Logarithms Practice Exam

1) Write the following in exponential form $\log_9 27 = \frac{3}{2}$ $9^{3/2} = 27$

2) Write each of the following in logarithmic form $16^{1/4} = 2$ $\log_{16} 2 = \frac{1}{4}$

Evaluate each of the following logarithms without the use of a calculator.

3) $\log_4 \frac{1}{2} = -\frac{1}{2}$

$4^x = \frac{1}{2}$

$(2^2)^x = 2^{-1}$

$2^{2x} = 2^{-1}$

$2x = -1$

4) $\log_8 4 = \frac{2}{3}$

$8^x = 4$

$(2^3)^x = 2^2$

$2^{3x} = 2^2$

$3x = 2$

5) $\log_3 81 = 4$

$3^x = 81$

$3^x = 3^4$

$x = 4$

6) $\log_4 0 =$ NO Solution

Cannot take log of 0 or a $-\#$

Write each of the following as the sum or difference of logarithms.

7) $\log \sqrt[4]{(x+1)^3 (x-2)^2}$

$\log \left[(x+1)^3 (x-2)^2\right]^{1/4}$

$\log (x+1)^{3/4} (x-2)^{2/4}$

$\log (x+1)^{3/4} + \log (x-2)^{2/4}$

$\frac{3}{4} \log (x+1) + \frac{2}{4} \log (x-2)$

8) $\log_5 \frac{6x^2}{11y^5 z} = \log_5 6x^2 - \log_5 11y^5 z$

$\log_5 6 + \log_5 x^2 - (\log_5 11 + \log_5 y^5 + \log_5 z)$

$\log_5 6 + \log_5 x^2 - \log_5 11 - \log_5 y^5 - \log_5 z$

$\log_5 6 + 2\log_5 x - \log_5 11 - 5\log_5 y - \log_5 z$

9) $\log_2 \frac{\sqrt[5]{3(x+2)^3}}{x-1}$

$\log_2 \frac{\left[3(x+2)^3\right]^{1/5}}{x-1}$

$\log_2 \frac{3^{1/5} (x+2)^{3/5}}{x-1}$

$\frac{1}{5}\log_2 3 + \frac{3}{5}\log_2 (x+2) - \log_2 (x-1)$

10) $\log_3 \frac{\sqrt{5x^5 y^3}}{\sqrt[3]{z^2}}$ $\log_3 \frac{(5x^5 y^3)^{1/2}}{(z^2)^{1/3}}$

$\log_3 \frac{5^{1/2} x^{5/2} y^{3/2}}{z^{2/3}}$

$\frac{1}{2}\log_3 5 + \frac{5}{2}\log_3 x + \frac{3}{2}\log_3 y - \frac{2}{3}\log_3 z$

Rewrite each of the following logarithmic expressions using a single logarithm.

11) $\frac{1}{3}\log 6 + \frac{1}{3}\log x + \frac{2}{3}\log y$

$\frac{1}{3}(\log 6 + \log x + 2\log y)$

$\frac{1}{3}\log 6xy^2$

$\log (6xy^2)^{1/3}$ or $\log \sqrt[3]{6xy^2}$

12) $\ln(x+3) - \ln(2x+5) + 2\ln(x-1)$

$\ln (x+3) - \ln (2x+5) + \ln (x-1)^2$

$\ln \frac{(x+3)(x-1)^2}{2x+5}$

13) $3\log_4 x - 5\log_4 y + 2\log_4 z$

$\log_4 x^3 - \log_4 y^5 + \log_4 z^2$

$\log_4 \frac{x^3 z^2}{y^5}$

14) $\log_3(x+2) + \log_3(x-2) - \log_3(x+4)$

$\log_3 \frac{(x+2)(x-2)}{x+4}$ or $\log_3 \frac{x^2-4}{x+4}$

Use the following information, to approximate the logarithm to 4 significant digits by using the properties of logarithms.

$\log_a 2 \approx 0.3562$, $\log_a 3 \approx 0.5646$, *and* $\log_a 5 \approx 0.8271$

15) $\log_a 18$

$18 = 2 \cdot 3^2$

$\log_a (2 \cdot 3^2)$

$\log_a 2 + \log_a 3^2$

$\log_a 2 + 2\log_a 3$

$(0.3562) + 2(0.5646)$

1.4854

16) $\log_a \frac{4}{9}$

$\frac{4}{9} = \frac{2^2}{3^2}$

$\log_a \frac{2^2}{3^2}$

$\log_a 2^2 - \log_a 3^2$

$2\log_a 2 - 2\log_a 3$

$2(0.3562) - 2(0.5646)$

-0.4168

17) $\log_a 100$

$100 = 2^2 \cdot 5^2$

$\log_a (2^2 \cdot 5^2)$

$\log_a 2^2 + \log_a 5^2$

$2\log_a 2 + 2\log_a 5$

$2(0.3562) + 2(0.8271)$

2.3666

Using a calculator, evaluate each of the following. Round all answers to three decimal places.

18) $\log_3 12 \approx 2.262$

$\frac{\log 12}{\log 3}$

19) $\log_6 17 \approx 1.581$

$\frac{\log 17}{\log 6}$

20) $\log_3 \frac{1}{5} \approx -1.465$

$\frac{\log 1/5}{\log 3}$

21) $\log_4 8 = \frac{3}{2}$ or 1.5

$\frac{\log 8}{\log 4}$

Solve each of the following logarithmic equations. **(Round any solutions with decimals to three decimal places)**
Always check for extraneous roots!!!

22) $\log_3(x+5) + \log_3(x+3) = \log_3 35$

one-to-one property → $\log_3(x^2+8x+15) = \log_3 35$

$x^2 + 8x + 15 = 35$

$-35 \quad -35$

$x^2 + 8x - 20 = 0$

$(x+10)(x-2) = 0$

$x+10=0 \quad x-2=0$

$x=-10 \quad x=2$

Does not work → $-\#$ inside log

$x = 2$

ck $x=2$

$\log_3 7 + \log_3 5 = \log_3 35$

$\log_3 35 = \log_3 35$ ✓

23) $2\log_3 x - \log_3(x-2) = 2$

$\log_3 \frac{x^2}{x-2} = 2$

Exp Form → $3^2 = \frac{x^2}{x-2}$ $(x-2)$

$(x-2)$

$9x - 18 = x^2$

$x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$x=6 \quad x=3$

ck $x=6$

$2\log_3 6 - \log_3 4 = 2$

$\log_3 36 - \log_3 4 = 2$

$\log_3 9 = 2$ ✓

ck $x=3$

$2\log_3 3 - \log_3 1 = 2$

$2(1) - 0 = 2$

$2 = 2$ ✓

$x=6 \quad x=3$

24) $\log_2(x+3)+\log_2(x-3)=4$

$\log_2(x^2-9)=4$

$2^4 = x^2-9$

$16 = x^2-9$

$-16 \quad -16$

$x^2-25=0$

$(x+5)(x-5)=0$

$x=-5 \quad x=5$

-5 does not work

$x=5$

ck $x=5$

$\log_2 8+\log_2 2=4$

$3+1=4$ ✓

25) $2-6\ln x=10$

$-2 \quad -2$

$\frac{-6\ln x}{-6} = \frac{8}{-6}$

$\ln x = -\frac{4}{3}$

Exp form $\rightarrow$ $e^{-4/3}=x$

$x=e^{-4/3}$ $\quad x\approx 0.264$

Solve each of the following exponential equations. Round solutions to three decimal places.

26) $12^{3x+1}=7^2$

$\log 12^{3x+1} = \log 7^2$

$(3x+1)\log 12 = 2\log 7$

$3x\log 12+\log 12 = 2\log 7$

$\frac{3x\log 12}{3\log 12} = \frac{2\log 7-\log 12}{3\log 12}$

$x=\frac{2\log 7-\log 12}{3\log 12}$ $\quad x\approx 0.189$

27) $12^{3x-2}=8^{5x+1}$

$\log 12^{3x-2} = \log 8^{5x+1}$

$(3x-2)\log 12 = (5x+1)\log 8$

$3x\log 12-2\log 12 = 5x\log 8+\log 8$

$3x\log 12-5x\log 8 = 2\log 12+\log 8$

$x(3\log 12-5\log 8) = 2\log 12+\log 8$

$x=\frac{2\log 12+\log 8}{3\log 12-5\log 8}$

$x\approx -2.396$

28) $2-4e^{2x-1}=12$

$-2 \quad -2$

$\frac{-4e^{2x-1}}{-4} = \frac{10}{-4}$

take ln both sides

$e^{2x-1} = -\frac{5}{2}$

$\ln e^{2x-1} = \ln -\frac{5}{2}$

$-\#$

No solution

Answer each of the following.

29) If you invest \$5000 in an account that pays 12% interest, compounded quarterly, how much would you have at the end of 15 years?

$A=$

$P=5000$

$r=.12$

$n=4$

$t=15$

$A=P\left(1+\frac{r}{n}\right)^{nt}$

$A=5000\left(1+\frac{.12}{4}\right)^{(4)(15)}$

$A=5000\left(1.03\right)^{60}$

$A=\$29,458.02$

30) How much would you have to invest in an account that pays 5% interest, compounded continuously, to have a balance of \$30,000 at the end of 15 years?

$A=30\,000$

$P=$

$r=.05$

$t=15$

$A=Pe^{rt}$

$30\,000 = Pe^{(.05)(15)}$

$30\,000 = Pe^{0.75}$

$\frac{30\,000}{e^{0.75}} = \frac{Pe^{0.75}}{e^{0.75}}$

$P=\frac{30\,000}{e^{0.75}}$

$P\approx \$14,171.00$

31) How long will it take for an investment of \$2,000 in an account that pays $4\frac{1}{2}$% interest compounded quarterly to become \$12,000.

$A=12\,000$

$P=2000$

$r=.045$

$n=4$

$t=$

$A=P\left(1+\frac{r}{n}\right)^{nt}$

$\frac{12\,000}{2000} = \frac{2000\left(1+\frac{.045}{4}\right)^{4t}}{2000}$

log of both sides $\rightarrow$

$\log 6 = \log\left(1+\frac{.075}{4}\right)^{4t}$

$\log 6 = 4t\log\left(1+\frac{.045}{4}\right)$

$t=\frac{\log 6}{4\log\left(1+\frac{.045}{4}\right)}$

$t\approx 40$ yrs