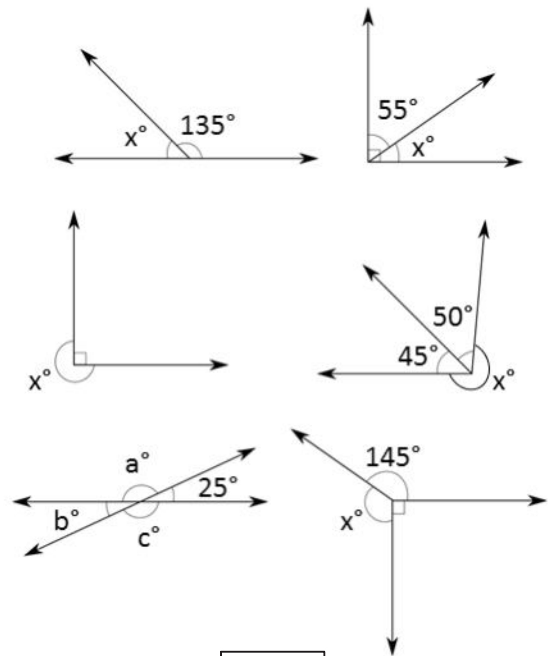
S: (Write 716,450.)

T: (Write 325 thousands 139 ones.) Subtract this number from 716,450 using the standard algorithm.

S: (Write 716,450 – 325,139 = 391,311 using the standard algorithm.)

Continue with the following possible sequence: 451,151 – 122,616 and 500,000 – 315,415.

Find the Unknown Angle (5 minutes)



Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 10.

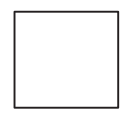
T: (Project the first unknown angle problem. Run a finger along the horizontal line.) This is a straight angle. On your personal white boards, write a number sentence to find the measure of $\angle x$.

S: (Write 180 – 135 = x. Below it, write $x°$ = 45°.)

Continue with the remaining unknown angle problems.

Classify the Quadrilateral (3 minutes)

Notes: This fluency exercise reviews Lesson 15.



T: (Project square.) How many sides does the polygon have?

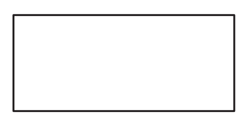
S: Four sides.

T: What's the name for polygons with four sides?

S: Quadrilateral.

T: Each angle in this quadrilateral is 90°. It also has four equal sides. What's a more specific name?

S: Square.



T: (Project second polygon.) Is this polygon a quadrilateral?

S: Yes.

T: Why?

S: Because it has four sides.

T: Is this quadrilateral a square?

S: No.

T: How do you know?

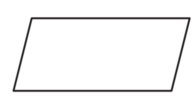
S: The sides are not the same length.

T: Each angle is 90°. What type of quadrilateral is it?

S: Rectangle.

T: Does a rectangle have two sets of parallel sides?

S: Yes.



T: (Project parallelogram.) Is this polygon a quadrilateral?

S: Yes.

T: This quadrilateral has two sets of parallel sides. Is it a rectangle?

S: No.

T: How do you know?
S: All four angles are not 90°.
T: What's the name of a quadrilateral with two sets of parallel sides that best defines this figure?
S: Parallelogram.
T: (Project trapezoid.) Is this polygon a quadrilateral?
S: Yes.

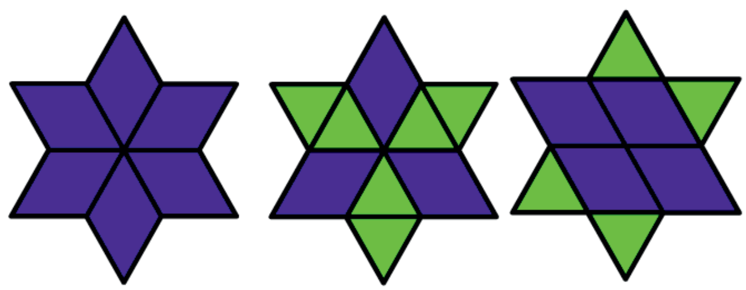
T: How do you know?
S: It has four sides.
T: Is it a rectangle?
S: No.
T: How do you know?
S: Each angle doesn't measure 90°.
T: Is it a parallelogram?
S: No.
T: How do you know?
S: It doesn't have two sets of parallel sides.
T: Classify this quadrilateral.
S: It's a trapezoid.
T: Describe its attribute.
S: It has at least one pair of parallel sides.

Application Problem (5 minutes)

Within the stars, find at least two different examples for each of the following. Explain which attributes you used to identify each.

- Equilateral triangles
- Trapezoids
- Parallelograms
- Rhombuses

Note: Identifying these polygons within the star serves as a review for identifying the shapes and introduces the students to drawing these shapes on triangular grid paper used during the Concept Development.



To find the equilateral triangles, I looked for triangles with equal side lengths.
To find the trapezoids, I looked for quadrilaterals with at least one set of parallel sides.
To find the parallelograms, I looked for quadrilaterals with two sets of parallel sides.
To find the rhombuses, I looked for quadrilaterals with equal side lengths.
I used my ruler to measure the sides.

Concept Development (33 minutes)

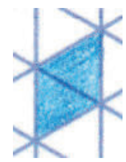
Materials: (T/S) Rectangular and triangular grid paper, ruler, right angle template (created in Lesson 2)

Problem 1: Construct a rhombus on a triangular grid.

T: On your triangular grid paper, all the small triangles are equilateral. Please shade in two triangles that share a side. (Allow students time to complete the task.) Talk to your partner. What do you notice about the side lengths and angle sizes of the larger shape you have shaded?

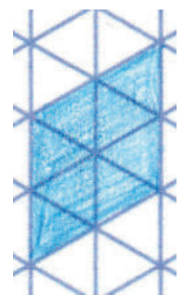
S: The sides are all the same length. → The acute angles are the same size because I know that the angles of equilateral triangles are equal. → The obtuse angles are the same size, too, because both of them are the sum of two of the equilateral triangle's angles.

T: Which of the following terms relate to this shape? (Write *quadrilateral, trapezoid, parallelogram, rectangle,* and *square*.)



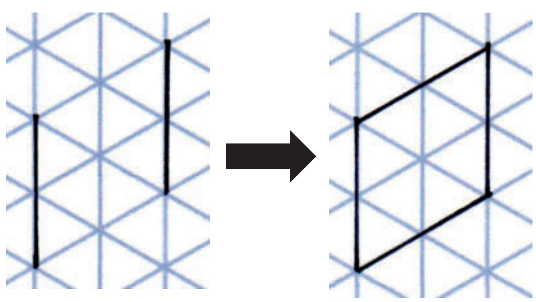
S: It's a quadrilateral because it has four straight sides. → It's a trapezoid because it has parallel sides. → It's a parallelogram because it has two sets of parallel sides. → It's like a square because it has equal sides, but it doesn't have right angles.

T: This is a rhombus, a parallelogram with four equal sides. Shade a larger rhombus on your triangular grid paper. (Allow students time to complete the task.)



T: Now that you have shaded two rhombuses, draw a pair of parallel segments that are the same length. Draw two more segments that are parallel and the same length. They must be drawn so that the endpoints of all four segments are connected. Now, you have a rhombus.

Circulate as the students draw, supporting them to construct a rhombus beginning with a pair of equal parallel sides.



Problem 2: Construct a rectangle on a triangular grid.

T: On the triangular grid, begin the construction of a rectangle just as we did yesterday with a pair of parallel segments. To begin, locate a pair of parallel lines on the triangular grid paper, and trace them. (See Step 1 below.)

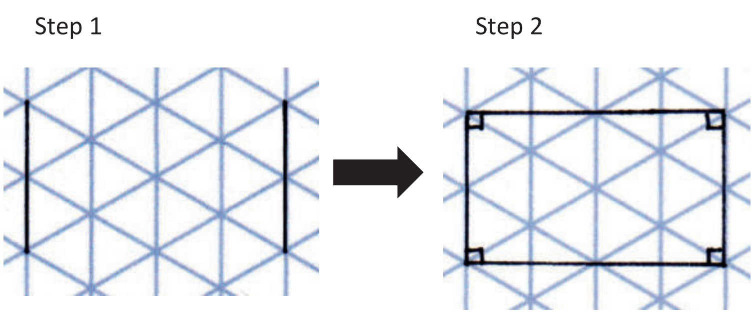
MP.5

T: Discuss with your partner: What did we do next to draw a rectangle?

S: We drew two segments perpendicular to these. → We can also draw one line that is perpendicular and then draw a line that is parallel to that third line. → I can connect the vertices on the grid paper and check to make sure the segments form four right angles.

T: Let's connect these two points on the grid. (Draw two horizontal lines.) Use your ruler and right angle template to verify the parallel lines and the perpendicular lines. Use right angle symbols. (See Step 2 below.)

S: That is tricky. I am going to try again. → Neat! I can see where I can connect vertices of the grid to form the other segments of the rectangle even though there wasn't a line on the grid to trace.



NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:

Constructing a rectangle on a triangular grid may be tricky for students working below grade level and others. Offer the following supports:

- Enlarge the grid.
- Download this triangular grid: http://gwydir.demon.co.uk/jo/tess/bigtri.htm. Students may draw segments in a word processing or drawing program, if beneficial. Offer a touch-screen (interactive white board), if available. (In the absence of virtual angle templates and straightedges, students may use the real tools with assistance.)
- Have students work in pairs. Encourage successful students to speak and demonstrate the steps.

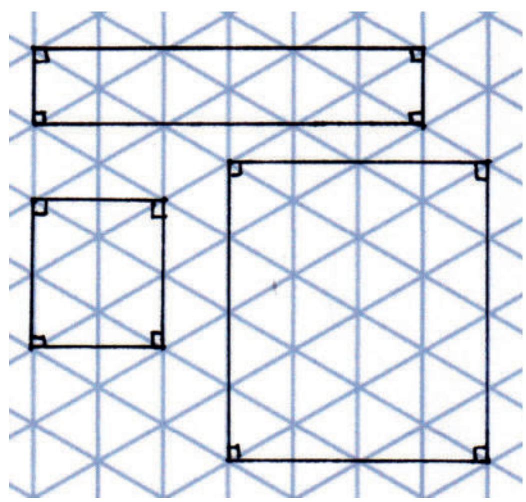
T: Try another one by beginning with drawing the perpendicular segments at the vertex of one of the equilateral triangles and extending to another vertex. Compare your rectangles with a partner's. Try another while you wait.

T: Talk to your partner about what was challenging about drawing the rectangle on the triangular grid paper.

S: (Discuss.)

T: What is another word for any parallelogram with four right angles?

S: A rectangle.



Other various rectangles

Problem 3: Construct non-rectangular parallelograms on a rectangular grid.

T: (Display a rectangular grid.) What polygons can you identify inside this rectangular grid? Shade them.

S: I see a square. → I can make a larger square by shading four smaller squares. → I see many rectangles.

T: Can you identify a parallelogram? Can you identify a trapezoid?

S: No, these lines are all perpendicular. → Squares and rectangles are parallelograms and trapezoids.

T: Visualize a parallelogram with no right angles. Let's begin constructing one. Identify a pair of parallel lines of equal lengths, and trace them. (See Step 1 below.) Next, can you use the grid to draw a third segment that cannot be traced on the rectangular grid? (See Step 2 below.)

S: Yes, I can connect these two points.

T: Now, use the grid again to connect two more points to draw the fourth segment, which is parallel to the third segment. (See Step 3 below.)

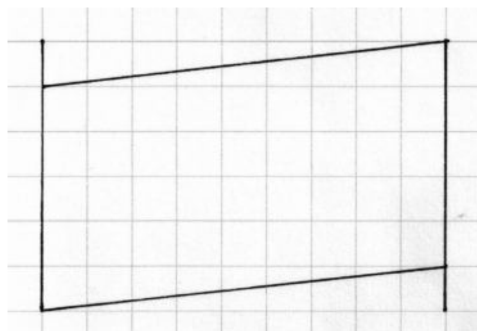
S: I think if I connect these two points here that the segments will be parallel.

T: Go ahead and try. What do you need to do to confirm the lines are parallel?

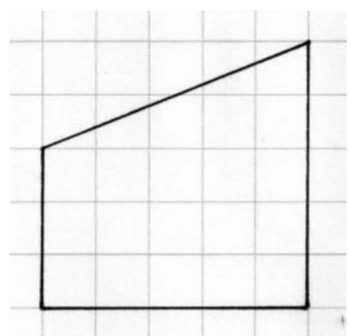
S: Use our right angle template and ruler.

T: Go ahead and do so.

Step 1



Steps 2 and 3



T: Talk to your partner about what changes could be made to your figure to make it a trapezoid that has only one set of parallel lines.

S: We could make the third and fourth segments not parallel.

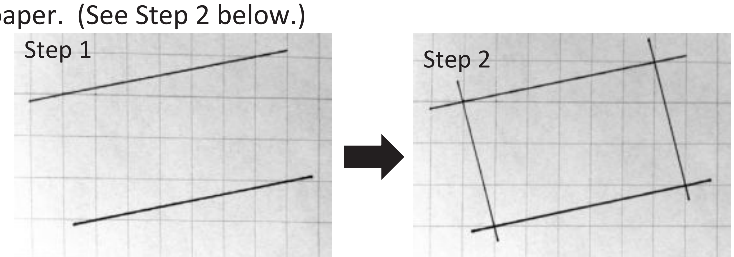
T: Construct a trapezoid that has only one set of parallel lines.

Students construct trapezoid on grid. (See example to the right.)

T: Construct another parallelogram. This time, draw your first segment from one vertex to another so that the segment does not trace the rectangular grid. (See Step 1, below.)

Students draw first segment on grid.

T: Draw a segment parallel to that. Work with your partner on how to construct the last set of parallel lines so that they do not trace the gridlines. Make sure the vertices of the parallelogram meet at the vertices of the grid paper. (See Step 2 below.)

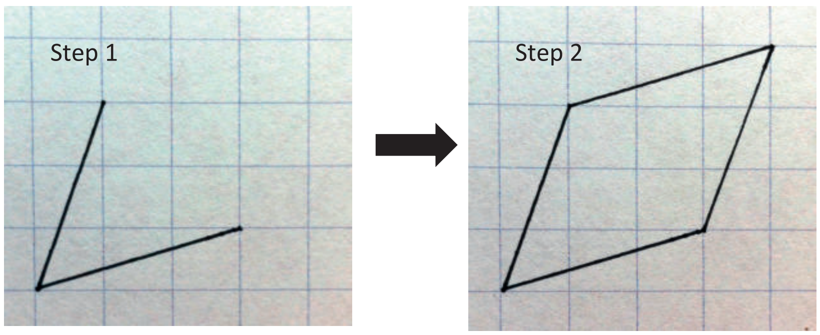


T: Discuss with your partner the challenges that you faced during the construction of this parallelogram.

S: It was tricky with the lines of the parallelogram not on the gridlines. → I had to make sure to use my tools to help me draw parallel lines. → I saw a pattern in the grid to help me draw the lines.

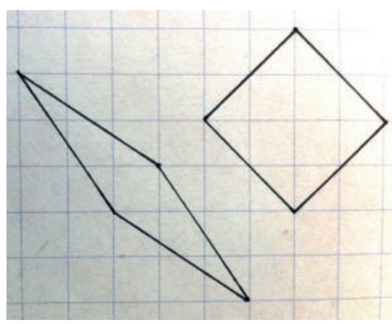
Extension: Construct rhombuses on a rectangular grid.

Draw the figure below step by step for students without identifying it as a rhombus. Ask them to copy it, and then, using their tools, verify what shape it is.



S: We drew a rhombus! All the sides measure the same, and it has two sets of parallel sides.

Have students draw two more rhombuses of different sizes.



Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

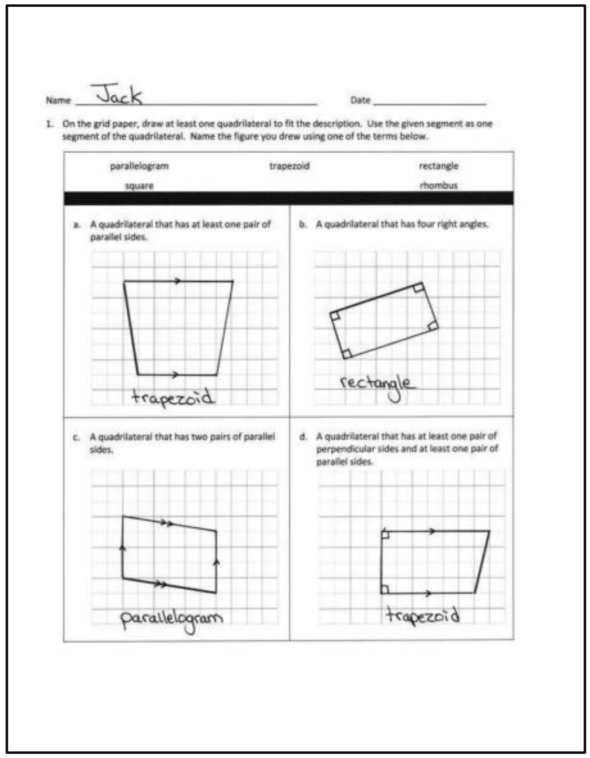
Lesson Objective: Reason about attributes to construct quadrilaterals on square or triangular grid paper.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What figure did you draw in Problem 1(a)? Why are there so many different shapes that can be constructed?
- How did the gridlines in Problem 1(b) help you to draw the right angles?
- How are the shapes in Problems 2(a) and 2(b) similar and different?
- How are the attributes of a rhombus and a rectangle similar? What two attributes distinguish a rhombus from a rectangle in Problem 3?
- Which grid is more challenging for you, the triangular or the square grid? Explain which quadrilaterals are easiest for you to draw on either grid. Why do you think that is so?



Name Jack Date

1. On the grid paper, draw at least one quadrilateral to fit the description. Use the given segment as one segment of the quadrilateral. Name the figure you drew using one of the terms below.

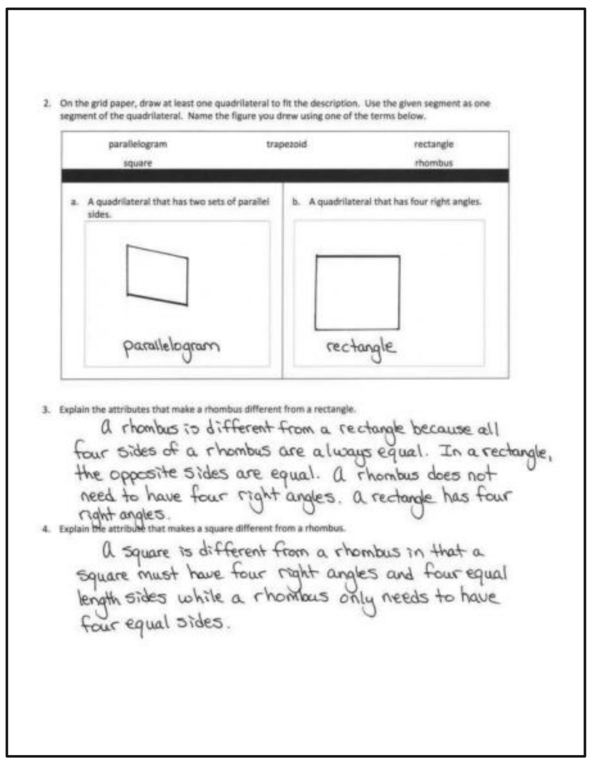
parallelogram trapezoid rectangle square rhombus

a. A quadrilateral that has at least one pair of parallel sides. trapezoid

b. A quadrilateral that has four right angles. rectangle

c. A quadrilateral that has two pairs of parallel sides. parallelogram

d. A quadrilateral that has at least one pair of perpendicular sides and at least one pair of parallel sides. trapezoid



2. On the grid paper, draw at least one quadrilateral to fit the description. Use the given segment as one segment of the quadrilateral. Name the figure you drew using one of the terms below.

parallelogram trapezoid rectangle square rhombus

a. A quadrilateral that has two sets of parallel sides. parallelogram

b. A quadrilateral that has four right angles. rectangle

3. Explain the attributes that make a rhombus different from a rectangle.

A rhombus is different from a rectangle because all four sides of a rhombus are always equal. In a rectangle, the opposite sides are equal. A rhombus does not need to have four right angles. A rectangle has four right angles.

4. Explain the attribute that makes a square different from a rhombus.

A square is different from a rhombus in that a square must have four right angles and four equal length sides while a rhombus only needs to have four equal sides.

Exit Ticket (3 minutes)

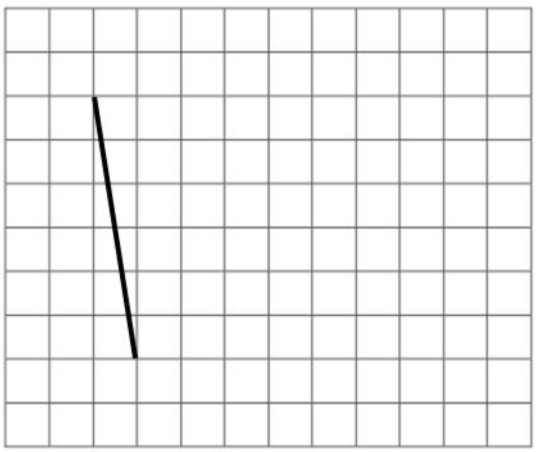
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ______________________________________ Date ____________________

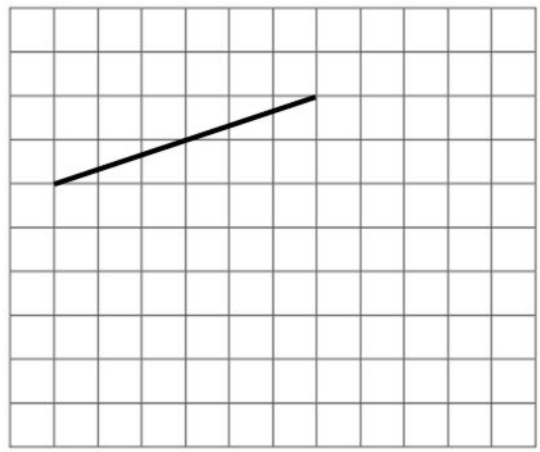
1. On the grid paper, draw at least one quadrilateral to fit the description. Use the given segment as one segment of the quadrilateral. Name the figure you drew using one of the terms below.

Parallelogram	Trapezoid	Rectangle
Square		Rhombus

a. A quadrilateral that has at least one pair of parallel sides.

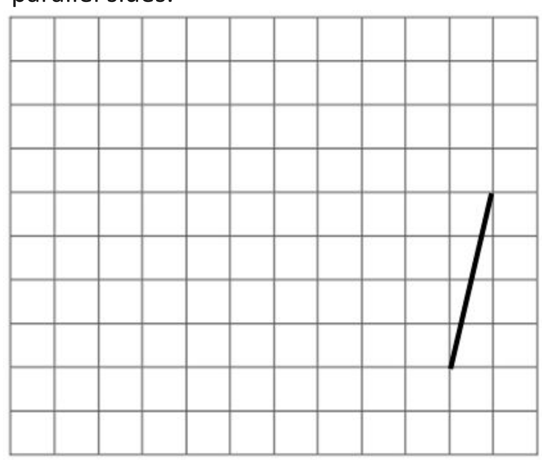


b. A quadrilateral that has four right angles.



c. A quadrilateral that has two pairs of parallel side

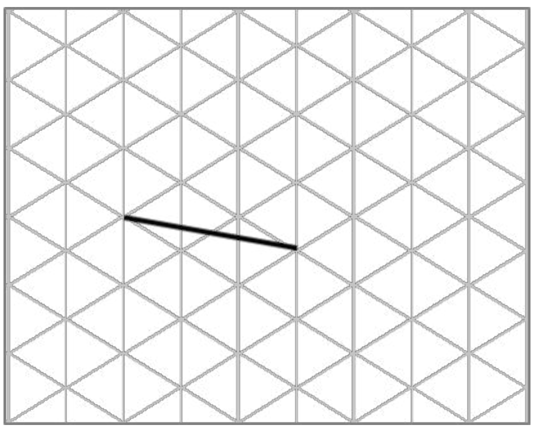
d. A quadrilateral that has at least one pair of perpendicular sides and at least one pair of parallel sides.



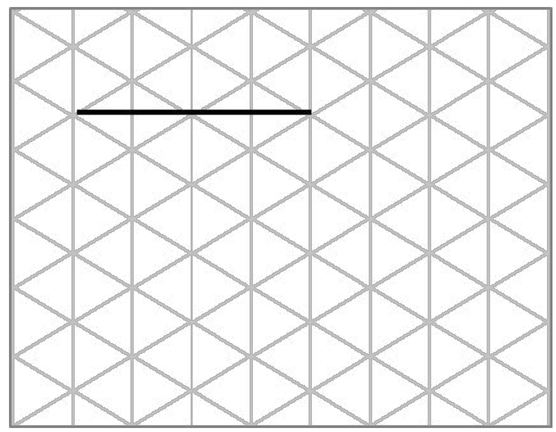
2. On the grid paper, draw at least one quadrilateral to fit the description. Use the given segment as one segment of the quadrilateral. Name the figure you drew using one of the terms below.

Parallelogram	Trapezoid	Rectangle
Square		Rhombus

a. A quadrilateral that has two sets of parallel sides.



b. A quadrilateral that has four right angles.

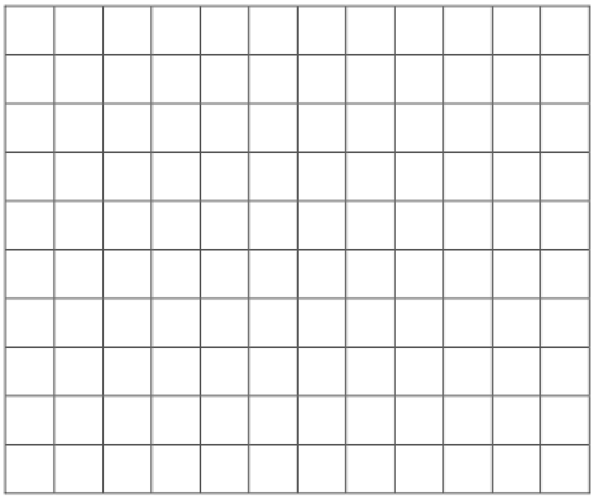


3. Explain the attributes that make a rhombus different from a rectangle.

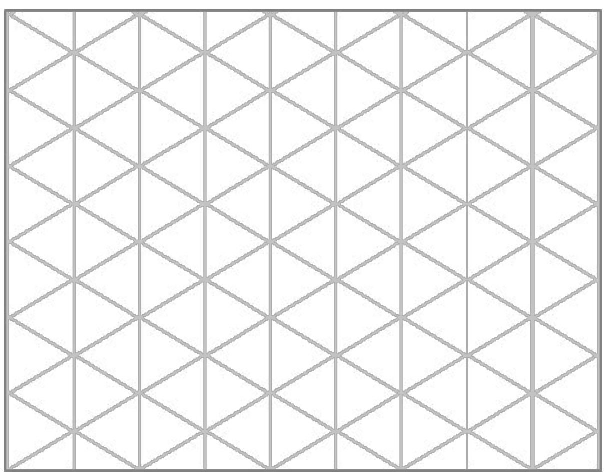
4. Explain the attribute that makes a square different from a rhombus.

Name ____ Date ______________________

1. Construct a parallelogram that does not have any right angles on a rectangular grid.



2. Construct a rectangle on a triangular grid.



Name ____ Date ______________________

Use the grid to construct the following. Name the figure you drew using one of the terms in the word box.

WORD BOX

Parallelogram

Trapezoid

Rectangle

Square

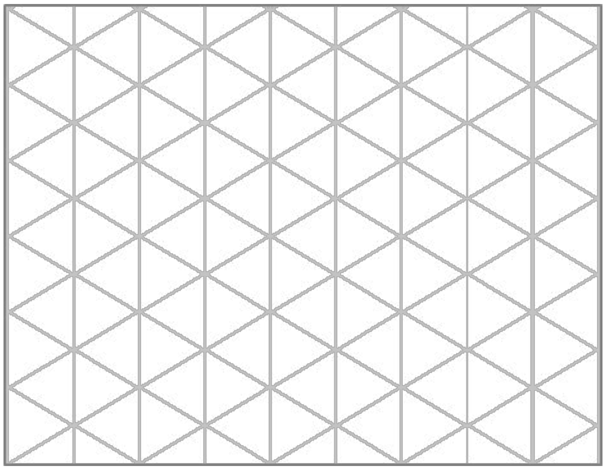
Rhombus

1. Construct a quadrilateral with only one set of parallel sides.
   Which shape did you create?

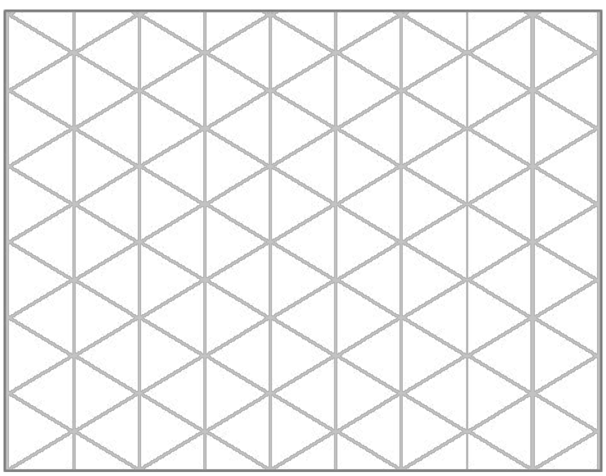
2. Construct a quadrilateral with one set of parallel sides and two right angles.
   Which shape did you create?

3. Construct a quadrilateral with two sets of parallel sides.
   Which shape did you create?

4. Construct a quadrilateral with all sides of equal length.
   Which shape did you create?



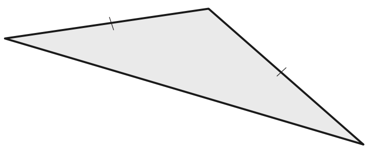
5. Construct a rectangle with all sides of equal length.
   Which shape did you create?



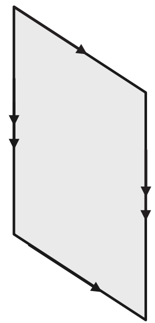
Name ____ Date ____________________

1. Find and draw all lines of symmetry in the following figures. If there are none, write "none."

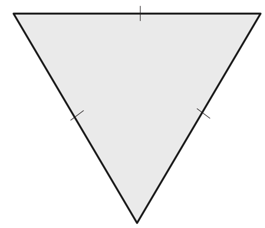
a.



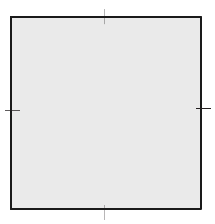
b.



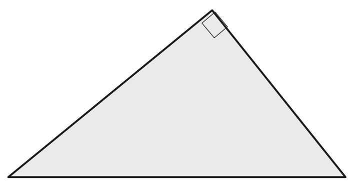
c.



d.



e.



f.

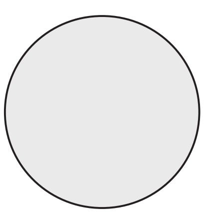
g. For each triangle listed below, state whether it is acute, obtuse, or right and whether it is isosceles, equilateral, or scalene.

Triangle a: ________________________ ________________________

Triangle c: ________________________ ________________________

Triangle e: ________________________ ________________________

h. How many lines of symmetry does a circle have? What point do all lines of symmetry for a given circle have in common?



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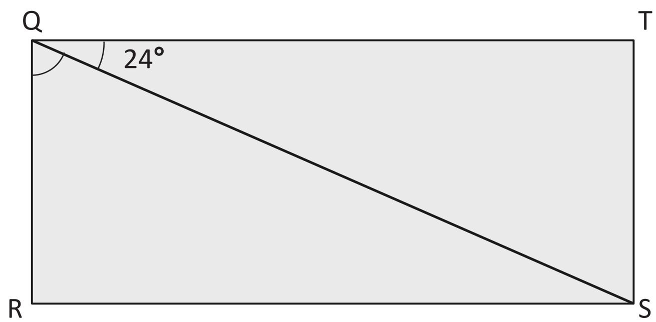
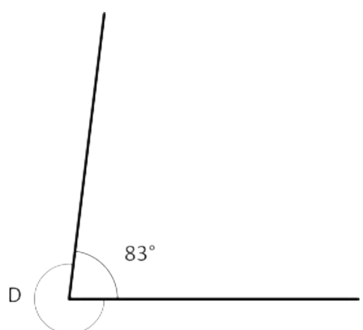
2. In the following figure, QRST is a rectangle. Without using a protractor, determine the measure of ∠RQS. Write an equation that could be used to solve the problem.

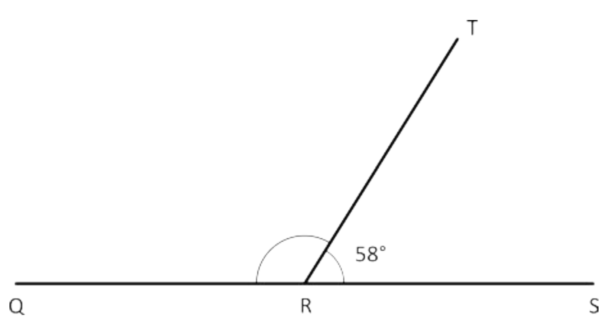


For each part below, explain how the measure of the unknown angle can be found without using a protractor.

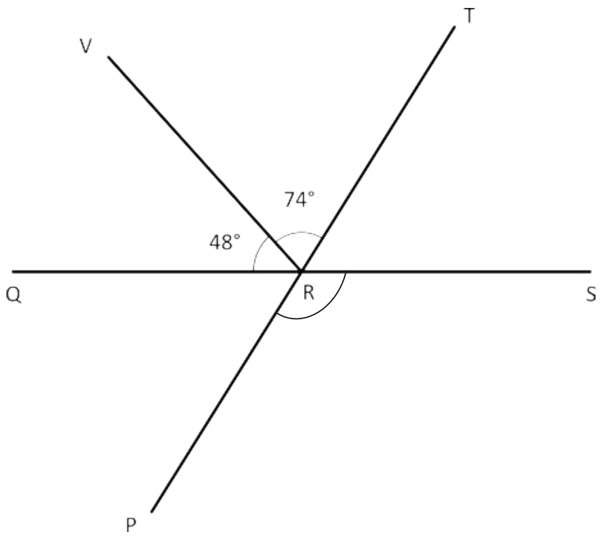
a. Find the measure of ∠D.



b. In this figure, Q, R, and S lie on a line. Find the measure of ∠*QRT*.



c. In this figure, Q, R, and S lie on a line, as do P, R, and T. Find the measure of ∠*PRS.*



3. Mike drew some two-dimensional figures.

Sketch the figures, and answer each part about the figures that Mike drew.

a. He drew a four-sided figure with four right angles. It is 4 cm long and 3 cm wide.

What type of quadrilateral did Mike draw?

How many lines of symmetry does it have?

b. He drew a quadrilateral with four equal sides and no right angles.

What type of quadrilateral did Mike draw?

How many lines of symmetry does it have?

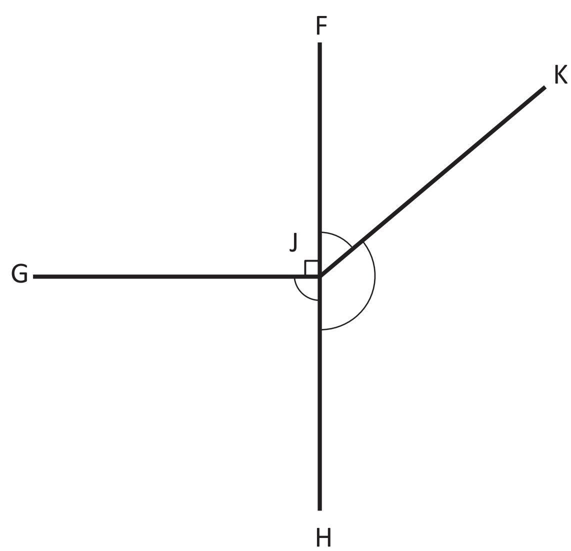
c. He drew a triangle with one right angle and sides that measure 6 cm, 8 cm, and 10 cm.

Classify the type of triangle Mike drew based on side length and angle measure.

How many lines of symmetry does it have?

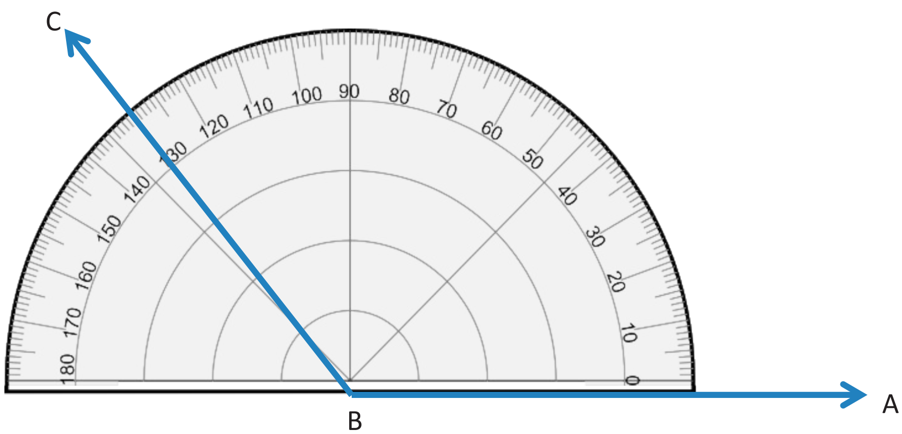
d. Using the dimensions given, draw the same shape that Mike drew in Part (c).

e. Mike drew this figure. Without using a protractor, find the sum of *∠FJK, ∠KJH, and ∠HJG.*

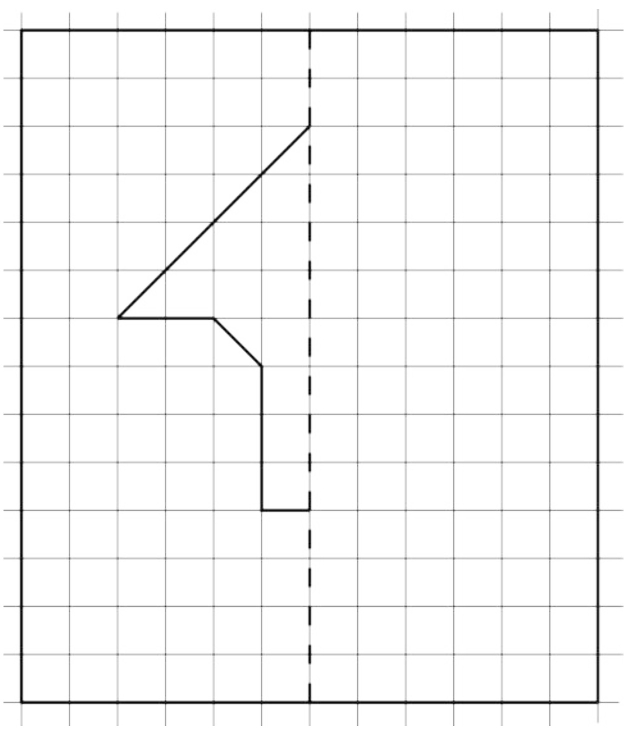


f. Points F, J, and H lie on a line. What is the measure of *∠KJH* if *∠FJK* measures 45°? Write an equation that could be used to determine the measure of *∠KJH.*

g. Mike used a protractor to measure ∠*ABC* as shown below and said the result was exactly 130°. Do you agree or disagree? Explain your thinking.



h. Below is half of a line-symmetric figure and its line of symmetry. Use a ruler to complete Mike's drawing.



End-of-Module Assessment Task Standards Addressed	Topics A–D

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right angles as a category, and identify right triangles.

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop on their way to proficiency. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	**STEP 1 Little evidence of reasoning without a correct answer. (1 Point)**	**STEP 2 Evidence of some reasoning without a correct answer. (2 Points)**	**STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)**	**STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)**
1 **4.G.2** **4.G.3**	The student correctly answers fewer than five of the eight parts and shows little to no reasoning.	The student correctly completes at least five of the parts but shows little evidence of reasoning in Part (h).	The student correctly completes six or seven of the eight parts, providing sufficient reasoning in Part (h). Or, the student answers all parts correctly but without solid reasoning in Part (h).	The student correctly draws all lines of symmetry, identifies figures with *none*, and answers Parts (g) and (h). a. 1 line. b. None. c. 3 lines. d. 4 lines. e. None. f. 2 lines. g. Triangle *a* is obtuse and isosceles. Triangle *c* is acute and equilateral. Triangle *e* is right and scalene. h. A circle has an infinite number of lines of symmetry. All lines of symmetry for a circle share the center point.

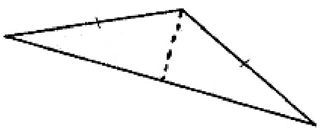
A Progression Toward Mastery				
2 **4.MD.7**	The student incorrectly determines the measure of ∠*RQS* and provides little to no reasoning.	The student shows some evidence of a correct equation or adequate reasoning but miscalculates the angle measure.	The student correctly identifies 66° with some evidence of a correct equation or adequate reasoning. Or, the student uses reasoning and an equation correctly but miscalculates the angle measure.	The student correctly identifies that ∠*RQS* and ∠*TQS* total 90°, so ∠*RQS* measures 66°, and includes an equation such as 24 + w = 90.
3 **4.MD.5** **4.MD.6** **4.MD.7**	The student correctly answers fewer than three parts, providing no reasoning.	The student correctly answers at least one of the three parts, providing little reasoning.	The student correctly finds the measures for two of the three parts, providing solid reasoning. Or, the student solves correctly all three parts but only provides some reasoning.	The student correctly answers all three parts with solid reasoning: a. ∠*D* = 277°. The number of degrees in a circle is 360, so ∠*D* is the difference between 83 and 360. b. ∠*QRT* = 122°. A line equals 180°, so ∠*QRT* must be equal to the difference between 180 and 58. c. ∠*PRS* = 122°. The measure of ∠*TRS* using $\overline{QRS}$ or ∠*QRP* using $\overline{PRT}$ is 58°, making ∠*PRS* equal to the difference between 180 and 58. The students may also determine that ∠*PRS* is equal to ∠*QRT* because of the two intersecting lines creating vertical angles. ∠*QRV* + ∠*VRT* = 122°. (Referencing vertical angles, although not necessary, is acceptable.)

A Progression Toward Mastery				
4 **4.MD.5** **4.MD.6** **4.MD.7** **4.G.1** **4.G.2** **4.G.3**	The student correctly answers fewer than four of the eight parts.	The student correctly answers four or five of the eight parts.	The student correctly answers six or seven of the eight parts.	The student correctly answers all eight parts: a. Rectangle: 2 lines. b. Rhombus: 2 lines. c. Right, scalene triangle: No lines. d. Drawing depicts a right triangle with sides measuring 6 cm, 8 cm, and 10 cm. e. 270°. f. 135°: 45 + b = 180 or 180 – 45 = b. g. Mike lined the bottom ray up with the bottom edge of the protractor and not with the line that measures to zero. h. Drawing depicts a line-symmetric figure.

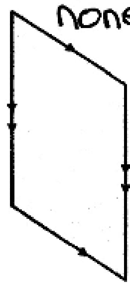
Name Jack Date ________

1. Find and draw all lines of symmetry in the following figures. If there are none, write "none."

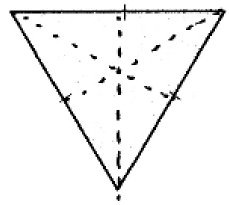
a.



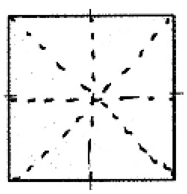
b. none



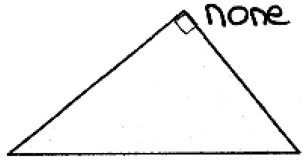
c.



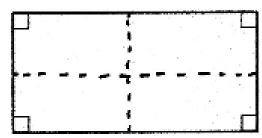
d.



e. none



f.



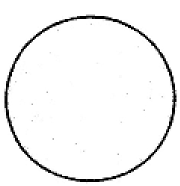
g. For each triangle listed below, state whether it is acute, obtuse, or right and whether it is isosceles, equilateral, or scalene.

Triangle a: obtuse isosceles

Triangle c: acute equilateral

Triangle e: right scalene

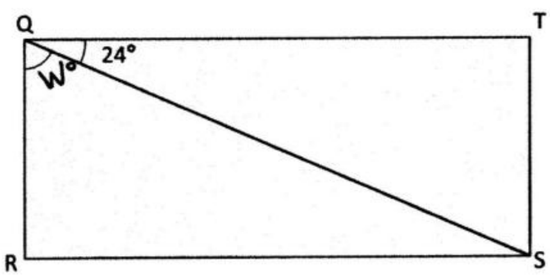
h. How many lines of symmetry does a circle have? What point do all lines of symmetry for a given circle have in common?



A circle has an infinite amount of lines of symmetry. All lines of symmetry for a circle pass through the center point.

2. In the following figure, QRST is a rectangle. Without using a protractor, determine the measure of ∠RQS.

   Write an equation that could be used to solve the problem.



$24° + W° = 90°$

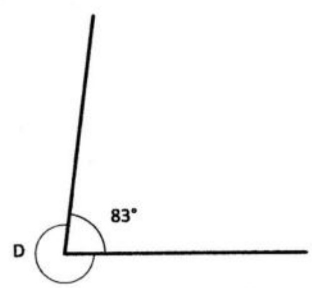
$90 - 24 = 66$

$W° = 66°$

∠RQS = 66°

3. For each part below, explain how the measure of the unknown angle can be found without using a protractor.

   a. Find the measure of ∠D.



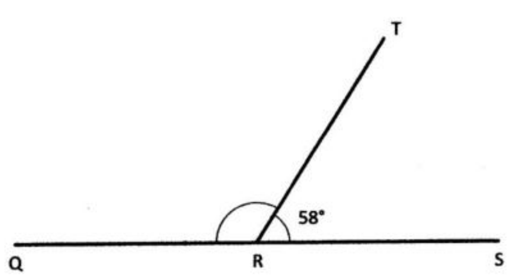
$360 - 83 = 277$

$83° + ∠D = 360°$

$∠D = 277°$

∠D is 277°. A circle measures 360°. If one angle is 83°, the other angle is the difference.

   b. In this figure, Q, R, and S lie on a line. Find the measure of ∠*QRT*.



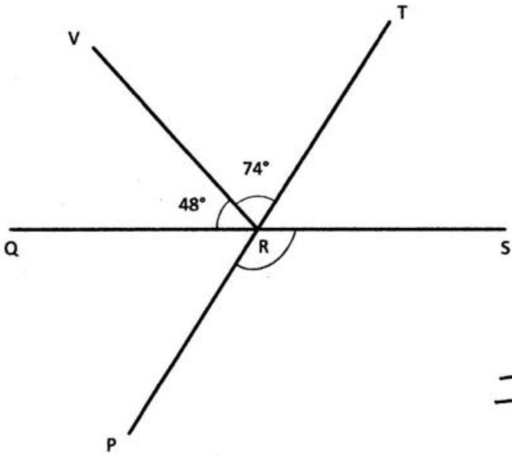
$58° + ∠QRT = 180°$

$180 - 58 = 122$

$∠QRT = 122°$

∠QRT is 122°. I know that because a line measures 180°, so ∠QRT and ∠TRS have to add to 180°.

c. In this figure, Q, R, and S lie on a line, as do P, R, and T. Find the measure of ∠*PRS*.



$48° + 74° + \angle TRS = 180°$

$48 + 74 = 122$ $\quad$ $180 - 122 = 58$ $\quad$ $\angle TRS = 58°$

$\angle TRS + \angle PRS = 180°$

$58° + \angle PRS = 180°$

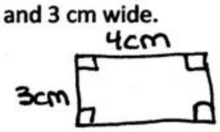
$\angle PRS = 122°$ $\quad$ $180 - 58 = 122$

Since Q, R, and S lie on a line, I know 48° + 74° + <TRS = 180°. That means <TRS = 58°. Since P, R, and T lie on a line, I know <TRS + <PRS = 180°. That means <PRS = 122°.

4. Mike drew some two-dimensional figures.

Sketch the figures and answer each part about the figures that Mike drew.

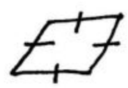
a. He drew a four-sided figure with four right angles. It is 4 cm long and 3 cm wide.



What type of quadrilateral did Mike draw? rectangle

How many lines of symmetry does it have? 2 lines of symmetry

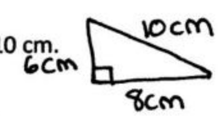
b. He drew a quadrilateral with four equal sides and no right angles.



What type of quadrilateral did Mike draw? rhombus

How many lines of symmetry does it have? 2 lines of symmetry

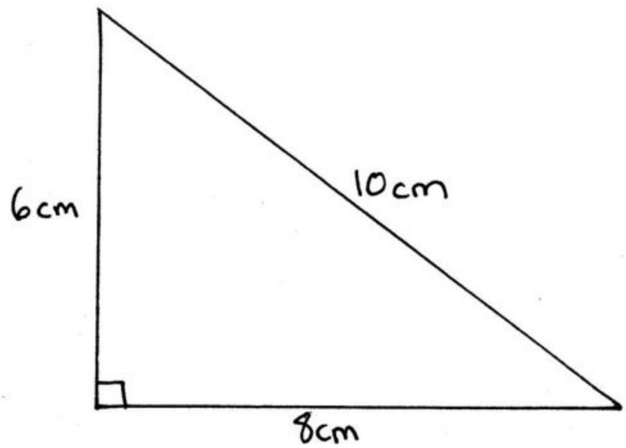
c. He drew a triangle with one right angle and sides that measure 6 cm, 8 cm, and 10 cm.



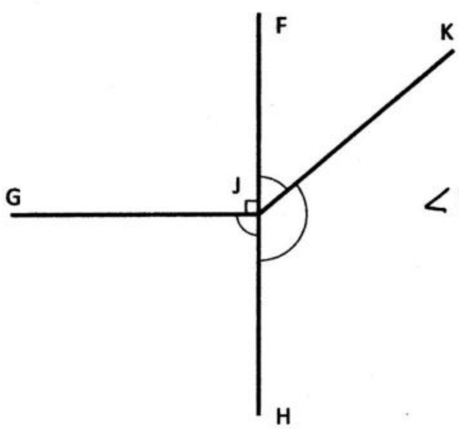
Classify the type of triangle Mike drew based on side length and angle measure. right triangle, scalene triangle

How many lines of symmetry does it have? no lines of symmetry

d. Using the dimensions given, draw the same shape Mike that drew in Part (c).



e. Mike drew this figure. Without using a protractor, find the sum of *∠FJK, ∠KJH, and ∠HJG.*



$360° - 90° = 270°$

$\angle FJK + \angle KJH + \angle HJG = 270°$

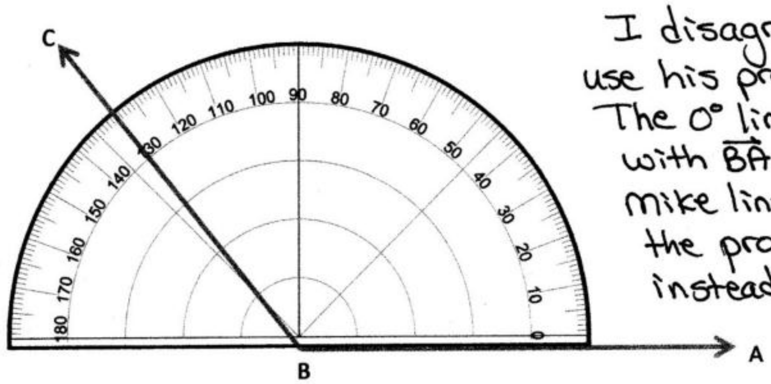
f. Points F, J, and H lie on a line. What is the measure of *∠KJH* if *∠FJK* measures 45°? Write an equation that could be used to determine the measure of *∠KJH.*

$45° + \angle KJH = 180°$

$$\begin{array}{r} 180 \\ -\ 45 \\ \hline 135 \end{array}$$

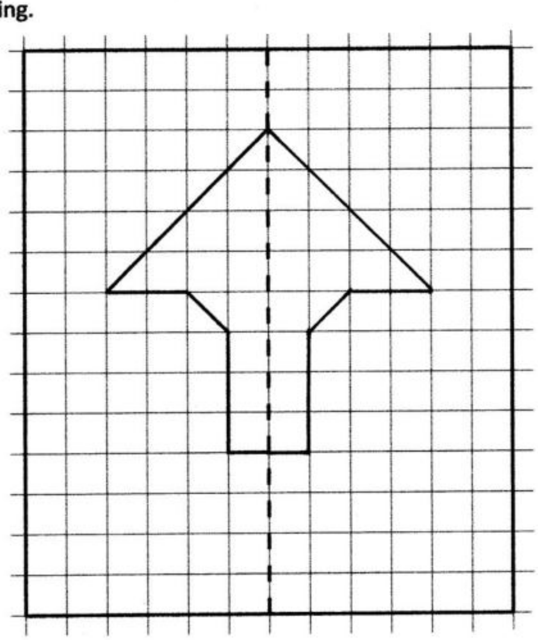
$\angle KJH = 135°$

g. Mike used a protractor to measure ∠*ABC* as shown below and said the result was exactly 130°. Do you agree or disagree? Explain your thinking.



I disagree. Mike didn't use his protractor correctly. The 0° line should match up with $\overrightarrow{BA}$, but it doesn't. Mike lined up the bottom of the protractor with $\overrightarrow{BA}$ instead.

h. Below is half of a line-symmetric figure and its line of symmetry. Use a ruler to complete Mike's drawing.



Answer Key

GRADE 4 • MODULE 4

Angle Measure and Plane Figures

Lesson 1

Problem Set

1. a. – f. Figure drawn accurately.
   g. Answers will vary.
2. a. – g. Figure drawn accurately.
   h. Answers will vary.
3. a. Points labeled; labels will vary.
   b. Answers will vary.

Exit Ticket

1. Words connect to corresponding pictures.
2. Answers will vary.

Homework

1. a. – f. Figure drawn accurately.
   g. Answers will vary.
2. a. – g. Figure drawn accurately.
   h. Answers will vary.
3. a. Points labeled; labels will vary.
   b. Answers will vary.

Lesson 2

Problem Set

1. a. Answer provided
   b. Less than; acute
   c. Equal to; right
   d. Greater than; obtuse
   e. Greater than; obtuse
   f. Equal to; right
   g. Greater than; obtuse
   h. Greater than; obtuse
   i. Less than; acute
   j. Less than; acute

2. Angles accurately identified and traced; points labeled; angles named; answers will vary.

3. a. Acute angle constructed; less than a right angle
   b. Right angle constructed; equal to a right angle
   c. Obtuse angle constructed; greater than a right angle

Exit Ticket

1. a. Right
   b. Acute
   c. Obtuse

2. a. C, G
   b. B, E
   c. A, D
   d. F, H

Homework

1. a. Answer provided
   b. Equal to; right
   c. Greater than; obtuse
   d. Greater than; obtuse
   e. Less than; acute
   f. Greater than; obtuse
   g. Equal to; right
   h. Less than; acute
   i. Greater than; obtuse
   j. Equal to; right

2. Angles accurately identified and traced; points labeled; angles named; answers will vary.

3. a. Acute angle constructed; less than a right angle
   b. Right angle constructed; equal to a right angle
   c. Obtuse angle constructed; greater than a right angle

Lesson 3

Problem Set

1. Perpendicular lines accurately traced.
2. Answers will vary.
3. Perpendicular segments accurately drawn.
4. a. Right angles accurately identified and marked; $\overline{BD} \perp \overline{CD}$; $\overline{CD} \perp \overline{CA}$; $\overline{CA} \perp \overline{AB}$
   b. No right angles.
   c. Right angle accurately identified and marked; $\overline{GE} \perp \overline{EF}$
   d. No right angles.
   e. Right angles accurately identified and marked; $\overline{AW} \perp \overline{WF}$; $\overline{WF} \perp \overline{FZ}$; $\overline{FZ} \perp \overline{ZA}$; $\overline{AZ} \perp \overline{AW}$
   f. No right angles.
   g. No right angles.
   h. Right angles accurately identified and marked; $\overline{VW} \perp \overline{WX}$; $\overline{WX} \perp \overline{XY}$; $\overline{YU} \perp \overline{UV}$
5. Right angles accurately identified and marked; 12 perpendicular pairs.
6. True; explanations will vary.

Exit Ticket

1. Right angles accurately identified and marked; $\overline{BC} \perp \overline{CD}$; $\overline{CD} \perp \overline{DE}$; $\overline{BA} \perp \overline{AE}$
2. Right angles accurately identified and marked; $\overline{MN} \perp \overline{MP}$

Homework

1. Perpendicular lines accurately traced.
2. Answers will vary.
3. Perpendicular segments accurately drawn.
4. a. Right angles accurately identified and marked; $\overline{AB} \perp \overline{BD}$; $\overline{BD} \perp \overline{DC}$; $\overline{AC} \perp \overline{CD}$
   b. No right angles.
   c. Right angle accurately identified and marked; $\overline{DO} \perp \overline{OG}$
   d. No right angles.
   e. No right angles.
   f. Right angles accurately identified and marked; $\overline{PO} \perp \overline{ON}$; $\overline{ON} \perp \overline{NM}$; $\overline{NM} \perp \overline{MP}$; $\overline{MP} \perp \overline{PO}$
   g. No right angles.
   h. Right angles accurately identified and marked; $\overline{UT} \perp \overline{TZ}$; $\overline{TZ} \perp \overline{ZY}$; $\overline{ZY} \perp \overline{YX}$; $\overline{YX} \perp \overline{XW}$; $\overline{WV} \perp \overline{VU}$
5. Right angles accurately identified and marked; 8 perpendicular pairs
6. True; explanations will vary.

Lesson 4

Problem Set

1. Parallel lines accurately traced
2. Answers will vary.
3. Parallel segments accurately drawn
4. a. Sides accurately identified and marked with arrows; $\overline{AC} \parallel \overline{BD}$
   b. Circled; sides accurately identified and marked with arrows; $\overline{HI} \parallel \overline{JK}$
   c. No parallel sides
   d. No parallel sides
   e. Circled; sides accurately identified and marked with arrows; $\overline{ZA} \parallel \overline{FW}$; $\overline{ZF} \parallel \overline{AW}$
   f. No parallel sides
   g. Circled; sides accurately identified and marked with arrows; $\overline{TO} \parallel \overline{RQ}$; $\overline{ST} \parallel \overline{QP}$; $\overline{RS} \parallel \overline{OP}$
   h. Circled; sides accurately identified and marked with arrows; $\overline{YX} \parallel \overline{VW}$
5. True; explanations will vary.
6. Explanations will vary.
7. Parallel lines constructed

Exit Ticket

1. Parallel
2. Perpendicular
3. Intersecting
4. Intersecting

Homework

1. Parallel lines accurately traced
2. Answers will vary.
3. Parallel segments accurately drawn
4. a. Sides accurately identified and marked with arrows; $\overline{AB} \parallel \overline{CD}$
   b. Circled; sides accurately identified and marked with arrows; $\overline{HI} \parallel \overline{JK}$
   c. No parallel sides
   d. No parallel sides
   e. No parallel sides
   f. Circled; sides accurately identified and marked with arrows; $\overline{OP} \parallel \overline{MN}$; $\overline{ON} \parallel \overline{PM}$
   g. Circled; sides accurately identified and marked with arrows; $\overline{TU} \parallel \overline{RQ}$; $\overline{ST} \parallel \overline{QP}$; $\overline{SR} \parallel \overline{UP}$
   h. Circled; sides accurately identified and marked with arrows; $\overline{TZ} \parallel \overline{XY}$; $\overline{TU} \parallel \overline{ZY}$; $\overline{WX} \parallel \overline{ZY}$
5. False; explanations will vary.
6. Explanations will vary.
7. Parallel lines constructed

Lesson 5

Problem Set

1. a. 135°, 180°, 225°, 270°, 315°, 360°

   b. 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, 330°, 360°

2. 90°, 180°, 270°, 360°; answers will vary.

3. 30°, 45°, 60°

4. 120°, 135°, 150°

5. $\frac{30}{360}, \frac{45}{360}, \frac{60}{360}, \frac{90}{360}, \frac{120}{360}, \frac{135}{360}, \frac{150}{360}, \frac{180}{360}, \frac{210}{360}, \frac{225}{360}, \frac{240}{360}, \frac{270}{360}, \frac{300}{360}, \frac{315}{360}, \frac{330}{360}, \frac{360}{360}$

6. 8

7. 12

8. Explanations will vary.

Exit Ticket

1. 4

2. 90°

3. $\frac{1}{360}$

4. Answers will vary.

Homework

1. a. 60°

   b. 130°

   c. 315°

   d. 120°

2. Explanations will vary.

Lesson 6

Problem Set

1. a. 36°
   b. 36°
   c. 90°
   d. 90°
   e. 36°
   f. 155°
   g. 155°
   h. 90°
   i. 90°
   j. 150°
2. a. 29°, 29°, 29°
   b. Answers will vary.
3. a. 180°
   b. 178°; explanations will vary.

Exit Ticket

1. 135°
2. 150°
3. 37°
4. 90°

Homework

1. a. 67°
   b. 78°
   c. 32°
   d. 60°
   e. 105°
   f. 153°
   g. 135°
   h. 65°
   i. 45°
   j. 118°
2. Explanations will vary.
3. a. 172°
   b. 180°; explanations will vary.

Lesson 7

Problem Set

1. 30° angle constructed
2. 65° angle constructed
3. 115° angle constructed
4. 135° angle constructed
5. 5° angle constructed
6. 175° angle constructed
7. 27° angle constructed
8. 117° angle constructed
9. 48° angle constructed
10. 132° angle constructed

Exit Ticket

1. 75° angle constructed
2. 105° angle constructed
3. 81° angle constructed
4. 99° angle constructed

Homework

1. 25° angle constructed
2. 85° angle constructed
3. 140° angle constructed
4. 83° angle constructed
5. 108° angle constructed
6. 72° angle constructed
7. 25° angle constructed
8. 155° angle constructed
9. 45° angle constructed
10. 135° angle constructed

Lesson 8

Problem Set

1. Fence, tree, barn
2. 270°
3. Full turn
4. Towards his house
5. Picture shows a 270° turn.
6. 4 quarter turns
7. 1 counter-clockwise or 3 clockwise quarter turns
8. West

Exit Ticket

1. 180°
2. East

Homework

1. House, fence, house
2. 360°
3. Opposite direction; explanations will vary.
4. Full turn
5. Picture shows a 180° turn.
6. 4 quarter turns
7. 2 quarter turns
8. West

Lesson 9

Problem Set

1. a. 4; 4, 90°; 90°, 90°, 90°, 90°
   b. 6; 360° ÷ 6 = 60°; 60°+ 60° + 60° + 60° +60° + 60° = 360°
   c. 3; 360° ÷ 3 = 120°; 120°, 120°, 120°
   d. 6; 360° ÷ 6 = 60°; 60°+ 60° + 60° + 60° +60° + 60° = 360°
   e. 3; 360° ÷ 3 = 120°; 120° + 120° + 120° = 360°
   f. 12; 360° ÷ 12 = 30°; 30° + 30° + 30° + 30° + 30° + 30° + 30° + 30° + 30° + 30° + 30° + 30° = 360°
2. a. 150°; 60° + 90° = 150°
   b. 180°; 60° + 120° = 180°
   c. 210°; 120° + 90° = 210°
3. a. 60°; 30° + 30° = 60°
   b. 210°; 120° + 90° = 210°
   c. 120°; 90° + 30° = 120°

Exit Ticket

1. Answers will vary.
2. Answers will vary.

Homework

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. Answers will vary.
5. a. Answer provided
   b. 30° + 60°; 90°
   c. 120° + 60° + 30°; 210°

Lesson 10

Problem Set

1. 45°; 45°
2. 20°, 70°, 90°; 70°
3. 110°; 110°
4. 83°, 97°, 180°; 97°
5. Equations will vary; 54°
6. Equations will vary; 12°
7. Equations will vary; 63°
8. a. – d. Figure accurately constructed.
   e. Answers will vary.
   f. Equations will vary.

Exit Ticket

Equations will vary; 60°

Homework

1. 55°; 55°
2. 62° + 28° = 90°; 28°
3. 35°; 35°
4. 16°, 164°, 180°; 164°
5. Equations will vary; 75°
6. Equations will vary; 35°
7. Equations will vary; 16°
8. a. – d. Figure accurately constructed.
   e. Answers will vary.
   f. Equations will vary.

Lesson 11

Problem Set

1. 340; 340
2. 270, 90; 270
3. 74, 90, 196, 360; 196
4. 90, 160, 110; 360; 110
5. Equations will vary; 160°; 20°
6. Equations will vary; 55°; 125°; 55°
7. Equations will vary; 36°; 54°; 144°

Exit Ticket

1. Equations will vary; 24°
2. Equations will vary; 156°
3. Equations will vary; 24°

Homework

1. 40; 40
2. 45, 315; 315
3. 115, 100, 145, 360; 145
4. 135, 145, 80, 360; 80
5. Equations will vary; 145°; 35°
6. Equations will vary; 125°; °125°; 55°
7. Equations will vary; 44°; 46°; 134°

Lesson 12

Problem Set

1. (a), (b), and (d) circled
2. a. Line of symmetry accurately drawn; 1
   b. Lines of symmetry accurately drawn; 4
   c. 0
   d. Lines of symmetry accurately drawn; 6
   e. Line of symmetry accurately drawn; 1
   f. 0
   g. Line of symmetry accurately drawn; 1
   h. Line of symmetry accurately drawn; 1
   i. Lines of symmetry accurately drawn; 4
3. Symmetric figures accurately drawn
4. Infinite; explanations will vary.

Exit Ticket

1. No; yes; no
2. 4 lines of symmetry accurately drawn

Homework

1. (a) and (c) circled
2. a. Line of symmetry accurately drawn; 1
   b. Lines of symmetry accurately drawn; 4
   c. Lines of symmetry accurately drawn; 8
   d. Line of symmetry accurately drawn; 5
   e. 0
   f. 0
   g. Lines of symmetry accurately drawn; 2
   h. Line of symmetry accurately drawn; 1
   i. Line of symmetry accurately drawn; 1
3. Symmetric figures accurately drawn
4. No; explanations will vary.

Lesson 13

Problem Set

1. a. Isosceles; obtuse
   b. Equilateral; acute
   c. Scalene; right
   d. Scalene; obtuse
2. $\angle A = \angle C$; explanations will vary.
3. a. Answers will vary.
   b. Each side length labeled as 10 cm
4. Answers will vary.
5. a. G, I, H
   b. Answers will vary.
6. No; explanations will vary.

Exit Ticket

1. Acute; isosceles; right
2. a. Right, scalene
   b. Obtuse, isosceles
   c. Acute, equilateral
3. Lines of symmetry accurately drawn in triangles (b) and (c)

Homework

1. a. Scalene; right
   b. Scalene; obtuse
   c. Isosceles; acute
   d. Equilateral; acute
2. a. $\angle A = \angle C$
   b. Answers will vary.
3. Answers will vary.
4. 5 cm
5. No; explanations will vary.
6. No; explanations will vary.

Lesson 14

Problem Set

1. Triangles accurately drawn; side lengths and angles labeled
2. Lines of symmetry accurately drawn in 1(a) and 1(d); explanations will vary.
3. False; explanations will vary.
4. True; explanations will vary.
5. True; explanations will vary.
6. True; explanations will vary.

Extension: True; explanations will vary.

Exit Ticket

1. Triangle accurately drawn with 1 line of symmetry
2. Triangle accurately drawn with no lines of symmetry
3. 2

Homework

1. Triangles drawn accurately; side lengths and angles labeled
2. Lines of symmetry accurately drawn in 1(a) and 1(c); explanations will vary.
3. True; explanations will vary.
4. False; explanations will vary.
5. True; explanations will vary.
6. False; explanations will vary.

Extension: False; explanations will vary.

Lesson 15

Problem Set

1. Figure accurately constructed; trapezoid
2. Figure accurately constructed; parallelogram
3. Figure accurately constructed; rectangle
4. Figure accurately constructed; square
5. a. Trapezoid
   b. Parallelogram
   c. Square
   d. Rectangle
6. Sides of equal length; explanations will vary.
7. Four right angles; explanations will vary.
8. Two sets of parallel sides; explanations will vary.

Exit Ticket

1. Figure accurately constructed
2. Four right angles; answers will vary.

Homework

1. a. Trapezoid
   b. Parallelogram
   c. Square
   d. Rectangle
2. Sides of equal length; explanations will vary.
3. Four right angles; explanations will vary.
4. Two parallel pairs; explanations will vary.
5. a. Figure accurately constructed; square
   b. Figure accurately constructed; parallelogram (or rectangle or square)
   c. Figure accurately constructed; trapezoid
   d. Figure accurately constructed; rectangle (or square)

Lesson 16

Problem Set

1. a. Figure accurately constructed; figures will vary; answers will vary.
   b. Figure accurately constructed; figures will vary; answers will vary.
   c. Figure accurately constructed; figures will vary; answers will vary.
   d. Figure accurately constructed; figures will vary; answers will vary.
2. a. Figure accurately constructed; figures will vary; answers will vary.
   b. Figure accurately constructed; figures will vary; answers will vary.
3. Answers will vary.
4. Answers will vary.

Exit Ticket

1. Parallelogram accurately constructed; figures will vary.
2. Rectangle accurately constructed; figures will vary.

Homework

1. Figure accurately constructed; figures will vary; trapezoid
2. Figure accurately constructed; figures will vary; trapezoid
3. Figure accurately constructed; figures will vary; parallelogram, rectangle, square, or rhombus
4. Figure accurately constructed; figures will vary; rhombus or square
5. Figure accurately constructed; figures will vary; square