Practice FTC Application for Chapter 6 Exam

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t + 1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

Solution on next page

(a) $\int_0^{12} H(t)\,dt = 70.570$ or 70.571

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(b) $H(6) - R(6) = -2.924$,
so the level of heating oil is falling at $t = 6$.

$1: \text{answer with reason}$

(c) $125 + \int_0^{12} (H(t) - R(t))\,dt = 122.025$ or 122.026

$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(d) The absolute minimum occurs at a critical point
or an endpoint.
$H(t) - R(t) = 0$ when $t = 4.790$ and $t = 11.318$.

The volume increases until $t = 4.790$, then
decreases until $t = 11.318$, then increases, so the
absolute minimum will be at $t = 0$ or at
$t = 11.318$.

$$125 + \int_0^{11.318} (H(t) - R(t))\,dt = 120.738$$

Since the volume is 125 at $t = 0$, the volume is
least at $t = 11.318$.

$3: \begin{cases} 1: \text{sets } H(t) - R(t) = 0 \\ 1: \text{volume is least at} \\ \quad\ t = 11.318 \\ 1: \text{analysis for absolute} \\ \quad\ \text{minimum} \end{cases}$