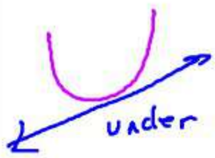
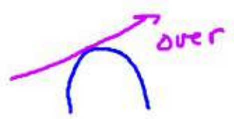
Let f be a function that has derivatives of all orders for all real numbers. Assume f(0) = 4, $f'(0) = 5$, $f''(0) = -8$, and $f'''(0) = 6$.

a. Write the third order Taylor Polynomial for f at x = 0 and use it to approximate f(.2).

$4 + 5x - \frac{8x^2}{2} + \frac{6x^3}{3!} = 4 + 5x - 4x^2 + x^3$



b. Write the second order Taylor polynomial for f', at x = 0



c. Write the fourth order Taylor polynomial for $\int_0^x f(t)dt$, at x = 0.

$\int_0^x f(t)\,dt = 4x + \frac{5}{2}x^2 - \frac{4}{3}x^3 + \frac{1}{4}x^4 + C \qquad f(0) = 4$

$4x + \frac{5}{2}x^2 - \frac{4}{3}x^3 + \frac{1}{4}x^4 + 4$

d. Determine if the linearization of f is an underestimate or overestimate near 0.

tangent line

$y = 4 + 5(x - 0)$

$f''(0) = -8 < 0$

f concave down at x=0

overestimate

p. 527 57

a. Write the first three nonzero terms and the general term of the Taylor Series generated by $f(x) = 5\sin\left(\frac{x}{2}\right)$ at x = 0.

c. What is the minimum number of terms of the series in part a needed to approximate f(x) on the interval (-2, 2) with an error not exceeding .1 in magnitude. Explain your answer.

$P_1(x) = 5\left(\frac{x}{2}\right)$

$\text{Error} \le \frac{5\left(\frac{x}{2}\right)^3}{3!} \rightarrow \frac{5\left(\frac{2}{2}\right)^3}{3!} = \frac{5}{3!} = \frac{5}{6}$

2 Terms

$P_3(x) = 5\left(\frac{x}{2}\right) - \frac{5\left(\frac{x}{2}\right)^3}{3!}$

$\text{Error} \le \frac{5\left(\frac{x}{2}\right)^5}{5!} = \frac{5\left(\frac{2}{2}\right)^5}{5!} = \frac{5}{5!} = \frac{5}{120}$

p. 492 #24

The Maclaurin Series for f(x) is $f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots \frac{x^n}{(n+1)!}$.

a. Find $f''(0)$ and $f^{10}(0)$.

b. Let g(x) = xf(x). Write the Maclaurin Series for g(x), showing the first three non-zero terms and the general term.

c. Write g(x) in terms of a familiar function without using series.

p. 500 #13

Find a formula for the truncation error if we use $P_6(x)$ to approximate $\frac{1}{1+2x}$ on (-.5, .5).

geometric $r = -2x$

$P_6(x) = 1 - 2x + 4x^2 - 8x^3 + 16x^4 - 32x^5 + 64x^6$

next term = error bound $\leq 128x^7$

p. 500 20

a. If cos(x) is replaced by $1-\frac{x^2}{2}$ and $|x|<.5$, what estimate can be made of the error?

$P_2(x) = 1 - \frac{x^2}{2}$

next term = error bound $\leq \frac{x^4}{4!}$ $\quad \frac{.5^4}{4!}$

b. Does $1-\frac{x^2}{2}$ tend to be too large or too small.

$-.5 < x < .5$

$P_2(x) = 1 - \frac{x^2}{2}$ $\quad f(x) = \cos x$

$P_2(.5) = 1 - \frac{.5^2}{2}$ $\quad f(.5) = \cos(.5)$

p. 500 #22

The approximation $\sqrt{1+x} \approx 1+\frac{x}{2}$ is used when x is small. Estimate the error when $|x|<.1$

$f(x) = (1+x)^{1/2}$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2} = \frac{-1}{4(1+x)^{3/2}}$

Build next term at

$x = -.1$	$x = 0$	$x = .1$
$\frac{f^2(-.1)x^2}{2!}$	$\frac{f^2(0)x^2}{2!}$	$\frac{f^2(.1)x^2}{2!}$
error bound $< \left\| \frac{\frac{-1}{4(.9)^{3/2}}(.1)^2}{2!} \right\|$	$\left\| \frac{-\frac{1}{4}x^2}{2!} \right\|$	$\left\| \frac{\frac{-1}{4(1.1)^{3/2}}(.1)^2}{2!} \right\|$

p. 527 #60

Let $f(x)=\dfrac{1}{x-2}$ at x = 3.

a. Write the first 4 terms and the general term of the Taylor Series generated by f(x) at x = 3.

b. Use the result in part (a) to find the fourth order polynomial and the general term of the series generated by $\ln|x-2|$ at x = 3.

c. Use the series in part (b) to compute a number that differs from ln(1.5) by less than 0.05. Justify your answer.

83. The Taylor Series for lnx, centered at x = 1, is $\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $.3 \le x \le 1.7$ is

(A) .030 (B) .039 (C) .145 (D) .153 (E) .529

2011 BC6

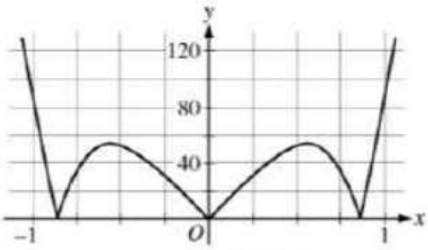
Let $f(x) = \sin(x^2) + \cos x$.

a. Write the first four nonzero terms of the Taylor series for sinx about x = 0, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about x = 0.

b. Write the first four nonzero terms of the Taylor series for cosx about x = 0. Use this series and the series for $\sin(x^2)$, found in part a, to write the first four nonzero terms of the Taylor series for f(x) about x = 0.

c. Find the value of $f^{(6)}(0)$.

d. Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = \left|f^{(5)}(x)\right|$ is shown.



Graph of $y = \left|f^{(5)}(x)\right|$

Let $P_4(x)$ be the fourth degree Taylor polynomial for f about x = 0. Using information from the graph of $y = \left|f^{(5)}(x)\right|$, shown above, show that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

2004 BC6

Let f be the function given by $f(x)=\sin\left(5x+\frac{\pi}{4}\right)$, and let P(x) be the third-degree Taylor polynomial for f about x = 0.

a) Find P(x).

b) Find the coefficient of x^{22} in the Taylor series about x = 0.

c) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right)-P\left(\frac{1}{10}\right)\right|<\frac{1}{100}$.

d) Let G be the function given $G(x)=\int_0^x f(t)dt$. Write the third-degree Taylor polynomial for G about x = 0.