

Create the 3rd order Maclaurin Series for $f(x) = \arctan(x)$ by using the Taylor Polynomial process

Page 17

Create the Maclaurin Series for $f(x) = \sin x$ by using the Taylor Polynomial process (5th order)

$$\begin{array}{ll} f(x) = \sin x & f(0) = 0 \\ f'(x) = \cos x & f'(0) = 1 \\ f''(x) = -\sin x & f''(0) = 0 \\ f^3(x) = -\cos x & f'''(0) = -1 \\ f^4(x) = \sin x & f^{(4)}(0) = 0 \\ f^5(x) = \cos x & f^{(5)}(0) = 1 \end{array}$$

$$P_5(x) = x - \frac{x^3}{3} + \frac{x^5}{5!}$$

Write the first four terms for $f(x) = \sin(x)$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!}$$

Find each of the following

1. $f^{(5)}(0) = 1$ 2. $f^{(13)}(0) = 1$ 3. $f^{(23)}(0) = -1$

$4n+1$ positive derivatives

$4n+3$ neg derivatives

$$f^{(99)}(0) = -1$$

$$\begin{array}{r} 4n+3 = 99 \\ -3 \quad -3 \\ \hline 4n = 96 \\ n = 24 \end{array}$$

Write the first four terms for $f(x) = \sin(x^4)$

$$x^4 - \frac{x^{12}}{3!} + \frac{x^{20}}{5!} - \frac{x^{28}}{7!}$$

Find each of the following

1. $f^{(4)}(0) = 4!$ 2. $f^{(12)}(0) =$ 3. $f^{(20)}(0) = \frac{20! \cdot 1}{5! \cdot 5!} \cdot 20!$

$$\begin{aligned} f(x) &= \sin(x^4) \\ f'(x) &= \cos(x^4) \cdot 4x^3 \\ f''(x) &= \end{aligned}$$

4th derivative
in front of
4th power

$$\frac{f^{(4)}(x) x^4}{4!} = \frac{(4!) x^4}{4!}$$

$$\frac{-x^{12}}{3!} = \frac{f^{(12)}(0) x^{12}}{12!}$$

$$\frac{-1}{3!} = \frac{f^{(12)}(0)}{12!}$$

$$\frac{-(2!)}{3!} = f^{(12)}(0)$$

$$\frac{x^{20}}{5!} = \frac{f^{(20)}(0) x^{20}}{20!}$$

$$\frac{1}{5!} = \frac{f^{(20)}(0)}{20!}$$

$$\frac{20!}{5!} = f^{(20)}(0)$$

$$\text{If } g(x) = \dots \frac{x^{12}}{12} \dots \text{ find } f^{12}(0) = \frac{1}{12} \cdot 12! = 11!$$

$$\text{If } g(x) = \dots \frac{(x-3)^{20}}{10} \dots \text{ find } f^{20}(3) = \frac{1}{10} \cdot 20! = \frac{20!}{10}$$

- ① Take derivatives
- ② Plug in center
- ③ Build polynomial

Find the 3rd order Taylor Polynomial centered at $x=2$

18) $f(x) = \frac{1}{x} = x^{-1}$ $f(2) = \frac{1}{2}$

$f'(x) = -x^{-2} = -\frac{1}{x^2}$ $f'(2) = -\frac{1}{4}$

$f''(x) = 2x^{-3} = \frac{2}{x^3}$ $f''(2) = \frac{2}{8} = \frac{1}{4}$

$f'''(x) = -6x^{-4} = -\frac{6}{x^4}$ $f'''(2) = -\frac{6}{16} = -\frac{3}{8}$

$$P_3(x-2) = \frac{1}{2}(x-2)^0 + \frac{-\frac{1}{4}(x-2)^1}{1!} + \frac{\frac{1}{4}(x-2)^2}{2!} - \frac{\frac{3}{8}(x-2)^3}{3!}$$

$f(x) = \frac{1}{x} \approx P_3(x-2) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$

19) $f(x) = \sin x$ at $x = \frac{\pi}{4}$ (center)

$f(x) = \sin x$ $f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$f'(x) = \cos x$ $f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$f''(x) = -\sin x$ $f''(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$f'''(x) = -\cos x$ $f'''(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$$P_3(x-\frac{\pi}{4}) = \frac{\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})^0}{0!} + \frac{\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})^1}{1!} - \frac{\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})^2}{2!} - \frac{\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})^3}{3!}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x-\frac{\pi}{4}) - \frac{\sqrt{2}}{4}(x-\frac{\pi}{4})^2 - \frac{\sqrt{2}}{12}(x-\frac{\pi}{4})^3$$

Sometimes if your center is not at zero, you do not need to build the polynomial. You can use the MacLaurin series and Substitution.

Construct the polynomial for $f(x) = e^{x-1}$ centered at $x = 1$ using derivatives.

Construct the polynomial for $f(x) = e^{x-1}$ centered at $x = 1$ using a MacLaurin series.