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What you'll Learn About
How to build a polynomial using derivatives

- $P(0) = 7$
- $P'(0) = 3$
- $P''(0) = 9$
- $P'''(0) = 15$
- $P^{(4)}(0) = 6$
- $P^{(5)}(0) = 4$
- $P^{(6)}(0) = 12$

Given the values of the following, construct the 6th degree Taylor Polynomial centered at $x = 0$

$$P(x) = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6$$

$$P(0) = a \quad a = 7$$

$$P'(x) = b + 2cx + 3dx^2 + 4ex^3 + 5fx^4 + 6gx^5$$

$$3 = b$$

$$P''(x) = 2c + 6dx + 12ex^2 + 20fx^3 + 30gx^4$$

$$P''(0) = 2c \quad 9 = 2c \quad c = \frac{9}{2}$$

$$P'''(x) = 6d + 24ex + 60fx^2 + 120gx^3$$

$$15 = 6d \quad d = \frac{15}{6}$$

$$P^{(4)}(x) = 24e + 120fx + 360gx^2$$

$$6 = 24e \quad e = \frac{6}{24}$$

$$P^{(5)}(x) = 120f + 720gx$$

$$4 = 120f \quad f = \frac{4}{120}$$

$$P^{(6)}(x) = 720g$$

$$12 = 720g \quad g = \frac{12}{720}$$

$$P_6(x) = 7 + 3x + \frac{9}{2}x^2 + \frac{15}{3!}x^3 + \frac{6}{4!}x^4 + \frac{4}{5!}x^5 + \frac{12}{6!}x^6$$

What would be the next 2 terms if $P^{(7)}(0) = 22$ and $P^{(8)}(0) = 50$?

$$\frac{22x^7}{7!} + \frac{50x^8}{8!} \quad \begin{matrix} \uparrow \\ 7^{\text{th}} \text{ derivative} \end{matrix}$$

$\frac{(\text{value of derivative})x^n}{n!}$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

Given the values of the following, construct the 4th degree Taylor Polynomial centered at $x = 0$

1. $P(0)=2$ $P'(0)=5$ $P''(0)=8$ $P'''(0)=11$ $P^{(4)}(0)=14$

$$P_4(x) = \frac{2x^0}{0!} + \frac{5x^1}{1!} + \frac{8x^2}{2!} + \frac{11x^3}{3!} + \frac{14x^4}{4!}$$

$$P_4(-2) = 2 + 5(-2) + \frac{8(-2)^2}{2!}$$

2. $P(0)=5$ $P'(0)=-2$ $P''(0)=7$ $P'''(0)=-4$ $P^{(4)}(0)=10$

$$P_5(x) = \frac{5x^0}{0!} - \frac{2x^1}{1!} + \frac{7x^2}{2!} - \frac{4x^3}{3!} + \frac{10x^4}{4!}$$

Given the values of the following, construct the 4th degree Taylor Polynomial centered at $x = 2$

3. $P(2)=2$ $P'(2)=5$ $P''(2)=8$ $P'''(2)=11$ $P^{(4)}(2)=14$

$$P_4(x-2) = \frac{2(x-2)^0}{0!} + \frac{5(x-2)^1}{1!} + \frac{8(x-2)^2}{2!} + \frac{11(x-2)^3}{3!} + \frac{14(x-2)^4}{4!}$$

$$P_4(x-2) = 2 + 5(x-2) + 4(x-2)^2 + \frac{11}{2}(x-2)^3 + \frac{7}{2}(x-2)^4$$

Given the values of the following, construct the 4th degree Taylor Polynomial centered at $x = -2$

4. $P(-2)=5$ $P'(-2)=-2$ $P''(-2)=7$ $P'''(-2)=-4$ $P^{(4)}(-2)=10$

$$P_4(x+2) = 5 - 2(x+2) + \frac{7(x+2)^2}{2} - \frac{4(x+2)^3}{3!} + \frac{10(x+2)^4}{4!}$$

3rd order = 3rd degree (Take 3 derivatives) → centered at $x=0$

Create the Maclaurin Series for $f(x) = e^x$ by using the Taylor Polynomial process

$$f(x) = e^x \quad f(0) = e^0 = 1$$

$$f'(x) = e^x \quad f'(0) = 1$$

$$f''(x) = e^x \quad f''(0) = 1$$

$$f'''(x) = e^x \quad f'''(0) = 1$$

$$e^x \approx P_3(x) = \frac{1x^0}{0!} + \frac{1x^1}{1!} + \frac{1x^2}{2!} + \frac{1x^3}{3!}$$

$$P_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

Create the Maclaurin Series for $f(x) = \sin x$ by using the Taylor Polynomial process