Functions and Their Graphs

1.5 Combinations of Functions

What You Should Learn

- Add, subtract, multiply, and divide functions.
- Find compositions of one function with another function.
- Use combinations of functions to model and solve real-life problems.

Arithmetic Combinations of Functions

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two *functions* can be combined to create new functions. When

$f(x) = 2x - 3$ and $g(x) = x^2 - 1$

you can form the sum, difference, product, and quotient of *f* and *g* as follows.

Arithmetic Combinations of Functions

$f(x) + g(x) = (2x - 3) + (x^2 - 1)$

Sum

$= x^2 + 2x - 4$

$f(x) - g(x) = (2x - 3) - (x^2 - 1)$

Difference

$= -x^2 + 2x - 2$

Arithmetic Combinations of Functions

$f(x) \cdot g(x) = (2x - 3)(x^2 - 1)$

Product

$= 2x^3 - 3x^2 - 2x + 3$

$$\frac{f(x)}{g(x)} = \frac{2x - 3}{x^2 - 1}, \quad x \neq \pm 1$$

Quotient

Arithmetic Combinations of Functions

The domain of an **arithmetic combination** of functions *f* and *g* consists of all real numbers that are common to the domains of *f* and *g*. In the case of the quotient

$$\frac{f(x)}{g(x)}$$

there is the further restriction that $g(x) \neq 0$.

Example 1 – *Finding the Sum of Two Functions*

Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$, find $(f + g)(x)$. Then evaluate the sum when $x = 2$.

Solution:

$(f + g)(x) = f(x) + g(x)$

$= (2x + 1) + (x^2 + 2x - 1)$

$= x^2 + 4x$

When $x = 2$, the value of this sum is

$(f + g)(2) = 2^2 + 4(2)$

$= 12.$

Compositions of Functions

Compositions of Functions

Another way of combining two functions is to form the **composition** of one with the other.

For instance, when $f(x) = x^2$ and $g(x) = x + 1$, the composition of *f* with *g* is

$$f(g(x)) = f(x + 1)$$
$$= (x + 1)^2.$$

This composition is denoted as $f \circ g$ and is read as "*f* composed with *g*".

Compositions of Functions

Definition of Composition of Two Functions

The **composition** of the function f with the function g is

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. (See Figure 1.50.)

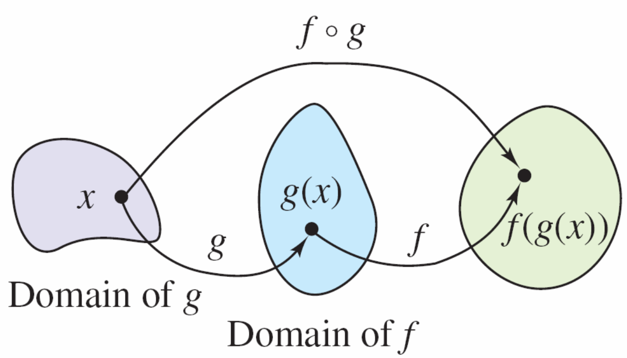

Figure 1.50

Example 5 – *Forming the Composition of f with g*

Find $(f \circ g)(x)$ for $f(x) = \sqrt{x}, x \geq 0,$ and $g(x) = x - 1, x \geq 1$.
If possible, find $(f \circ g)(2)$ and $(f \circ g)(0)$.

Solution:

The composition of *f* with *g* is

$(f \circ g)(x) = f(g(x))$ — Definition of $f \circ g$

$= f(x - 1)$ — Definition of $g(x)$

$= \sqrt{x - 1}, \quad x \geq 1.$ — Definition of $f(x)$

Example 5 – *Solution*

cont'd

The domain of $f \circ g$ is $[1, \infty)$ (See Figure 1.51).

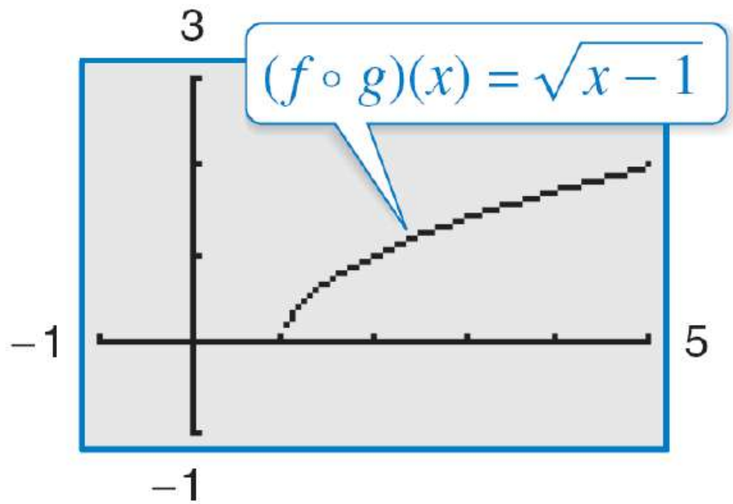

Figure 1.51

So,

$(f \circ g)(2) = \sqrt{2 - 1}$

is defined, but $(f \circ g)(0)$ is not defined because 0 is not in the domain of $f \circ g$.

Application

Example 10 – *Bacteria Count*

The number N of bacteria in a refrigerated petri dish is given by

$$N(T) = 20T^2 - 80T + 500, \quad 2 \le T \le 14$$

where T is the temperature of the petri dish (in degrees Celsius). When the petri dish is removed from refrigeration, the temperature of the petri dish is given by

$$T(t) = 4t + 2, \quad 0 \le t \le 3$$

where t is the time (in hours).

a. Find the composition $N(T(t))$ and interpret its meaning in context.

Example 10 – *Bacteria Count*

cont'd

b. Find the number of bacteria in the petri dish when $t = 2$ hours.

c. Find the time when the bacteria count reaches 2000.

Solution:

a. $N(T(t)) = 20(4t + 2)^2 - 80(4t + 2) + 500$

$= 20(16t^2 + 16t + 4) - 320t - 160 + 500$

$= 320t^2 + 320t + 80 - 320t - 160 + 500$

$= 320t^2 + 500$

The composite function $N(T(t))$ represents the number of bacteria as a function of the amount of time the petri dish has been out of refrigeration.

Example 10 – *Solution*

cont'd

b. When $t = 2$ the number of bacteria is

$$N = 320(2)^2 + 420$$

$$= 1280 + 420$$

$$= 1700.$$

c. The bacteria count will reach $N = 2000$ when $320t^2 + 420 = 2000$.

You can solve this equation for t algebraically as follows.

$$320t^2 + 420 = 2000$$

$$320t^2 = 1580$$

Example 10 – *Solution*

cont'd

$$t^2 = \frac{79}{16}$$

$$t = \frac{\sqrt{79}}{4} \quad \Rightarrow \quad t \approx 2.22 \text{ hours}$$

So, the count will reach 2000 when $t \approx 2.22$ hours. Note that the negative value is rejected because it is not in the domain of the composite function.

Example 10 – *Solution*

cont'd

To confirm your solution, graph the equation $N = 320t^2 + 420$ as shown in Figure 1.54. Then use the *zoom* and *trace* features to approximate $N = 2000$ when $t \approx 2.22$ as shown in Figure 1.55.

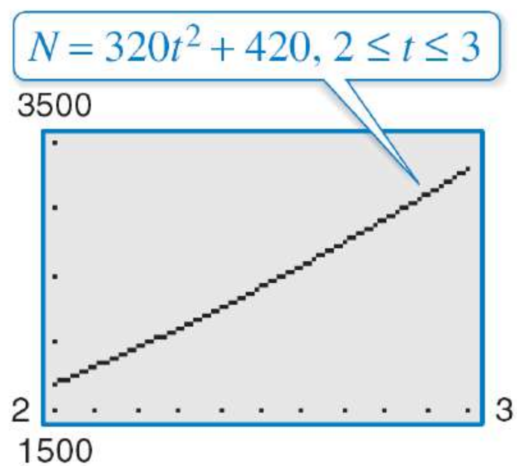

Figure 1.54

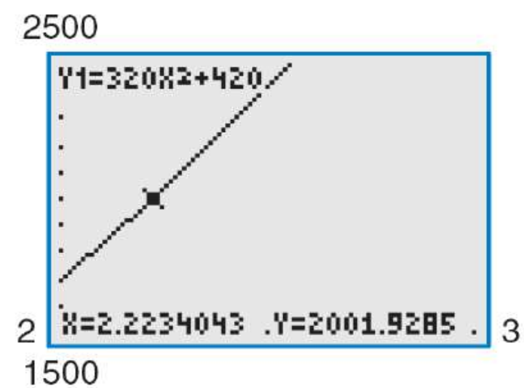

Figure 1.55