TRANSFORMATIONS

Composition of Isometries & Dilations

Essential Questions: How can you change a figures position without changing its size or shape? How can you change a figures size without changing its shape? How can you represent a transformation in the coordinate plane? How do you recognize congruence and similarity mappings?

Compositions

The term *isometry* means same distance. An **isometry** is a transformation that preserves distance, or length. So, translations, reflections, and rotations are isometries.

take note

Theorem 9-1

The composition of two or more isometries is an isometry.

There are only four kinds of isometries.

Goal: To identify and classify compositions of isometries.

Essential Understanding: All isometries can be expressed as a composition of reflections.

Compositions

take note

Theorem 9-2 Reflections Across Parallel Lines

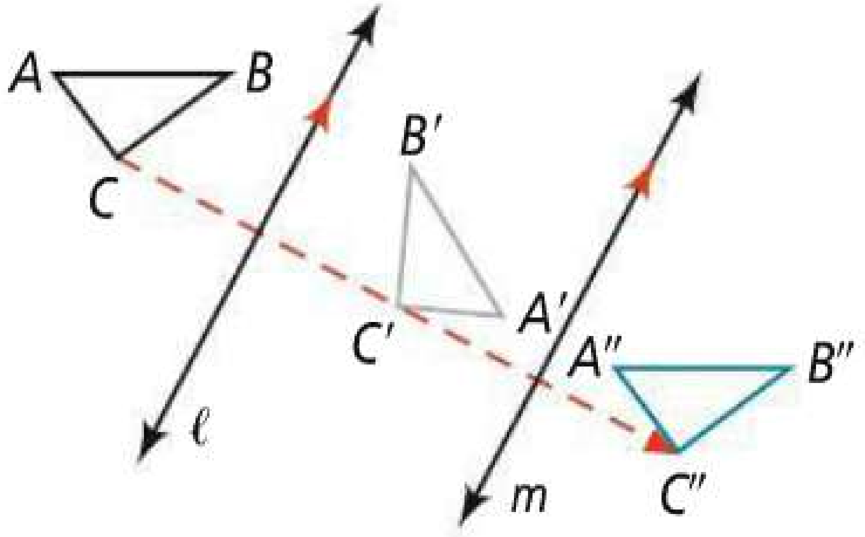
A composition of reflections across two parallel lines is a translation.

You can write this composition as

$(R_m \circ R_\ell)(\triangle ABC) = \triangle A''B''C''$

or $R_m(R_\ell(\triangle ABC)) = \triangle A''B''C''$.

$\overline{AA''}$, $\overline{BB''}$, and $\overline{CC''}$ are all perpendicular to lines ℓ and m.



Goal: To identify and classify compositions of isometries.

Essential Understanding: All isometries can be expressed as a composition of reflections.

Compositions

Problem 1 **Composing Reflections Across Parallel Lines**

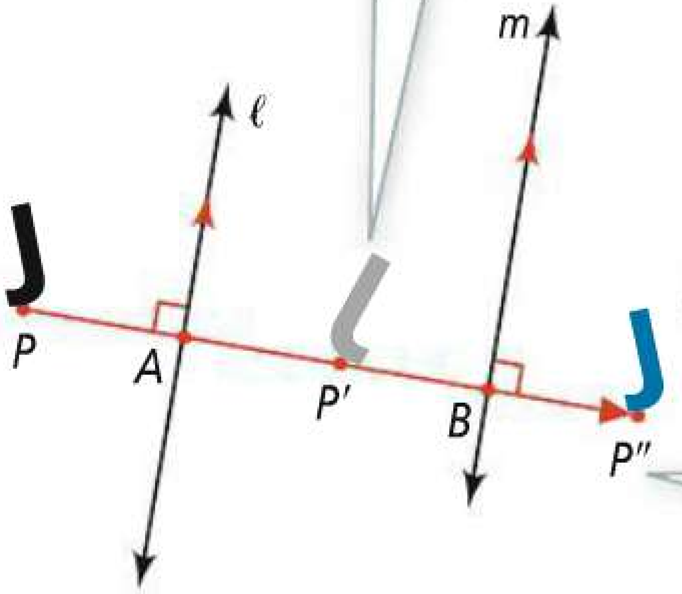
What is $(R_m \circ R_\ell)(J)$? What is the distance of the resulting translation?

Think

How do you know that $PA = AP'$, $P'B = BP''$, and $\overleftrightarrow{AB} \perp \ell$?

All three statements are true by the definition of reflection across a line.

Step 1 Reflect J across ℓ. $PA = AP'$, so $PP' = 2AP'$.



Step 2 Reflect the image across m. $P'B = BP''$, so $P'P'' = 2P'B$.

P moved a total distance of $2AP' + 2P'B$, or $2AB$.

Goal: To identify and classify compositions of isometries.

Essential Understanding: All isometries can be expressed as a composition of reflections.

Compositions

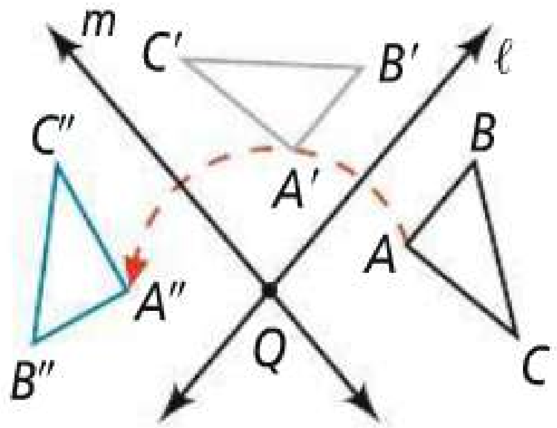
take note

Theorem 9-3 Reflections Across Intersecting Lines

A composition of reflections across two intersecting lines is a rotation.

You can write this composition as $(R_m \circ R_\ell)(\triangle ABC) = \triangle A''B''C''$ or $R_m(R_\ell(\triangle ABC)) = \triangle A''B''C''$.

The figure is rotated about the point where the two lines intersect. In this case, point Q.



Goal: To identify and classify compositions of isometries.

Essential Understanding: All isometries can be expressed as a composition of reflections.

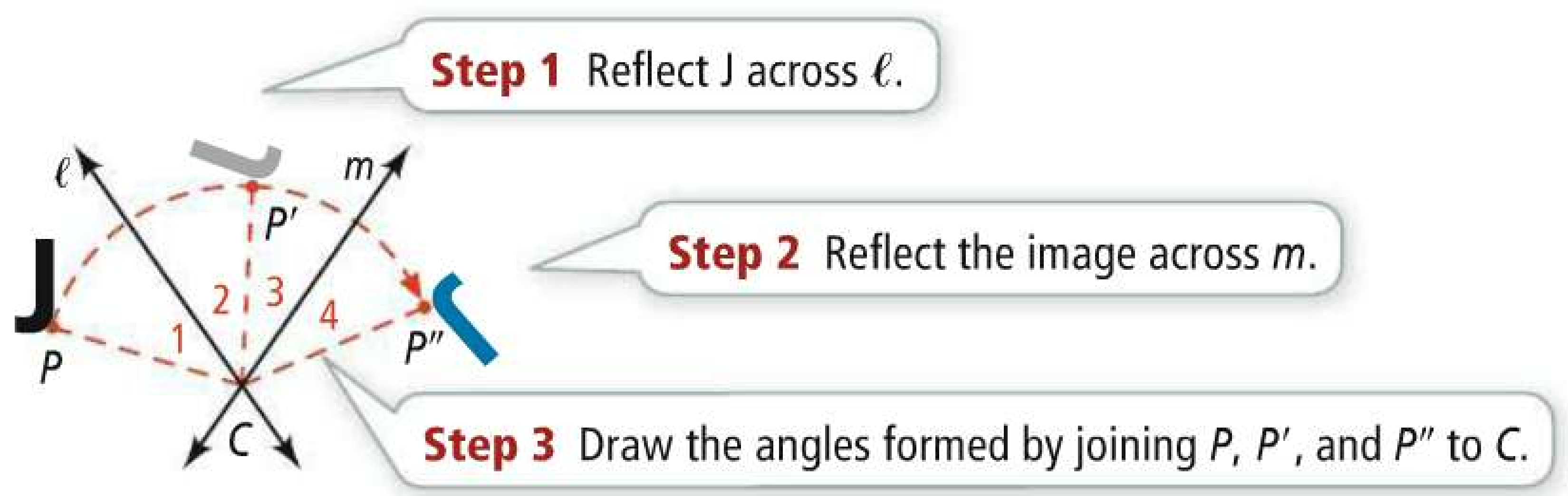
Compositions

Problem 2 Composing Reflections Across Intersecting Lines

Lines ℓ and m intersect at point C and form a 70° angle. What is $(R_m \circ R_\ell)(\text{J})$? What are the center of rotation and the angle of rotation for the resulting rotation?

Think

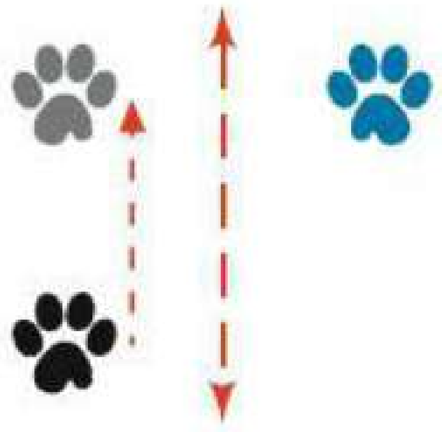
How do you show that $m\angle 1 = m\angle 2$?
If you draw $\overline{PP'}$ and label its intersection point with line ℓ as A, then $PA = P'A$ and $PP' \perp \ell$. So, by the Converse of the Angle Bisector Theorem, $m\angle 1 = m\angle 2$.



Goal: To ... of isometries.
Essential Understanding: All isometries can be expressed as a composition of reflections.

Compositions

Any composition of isometries can be represented by either a reflection, translation, rotation, or glide reflection. A **glide reflection** is the composition of a translation (a glide) and a reflection across a line parallel to the direction of translation. You can map a left paw print onto a right paw print with a glide reflection.

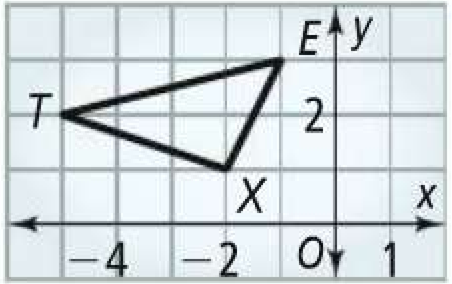


Goal: To identify and classify compositions of isometries.

Essential Understanding: All isometries can be expressed as a composition of reflections.

Problem 3 Finding a Glide Reflection Image

Coordinate Geometry What is $(R_{x=0} \circ T_{<0,\,-5>})(\triangle TEX)$?



Know
- The vertices of $\triangle TEX$
- The translation rule
- The line of reflection

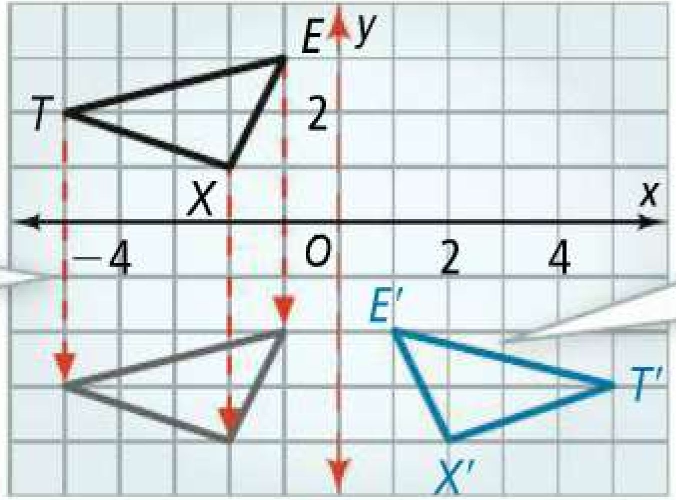
Need

The image of $\triangle TEX$ for the glide reflection

Plan

First use the translation rule to translate $\triangle TEX$. Then reflect the translation image of each vertex across the line of reflection.

Use the translation rule $T_{<0,\,-5>}(\triangle TEX)$ to move $\triangle TEX$ down 5 units.



Reflect the image of $\triangle TEX$ across the line $x = 0$.

Goal: To identify and classify compositions of isometries.

Essential Understanding: All isometries can be expressed as a composition of reflections.

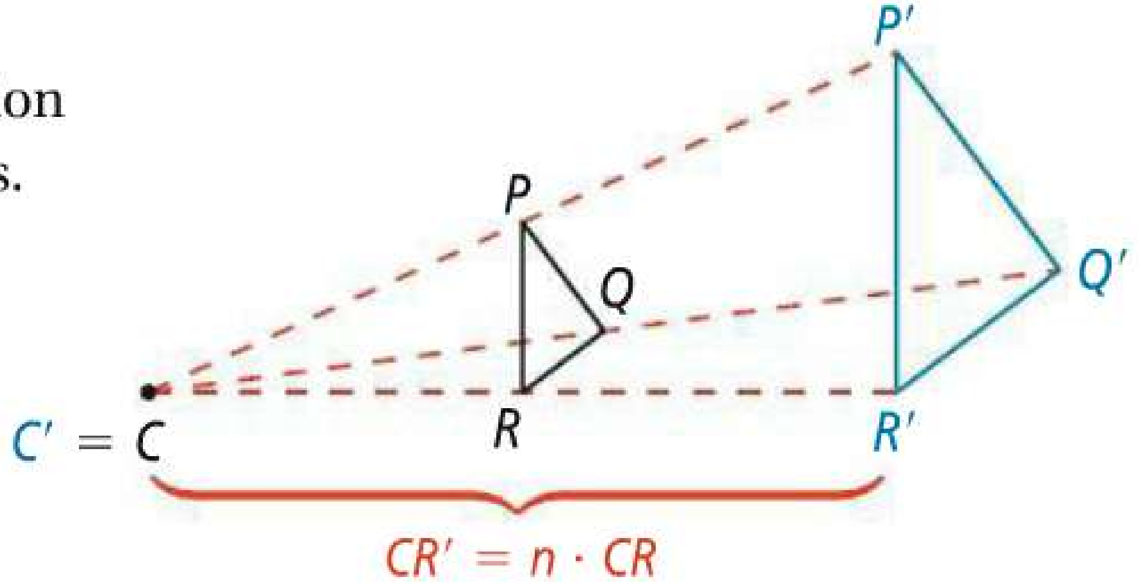
Dilations

take note

Key Concept **Dilation**

A **dilation** with **center of dilation** C and **scale factor** n, $n > 0$, can be written as $D_{(n,\,C)}$. A dilation is a transformation with the following properties.

- The image of C is itself (that is, $C' = C$).
- For any other point R, R' is on $\overrightarrow{CR}$ and $CR' = n \cdot CR$, or $n = \frac{CR'}{CR}$.
- Dilations preserve angle measure.

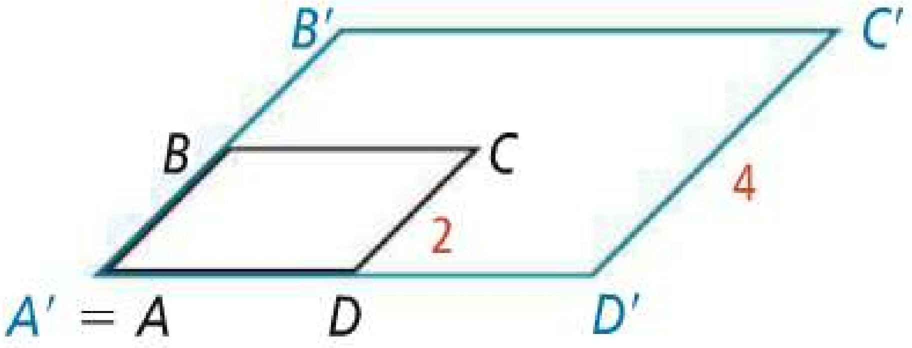


Goal: To understand dilation images of figures.

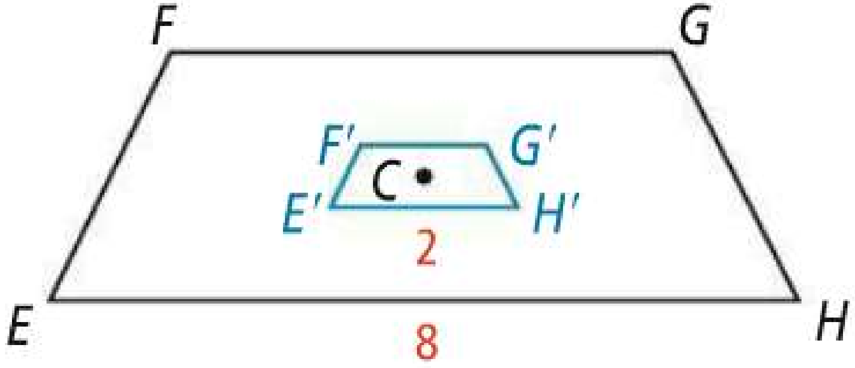
Essential Understanding: You can use a scale factor to make an enlargement or reduction of a figure that will be similar to the figure.

Dilations

The scale factor *n* of a dilation is the ratio of a length of the image to the corresponding length in the preimage, with the image length always in the numerator. For the figure shown on page 587, $n = \frac{CR'}{CR} = \frac{R'P'}{RP} = \frac{P'Q'}{PQ} = \frac{Q'R'}{QR}$.



Enlargement
center *A*, scale factor 2



Reduction
center C, scale factor $\frac{1}{4}$

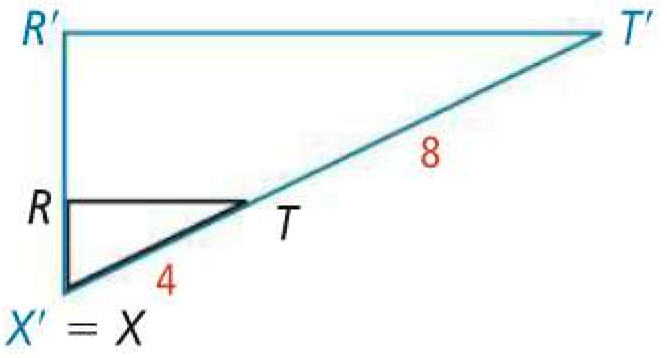
Goal: To understand dilation images of figures.

Essential Understanding: You can use a scale factor to make an enlargement or reduction of a figure that will be similar to the figure.

Dilations

Problem 1 **Finding a Scale Factor**

Multiple Choice Is $D_{(n,X)}(\triangle XTR) = \triangle X'T'R'$ an enlargement or a reduction? What is the scale factor n of the dilation?



Think

Why is the scale factor not $\frac{4}{12}$, or $\frac{1}{3}$?

The scale factor of a dilation always has the image length (or the distance between a point on the image and the center of dilation) in the numerator.

X′ = X

The image is larger than the preimage, so the dilation is an enlargement.

Use the ratio of the lengths of corresponding sides to find the scale factor.

$$n = \frac{X'T'}{XT} = \frac{4+8}{4} = \frac{12}{4} = 3$$

$\triangle X'T'R'$ is an enlargement of $\triangle XTR$, with a scale factor of 3.

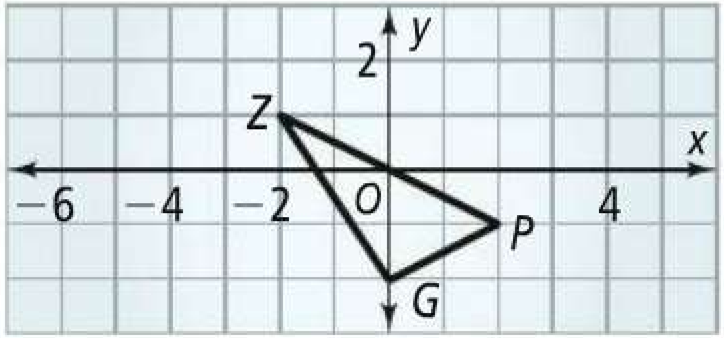
Goal: To understand dilation images of figures.

Essential Understanding: You can use a scale factor to make an enlargement or reduction of a figure that will be similar to the figure.

Dilations

Problem 2 **Finding a Dilation Image**

What are the coordinates of the vertices of $D_2(\triangle PZG)$? Graph the image of $\triangle PZG$.



Think

Will the vertices of the triangle move closer to (0, 0) or farther from (0, 0)?

The scale factor is 2, so the dilation is an enlargement. The vertices will move farther from (0, 0).

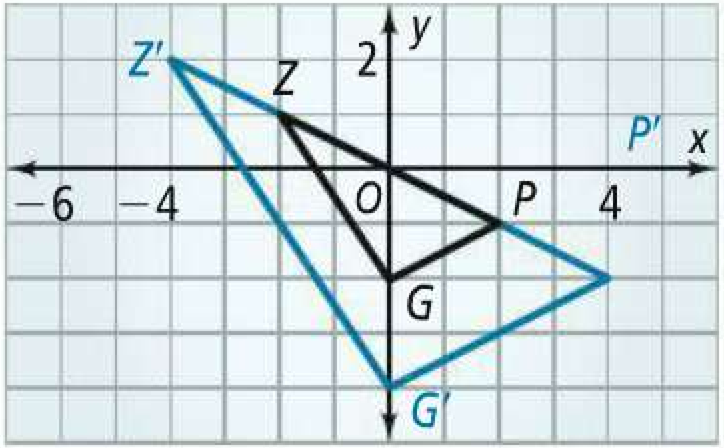
Identify the coordinates of each vertex. The center of dilation is the origin and the scale factor is 2, so use the dilation rule $D_2(x, y) = (2x, 2y)$.

$D_2(P) = (2 \cdot 2, 2 \cdot (-1))$, or $P'(4, -2)$.

$D_2(Z) = (2 \cdot (-2), 2 \cdot 1)$, or $Z'(-4, 2)$.

$D_2(G) = (2 \cdot 0, 2 \cdot (-2))$, or $G'(0, -4)$.

To graph the image of $\triangle PZG$, graph P', Z', and G'. Then draw $\triangle P'Z'G'$.



Goal: To understand dilation images of figures.

Essential Understanding: You can use a scale factor to make an enlargement or reduction of a figure that will be similar to the figure.

Compositions & Dilations

- Homework:
  - Worksheet 9.4/9.6
    - All Problems
    - Quiz Tomorrow

Goal: To identify rigid motion. To find translations images of figures.

Essential Understanding: You can change a figures position so that the image if the figure is congruent to the original figure.