r : common ratio

$n=0 \quad a_n = a_0(r)^n$

$n=1 \quad a_n = a_1(r)^{n-1}$

What you'll Learn About
What a geometric series is and whether or not the series converges or diverges
The nth term test for divergence

$y = a \cdot b^x$

$y = 4 \cdot 2^x$

$a_n = 4 \cdot 2^n$

Start at $n=0$

Geometric Series Converges when

$|r| < 1$

$-1 < r < 1$

$\boxed{S = \frac{a}{1-r}}$

$\text{Sum} = \frac{1^{st}\text{ term}}{1 - \text{common ratio}}$

Rational Function

Given the first 4 terms of the Geometric Series: a) Write the general term of the series, b) Write the power series and c) Find the sum of the series, if possible

A) $4+8+16+32+\cdots + 4(2)^{n-1} + \cdots = \sum_{n=1}^{\infty} 4(2)^{n-1} \rightarrow$ Diverges

n=1 n=2 n=3 n=4

general term

Power series

B) $1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\cdots + 1\left(-\frac{1}{4}\right)^{n-1} + \cdots = \sum_{n=1}^{\infty}\left(-\frac{1}{4}\right)^{n-1} = .8$

Converges to

$r = -\frac{1}{4}$

$S = \frac{1}{1-\left(-\frac{1}{4}\right)} = \frac{1}{\frac{5}{4}}$

C) $5+15+45+135+\cdots + 5(3)^{n-1} + \cdots = \sum_{n=1}^{\infty} 5(3)^{n-1}$ = Diverges

$r = 3 > 1$

D) $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots + 1\left(-\frac{1}{2}\right)^{n-1} + \cdots = \sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n-1} = \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{2}{3}$

	Given the first 4 terms of the Geometric Series; a) Write the general term of the series, b) Write the power series, c) Find the equation for the sum of the series, and d) the Interval and Radius of Convergence of the series.

$A)\quad 1+5x+25x^2+125x^3+\cdots+1(5x)^{n-1}+\cdots=\sum_{n=1}^{\infty}(5x)^{n-1}=\frac{1}{1-5x}$

Interval of Convergence

$-1<r<1$

$\frac{-1}{5}<\frac{5x}{5}<\frac{1}{5}$

$\boxed{-\frac{1}{5}<x<\frac{1}{5}}$ I.O.C.

Radius of Convergence $=\frac{1}{5}$

$x=\frac{1}{10}\quad 1+\frac{5}{10}+\frac{25}{100}+\frac{125}{1000}+\cdots+(1)\left(\frac{5}{10}\right)^{n-1}+\cdots\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)=\frac{1}{1-5\left(\frac{1}{2}\right)}$

signs alternate

$B)\quad 1-(x-1)+(x-1)^2-(x-1)^3+\cdots+\underbrace{(-1)^n}_{\text{general term}}(x-1)^n+\cdots=\sum_{n=0}^{\infty}(-1)^n(x-1)^n$

power series

$1\cancel{(-(x-1))}^{n-1}$

$1(-(x-1))^n$

$(-1(x-1))^n$

$(-1)^n(x-1)^n$

R.O.C: 1

I.O.C: $-1<r<1$

$\frac{-1}{-1}<\frac{-(x-1)}{-1}<\frac{1}{-1}$

$1>x-1>-1$

$+1\quad +1\quad +1$

$2>x>0$

$=\frac{1}{1-(-(x-1))}$

$=\frac{1}{1+x-1}$

$=\frac{1}{x}$