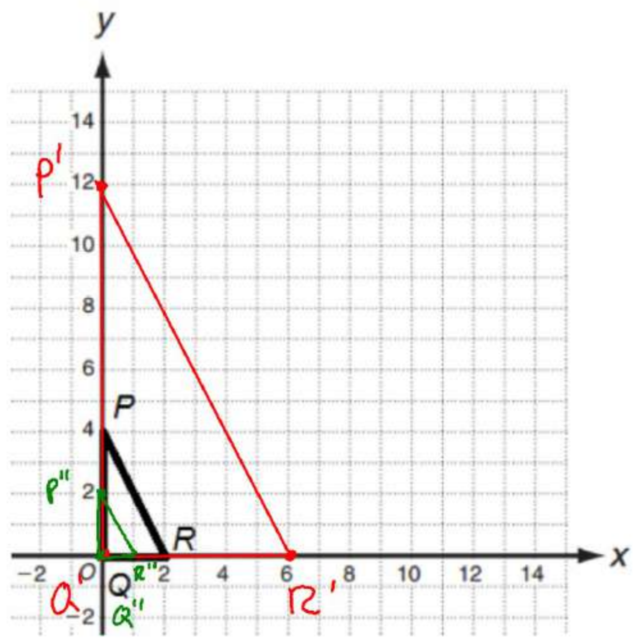
Use the graph at right to explore similarity in the coordinate plane.



1. Write down the vertices of ΔPQR.

   $P($ 0 , 4 $)$, $Q($ 0 , 0 $)$, $R($ 2 , 0 $)$

2. Multiply each coordinate of each vertex of ΔPQR by 3. Then graph $\Delta P'Q'R'$ with these new vertices. How is $\Delta P'Q'R'$ related to ΔPQR ?

   $P(0,4) \rightarrow P'(0\cdot 3, 4\cdot 3)$

   $P'($ 0 , 12 $)$, $Q'($ 0 , 0 $)$, $R'($ 6 , 0 $)$

   $Q' \rightarrow Q(0\cdot 3, 0\cdot 3)$

3. Now multiply each coordinate of each vertex of ΔPQR by ½ . Then graph $\Delta P''Q''R''$ with these new vertices. How is $\Delta P''Q''R''$ related to ΔPQR ?

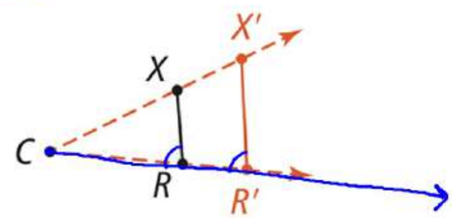
   $P''($ 0 , 2 $)$, $Q''($ 0 , 0 $)$, $R''($ 1 , 0 $)$

$P'' \rightarrow P(0\cdot \frac{1}{2}, 4\cdot \frac{1}{2})$ $R'' \rightarrow (2\cdot \frac{1}{2}, 0\cdot \frac{1}{2})$

$Q'' \rightarrow Q(0\cdot \frac{1}{2}, 0\cdot \frac{1}{2})$

Dilations

A dilation $D_{(n,\ C)}$ is a transformation that has center of dilation C and scale factor n, where $n > 0$, with the following properties:



- Point R maps to R' in such a way that R' is on $\overrightarrow{CR}$ and $CR' = n \bullet CR$.
- Each length in the image is n times the corresponding length in the preimage (i.e., $X'R' = n \bullet XR$).
- The image of the center of dilation is the center itself (i.e., $C' = C$).
- If $n > 1$, the dilation is an *enlargement.*
- If $0 < n < 1$, the dilation is a *reduction.*
- Every angle is congruent to its image under the dilation.

On a coordinate plane, the notation D_n describes the dilation with the origin as center of dilation.

Scale Factor → Ratio of Sides (K)

$D_{(n,c)}$

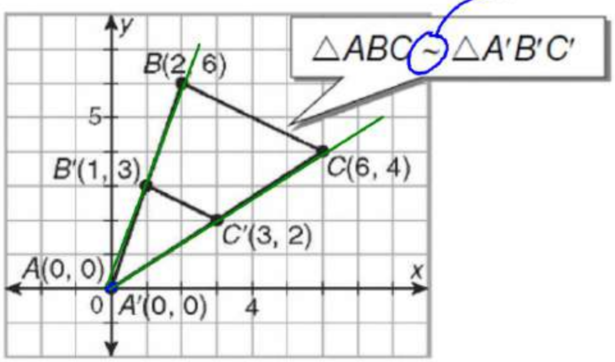
↳ scale factor

K or $n > 1$ Enlargement

K or n $0 < x < 1$ Reduce

A dilation is a transformation that changes the Size of a figure but not its Shape. The preimage and image are always similar. A Scale Factor describes how much a figure is enlarged or reduced.

similar



Example 1. Triangle ABC above has vertices $A(0, 0)$, $B(2, 6)$, and $C(6, 4)$. Find the coordinates of the vertices of the image after a dilation with a scale factor ½ .

Preimage ΔABC		Image $\Delta A'B'C'$
$A(0,0)$	$\rightarrow (\frac{1}{2}\cdot 0, \quad \frac{1}{2}\cdot 0) \rightarrow$	$A'(0,0)$
$B(2,6)$	$\rightarrow (\frac{1}{2}\cdot 2, \quad \frac{1}{2}\cdot 6) \rightarrow$	$B'(1,3)$
$C(6,4)$	$\rightarrow (\frac{1}{2}\cdot 6, \quad \frac{1}{2}\cdot 4) \rightarrow$	$C'(3,2)$

Rectangle $A'B'C'D'$ is a dilation with center P of $ABCD$. How are the side lengths and angle measures of $ABCD$ related to those of $A'B'C'D'$?

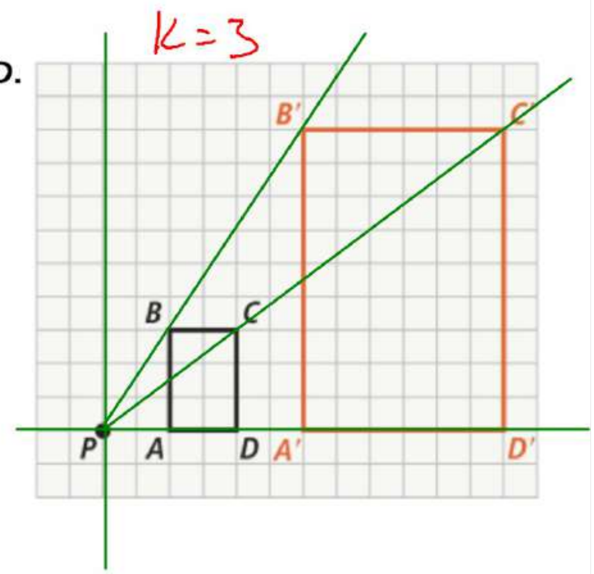
SOLUTION

A (2,0) A' (6,0)

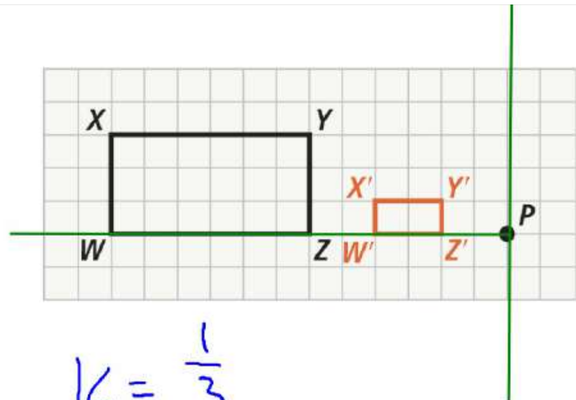
B (2,3) B' (6,9)

C (4,3) C' (12,9)

D (4,0) D' (12,0)



2. Rectangle $W'X'Y'Z'$ is a dilation with center P of $WXYZ$. How are the side lengths and angle measures of the two figures related?



Enter your answer

$W(-12,0)$ $W'(-4,0)$

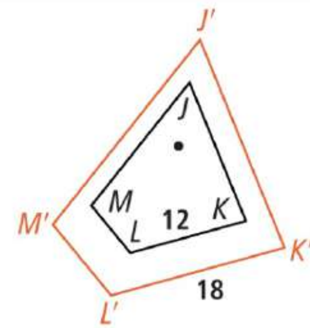
$X(-12,3)$ $X'(-4,1)$

$Y(-6,3)$ $Y'(-2,1)$

$Z(-6,0)$ $Z'(-2,0)$

$k = \frac{1}{3}$

Quadrilateral $J'K'L'M'$ is a dilation of $JKLM$. What is the scale factor?



SOLUTION

$K = 4, 6, 3.2$

$\frac{1}{2}$

-6

1.5

Carrot

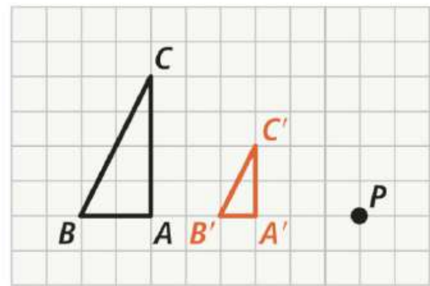
$\frac{12}{18} = \frac{2}{3}$

$\frac{18}{12} = \frac{3}{2}$

3. Consider the dilation shown.

a. Is the dilation an enlargement or a reduction?

Enter your answer Reduction



$A(-6,0)$ $A'(3,0)$

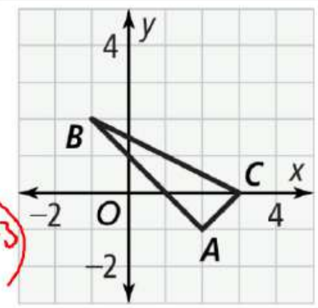
$\frac{-3}{-6} = \frac{1}{2}$

b. What is the scale factor? $K = \frac{1}{2}$

Dilate a Figure With Center at the Origin

Scale Factor (K) = 3

What are the vertices of $D_3(\triangle ABC)$?



The notation $D_3(\triangle ABC)$ means the image of $\triangle ABC$ after a dilation centered at the origin, with scale factor 3.

$A(2,-1) \rightarrow A'(2\cdot 3, -1\cdot 3) = (6,-3)$

SOLUTION

$B(-1,2)$ $B'(-3,6)$

$C(3,0)$ $C'(9,0)$

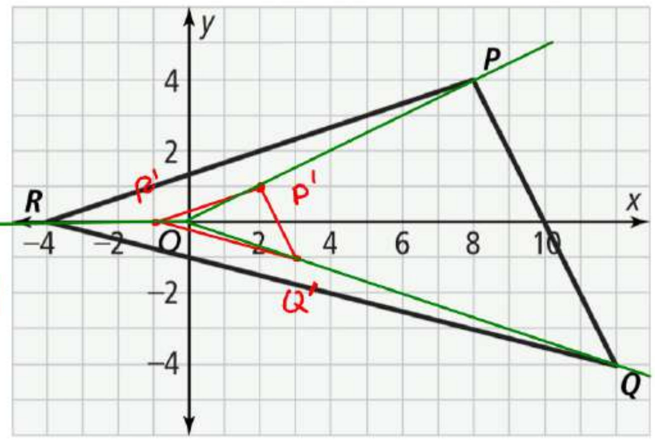
4. Use $\triangle PQR$.

a. What are the vertices of $D_{\frac{1}{4}}(\triangle PQR)$? $k = \frac{1}{4}$

$P(8, 4) \rightarrow P'\left(8 \cdot \frac{1}{4}, 4 \cdot \frac{1}{4}\right) = (2, 1)$

$Q(12, -4) \rightarrow Q\left(12 \cdot \frac{1}{4}, -4 \cdot \frac{1}{4}\right) = (3, -1)$

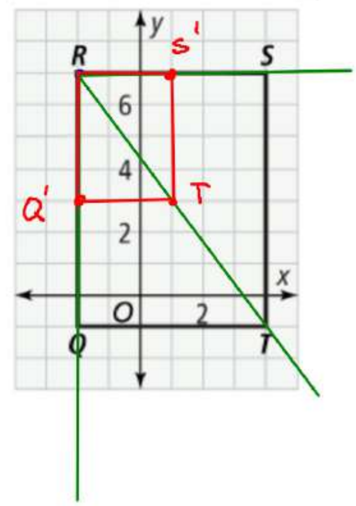
$R(-4, 0) \rightarrow R'(-1, 0)$



b. How are the distances to the origin from each image point related to the distance to the origin from each corresponding preimage point?

What are the vertices of $D_{(\frac{1}{2},\ R)}(QRST)$?

SOLUTION



Preimage Point	Change From R(-2,7)		Half of the Change from R(-2,7)		Add to R (-2, 7)	Image Point
	Horiz	Vert	Horiz	Vert		
Q (-2, -1)	0	-8	0	-4		(-2,3)
S (4, 7)	6	0	3	0		(1,7)
T (4, -1)	6	-8	3	-4		(1,3)

R (-2,7)