IM 1 12-7

Compostitions of Transformations

Homework

Read Sec 12-7

12-7 p922 11-17

No Khan Academy

Objective

Students draw glide reflections, and reflections across parallel or intersecting lines.

Isometries

Transformations

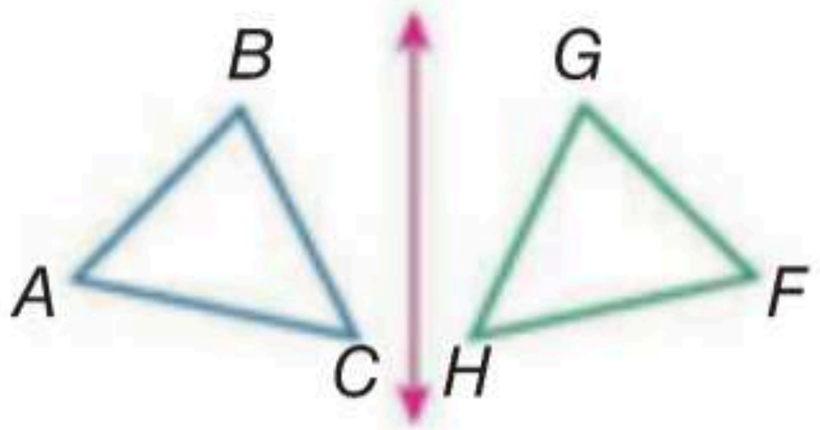
We have completed 3 basic forms of rigid transformations, or **isometries**.

In rigid transformations (**isometries**) the resulting **image** is congruent to the original **pre-image**.

KeyConcept Reflections, Translations, and Rotations

A **reflection** or *flip* is a transformation over a line called the *line of reflection*. Each point of the preimage and its image are the same distance from the line of reflection.

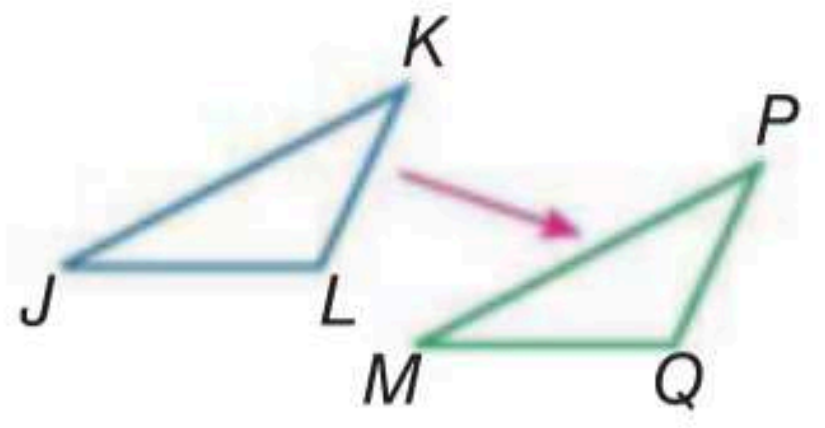
Example



$\triangle ABC \longrightarrow \triangle FGH$

A **translation** or *slide* is a transformation that moves all points of the original figure the same distance in the same direction.

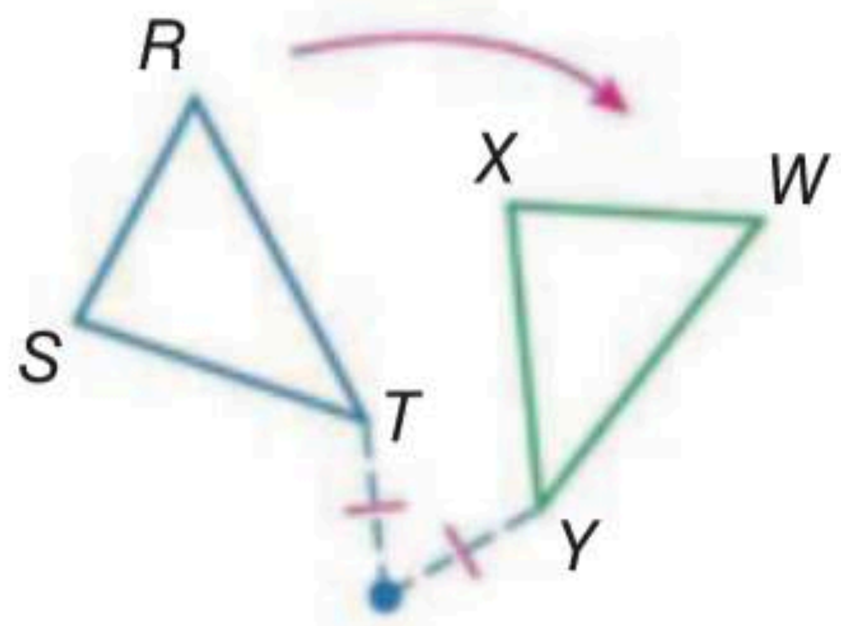
Example



$\triangle JKL \longrightarrow \triangle MPQ$

A **rotation** or *turn* is a transformation around a fixed point called the *center of rotation*, through a specific angle, and in a specific direction. Each point of the original figure and its image are the same distance from the center.

Example



$\triangle RST \longrightarrow \triangle WXY$

Composition of Rigid Transformations

We can combine rigid transformations (isometries) by completing one transformation followed by one or more additional transformations.

Compositions of rigid transformations result in a single image that is congruent to the original pre-image.
In other words, the composition of isometries is an isometry.

The composition of a **reflection** and a **translation** is called a **Glide Reflection**. The translation vector is parallel to the line of reflection.

The composition of **two reflections** over **parallel lines** is the same as a single **translation**. We need to be cautious when reflecting across parallel lines to be sure to translate in the correct direction and the correct distance.

The composition of **two reflections** over **intersecting lines** is the same as a single **rotation** with the point of intersection the center of rotation.

Students draw glide reflections, and reflections across parallel or intersecting lines.

Glide Reflection

Theorem 14.7 Composition of Isometries

The composition of two (or more) isometries is an isometry.

When performing a composition of rigid transformations (isometries) it is most useful to draw a picture.

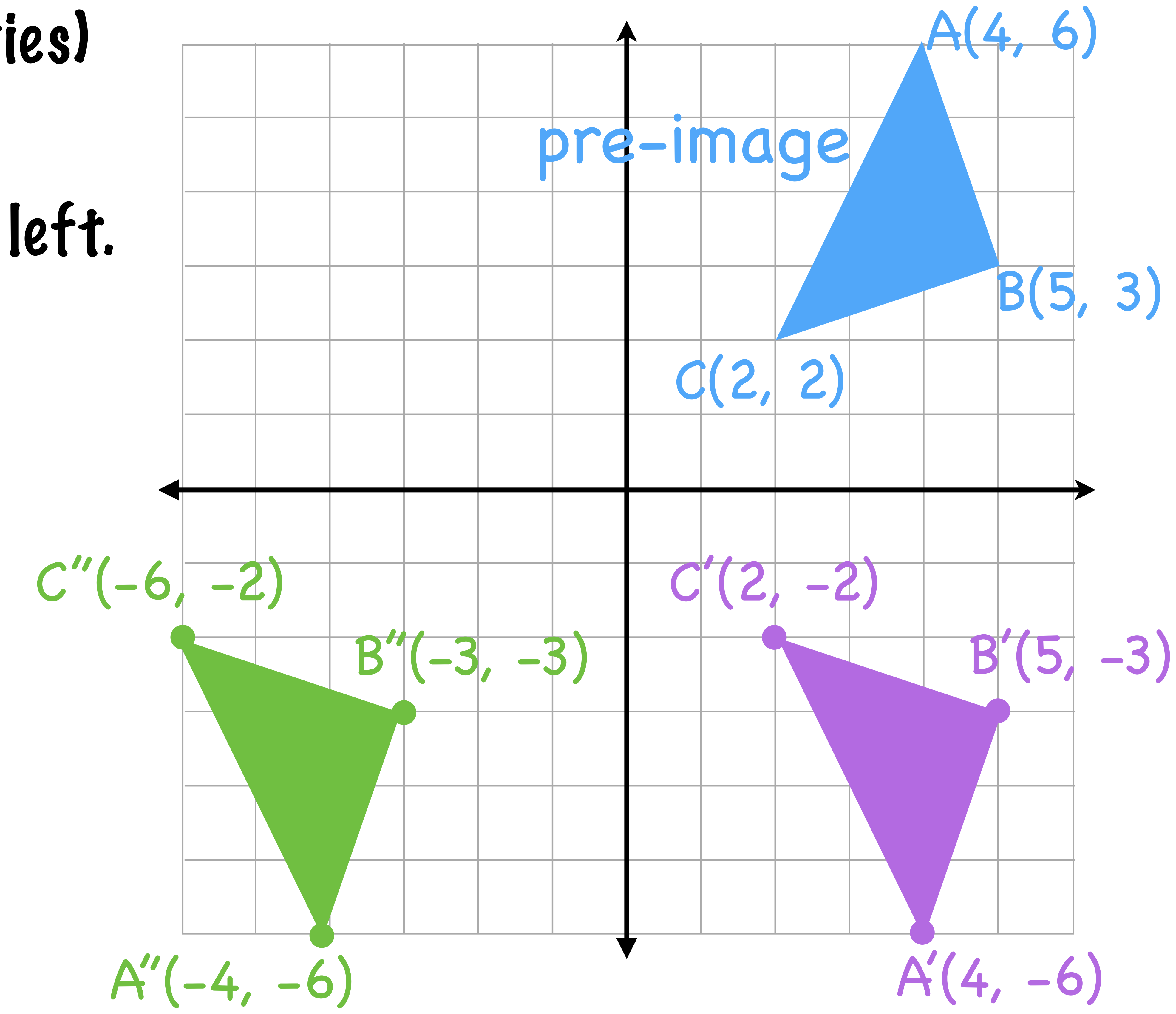
Let's reflect ΔABC over the x-axis, then translate the image 8 units left.

Reflect ΔABC over the x-axis, $(x, y) \rightarrow (x, -y)$

Translate ΔA′B′C′ 8 units left, $(x, y) \rightarrow (x-8, y)$

We can now write the rule that describes the composition of transformations. Note that the result is not one of the transformations but there is a rule.

$$(x, y) \rightarrow (x-8, -y)$$



Students draw glide reflections, and reflections across parallel or intersecting lines.

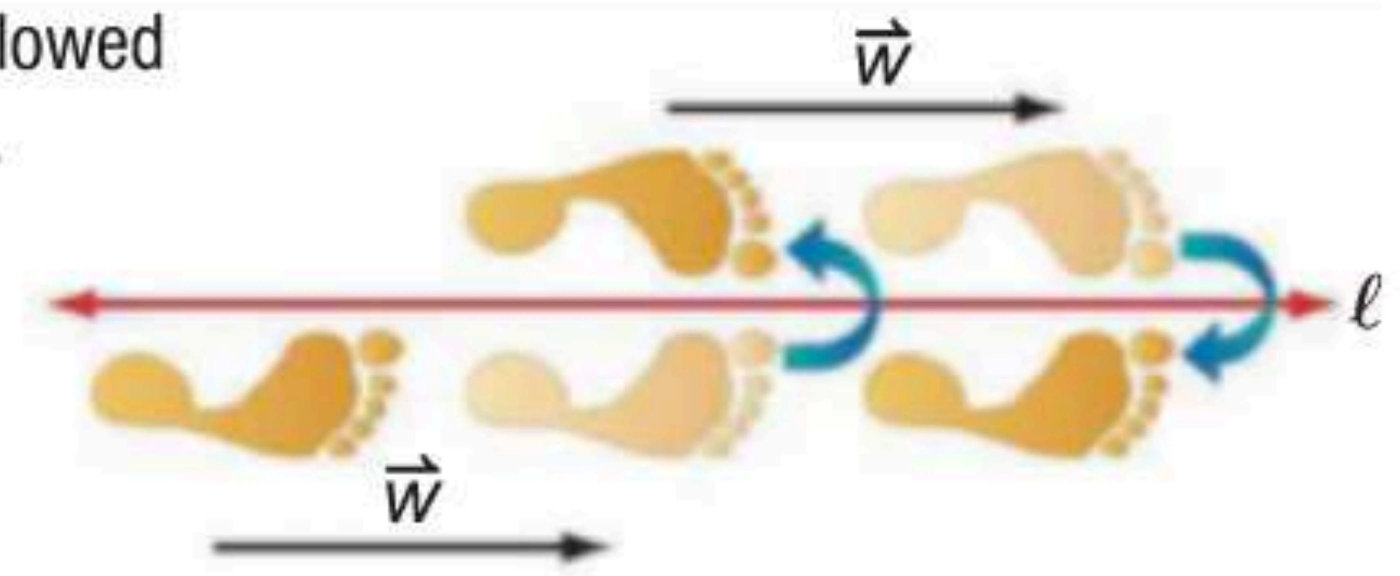
Glide Reflection

KeyConcept Glide Reflection

A **glide reflection** is the composition of a translation followed by a reflection in a line parallel to the translation vector.

Example

The glide reflection shown is the composition of a translation along $\vec{w}$ followed by a reflection in line ℓ.



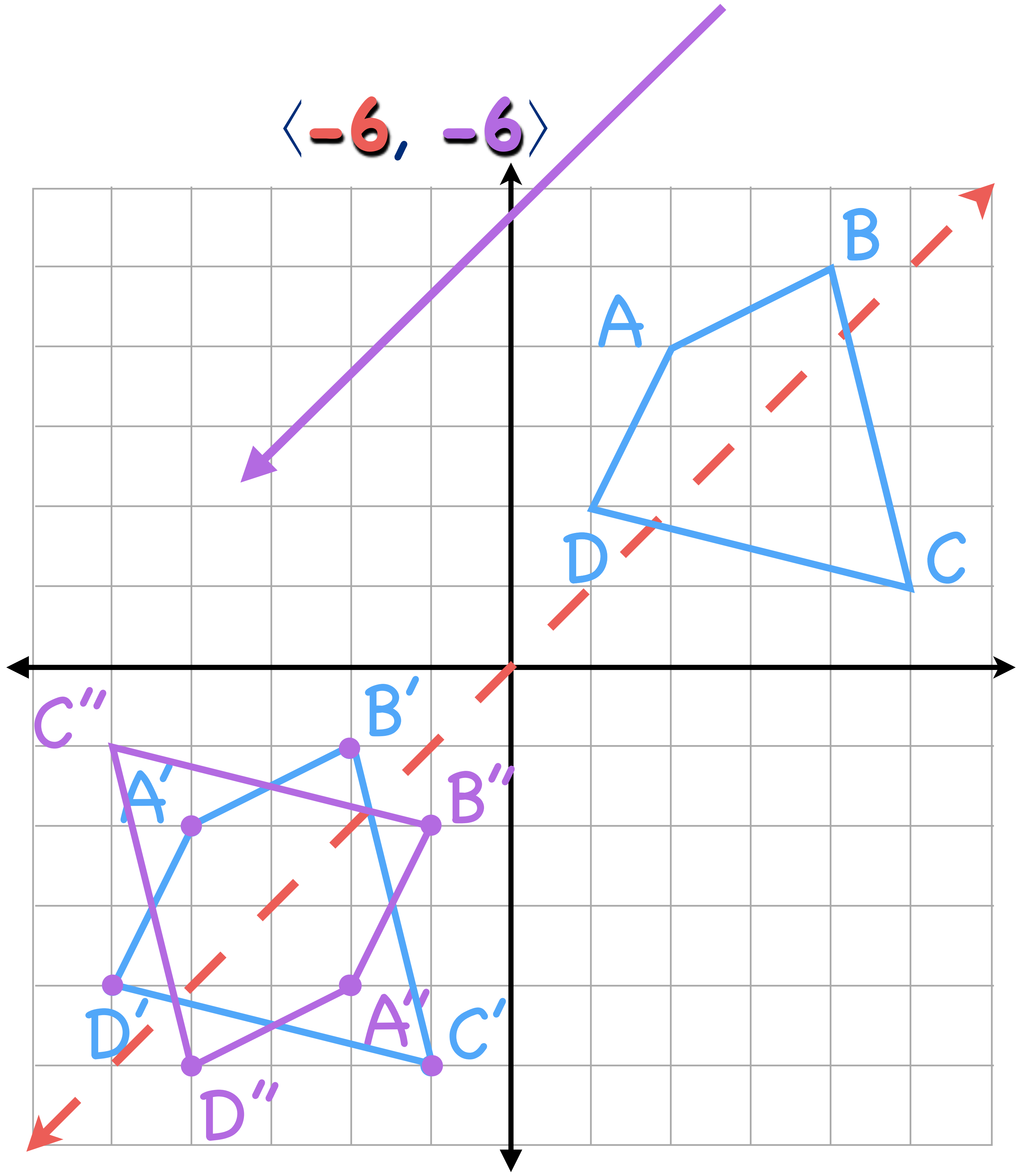
Translate the figure ⟨-6, -6⟩, then reflect across the line y = x.

Translate ABCD ⟨-6, -6⟩, (x, y) ➝ (x-6, y-6)

Reflect ABCD across the line of reflection y = x, (x, y) ➝ (y, x)

The rule for the composition is (x, y) ➝ (y-6, x-6)

When doing a glide reflection, the **order** of the transformations (translation, reflection) **does not matter**.



Reflections and Rotation

Now would be a good time to review the function notation for the rules of reflection and rotation.

Reflecting across x-axis in function arrow notation $(x, y) \rightarrow (x, -y)$

Reflecting across y-axis in function arrow notation $(x, y) \rightarrow (-x, y)$

Reflecting across $y = x$ in function arrow notation $(x, y) \rightarrow (y, x)$

Reflecting across $y = -x$ in function arrow notation $(x, y) \rightarrow (-y, -x)$

90° rotation in function arrow notation $(x, y) \rightarrow (-y, x)$

180° rotation in function arrow notation $(x, y) \rightarrow (-x, -y)$

270° rotation in function arrow notation $(x, y) \rightarrow (y, -x)$

Students draw glide reflections, and reflections across parallel or intersecting lines.

Reflection and Rotation

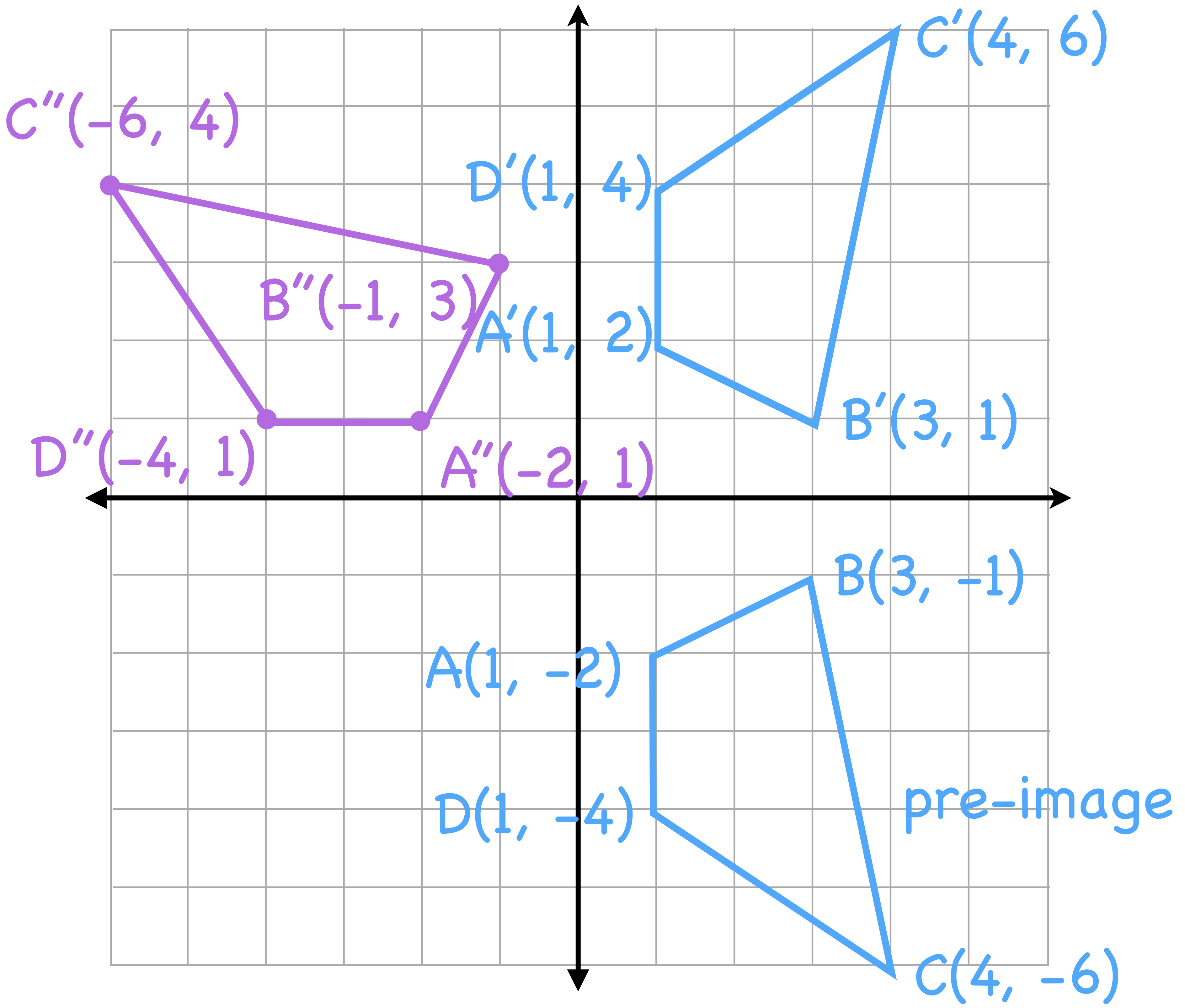
We can combine reflections and rotations, reflect the figure over the x-axis and rotate 90° around the origin.

Reflect across x-axis, $(x, y) \rightarrow (x, -y)$

90° rotation around origin (),)) $(x, y) \rightarrow (-y, x)$

The rule for the composition is $(x, y) \rightarrow (y, x)$

Look familiar? It is the reflection across the line $y = x$.



Students draw glide reflections, and reflections across parallel or intersecting lines.

Reflections Across Parallel Lines

Now we can do multiply reflections across parallel lines. We will restrict ourselves to horizontal and vertical lines of reflection.

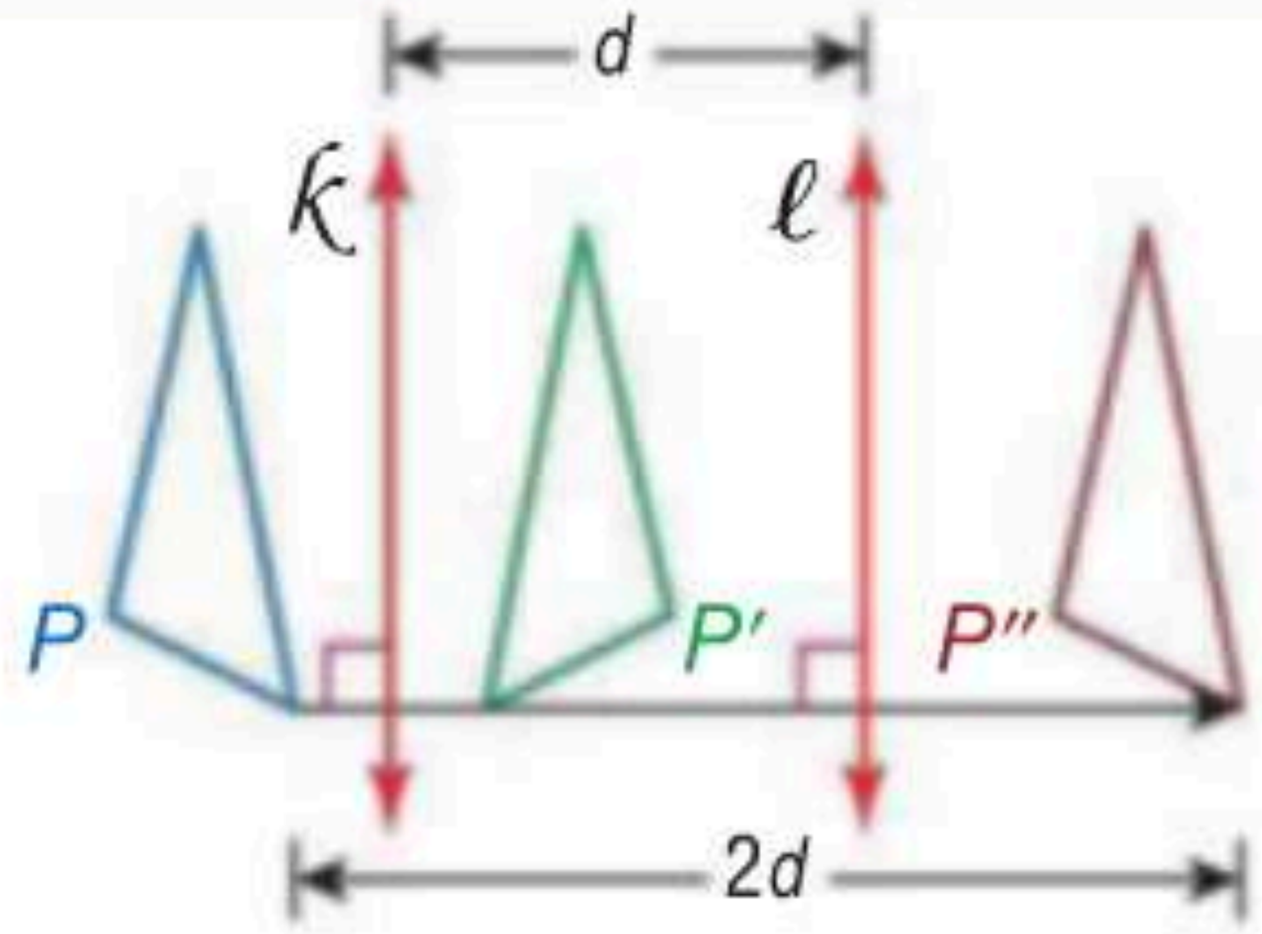
Be careful to note which line is the first line of reflection. Do not assume the line closest to the pre-image is the first line of reflection. Unlike with glide reflections, order matters with repeated reflections.

A composition of reflections across parallel lines results in a translation that is double the distance between the parallel lines.

Theorem 14.8 Reflections in Parallel Lines

The composition of two reflections in parallel lines can be described by a translation vector that is

- perpendicular to the two lines, and
- twice the distance between the two lines.



Double Reflection Across Parallel Lines

Reflect ΔABC across the lines y = 2 and y = -2

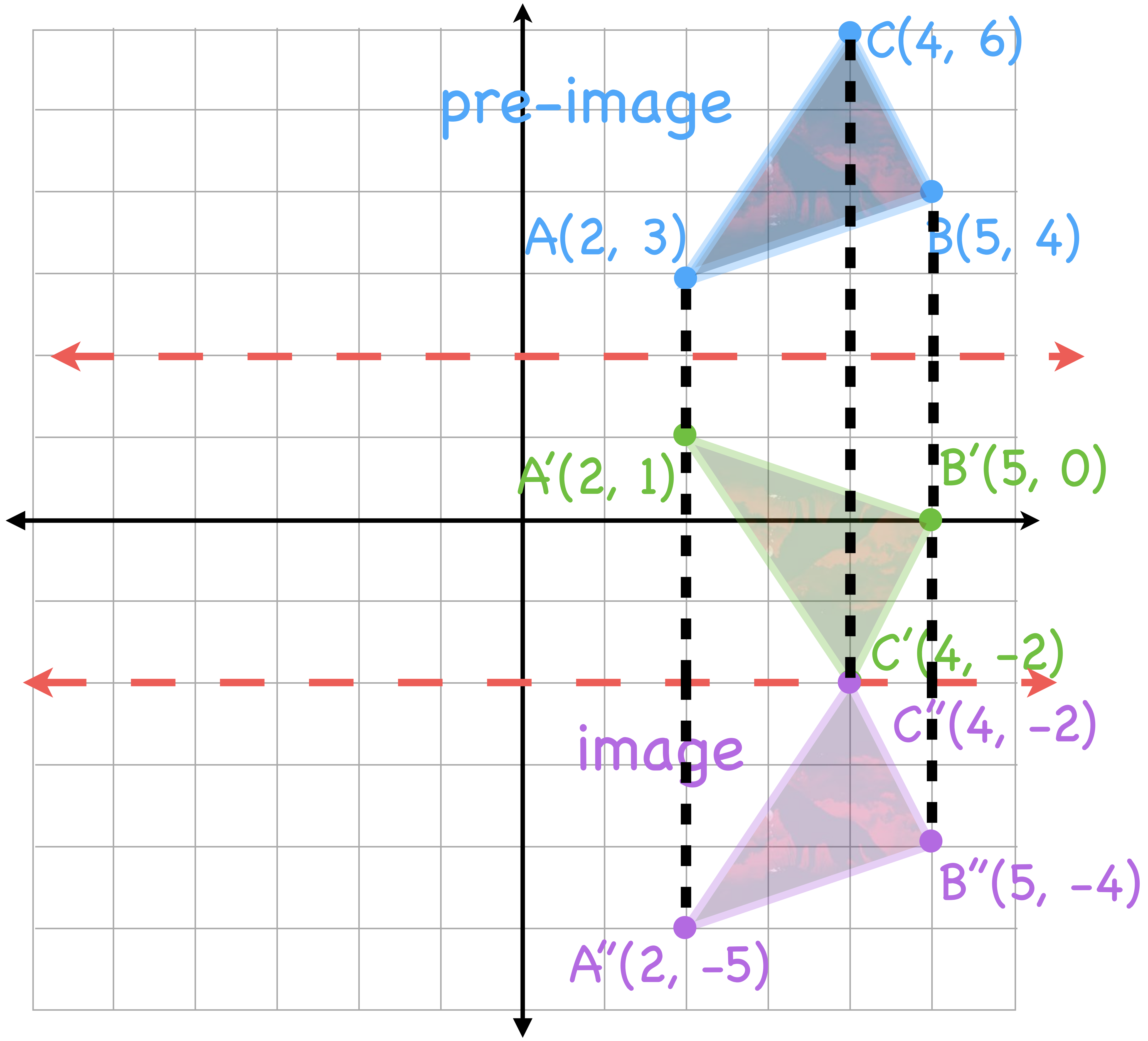
Reflect ΔABC across the line y = 2

Reflect ΔA′B′C′ across the line y = -2

The composition of the two reflections is the translation

$$(x, y) \rightarrow (x, y-8)$$

The reflections result in a vertical translation of -8 which is twice the distance between the two lines of reflection.



Double Reflection Across Parallel Lines

Reflect ΔABC across the lines x = 1 and x = -2

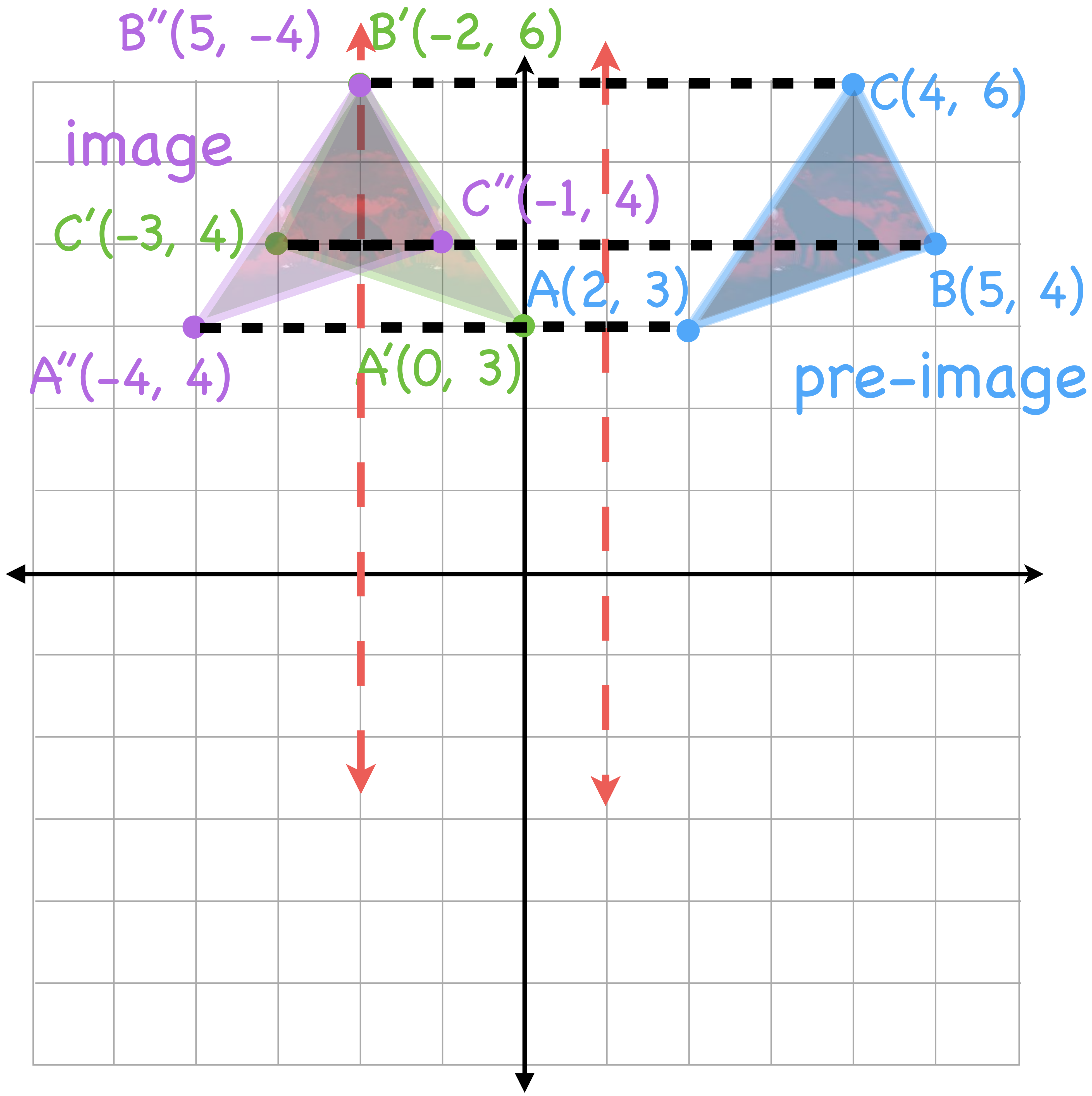
Reflect ΔABC across the lines x = 1

Reflect ΔA′B′C′ across the line x = 2

The composition of the two reflections is the translation

$(x, y) \rightarrow (x-6, y)$

The reflections result in a horizontal translation of -6 which is twice the distance between the two lines of reflection.



Reflections Across Intersecting Lines

This time let's reflect ΔABC across the lines y-axis and then the x-axis.

Reflect ΔABC across the y-axis (x = 0)

Reflecting across y-axis in arrow notation $(x, y) \rightarrow (-x, y)$

Reflect ΔA′B′C′ across the x-axis (y = 0)

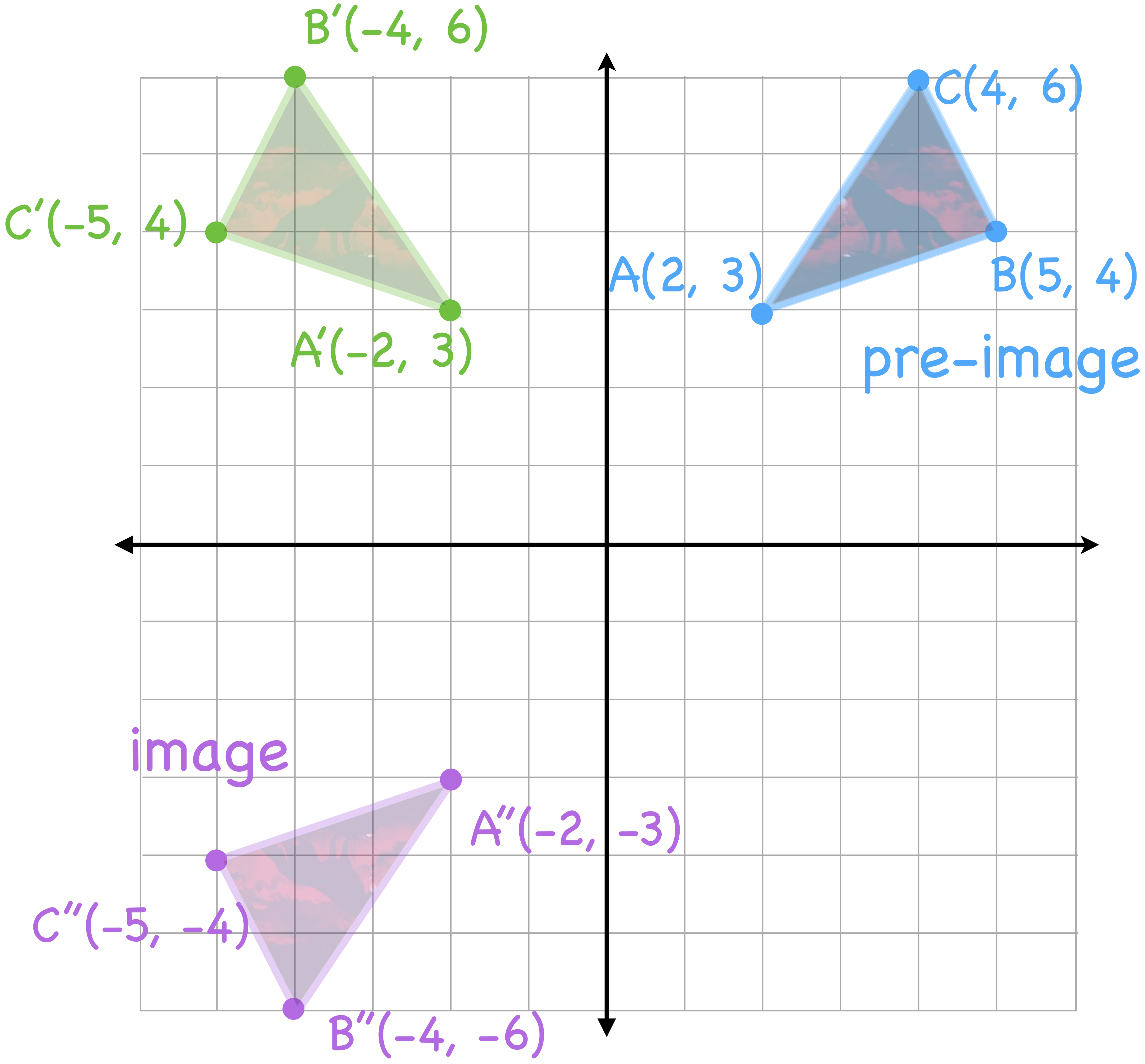
Reflecting across x-axis in arrow notation $(x, y) \rightarrow (x, -y)$

The composition of the two reflections results in the mapping

$$(x, y) \rightarrow (-x, -y)$$

180° rotation in function arrow notation $(x, y) \rightarrow (-x, -y)$

The composition of the two reflections across the x-axis and y-axis is a rotation of 180°

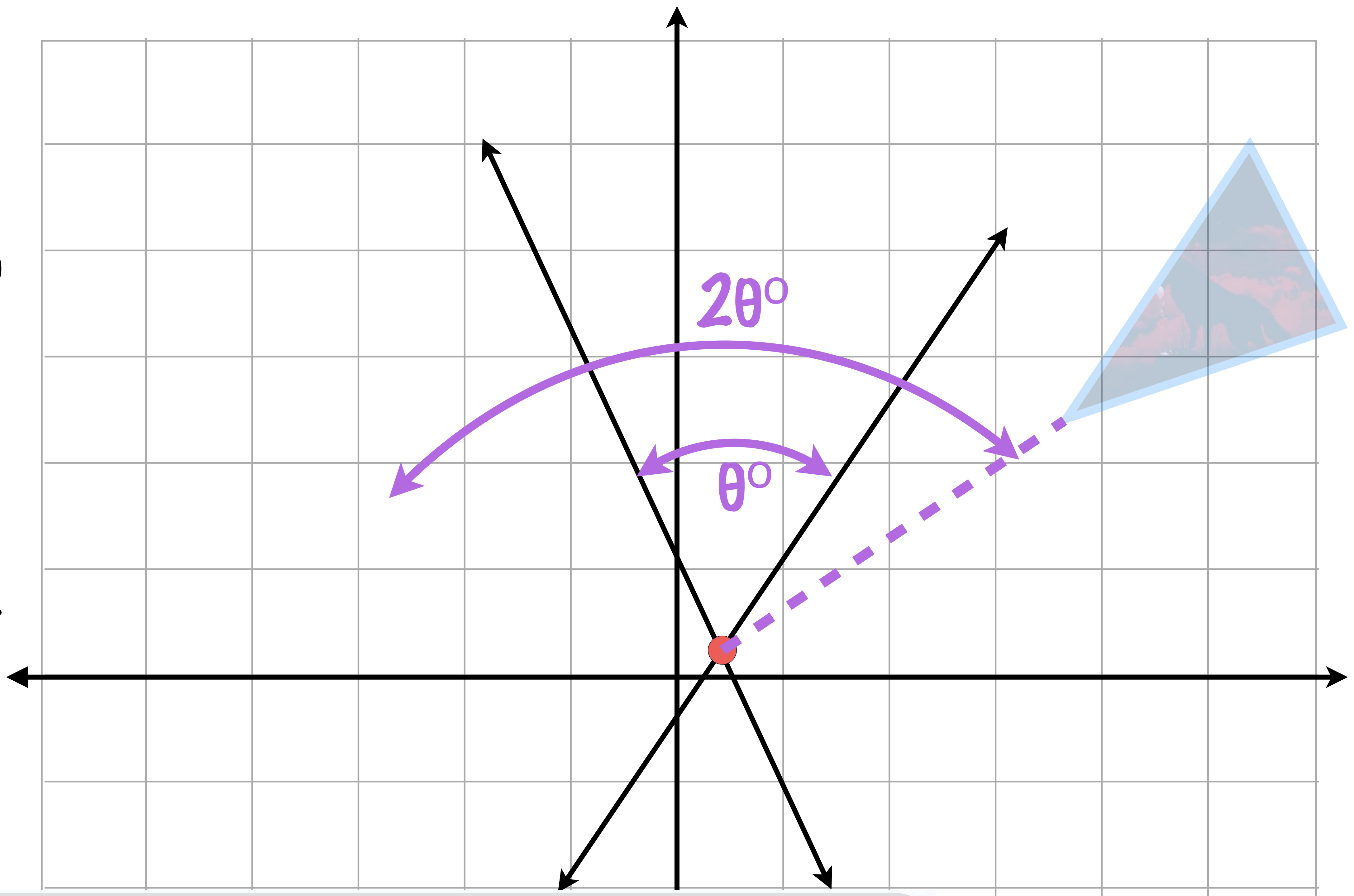


Reflections Across Intersecting Lines

That leads to the rule (theorem) for reflecting across intersecting lines.

In the previous slide we rotated around two intersecting lines, x = 0 and y = 0. The resulting rotation was 180° around the origin. The origin is the intersection of the two lines, and the two lines intersect in a 90° angle.

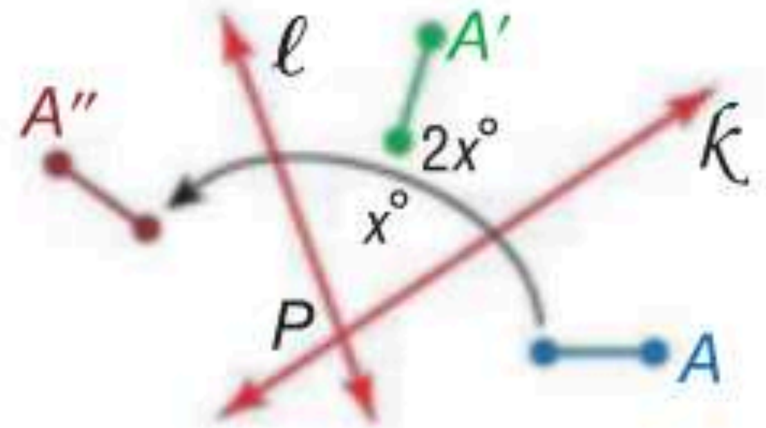
If you compose two reflections over lines that intersect, then the resulting image is a rotation of twice the angle between the intersecting lines, and the center of rotation is the point of intersection.



Theorem 14.9 Reflections in Intersecting Lines

The composition of two reflections in intersecting lines can be described by a rotation

- about the point where the lines intersect and
- through an angle that is twice the measure of the acute or right angle formed by the lines.



Reflections Across Intersecting Lines

Rotate the figure around the intersecting lines shown.

Step 1 is to determine the angle between the lines.

The angle is 110°

Step 2 Rotate the vertices twice the angle.

The rotation is 220°