2016 *Mathematics Standards of Learning*

Algebra II

Overview of Changes from 2009 to 2016

Related documents available on VDOE Mathematics 2016 webpage

Purpose

- Overview of the 2016 *Mathematics Standards of Learning* and the *Curriculum Framework*
- Highlight information included in the Essential Knowledge and Skills and the Understanding the Standard sections of the *Curriculum Framework*

Agenda

- Overview and Implementation Timeline
- Resources Currently Available
  - Crosswalk (Summary of Revisions)
  - Standards and Curriculum Frameworks
- Comparison of 2009 to 2016 Standards
  - Expressions and Operations
  - Equations and Inequalities
  - Functions
  - Statistics

Implementation Timeline

2016-2017 School Year – Curriculum Development
VDOE staff provides a summary of the revisions to assist school divisions in incorporating the new standards into local written curricula for inclusion in the taught curricula during the 2017-2018 school year.

2017-2018 School Year – Crossover Year
2009 Mathematics Standards of Learning and 2016 Mathematics Standards of Learning are included in the written and taught curricula. Spring 2018 Standards of Learning assessments measure the 2009 Mathematics Standards of Learning and include field test items measuring the 2016 Mathematics Standards of Learning.

2018-2019 School Year – Full-Implementation Year
Written and taught curricula reflect the 2016 Mathematics Standards of Learning. Standards of Learning assessments measure the 2016 Mathematics Standards of Learning.

2016 SOL Revisions

- Improve the vertical progression of mathematics content
- Ensure developmental appropriateness of student expectations
- Increase support for teachers in mathematics content
- Clarify expectations for teaching and learning
- Improve precision and consistency in mathematical language and format
- Ensure proficiency of elementary students in computational skills

Support for Teachers

- Significant additions to the Understanding the Standard column including
  - Definitions
  - Explanations
  - Examples
  - Instructional connections
- Improvements in precision, clarity, and consistency in language K-12
- Indicators of SOL sub-bullet added to each bullet within the Essential Knowledge and Skills

Algebra II **Strand: Functions**

AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include

a) domain, range, and continuity;
b) intervals in which a function is increasing or decreasing;
c) extrema;
d) zeros;
e) intercepts;
f) values of a function for elements in its domain;
g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;
h) end behavior;
i) vertical and horizontal asymptotes;
j) inverse of a function; and
k) composition of functions, algebraically and graphically.

Understanding the Standard	Essential Knowledge and Skills
• The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by a function. • Discontinuous domains and ranges include those with removable (holes) and nonremovable (asymptotes) discontinuities. • A function can be described on an interval as increasing, decreasing, or constant over a specified interval or over the entire domain of the function. • A function, $f(x)$, is increasing over an interval if the values of $f(x)$ consistently increase over the interval as the x values increase. • A function, $f(x)$, is decreasing over an interval if the values of $f(x)$ consistently decrease over the interval as the x values increase. • A function, $f(x)$, is constant over an interval if the values of $f(x)$ remain constant over the interval as the x values increase.	• For any x value in the domain of f, determine $f(x)$. (f) • Represent relations and functions using verbal descriptions, tables, equations, and graphs. Given one representation, represent the relation in another form. (g) • Describe the end behavior of a function. (h) • Determine the equations of vertical and horizontal asymptotes of functions (rational, exponential, and logarithmic). (i) • Determine the inverse of a function (linear, quadratic, cubic, square root, and cube root). (j) • Graph the inverse of a function as a reflection over the line $y = x$. (j) • Determine the composition of two functions algebraically and graphically. (k) • Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f, g, h, i, j, k)

Overview of Changes

Reporting Category	# of Standards (2009)	# of Standards (2016)
Expressions and Operations	3	2
Equations and Inequalities	2	2
Functions	3	4
Statistics	4	4
Total	12	12

Mathematics Process Goals for Students

"The content of the mathematics standards is intended to support the five process goals for students"

- 2009 and 2016 *Mathematics Standards of Learning*

Standards of Learning Curriculum Frameworks

Introduction includes:

- Mathematical Process Goals for Students
- **Instructional Technology**
- **Computational Fluency**
- **Algebra Readiness**
- **Equity**

Algebra II – Crosswalk (Summary of Revisions): 2016 *Mathematics Standards of Learning and Curriculum Framework*

Additions (2016 SOL)

- All.7a – Analyze the continuity of functions
- All.7c – Determine the extrema of a function
- All.7f – Determine values of a function for elements in its domain
- All.7g – Make connections between and among multiple representations of a function

Deletions from Algebra II (2009 SOL)

- All.3 – Identification of field properties valid for the complex numbers [SOL and EKS] removed but application of the properties is still expected
- All.3 EKS – Hierarchy of subsets of complex numbers
- All.7 EKS – Convert between logarithmic and exponential forms of an equation
- All.9 EKS – Determine a logarithmic curve of best fit

Parameter Changes/Clarifications (2016 SOL)

- All.1a EKS – Simplify rational algebraic expressions limited to linear and quadratic
- All.1b EKS – Simplification of radicals may include rationalizing deno
- All.1c EKS – Clarified that factoring polynomials should include those [illegible] variables with no more than four terms over the set of integers with factors that are constant, linear, or quadratic
- All.3c EKS – Limited solving rational equations to linear and quadratic with real solutions containing factorable algebraic expressions algebraically and graphically
- All.3 and All.7 US – Clarified that solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation.
- All.6b EKS – Transform exponential and logarithmic functions (given a graph) limited to single transformations
- All.7a,d,e EKS – Clarified that examples for identifying domain, range, zeros, and intercepts should include graphs with discontinuities
- All.7b EKS – Identify intervals on which the function is increasing or decreasing limited to linear, quadratic, absolute value, square root, cure root, polynomial, exponential, and logarithmic functions
- All.7i EKS – Determine equations of vertical and horizontal asymptotes limited to rational, exponential, and logarithmic functions
- All.7j EKS– Determine inverse of a function limited to linear, quadratic, cubic, square root, and cube root functions
- All.7k EKS – Clarified that determining composition of functions includes both algebraic and graphical approaches
- All.8 EKS – Given zeros, write the equation of a polynomial limited to polynomial functions in factored form
- All.9 EKS – Determine a curve of best fit limited to quadratic and exponential functions
- All.11 EKS – Interpret variation, standard deviation, and z-scores were moved from Algebra I and are now only included in AFDA.7 and All.11; mean absolute deviation is no longer addressed

Moves within Algebra II (2009 SOL to 2016 SOL)

- All.1c – Write radical expressions as expressions containing rational exponents and vice versa [Moved to All.1b EKS]
- All.1d – Factor polynomials [Moved to All.1c]
- [illegible] [Moved to "Functions" strand as All.5]
- All.3 – Moved to All.2]
- All.4 EKS - Recognize that the [illegible]ula can be derived by applying completion of squares to any [illegible]on in standard form [Moved to All.3 US]
- All.4 – [Moved to All.3]; Use of graphing calculator [Moved to All.3 EKS]
- All.5 – [Moved to All.4]; Use of graphing calculator [Moved to All.4 EKS]
- All.6 – Use of graphing calculator [Moved to All.6 EKS]
- All.7 – Use of graphing calculator [Moved to All.7 EKS]
- All.7b – Zeros [Moved to All.7d]
- All.7c – Intercepts [Moved to All.7e]
- All.7d – Intervals on which a function is increasing or decreasing [Moved to All.7b]
- All.7e – Asymptotes [Moved to All.7i]
- All.7f – End behavior [Moved to All.7h]
- All.7g – Inverse of a function [Moved to All.7j]
- All.7h – Composition of functions [Moved to All.7k]

2009

2009

2016

EKS = Essential Knowledge and Skills, referring to the column on the right side of the Curriculum Framework
EU = Essential Understandings, referring to the column on the left side of the Curriculum Framework

2009 SOL	2016 SOL
Expressions and Operations	
AII.1 The student, given rational, radical, or polynomial expressions, will a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; c) write radical expressions as expressions containing rational exponents and vice versa. [Moved to EKS] and d) factor polynomials completely.	AII.1 The student will a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; and c) factor polynomials completely in one or two variables.
AII.2 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n. [Moved to "Functions" strand as AII.5]	
AII.3 The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of i, and identify field properties that are valid for the complex numbers.	AII.2 The student will perform operations on complex numbers and express the results in simplest form using patterns of the powers of i.
Equations and Inequalities	
AII.4 The student will solve, algebraically and graphically, a) absolute value equations and inequalities; b) quadratic equations over the set of complex numbers; c) equations containing rational algebraic expressions; and d) equations containing radical expressions. Graphing calculators will be used for solving and for confirming the algebraic solutions. [Moved to EKS]	AII.3 The student will solve a) absolute value linear equations and inequalities; b) quadratic equations over the set of complex numbers; c) equations containing rational algebraic expressions; and d) equations containing radical expressions.
AII.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions. [Moved to EKS]	AII.4 The student will solve systems of linear-quadratic and quadratic-quadratic equations, algebraically and graphically.
Functions	
	AII.5 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first n terms, determining the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n. [Moved from "Expressions and Operations" strand, AII.2]

EXPRESSIONS AND OPERATIONS

2009 SOL	2016 SOL
AII.1 The student, given rational, radical, or polynomial expressions, will a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; c) write radical expressions as expressions containing rational exponents and vice versa; [Moved to EKS] and d) factor polynomials completely.	AII.1 The student will a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; and c) factor polynomials completely in one or two variables.

Revisions:

- Write radical expressions as expressions containing rational exponents and vice versa from the 2009 SOL AII.1c moved to SOL AII.1b EKS
- SOL AII.1a EKS - Simplifying rational expressions are limited to linear and quadratic expressions
- SOL AII.1b EKS - Simplification of radical expressions may include rationalizing denominators
- SOL AII.1c EKS - Clarification included that factoring polynomials should include those in one or two variables with no more than four terms over the set of integers with factors that are constant, linear or quadratic

Revisions:

- **2009 SOL AII.2 was moved to the "Functions" strand in the 2016 standards as AII.5**

2009 SOL	2016 SOL
AII.3 The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of i, and identify field properties that are valid for the complex numbers.	AII.2 The student will perform operations on complex numbers and express the results in simplest form using patterns of the powers of i.

Revisions:

- The identification of field properties valid for the complex numbers that was included in the 2009 SOL AII.3 SOL and Essential Knowledge and Skills section was removed, but application of the properties is still expected
- 2009 SOL AII.3 Essential Knowledge and Skill of placing numbers in a hierarchy of subsets of the complex numbers is no longer included in the 2016 SOL AII.2

EQUATIONS AND INEQUALITIES

2009 SOL	2016 SOL
AII.4 The student will solve, algebraically and graphically, a) absolute value equations and inequalities; b) quadratic equations over the set of complex numbers; c) equations containing rational algebraic expressions; and d) equations containing radical expressions. Graphing calculators will be used for solving and for confirming the algebraic solutions. [Moved to EKS]	AII.3 The student will solve a) absolute value linear equations and inequalities; b) quadratic equations over the set of complex numbers; c) equations containing rational algebraic expressions; and d) equations containing radical expressions.

Revisions:

- **SOL AII.3c EKS - Limits solving rational equations to linear and quadratic with real solutions containing factorable algebraic expressions algebraically and graphically**
- **SOL AII.3 Understanding the Standard - Clarified that solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation**
- **SOL AII.3 Understanding the Standard - Includes recognizing that the quadratic formula can be derived by applying completion of squares to any quadratic equation in standard form; this was moved from 2009 SOL AII.5 Essential Knowledge and Skills**

2009 SOL	2016 SOL
AII.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions. [Moved to EKS]	AII.4 The student will solve systems of linear-quadratic and quadratic-quadratic equations, algebraically and graphically.

Revisions:

- The phrase about graphing calculators included in 2009 SOL AII.5 was moved to the Essential Knowledge and Skills section of the 2016 SOL AII.4 and refers to "graphing utilities"
- 2009 SOL AII.5 EKS to recognize that the quadratic formula can be derived by applying completion of squares to any quadratic equation in standard form moved to 2016 SOL AII.3 Understanding the Standard section

FUNCTIONS

Revisions:

- **Real-world problems are now referred to as practical problems in the 2016 standards**

2009 SOL	2016 SOL
AII.6 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions. [Moved to EKS]	AII.6 For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, the student will a) recognize the general shape of function families; and b) use knowledge of transformations to convert between equations and the corresponding graph of functions.

Revisions:

- The phrase about graphing calculators included in 2009 SOL AII.6 was moved to the Essential Knowledge and Skills section of the 2016 SOL AII.6 and refers to "graphing utilities"
- SOL AII.6b EKS - Transforming exponential and logarithmic functions (given a graph) is now limited to single transformations

2009 SOL	2016 SOL
AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include **a) domain and range, including limited and discontinuous domains and ranges;** **b) zeros;** **c) *x*- and *y*-intercepts;** **d) intervals in which a function is increasing or decreasing;** **e) asymptotes;** **f) end behavior;** **g) inverse of a function; and** **h) composition of multiple functions.** **Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]**	**AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include** **a) domain, range, and continuity;** **b) intervals in which a function is increasing or decreasing;** **c) extrema;** **d) zeros;** **e) intercepts;** **f) values of a function for elements in its domain;** **g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;** **h) end behavior;** **i) vertical and horizontal asymptotes;** **j) inverse of a function; and** **k) composition of functions algebraically and graphically.**

Revisions:

- **The phrase about graphing calculators included in 2009 SOL AII.7 was moved to the Essential Knowledge and Skills section of the 2016 SOL AII.7 and refers to "graphing utilities"**
- **SOL AII.7 - Analyze the continuity of functions added to SOL AII.7a; determine the extrema of a function added as SOL AII. 7c; determine values of a function for elements in its domain added as SOL AII.7f; and make connections between and among multiple representations of a function added as SOL AII.7g**
- **SOL AII.7a,d,e EKS - Clarification provided that examples for identifying domain, range, zeros, and intercepts should include graphs with discontinuities**
- **SOL AII.7b EKS - Identify intervals on which the function is increasing or decreasing limited to linear, quadratic, absolute value, square root, cure root, polynomial, exponential, and logarithmic functions**

2009 SOL	2016 SOL
AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include a) domain and range, including limited and discontinuous domains and ranges; b) zeros; c) *x*- and *y*-intercepts; d) intervals in which a function is increasing or decreasing; e) asymptotes; f) end behavior; g) inverse of a function; and h) composition of multiple functions. Graphing calculators will be used as a tool to assist in investigation of functions. [Moved to EKS]	AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include a) domain, range, and continuity; b) intervals in which a function is increasing or decreasing; c) extrema; d) zeros; e) intercepts; f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; h) end behavior; i) vertical and horizontal asymptotes; j) inverse of a function; and k) composition of functions algebraically and graphically.

Revisions (con't):

- SOL AII.7i EKS - Determine equations of vertical and horizontal asymptotes limited to rational, exponential, and logarithmic functions
- SOL AII.7j EKS - Determine inverse of a function limited to linear, quadratic, cubic, square root, and cube root functions
- SOL AII.7k EKS - Clarified that determining composition of functions includes both algebraic and graphical approaches
- 2009 SOL AII.7 EKS to convert between logarithmic and exponential forms of an equation is no longer included in 2016 SOL AII.7

2009 SOL	2016 SOL
AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.	AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

Revisions:

- **SOL AII.8 EKS - If students are given the zeros of a polynomial function and asked to write the equation of the polynomial function, the equation will be limited to defining the polynomial function in factored form**

STATISTICS

2009 SOL	2016 SOL
AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.	AII.9 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of quadratic and exponential functions.

Revisions:

- SOL 2016 AII.9 no longer includes logarithmic functions when finding the curve of best fit
- Real world problems are now referred to as practical problems in 2016

2009 SOL	2016 SOL
AII.10 The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.	AII.10 The student will represent, create, and solve problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

Revisions:

- Real world problems are now referred to as practical problems in 2016

2009 SOL	2016 SOL
AII.11 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.	AII.11 The student will a) identify and describe properties of a normal distribution; b) interpret and compare z-scores for normally distributed data; and c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

Revisions:

- SOL AII.11 EKS – includes interpret variation, standard deviation, and z-scores [moved from Algebra I]
- Mean absolute deviation is no longer addressed in the 2016 Mathematics Standards of Learning

2009 SOL	2016 SOL
AII.12 The student will compute and distinguish between permutations and combinations and use technology for applications.	AII.12 The student will compute and distinguish between permutations and combinations.

Revisions:

- The phrase about technology included in 2009 SOL AII.12 was moved to the Essential Knowledge and Skills section of the 2016 standard and refers to "graphing utilities"

Questions?
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