Find the polynomial function with the given degree and given zeros

Degree = 3, Zeros = 0, 4, 5

Zeros $x=0 \quad x=4 \quad x=5$

Factors $x(x-4)(x-5)$

$x(x^2-9x+20)$

$f(x) = x^3-9x^2+20x$

$5(x)=\left(\frac{4}{5}\right)$

$5x=4$

$5x-4=0$

Degree = 3, Zeros = -3, 4/5, 5/2

$x=-3 \quad x=\frac{4}{5} \quad x=\frac{5}{2}$

$(x+3)(5x-4)(2x-5)$

$(x+3)(10x^2-33x+20)$

$$y = \begin{array}{r} 10x^3-33x^2+20x \\ \underline{\quad\quad 30x^2-99x+60} \\ 10x^3-3x^2-79x+60 \end{array}$$

Using only algebraic methods, find the cubic function with the given table of values

x	-4	-2	0	5
f(x)	0	0	100	0

Zeros $x=-4$ $x=-2$ $x=5$

$$y = a(x+4)(x+2)(x-5)$$

Solve for a

$$100 = a(0+4)(0+2)(0-5)$$

$$100 = -40a$$

$$a = \frac{100}{-40} = -\frac{10}{4} = -\frac{5}{2}$$

$$y = -\frac{5}{2}(x+4)(x+2)(x-5)$$

$$= -\frac{5}{2}(x+4)(x^2-3x-10)$$

$$= -\frac{5}{2}(x^3+x^2-22x-40)$$

$$-\frac{5}{2}x^3-\frac{5}{2}x^2+55x+100$$

x^3-3x^2-10x
$4x^2-12x-40$

State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept.

a) $f(x) = x(x + 2)^2$

Degree = 3

Zeros	mult	
$x = 0$	1	cross
$x = -2$	2	touch

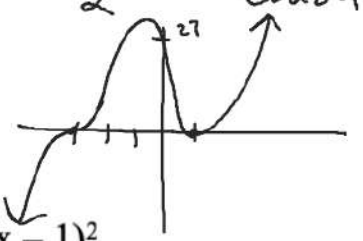
y-intercept (0,0)

b) $f(x) = (x + 3)^3(x - 1)^2$

D = 5

Zeros	mult	
$x = -3$	3	Cross
$x = 1$	2	touch

y-int (0,27)

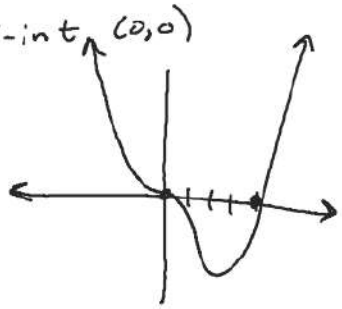


c) $f(x) = x^3(x - 4)$

Degree = 4

Zeros	mult	
$x = 0$	3	Cross
$x = 4$	1	Cross

y-int (0,0)

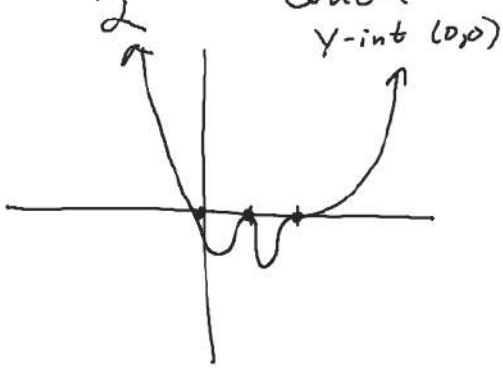


d) $f(x) = 3x(x - 2)^3(x - 1)^2$

Degree = 6

Zeros	mult	
$x = 0$	1	Cross
$x = 2$	3	Cross
$x = 1$	2	touch

y-int (0,0)



Start of 3.6

Use the Rational Zeros Theorem to write a list of all potential rational zeros and then determine which ones. if any, are zeros,

$f(x) = 3x^3 + 4x^2 - 5x - 2$

1	3	4	-5	-2
		3	7	2
-2	3	7	2	0
		-6	-2	
	3	1	0	

$3x+1=0$

Last term (p) $= \pm 1, \pm 2$

1st term (q) $= \pm 1, \pm 3$

List of all possible zeros

$\frac{p}{q} = \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

$3x^2+7x+2$

$(x-1)(3x+1)(x+2)$

$x=1 \quad x=-\frac{1}{3} \quad x=-2$

$f(x) = x^3 - 3x^2 + 1$

1	1	-3	0	1
		1	-2	-2
	1	-2	-2	-1

$p = \pm 1$

$q = \pm 1$

Possible zeros ± 1

Finding Rational Zeros

1) List all possible rational zeros p/q where q is the leading coefficient and p is the constant

2) Use your calculator to find the zeros and then use synthetic division and algebra to prove that the zeros that you chose are rational zeros

Use the Rational Zeros Theorem to write a list of all potential rational zeros and then determine which ones, if any, are zeros.

$f(x) = x^3 - 6x^2 + 7x + 4$

$p = 4 \quad \pm 1, \pm 2, \pm 4$

$q = 1 \quad \pm 1$

$\frac{p}{q}$ List of possible rational zeros $\pm 1, \pm 2, \pm 4$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{2}{2(1)} \pm \frac{\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$

$1 \pm \frac{\sqrt{4-(-4)}}{2}$

$1 \pm \frac{\sqrt{8}}{2}$

4	1	-6	7	4
		4	-8	-4
	1	-2	-1	0

$x^2 - 2x - 1$

$x = 4$ → Rational

$x = 1 + \frac{\sqrt{8}}{2}$ → Irrational

$x = 1 - \frac{\sqrt{8}}{2}$ → Irrational

$f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$

$(2x^3 + x^2) - 4x - 2$

$x^2(2x+1) - 2(2x+1)$

4	2	-7	-8	14	8
		8	4	-16	-8
$-\frac{1}{2}$	2	1	-4	-2	0
		-1	0	2	
	2	0	-4	0	

$x = 4, \; x = -\frac{1}{2}$

$x = \sqrt{2}, -\sqrt{2}$

$2x^2 - 4 = 0$

$\frac{2x^2}{2} = \frac{4}{2}$

$\sqrt{x^2} = \sqrt{2}$

$x = \pm\sqrt{2}$

3.5 extra practice

49, 51, 54, 56

What you'll Learn About

Write the polynomial in standard form, and identify the zeros of the function and the x-intercepts.

a) $(x - 4i)(x + 4i)$

b) $(x - 3)(x - \sqrt{4}i)\ (x + \sqrt{4}i)$

c) $x(x - 3)(x - 2 - i)\ (x - 2 + i)$

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include those listed.

a) 2, 5i, and -6i

b) -2, 3, and $2 - i$

c) -4, $2 + 3i$

	Write a polynomial function of minimum degree in standard form with real coefficients whose zeros and their multiplicities include those listed. Then sketch a graph and discuss what you notice. a) 3 (multiplicity 2), - 4 (multiplicity 3) b) 3 (multiplicity 3), - 4 (multiplicity 1) c) 5 (multiplicity 2), $2 + i$ (multiplicity 1)	

Find all of the zeros and write a linear factorization of the function

28) $f(x) = x^3 - 10x^2 + 44x - 69$

A) $f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$

Find all of the zeros and write a linear factorization of the function

$y = 3x^4 + 4x^3 + 2x^2 - x - 2$

Using the given zero find all of the zeros and write a linear factorization

33) $f(x) = x^4 - 2x^3 - x^2 + 6x - 6$ zero: $1 + i$

Write the function as a product of linear and irreducible quadratic factors all with real coefficients.

42) $f(x) = x^4 - 2x^3 + x^2 - 8x - 12$

$f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$