(−5, 2)

(4, −4)

Graph of $f$

3. The function $f$ is defined on the closed interval $[-5, 4]$. The graph of $f$ consists of three line segments and is shown in the figure above. Let $g$ be the function defined by $g(x) = \int_{-3}^{x} f(t)\, dt$.

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of $g$ both increasing and concave down? Give a reason for your answer.

(c) The function $h$ is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.

(d) The function $p$ is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of $p$ at the point where $x = -1$.

| $t$ (minutes) | 0 | 2 | 5 | 8 | 12 |
|---|---|---|---|---|---|
| $v_A(t)$ (meters/minute) | 0 | 100 | 40 | −120 | −150 |

4. Train $A$ runs back and forth on an east-west section of railroad track. Train $A$'s velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time $t$ is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

(a) Find the average acceleration of train $A$ over the interval $2 \le t \le 8$.

(b) Do the data in the table support the conclusion that train $A$'s velocity is $-100$ meters per minute at some time $t$ with $5 < t < 8$? Give a reason for your answer.

(c) At time $t = 2$, train $A$'s position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train $A$, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

(d) A second train, train $B$, travels north from the Origin Station. At time $t$ the velocity of train $B$ is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train $A$ and train $B$ is changing at time $t = 2$.

| $x$ | $-2$ | $-2 < x < -1$ | $-1$ | $-1 < x < 1$ | $1$ | $1 < x < 3$ | $3$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 12 | Positive | 8 | Positive | 2 | Positive | 7 |
| $f'(x)$ | $-5$ | Negative | 0 | Negative | 0 | Positive | $\frac{1}{2}$ |
| $g(x)$ | $-1$ | Negative | 0 | Positive | 3 | Positive | 1 |
| $g'(x)$ | 2 | Positive | $\frac{3}{2}$ | Positive | 0 | Negative | $-2$ |

5. The twice-differentiable functions $f$ and $g$ are defined for all real numbers $x$. Values of $f$, $f'$, $g$, and $g'$ for various values of $x$ are given in the table above.

(a) Find the $x$-coordinate of each relative minimum of $f$ on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value $c$, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function $h$ is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x)) g'(x)\, dx$.

6. Consider the differential equation $\dfrac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function $f$ is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.
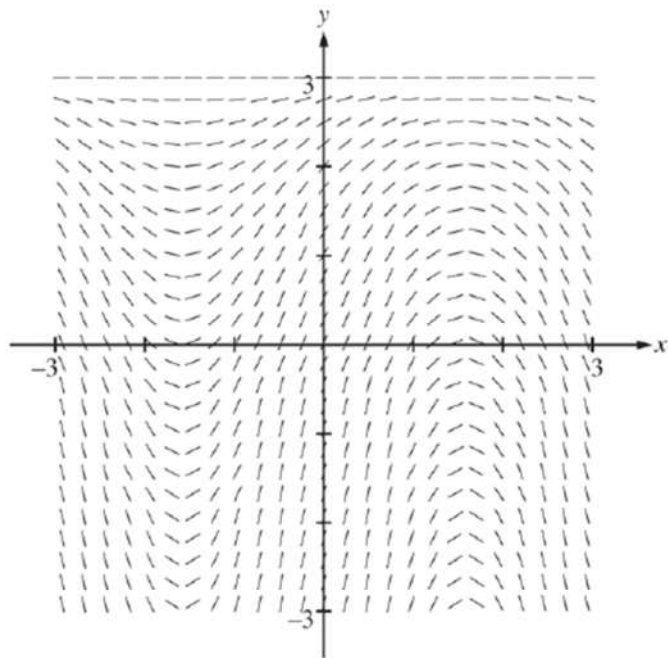


(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.