Interactive Classroom

Glencoe

ALGEBRA 2

LESSON 4–1 Graphing Quadratic Functions

Mc Graw Hill

Lesson Menu

5-Minute Check

Over Chapter 3

CheckPoint

1 Find $\begin{bmatrix} 6 & -5 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 4 & 7 \\ 3 & -5 & 1 \end{bmatrix}$.

A. $\begin{bmatrix} 5 & -9 \\ -3 & 9 \end{bmatrix}$

B. $\begin{bmatrix} 2 & -12 \\ 5 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 5 & -9 & -7 \\ -3 & 9 & -1 \end{bmatrix}$

D. impossible

5-Minute Check

Over Chapter 3

CheckPoint

2 Find $\begin{bmatrix} 6 & 0 \\ -4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -3 & 3 \end{bmatrix}$.

A. $\begin{bmatrix} 3 & 0 \\ 4 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 6 & 12 \\ -7 & -5 \end{bmatrix}$

C. $\begin{bmatrix} 6 & 0 \\ 12 & 3 \end{bmatrix}$

D. impossible

5-Minute Check

Over Chapter 3

CheckPoint

Standardized Test Practice

5 Janice bought three slices of pizza and one soft drink for $4.70. Jacob bought six slices of pizza and two soft drinks for $9.40. Which of the following matrix equations could be used to find the cost of one slice of pizza p and one soft drink s?

A. $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} p \\ s \end{bmatrix} = \begin{bmatrix} 4.7 \\ 9.4 \end{bmatrix}$

B. $\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} \cdot \begin{bmatrix} p \\ s \end{bmatrix} = \begin{bmatrix} 4.7 \\ 9.4 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} p \\ s \end{bmatrix} = \begin{bmatrix} 4.7 \\ 9.4 \end{bmatrix}$

D. $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} s \\ p \end{bmatrix} = \begin{bmatrix} 4.7 \\ 9.4 \end{bmatrix}$

Content Standards

A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Mathematical Practices

1 Make sense of problems and persevere in solving them.

Then

You identified and manipulated graphs of functions.

Now

- Graph quadratic functions.
- Find and interpret the maximum and minimum values of a quadratic function.

New Vocabulary

- quadratic function
- quadratic term
- linear term
- constant term
- parabola
- axis of symmetry
- vertex
- maximum value
- minimum value

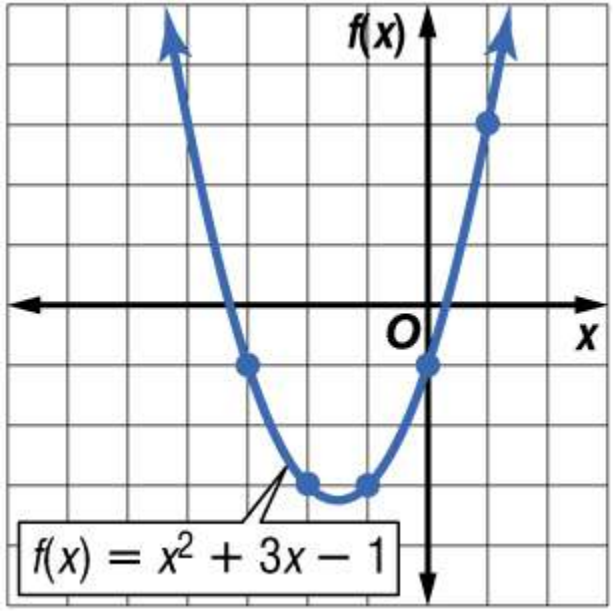
EXAMPLE 1 Graph a Quadratic Function by Using a Table

Graph $f(x) = x^2 + 3x - 1$ by making a table of values.

Choose integer values for x and evaluate the function for each value. Graph the resulting coordinate pairs and connect the points with a smooth curve.

Answer:

x	$x^2 + 3x - 1$	$f(x)$	(x, y)
-3	$(-3)^2 + 3(-3) - 1$	-1	$(-3, -1)$
-2	$(-2)^2 + 3(-2) - 1$	-3	$(-2, -3)$
-1	$(-1)^2 + 3(-1) - 1$	-3	$(-1, -3)$
0	$(0)^2 + 3(0) - 1$	-1	$(0, -1)$
1	$(1)^2 + 3(1) - 1$	3	$(1, 3)$

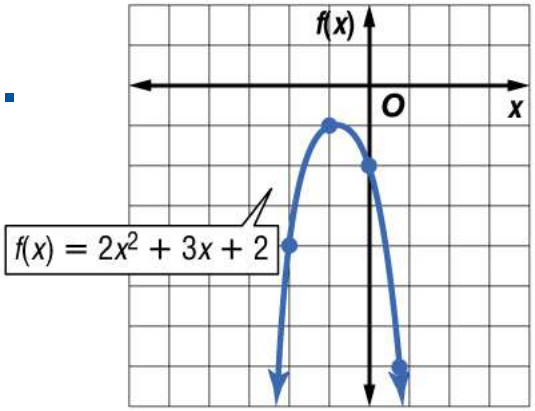


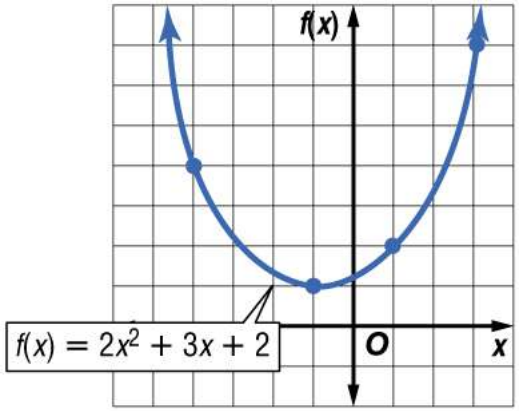
EXAMPLE 1 Check Your Progress

CheckPoint

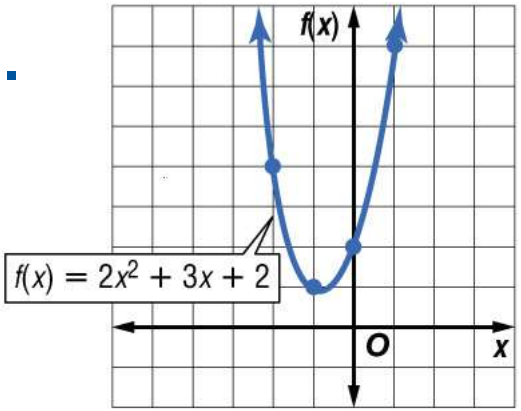
Which graph is the graph of $f(x) = 2x^2 + 3x + 2$?

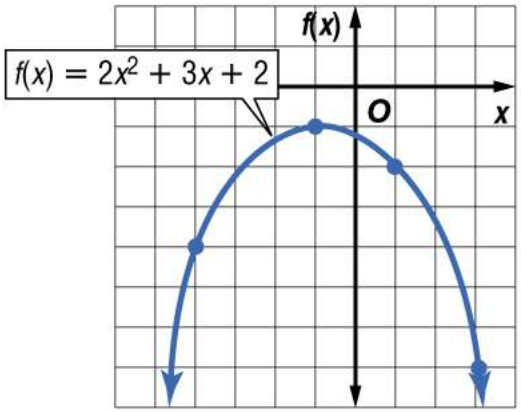
A.B.





C.D.





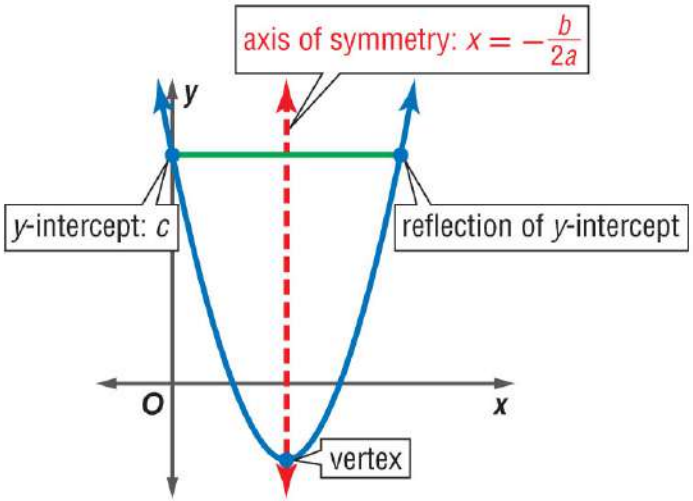
KeyConcept Graph of a Quadratic Function—Parabola

Words

Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.

- The y-intercept is $a(0)^2 + b(0) + c$ or c.
- The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.

Model



EXAMPLE 2 Axis of Symmetry, *y*-intercept, and Vertex

A. Consider the quadratic function $f(x) = 2 - 4x + x^2$. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

Begin by rearranging the terms of the function so that the quadratic term is first, the linear term is second and the constant term is last. Then identify *a*, *b*, and *c*.

$$f(x) = ax^2 + bx + c$$

$$f(x) = 2 - 4x + x^2 \rightarrow f(x) = 1x^2 - 4x + 2$$

$a = 1$, $b = -4$, $c = 2$

The *y*-intercept is 2.

EXAMPLE 2 Axis of Symmetry, *y*-intercept, and Vertex

Use *a* and *b* to find the equation of the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

$x = -\frac{-4}{2(1)}$ $a = 1$, $b = -4$

x=2Simplify.

Answer: The *y*-intercept is 2. The equation of the axis of symmetry is $x = 2$.Therefore, the *x*-coordinate of the vertex is 2.

EXAMPLE 2 Axis of Symmetry, *y*-intercept, and Vertex

B. Consider the quadratic function $f(x) = 2 - 4x + x^2$. Make a table of values that includes the vertex.

Choose some values for x that are less than 2 and some that are greater than 2. This ensures that points on each side of the axis of symmetry are graphed.

Answer:

x	$x^2 - 4x + 2$	$f(x)$	$(x, f(x))$
0	$0^2 - 4(0) + 2$	2	(0, 2)
1	$1^2 - 4(1) + 2$	−1	(1, −1)
2	$2^2 - 4(2) + 2$	−2	(2, −2)
3	$3^2 - 4(3) + 2$	−1	(3, −1)
4	$4^2 - 4(4) + 2$	2	(4, 2)

← Vertex

EXAMPLE 2 Axis of Symmetry, *y*-intercept, and Vertex

C. Consider the quadratic function $f(x) = 2 - 4x + x^2$. Use the information from parts A and B to graph the function.

Graph the vertex and *y*-intercept.

Then graph the points from your table, connecting them with a smooth curve.

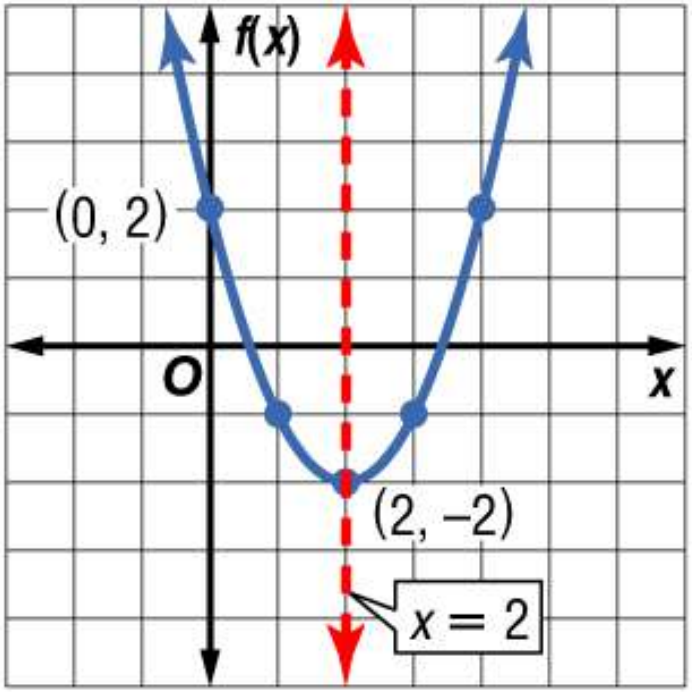
As a check, draw the axis of symmetry, $x = 2$, as a dashed line.

The graph of the function should be symmetrical about this line.

EXAMPLE 2

Axis of Symmetry, *y*-intercept, and Vertex

Answer:



EXAMPLE 2 Check Your Progress

CheckPoint

A. Consider the quadratic function $f(x) = 3 - 6x + x^2$. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

A. y-intercept = 3, axis of symmetry: $x = -3$, x-coordinate = -3

B. y-intercept = -3, axis of symmetry: $x = 3$, x-coordinate = 3

C. y-intercept = 3, axis of symmetry: $x = 3$, x-coordinate = 3

D. y-intercept = -3, axis of symmetry: $x = -3$, x-coordinate = -3

EXAMPLE 2 Check Your Progress

CheckPoint

B. Consider the quadratic function $f(x) = 3 - 6x + x^2$. Make a table of values that includes the vertex.

A.

x	0	1	2	3	4	5
$f(x)$	−3	2	5	6	5	2

B.

x	0	1	2	3	4	5
$f(x)$	−2	−5	−6	−5	−2	3

C.

x	0	1	2	3	4	5
$f(x)$	6	3	−2	−5	−6	−5

D.

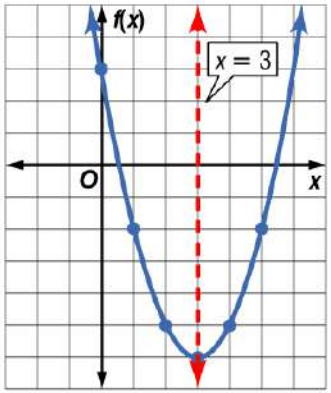
x	0	1	2	3	4	5
$f(x)$	3	−2	−5	−6	−5	−2

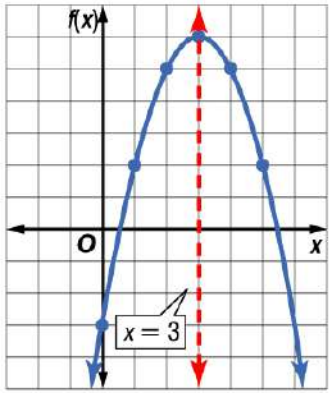
EXAMPLE 2 Check Your Progress

CheckPoint

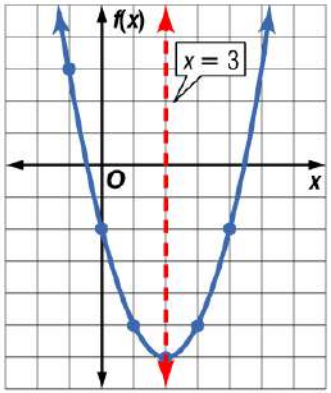
C. Consider the quadratic function $f(x) = 3 - 6x + x^2$. Use the information from parts A and B to graph the function.

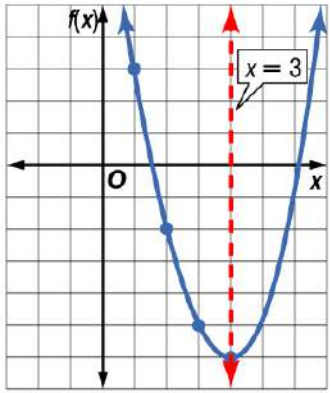
A.B





C.D





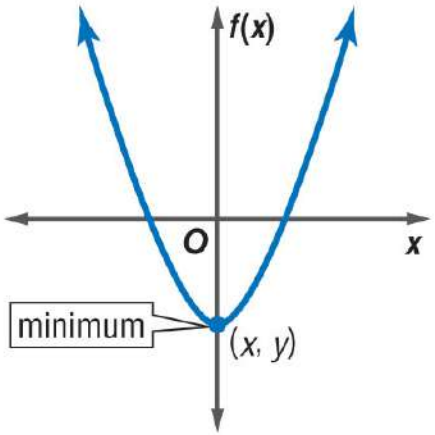
KeyConcept Maximum and Minimum Value

Words The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$,

- opens up and has a minimum value when $a > 0$, and
- opens down and has a maximum value when $a < 0$.

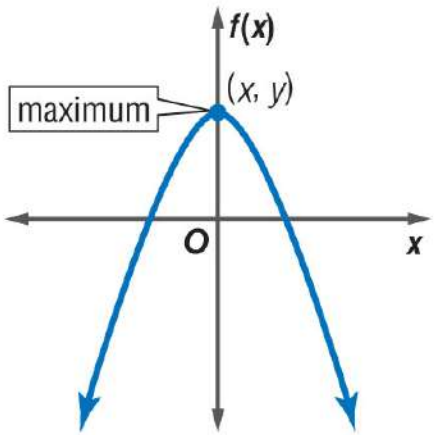
Model

a is positive.



The y-coordinate is the minimum value.

a is negative.



The y-coordinate is the maximum value.

EXAMPLE 3 Maximum or Minimum Values

A. Consider the function $f(x) = -x^2 + 2x + 3$. Determine whether the function has a maximum or a minimum value.

For this function, $a = -1$, $b = 2$, and $c = 3$.

Answer: Since $a < 0$, the graph opens down and the function has a maximum value.

EXAMPLE 3 Maximum or Minimum Values

B. Consider the function $f(x) = -x^2 + 2x + 3$. State the maximum or minimum value of the function.

The maximum value of this function is the *y*-coordinate of the vertex.

The *x*-coordinate of the vertex is $-\frac{2}{2(-1)}$ or 1.

Find the *y*-coordinate of the vertex by evaluating the function for $x = 1$.

$f(x) = -x^2 + 2x + 3$ Original function

$f(1) = -(1)^2 + 2(1) + 3$ or 4 $x = 1$

Answer: The maximum value of the function is 4.

EXAMPLE 3 Maximum or Minimum Values

C. Consider the function $f(x) = -x^2 + 2x + 3$. State the domain and range of the function.

The domain is all real numbers.

The range is all real numbers less than or equal to the maximum value.
That is, $\{f(x) | f(x) \leq 4\}$.

Answer : $D = \{\text{all real numbers}\}$; $R = \{f(x) | f(x) \leq 4\}$

EXAMPLE 3 Check Your Progress

CheckPoint

A. Consider the function $f(x) = x^2 + 4x - 1$. Determine whether the function has a maximum or a minimum value.

A. maximum

B. minimum

C. both

D. none

EXAMPLE 3 Check Your Progress

CheckPoint

B. Consider the function $f(x) = x^2 + 4x - 1$. What is the maximum or minimum value of the function?

A. –5

B. –1

C. 5

D. none

EXAMPLE 3 Check Your Progress

CheckPoint

C. Consider the function $f(x) = x^2 + 4x - 1$. What are the domain and range of the function?

A. $D = \{\text{all real numbers}\}$;
$R = \{f(x) \mid f(x) \geq -5\}$

B. $D = \{\text{all real numbers}\}$;
$R = \{f(x) \mid f(x) \leq -5\}$

C. $D = \{x \geq -5\}$;
$R = \{\text{all real numbers}\}$

D. $D = \{x \leq -5\}$;
$R = \{\text{all real numbers}\}$

Real-World Example 4

Quadratic Equations in the Real World

A. ECONOMICS A souvenir shop sells about 200 coffee mugs each month for $6 each. The shop owner estimates that for each $0.50 increase in the price, he will sell about 10 fewer coffee mugs per month. How much should the owner charge for each mug in order to maximize the monthly income from their sales?

Words Income equals number of mugs times price.

Variable Let x = the number of $0.50 price increases. Let $I(x)$ equal the income as a function of x.

Real-World Example 4

Quadratic Equations in the Real World

Income is number of mugs times price per mug.

Equation $I(x)=(200 - 10x)\bullet(6 + 0.50x)$

$I(x)=(200 - 10x) \bullet (6 + 0.50x)$

$=200(6) + 200(0.50x) - 10x(6) - 10x(0.50x)$

$=1200 + 100x - 60x - 5x^2$ Multiply.

$=1200 + 40x - 5x^2$ Simplify.

$=-5x^2 + 40x + 1200$ Write in $ax^2 + bx + c$ form.

Real-World Example 4

Quadratic Equations in the Real World

$l(x)$ is a quadratic function with $a = -5$, $b = 40$, and $c = 1200$. Since $a < 0$, the function has a maximum value at the vertex of the graph. Use the formula to find the x-coordinate of the vertex.

For $x = -\frac{b}{2a}$, x-coordinate of the vertex

$= -\frac{40}{2(-5)}$ $a = -5$, $b = 40$

Simplify. $= 4$

Real-World Example 4

Quadratic Equations in the Real World

This means that the shop should make 4 price increases of $0.50 to maximize their income.

Answer: The mug price should be $6 + $0.50(4) or $8.

Real-World Example 4

Quadratic Equations in the Real World

B. ECONOMICS **A souvenir shop sells about 200 coffee mugs each month for $6 each. The shop owner estimates that for each $0.50 increase in the price, he will sell about 10 fewer coffee mugs per month. What is the maximum monthly income the owner can expect to make from these items?**

To determine the maximum income, find the maximum value of the function by evaluating $I(x)$ for $x = 4$.

$I(x) = -5x^2 + 40x + 1200$ Income function

$= -5(4)^2 + 40(4) + 1200$ $x = 4$

$= \$1280$ Use a calculator.

Answer: Thus, the maximum income is $1280.

Real-World Example 4 | Check Your Progress | CheckPoint

A. ECONOMICS A sports team sells about 100 coupon books for $30 each during their annual fundraiser. They estimate that for each $0.50 decrease in the price, they will sell about 10 more coupon books. How much should they charge for each book in order to maximize the income from their sales?

A. $29.50

B. $20.00

C. $17.50

D. $15.00

Real-World Example 4 | Check Your Progress | CheckPoint

B. ECONOMICS A sports team sells about 100 coupon books for $30 each during their annual fundraiser. They estimate that for each $0.50 decrease in the price, they will sell about 10 more coupon books. What is the maximum income the team can expect to make from these items?

A. $3123.75

B. $5843.75

C. $6125.00

D. $12,250.00

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ALGEBRA 2

Click the mouse button to return to the lesson menu.

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