Composition and Inverse Functions DAY 5

Finding Domain Analytically:

DOMAIN WHEN X IS IN THE DENOMINATOR If in the form: $\frac{1}{x}$, **Domain:** $\Re, x \neq 0$ The denominator cannot equal 0. Set up an equation with the denominator equal to zero and solve.	$f(x)=\frac{3}{4x-1}$
DOMAIN OF A RADICAL FUNCTION If in the form: y = $\sqrt{x}$, **Domain: x ≥ 0** The radicand is always positive. Set the radicand ≥ 0 and solve.	$y=\sqrt{3x+5}$
WHEN A RADICAL IS IN THE DENOMINATOR If in the form: $\frac{1}{\sqrt{x}}$ **Domain:** $\Re, x > 0$ The denominator cannot equal 0. Set up an equation with the denominator equal to zero and solve.	$y=\frac{4}{\sqrt{2x-1}}$

Function Composition $f(x)=x+3$ $g(x)=x^2$ 1) Find $g(f(x))$ or $(g \circ f)(x)$. **First, substitute the inside function for f(x).** $g(f(x))$ $= g(x+3)$ **Second, substitute the function (f) into the function (g) for x.** $=(x+3)^2$ **Third, Simplify.** $=(x+3)(x+3)$ $=x^2+3x+3x+9$ $=x^2+6x+9$	f(x) = 2x + 1 g(x) = x – 5 Find f(g(x)). Find g(f(x)).

Inverse Functions

Given two functions *f* and *g*, the composite function, denoted by $f \circ g$ (read as "f of g" or "*f* composed with *g*"), is defined by $f(g(x)) = g(f(x)) = x$.

The domain of $f \circ g$ is the set of all numbers *x* in the domain of *g* such that *g(x)* is in the domain of *f*.

Table

A function's inverse, f^{-1}, will map the output values back the input values.

f(x)

x	y
1	14
2	15
3	16

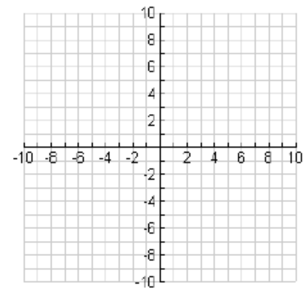
$f^{-1}(x)$

x	y
14	1
15	2
16	3

* *The x and y values are switched.*

On a Graph

The inverse will be the original graph's reflection over the line y = x.



Find the Inverse Function

Find the inverse of $f(x) = 2x^2 + 5$

First, Switch the x and y.

$$x = 2y^2 + 5$$

Second, Solve for y.

$$x - 5 = 2y^2$$

$$\frac{x-5}{2} = y^2$$

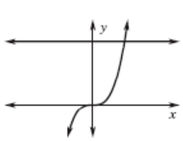
$$\sqrt{\frac{x-5}{2}} = y$$

Is the Inverse a Function?

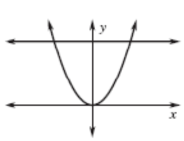
HORIZONTAL LINE TEST

The inverse of a function *f* is also a function if and only if no horizontal line intersects the graph of *f* more than once.

Function



Not a function



Verifying two functions are inverses!

$$f(x) = x^2 + 1 \qquad g(x) = \sqrt{x-1}$$

Determine if the two functions are inverses.

First, find f(g(x)).

$$= f\left(\sqrt{x-1}\right)$$

$$= \left(\sqrt{x-1}\right)^2 + 1$$

$$= x - 1 + 1$$

$$= x$$

Second, find g(f(x)).

$$= g\left(x^2 + 1\right)$$

$$= \sqrt{x^2 + 1 - 1}$$

$$= \sqrt{x^2}$$

$$= x$$

Lastly, verify that both composition functions equal x!

Given $f(x) = 3x + 2$ and $g(x) = 2x^2 - 1$.

1. Find $(f \circ g)(4)$	2. Find $(f \circ g)(x)$ Then evaluate your composition function at 4.
3. Find $(f \circ f)(1)$	4. Find $(f \circ f)(x)$ Then evaluate your composition function at 1.

5. The surface area *S* (in square meters) of a hot air balloon is given by $S(r) = 4\pi r^2$ where *r* is the radius of the balloon (in meters). If the radius *r* is increased with time *t* (in seconds) according to the formula $r(t) = \frac{2}{3}t^3,\ t \geq 0$, find the surface area *S* of the balloon as a function of the time *t*.

The function *f* is one-to-one. Find its inverse then state the domain of *f* and find its range using f^{-1}.

6. $f(x)=\dfrac{4}{2-x}$

Step 1: Switch x and y.

Step 2: Solve for y.

Step 3: Write the inverse as $f^{-1}(x)$.

Step 4: Find the domain of f(x).

Step 5: Find the domain of $f^{-1}(x)$, which is the range of f(x).

7. $f(x)=\dfrac{3x+1}{-x}$

Verify that the functions are inverses of each other by showing that f(g(x)) = x and g(f(x)) = x.

8. $f(x)=2x+6$ and $g(x)=\frac{1}{2}x-3$

9. $f(x)=\frac{x-5}{2x+3}$ and $g(x)=\frac{3x+5}{1-2x}$

PRACTICE **DAY 5**

Given that $f(x) = 3x + 2$ and $g(x) = 2x^2 - 1$.

1. Find $(f \circ g)(x)$	2. $(g \circ f)(x)$
3. Find $(g \circ f)(2)$	4. $(g \circ g)(0)$

5. The volume *V* of a right circular cylinder of height *h* and radius *r* is $V = \pi r^2 h$. If the height is twice the radius, express the volume *V* as a function of *r*.

6. The head circumference C of a child is related to the height *H* of the child (both in inches) through the function $H(c) = 2.15C - 10.53$.

   (a) Express the head circumference C as a function of height *H*.

   (b) Predict the head circumference of a child who is 26 inches tall.