

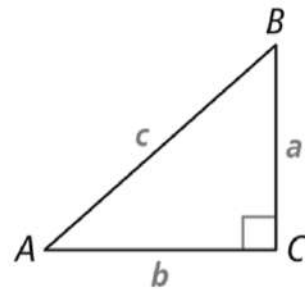
## Section 8.1 Pythagorean Theorem and Special Right Triangles

## Pythagorean Theorem

If a triangle is a right triangle, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

PROOF: SEE EXAMPLE 1.

If...  $\triangle ABC$  is a right triangle.



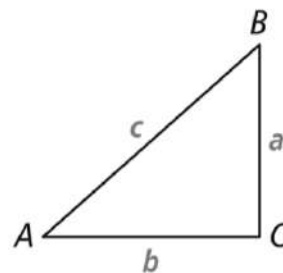
Then...  $a^2 + b^2 = c^2$

## Converse of the Pythagorean Theorem

If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.

PROOF: SEE EXERCISE 17.

If...  $a^2 + b^2 = c^2$



Then...  $\triangle ABC$  is a right triangle.

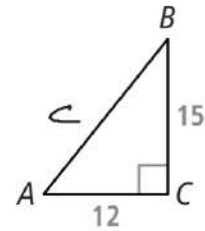
Comparing  $a^2$ ,  $b^2$ , and  $c^2$

$$a^2 + b^2 = c^2 \quad \text{Right Triangle}$$

$$a^2 + b^2 > c^2 \quad \text{Acute Triangle}$$

$$a^2 + b^2 < c^2 \quad \text{Obtuse Triangle}$$

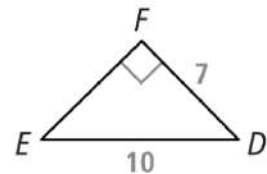
Use the figure shown. Find  $AB$ .



Enter your answer.

$$12^2 + 15^2 = c^2$$
$$144 + 225 = c^2$$
$$369 = c^2$$
$$c = \sqrt{369} \approx 19.2$$

Use the figure shown. Find  $EF$ .



Enter your answer.

$$a^2 + 7^2 = 10^2$$
$$a^2 + 49 = 100$$
$$a^2 = 51$$
$$a = \sqrt{51} \approx 7.141$$

Classify the triangle with the given sides as acute, obtuse, or right.

$$2, 3, 4$$

$$2^2 + 3^2 \stackrel{?}{=} 4^2$$

$$4 + 9$$

$$13 < 16$$

obtuse

$$3, 4, 5$$

$$3^2 + 4^2 \stackrel{?}{=} 5^2$$

$$9 + 16$$

$$25 = 25$$

Right

$$4, 5, 6$$

$$4^2 + 5^2 \stackrel{?}{=} 6^2$$

$$16 + 25$$

$$41 > 36$$

acute

$$1.2, 2.0, 2.5$$

$$1.2^2 + 2.0^2 \stackrel{?}{=} 2.5^2$$

$$1.44 + 4$$

$$5.44 < 6.25$$

obtuse

$$0.6, 0.8, 0.9$$

$$(.6)^2 + (.8)^2 \stackrel{?}{=} (.9)^2$$

$$.36 + .64$$

$$1 > .81$$

Acute

$$1, 1, \sqrt{2}$$

$$1^2 + 1^2 \stackrel{?}{=} (\sqrt{2})^2$$

$$1 + 1 = 2$$

$$2 = 2$$

Right

$$1, \sqrt{2}, 2$$

$$1^2 + (\sqrt{2})^2 \stackrel{?}{=} 2^2$$

$$1 + 2$$

$$3 < 4$$

obtuse

$$\sqrt{2}, 2, \sqrt{5}$$

$$(\sqrt{2})^2 + 2^2 \stackrel{?}{=} (\sqrt{5})^2$$

$$2 + 4$$

$$6 > 5$$

Acute

$$11, 14, 17$$

$$11, 12, 17$$

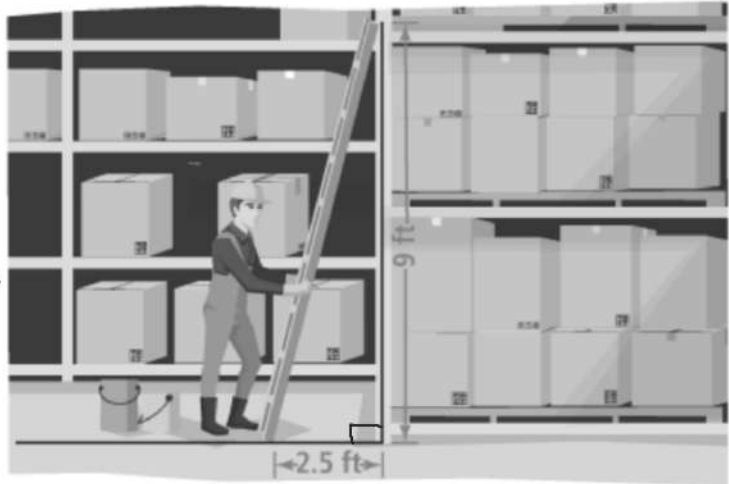
$$12, 35, 37$$

$$1, 3, 1$$

A. To satisfy safety regulations, the distance from the wall to the base of a ladder should be at least one-fourth the length of the ladder. Did Drew set up the ladder correctly?

**SOLUTION**

$$\begin{aligned}(2.5)^2 + 9^2 &= (\text{Ladder})^2 \\ 6.25 + 81 &= L^2 \\ 87.25 &= L^2 \\ L &= 9.34 \text{ ft} \\ &\hookrightarrow 2.33\end{aligned}$$



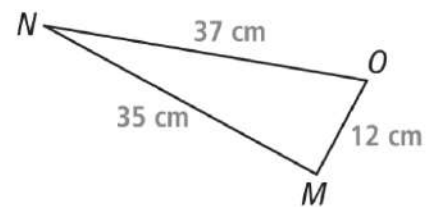
Is  $\triangle MNO$  a right triangle? Explain.

Enter your answer  $12^2 + 35^2 \stackrel{?}{=} 37^2$

$$144 + 1225 = 1369$$

$$1369 \checkmark = 1369$$

Yes  $\triangle MNO$  is a Rt  $\triangle$





## Special Right Triangles

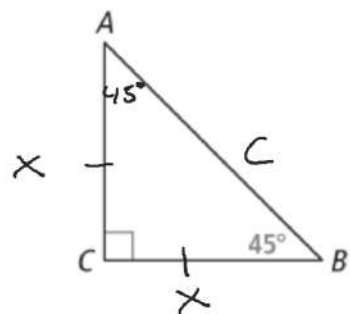
45° - 45° - 90°

Is there a relationship between the lengths of  $\overline{AB}$  and  $\overline{AC}$  in  $\triangle ABC$ ? Explain.

Legs are congruent

**SOLUTION**

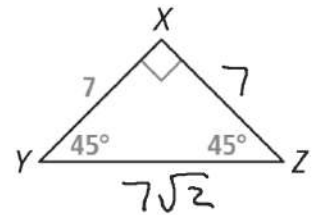
Hypotenuse = Leg  $\cdot \sqrt{2}$



$$\begin{aligned}x^2 + x^2 &= c^2 \\ \sqrt{2x^2} &= \sqrt{c^2} \\ c &= x\sqrt{2}\end{aligned}$$

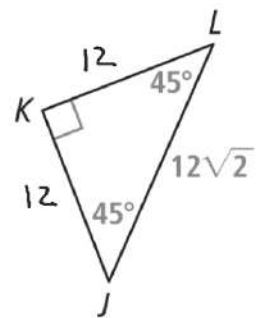
3. Find the side lengths of the  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle.

a. What are  $XZ$  and  $YZ$ ?



3. Find the side lengths of the  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle.

b. What are  $JK$  and  $LK$ ?

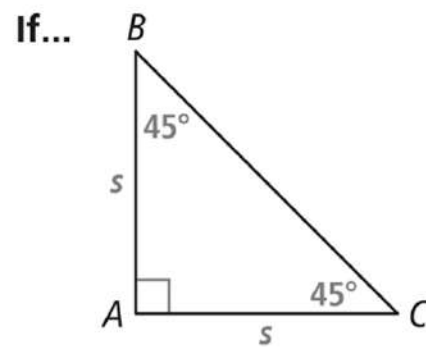


Enter your answer.

## 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the legs are congruent and the length of the hypotenuse is  $\sqrt{2}$  times the length of a leg.

PROOF: SEE EXERCISE 18.



Then...  $BC = s\sqrt{2}$

## Special Right Triangles

30° - 60° - 90°

Short Leg - Half the Hypotenuse

Long Leg - Short Leg times  $\sqrt{3}$

Hypotenuse -

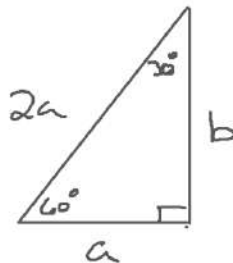
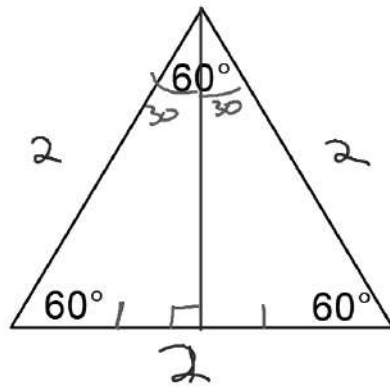
$$a^2 + b^2 = (2a)^2$$

$$a^2 + b^2 = 4a^2$$

$-a^2$                        $-a^2$

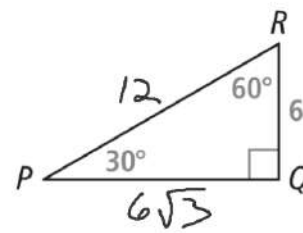
$$b^2 = 3a^2$$

$$b = \sqrt{3a^2}$$
$$= a\sqrt{3}$$



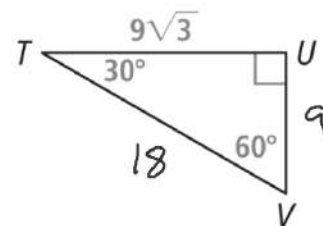
a. What are  $PQ$  and  $PR$ ?

Enter your answer.



What are  $UV$  and  $TV$ ?

Enter your answer.

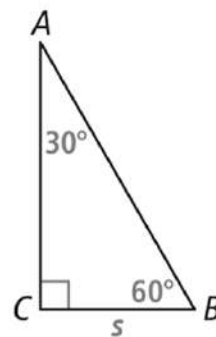


## 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the short leg. The length of the long leg is  $\sqrt{3}$  times the length of the short leg.

PROOF: SEE EXERCISE 19.

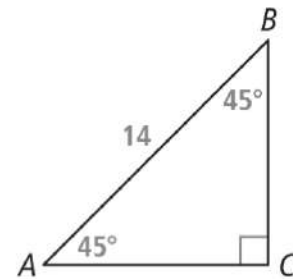
If...



Then...  $AC = s\sqrt{3}$ ,  $AB = 2s$

b. What are AC and BC?

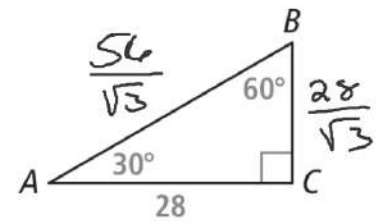
Enter your answer.  
 $AC = BC = \frac{14}{\sqrt{2}}$



$$\left(\frac{28}{\sqrt{3}}\right)\left(\frac{2}{1}\right)$$

What are AB and BC?

Enter your answer.



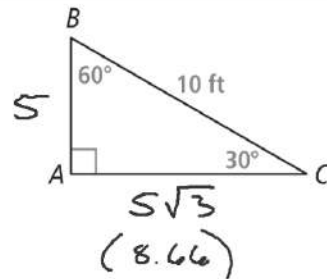
A. Alejandro needs to make both the horizontal and vertical supports,  $\overline{AC}$  and  $\overline{AB}$ , for a ramp. Is one 12-foot board long enough for both supports? Explain.

↳ No more than 12'

**SOLUTION**

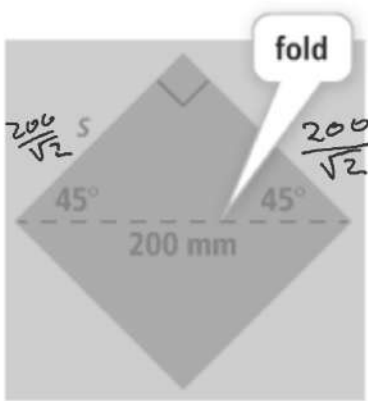
$$5 + 8.66$$

$$13.66 > 12$$





**B. Olivia starts an origami paper crane by making the 200-mm diagonal fold. What are the side length and area of the paper square?**



$$\begin{aligned} A &= \left(\frac{200}{\sqrt{2}}\right)^2 \\ &= \frac{40,000}{2} \\ &= 20,000 \text{ mm}^2 \end{aligned}$$

If  $RS = 35$ ,  $ST = 37$ , and  $RT = 71$ , is  $\triangle RST$  a right triangle? Explain.

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$$2594 < 5041$$

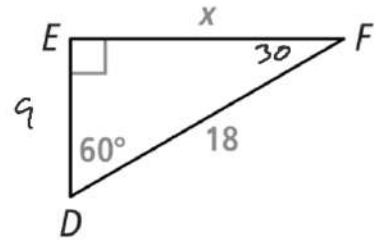
If  $RS = 40$ ,  $ST = 41$ , and  $RT = 11$ , is  $\triangle RST$  a right triangle? Explain.

$$40^2 + 11^2 \stackrel{?}{=} 41^2$$

If  $RS = 20$ ,  $ST = 21$ , and  $RT = 29$ , is  $\triangle RST$  a right triangle? Explain.

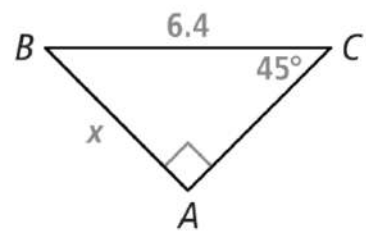
Find the value of  $x$ .

Enter your answer.



Find the value of  $x$ .

Enter your answer.



9. Charles wants to hang the pennant shown vertically between two windows that are 19 inches apart. Will the pennant fit? Explain.



Enter your answer.



