

What you'll Learn About

- Definition of the derivative
- Notation

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Use the substitution $h = x - a$ to create ^{another} the definition of the derivative $\Delta x = h$

A₁) Set-up a formula for the slope of $f(x) = x^2$ at $x = -1$

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} = \frac{(x+1)(x-1)}{(x+1)} = -2$$

$$\lim_{h \rightarrow 0} \frac{(h-1)^2 - 1}{h} = \lim_{h \rightarrow 0} \frac{(h-1) - 1}{h} = \frac{h^2 - 2h + 1 - 1}{h} = \frac{h^2 - 2h}{h} = h - 2$$

A₂) Use the substitution $h = x - a$ to set-up the definition of the derivative

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

B₁) Set-up a formula for the slope of $f(x) = \frac{1}{x-2}$ at $x = 4$

$$\lim_{x \rightarrow 4} \frac{\frac{1}{x-2} - \frac{1}{2}}{x-4}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{4+h-2} - \frac{1}{4-2}}{h}$$

B₂) Use the substitution $h = x - a$ to set-up the definition of the derivative

$$h = x - 4 \implies h + 4 = x$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{(h+4)-2} - \frac{1}{2}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{h+2} - \frac{1}{2}}{h}$$

$$\frac{1}{x-2} = f(x)$$

$$\frac{1}{5-2} = f(5)$$

$$\frac{1}{(x+h)-2} = f(x+h)$$

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Given a definition of the derivative (slope) find the function that you are taking the derivative of and the point you are finding the derivative (slope) at

A) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

$$f(x) = \sqrt{x}$$

$$x = 4 \quad y = 2$$

B) $\lim_{x \rightarrow 2} \frac{\ln x - \ln 2}{x - 2}$

Handwritten annotations: $f(x)$ above $\ln x$, $f(a)$ above $\ln 2$, and a below 2 .

$$f(x) = \ln x \quad (2, \ln 2)$$

C) $\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h}$

Handwritten annotations: $f(x+h)$ above $(2+h)^3$, $f(x) \rightarrow f(2)$ above 8 .

$$f(x) = x^3 \quad (2, 8)$$

D) $\lim_{h \rightarrow 0} \frac{\frac{2}{3+h} - \frac{2}{3}}{h}$

$$\rightarrow f(x) = \frac{2}{x} \quad \left(3, \frac{2}{3}\right)$$

Another Definition: $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$