AP Calc BC Spring Break Exam Answers

PART A – NO CALCULATOR ALLOWED

1. $v(t)=\langle t^2,5t\rangle \quad a(t)=\langle 2t,5\rangle \quad a(3)=\langle 6,5\rangle$ $\boxed{\text{B}}$

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2. $\int xe^{x^2}\,dx=\frac{1}{2}e^{x^2}+C$ $\boxed{\text{A}}$

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3. $\lim\limits_{x\to 0}\frac{\sin x\cos x}{x}=\lim\limits_{x\to 0}\frac{2\sin x\cos x}{2x}=\lim\limits_{x\to 0}\frac{\sin(2x)}{2x}=1$ $\boxed{\text{C}}$

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4. $\sum\limits_{n=1}^{\infty}\frac{e^n}{n!}$ Ratio Test $\lim\limits_{n\to\infty}\left(\frac{e^{n+1}}{(n+1)!}\bullet\frac{n!}{e^n}\right)=\lim\limits_{n\to\infty}\left(\frac{e^n\cdot e}{(n+1)n!}\bullet\frac{n!}{e^n}\right)=\lim\limits_{n\to\infty}\left(\frac{e}{n+1}\right)=0<1$ $\boxed{\text{D}}$

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5. $x=\sin(t^3)$ and $y=e^{5t}$ $\frac{dx}{dt}=3t^2\cos t^2$ $\frac{dy}{dt}5e^{5t}$ $L=\int_0^{\pi}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_0^{\pi}\sqrt{9t^4\cos^2(t^3)+25e^{10t}}\,dt$ $\boxed{\text{C}}$

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$$

6. $\boxed{\text{A}}$

I. f has a limit at $x=2$. True II. f is continuous at $x=2$. False III. f is differentiable at $x=2$. False

$\lim\limits_{x\to 2^-}f(x)=\lim\limits_{x\to 2^-}\frac{(x+2)(x-2)}{(x-2)}=4$ $\lim\limits_{x\to 2}f(x)=4\neq f(2)=1$ f is not continuous, so f is not differentiable

$\lim\limits_{x\to 2^+}f(x)=\lim\limits_{x\to 2^+}\frac{(x+2)(x-2)}{(x-2)}=4$

$\lim\limits_{x\to 2}f(x)=\lim\limits_{x\to 2^-}f(x)=\lim\limits_{x\to 2^+}f(x)=4$

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7. Given that $y(1)=-3$ and $\frac{dy}{dx}=2x+y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x=1$?

$y(1.5)=y(1)+(0.5)(2(1)+(-3))=-3+(0.5)(-1)=-3.5$

$y(2)=y(1.5)+(0.5)(2(1.5)+(-3.5))=-3.5+(0.5)(-0.5)=-3.75$ $\boxed{\text{D}}$

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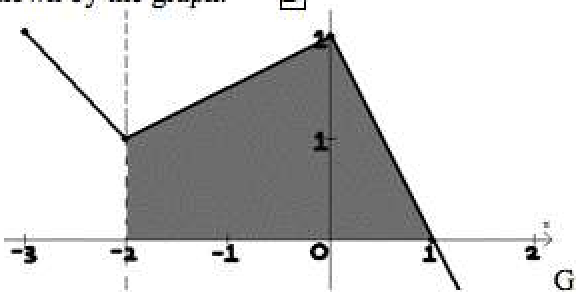
8. Left Riemann Sum:

$\int_2^{13}f(x)dx\approx(3-2)(6)+(5-3)(-2)+(8-5)(-1)+(13-8)(3)=6+(-4)+(-3)+15=14$ $\boxed{\text{B}}$

9. $g(x)=\int_{-2}^{x} f(t)dt \quad g'(x)=f(x)$ So g is greatest when g' or f changes from + to - . This occurs at $x=1$.

So, g(1) is greatest, as shown by the graph. $\boxed{\text{D}}$



Graph of f

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10. $x^2+xy+y^2=7 \quad 2x+x\dfrac{dy}{dx}+y+2y\dfrac{dy}{dx}=0 \Rightarrow \dfrac{dy}{dx}=\dfrac{-2x-y}{x+2y} \Rightarrow m_{(2,1)}=\dfrac{-4-1}{2+2}=\dfrac{-5}{4}$ $\boxed{\text{B}}$

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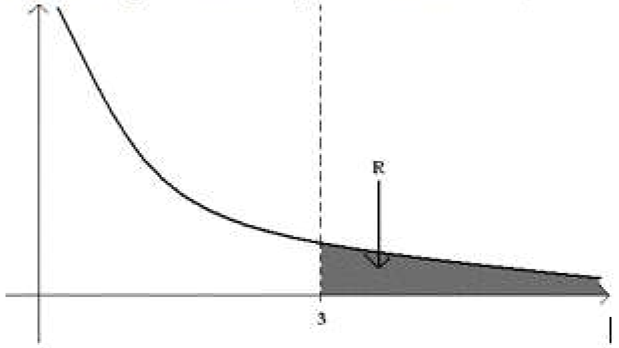
11. R is the region between $y=e^{-2x}$ and x-axis, $x\geq 3$



$$R=\lim_{b\to\infty}\int_{3}^{b} e^{-2x}\,dx=\lim_{b\to\infty}\left(-\frac{1}{2}\right)e^{-2x}\Big|_{3}^{b}=-\frac{1}{2}\lim_{b\to\infty}\left(\frac{1}{e^{2b}}-\frac{1}{e^{6}}\right)=\frac{1}{2e^{6}}$$ $\boxed{\text{A}}$

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12. $\displaystyle\sum_{n=1}^{\infty}\frac{e^n x^n}{n!}$ Ratio Test $\displaystyle\lim_{n\to\infty}\frac{e^{n+1}\bullet x^{n+1}}{(n+1)!}\bullet\frac{n!}{e^n x^n}=\lim_{n\to\infty}\frac{e^n\bullet e\bullet x^n\bullet x}{(n+1)(n!)}\bullet\frac{n!}{e^n x^n}=\lim_{n\to\infty}|x|\left(\frac{1}{n+1}\right)=0<1$

Therefore, Converges $\boxed{\text{D}}$

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13. $\displaystyle\int_{1}^{e}\frac{x^2+1}{x}dx=\int_{1}^{e}x+\frac{1}{x}\,dx=\frac{x^2}{2}+\ln x\Big|_{1}^{e}=\left(\frac{e^2}{2}+\ln e\right)-\left(\frac{1}{2}+\ln 1\right)=\frac{e^2+1}{2}$ $\boxed{\text{B}}$

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14. The graph of f changes concavity in (0,2) because the second derivative goes from positive to negative on this interval. $\boxed{\text{E}}$

15. $f(x)=(\ln x)^2$, $f''(\sqrt{e})=?$ $f'(x)=\dfrac{2(\ln x)}{x}$ $\rightarrow$ $f''(x)=\dfrac{x\left(\dfrac{2}{x}\right)-2\ln x}{x^2}=\dfrac{2-2\ln x}{x^2}$ $f''(\sqrt{e})=\dfrac{1}{e}$ $\boxed{\textbf{A}}$

16. $\sum\limits_{n=1}^{\infty}\left(\dfrac{2}{x^2+1}\right)^n$ Geometric Series $-1<\dfrac{2}{x^2+1}<1 \rightarrow x^2+1>2 \rightarrow x^2>1 \rightarrow |x|>1 \rightarrow x>1, x<-1$ only $\boxed{\textbf{D}}$

17. $f(x)=h(x^2-3) \rightarrow f'(x)=(2x)h'(x^2-3) \rightarrow f'(2)=4h'(1)$ $\boxed{\textbf{B}}$

18. $y=x^2+3x+1$ line $x+y=k \rightarrow y=-x+k \rightarrow m=-1$

So, $\dfrac{dy}{dx}=2x+3=-1 \rightarrow x=-2 \rightarrow y=(-2)^2+3(-2)+1=4-6+1=-1 \rightarrow k=-2-1=-3$ $\boxed{\textbf{A}}$

19. $\displaystyle\int\frac{7x}{(2x-3)(x+2)}dx=\int\frac{A}{2x-3}+\frac{B}{x+2}dx$ Cover-Up Method

When $x=\dfrac{3}{2}$: $A=\dfrac{7(3/2)}{(3/2)+2}=3$ When $x=-2$: $\mathrm{B}=\dfrac{7(-2)}{2(-2)-3}=2$

$\displaystyle\int\frac{3}{2x-3}+\frac{2}{x+2}dx=\frac{3}{2}\ln|2x-3|+2\ln|x+2|+C$ $\boxed{\textbf{A}}$

20. $e^x=1+x+\dfrac{x^2}{2!}+\ldots+\dfrac{x^n}{n!}+\ldots$

If $x=\ln 2$ this becomes $e^{\ln 2}=1+\ln 2+\dfrac{(\ln 2)^2}{2!}+\ldots+\dfrac{(\ln 2)^n}{n!}+\ldots$ and $e^{\ln 2}=2$ $\boxed{\textbf{C}}$

21. Velocity of particle increasing $\Rightarrow \dfrac{dv}{dt}>0 \Rightarrow \dfrac{d^2x}{dt^2}>0 \Rightarrow$ graph of $x(t)$ is concave up when $0<t<2$ $\boxed{\textbf{A}}$

22. $\displaystyle\int_0^1 f'(x)g(x)dx=5$ Integration by parts: $\displaystyle\int_0^1 u\,dv=uv\Big|_0^1-\int_0^1 v\,du$ $\quad u=f(x) \quad v=g(x)$; $du=f'(x)dx \quad dv=g'(x)dx$

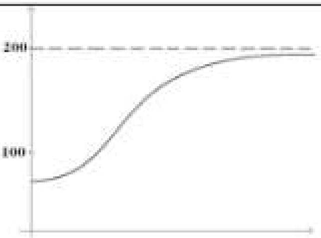
$\displaystyle\int_0^1 f(x)g'(x)dx=f(x)g(x)\Big|_0^1-\int_0^1 g(x)f'(x)dx=f(1)g(1)-f(0)g(0)-5=4(3)-2(-4)-5=15$ $\boxed{\text{E}}$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

23. $$\sin(2x) = 2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \ldots \qquad \boxed{E}$$

$$x\sin(2x) = 2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!} + \ldots$$

Using the following model formula for logistic growth: $\frac{dP}{dt} = kP(A-P)$



Carrying Capacity = A = 200

24. $$\frac{dP}{dt} = kP(A-P) = kP(200-P) = k(200P - P^2)$$

Only one of the 5 choices can be put into the form $k(200P - P^2)$:

$$\frac{dP}{dt} = .2P - .001P^2 = .001(200P - P^2) \rightarrow k = .001$$

$\boxed{A}$

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25. $$f'(x) = \begin{cases} c & \text{for } x \le 2 \\ 2x - c & \text{for } x > 2 \end{cases} \qquad c = 2x - c \;\Rightarrow\; c = 4 - c \;\Rightarrow\; c = 2$$

So, $cx + d = x^2 + cx$ → $2c + d = 4 - 2c \quad d = -4$

$\therefore c + d = 2 + (-4) = -2$ $\boxed{B}$

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26. $\sin^2\theta = 0$ → $\sin^{-1} 0$ when $\theta = 0$ and π

$$A = 2\left(\frac{1}{2}\int r^2 d\theta\right) \rightarrow A = \int_0^\pi \left(\sin^2\theta\right)^2 d\theta \rightarrow A = \int_0^\pi \sin^4\theta\, d\theta \qquad \boxed{D}$$

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27. $\frac{dy}{dx} = y^2 - 1$

y	-2	-1	0	1	2
dy/dx	3	0	-1	0	3

Look at graph $\boxed{A}$

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28. $y = x^2 - x \quad \frac{dx}{dt} > 0 \quad \frac{dy}{dt} = ?\ (2,2)$ Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 2\sqrt{10}$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (2\sqrt{10})^2 = 40 \rightarrow \left(\frac{dy}{dt}\right)^2 = 40 - \left(\frac{dx}{dt}\right)^2$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 2x - 1 \rightarrow \frac{dy}{dt} = (2x-1)\frac{dx}{dt}$$

$$\rightarrow \left(\frac{dy}{dt}\right)^2 = (2x-1)^2\left(\frac{dx}{dt}\right)^2 = 40 - \left(\frac{dx}{dt}\right)^2 \rightarrow \text{plug } x = 2 \rightarrow 9\left(\frac{dx}{dt}\right)^2 = 40 - \left(\frac{dx}{dt}\right)^2$$

$$\rightarrow 10\left(\frac{dx}{dt}\right)^2 = 40 \rightarrow \frac{dx}{dt} = 2 \quad \text{So, } \frac{dy}{dt} = (2(2)-1)(2) = 6 \qquad \boxed{D}$$

PART B – CALCULATOR ALLOWED

76. 1st derivative test

f'(x)	−	0		+		0		−
		a		b		c		d

2nd derivative test

f''(x)	+		+	0	−		−	
		a		b		c		d

Point of inflection at $(b, f(b)) =$ $\boxed{\text{D}}$

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77. $\int_0^5 R(t)dt \approx 47.193m^3$ $\boxed{\text{D}}$

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78. $\lim\limits_{x\to-2^-} = 1$ ok $\lim\limits_{x\to-2^+} = 1$ ok $\lim\limits_{x\to0^-} = 1$ ok $\lim\limits_{x\to0^+} = 1$ ok → -2 and 0 only $\boxed{\text{C}}$

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79. If $\sum\limits_{n=1}^{\infty} a_n$ converges to k, by the integral test, $\int_1^{\infty} f(x)dx$ converges $\boxed{\text{D}}$

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80. $f'(x) = x^2\cos(x^2)$ →

graph $f''(x)$ using your calculator - it changes signs (crosses x-axis) 5 times from (-2 to 2) $\boxed{\text{E}}$

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81. $\int_a^c f(x) - g(x)dx = \int_a^c f(x) - \int_a^c g(x)$ $\int_a^c f(x) = P - Q$ $\int_a^c g(x) = R - S$ (P-Q)-(R-S)= P-Q-R+S $\boxed{\text{B}}$

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82. If $\sum\limits_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all ***n***, which is true? By the direct comparison test, $\sum\limits_{n=1}^{\infty} b_n$ diverges $\boxed{\text{E}}$

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83. What is area enclosed by $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$

$$\int_1^5 \left|(x^3 - 8x^2 + 18x - 5) - (x + 5)\right|dx = 11.833$$ $\boxed{\text{B}}$

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84. $f(3) = 2, f'(3) = -1, f''(3) = 6, f'''(3) = 12$

$$2 - \frac{(x-3)}{1!} + \frac{6(x-3)^2}{2!} + \frac{12(x-3)^3}{3!} = 2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$$ $\boxed{\text{A}}$

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85. The graph of $v(t) = 3t^4 - 11t^2 + 9t - 2$ on $-3 \le t \le 3$

changes signs twice $\boxed{\text{C}}$

86. $\frac{dy}{dx}=2x$ and $y=3$ when $x=2$ $\int dy=\int 2xdx \to y=x^2+C \to 3=4+C \to C=-1$

$\Rightarrow y=x^2-1 \Rightarrow f(3)=3^2-1=8$ $\boxed{\text{C}}$

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87. $x(3)=\underbrace{x(0)}_{2}+\int_0^3 (1+t^2)^{1/3}dt=6.512$ $\boxed{\text{D}}$

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88. $g(x)=\int_2^x f(t)dt$ and $f>0$ and $f'<0$ $g'(x)=f(x)$ so $g''(x)=f'(x)$

$g(2)=\int_2^2 f(t)dt=0$ Since $f>0,\ g'>0$ so g is increasing so $g(1)<g(2) \Rightarrow g(1)=-2 \Rightarrow$ A or B

and $f'<0$ so $g''<0 \Rightarrow g'$ is decreasing $\Rightarrow \frac{g(3)-g(2)}{3-2}<\frac{g(2)-g(1)}{2-1} \Rightarrow \boxed{\text{A}}$

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89. No c, $-2<c<2$ for which $f'(c)=0$

Therefore, Rolle's Therom is violated. So either *f* not continuous or not differentiable. Since it is given that *f* is continuous, then it is not everywhere differentiable $\therefore f'(k)$ does not exist. $\boxed{\text{E}}$

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90. $h'(2)=f'(g(2))g'(2)=f'(-1)(2)=3(2)=6$ $\boxed{\text{D}}$

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91. $f(x)=\int_{1/3}^x \cos\left(\frac{1}{t^2}\right)dt$, $\frac{1}{3}\le x\le 1$

$f'(x)=\cos\left(\frac{1}{x^2}\right)=0$ The graph of f' from positive to negative at $x=.461$ $\boxed{\text{D}}$

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92. $B(x)=g(f(x))$

$B'(x)=g'(f(x))\bullet f'(x)$

$B'(-3)=g'(f(-3))\bullet f'(-3)=g'(1)\left(\frac{1}{3}\right)=\left(-\frac{1}{2}\right)\left(\frac{1}{3}\right)=-\frac{1}{6}$ $\boxed{\text{B}}$